

Dear Xiaofeng Wang,
 your phase-noise equation (3) now makes more sense:

$$L\{\Delta\omega\} = 10 \cdot \log \left\{ \frac{2 F k T}{P_s} \cdot \left[1 + \left(\frac{\omega_0}{2 Q_L \Delta\omega} \right)^2 \right] \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{\Delta\omega} \right) \right\} \quad [\text{dBc/Hz}]$$

but there are still inaccuracies. Your equation includes both phase and amplitude noise for both sidebands. Although both sidebands were originally used by Leeson, today it is usual to report just one sideband and phase-noise only. The final result is therefore 4 times or -6dB smaller:

$$L\{\Delta f\} = 10 \cdot \log \left\{ \frac{F k T}{2 P_s} \cdot \left[1 + \left(\frac{\omega_0}{2 Q_L \Delta\omega} \right)^2 \right] \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{\Delta\omega} \right) \right\} \quad [\text{dBc/Hz}]$$

as usually reported, for example:

https://en.wikipedia.org/wiki/Leeson%27s_equation

Since the units are [dBc/Hz], it makes sense to use the symbol $L\{\Delta f\}$ in place of $L\{\Delta\omega\}$. The latter suggests an incorrect result in [dBc/(rd/s)] !

Further, for logarithm with the base 10 we use the symbol "log" opposite to "lg" in your text. Finally, the expression under the logarithm has units [Hz⁻¹] in many published formulas. Please note that the logarithm of [Hz⁻¹] can not be computed. To stay accurate, the expression under the logarithm should be multiplied by 1Hz and the equation has to be rewritten as:

$$L\{\Delta f\} = 10 \cdot \log \left\{ \frac{F k T}{2 P_s} \cdot \left[1 + \left(\frac{\omega_0}{2 Q_L \Delta\omega} \right)^2 \right] \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{\Delta\omega} \right) \cdot 1\text{Hz} \right\} \quad [\text{dBc/Hz}]$$

Please note that F should not be an empiric factor. When the resonator losses are at room temperature (passive LC

circuit like in your case) and there are no design mistakes in the bias circuitry, F (in linear units, not in dB!) is the noise figure of the active device used under large-signal conditions. On the other hand, using noisy active filters or even more noisy opto-electronic delay lines in your feedback, the overall noise shall be computed in a slightly more complicated way.

Finally, please specify what does 20kHz mean in your text.

Does this mean $\Delta f_{1/f^3} = 20\text{kHz} = 20000\text{cycles/s}$ or

$\Delta \omega_{1/f^3} = 20\text{kHz} = 20000\text{rd/s}$?