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Rayleigh distribution

A continuous [probability distribution](#) with density

$$p(\boldsymbol{x})=\begin{cases}\frac{\boldsymbol{x}}{\sigma^2}e^{-\boldsymbol{x}^2/2\,\sigma^2}, & \boldsymbol{x}>0,\\0, & \boldsymbol{x}\leq 0,\end{cases}$$

depending on a [scale parameter](#) $\sigma>0$. A Rayleigh distribution has positive asymmetry; its unique mode is at the point $\boldsymbol{x}=\sigma$. All moments of a Rayleigh distribution are finite, the mathematical expectation and variance being $\sigma\sqrt{\pi/2}$ and $2\sigma^2\left(1-\pi/4\right)$, respectively. The distribution function of a Rayleigh distribution has the form

$$f(\boldsymbol{x})=\begin{cases}1-e^{-\boldsymbol{x}^2/2\,\sigma^2}, & \boldsymbol{x}>0,\\0, & \boldsymbol{x}\leq 0\,.\end{cases}$$

A Rayleigh distribution is a special case of the distribution with density

$$\frac{2}{2^{n/2}\,\sigma^n\,\Gamma\left(n/2\right)}\boldsymbol{x}^{n-1}e^{-\boldsymbol{x}^2/2\,\sigma^2}$$

when $n=2$; hence, when $\sigma=1$ the Rayleigh distribution coincides with the distribution of the square root of a random variable which has the "[chi-squared](#)" [distribution](#) with two degrees of freedom. In other words, a Rayleigh distribution can be interpreted as the distribution of the length of a vector in a plane Cartesian coordinate system, the coordinates of which are independent and have the [normal distribution](#) with parameters 0 and σ^2 . In the three-dimensional space the [Maxwell distribution](#) plays a role analogous to the Rayleigh distribution.

A Rayleigh distribution is mainly applied in target theory and statistical communication theory. It was first considered by Lord Rayleigh in 1880 as the distribution of the amplitude resulting from the addition of harmonic oscillations.

References

[1] J.W. [Lord Rayleigh] Strutt, "Wave theory of light" , Moscow-Leningrad (1940) (In Russian; translated from English)

Comments

References

[a1] A. Papoulis, "Probability, random variables and stochastic processes" , McGraw-Hill (1965)

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