Unified Presentation of 1/f Noise in Electronic Devices: Fundamental 1/f Noise Sources

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This review represents 1/f noise in electronic devices in terms of the Hooge parameter, a parameter that is currently being used in the noise characterization of various devices. The review is based on the quantum theory of 1/f noise, which is applicable to all collision-dominated systems involving mobility, diffusion, and cross-section fluctuations. It applies to collision-free devices (pentodes, vacuum photodiodes) and collision-dominated devices (like vacuum tubes, Schottky barrier diodes, n+-p diodes, p-i-n diodes, n+-p-n and p+-n-p BITs, n-channel and p-channel SiJFETs, and p-MOS devices operating under strong inversion). The approach is in itself not new: what is new, however, is its generalization to all collision-dominated systems involving mobility, diffusion, and cross-section fluctuations. It applies to collision-free devices (pentodes, vacuum photodiodes) and collision-dominated devices (like vacuum tubes, Schottky barrier diodes, n+-p diodes, p-i-n diodes, n+-p-n and p+-n-p BITs, n-channel and p-channel SiJFETs, and p-MOS devices operating under strong inversion). The approach is in itself not new: what is new, however, is its generalization to all collision-dominated systems involving mobility, diffusion, and cross-section fluctuations. It applies to collision-free devices (pentodes, vacuum photodiodes) and collision-dominated devices (like vacuum tubes, Schottky barrier diodes, n+-p diodes, p-i-n diodes, n+-p-n and p+-n-p BITs, n-channel and p-channel SiJFETs, and p-MOS devices operating under strong inversion). The approach is in itself not new: what is new, however, is its generalization to all collision-dominated systems involving mobility, diffusion, and cross-section fluctuations. It applies to collision-free devices (pentodes, vacuum photodiodes) and collision-dominated devices (like vacuum tubes, Schottky barrier diodes, n+-p diodes, p-i-n diodes, n+-p-n and p+-n-p BITs, n-channel and p-channel SiJFETs, and p-MOS devices operating under strong inversion). The approach is in itself not new: what is new, however, is its generalization to all collision-dominated systems involving mobility, diffusion, and cross-section fluctuations. It applies to collision-free devices (pentodes, vacuum photodiodes) and collision-dominated devices (like vacuum tubes, Schottky barrier diodes, n+-p diodes, p-i-n diodes, n+-p-n and p+-n-p BITs, n-channel and p-channel SiJFETs, and p-MOS devices operating under strong inversion).
devices. In most cases the predictions made by Handel's theory are verified. This does not necessarily indicate that the mathematical derivation of these predictions are correct; this remains open to discussion.

II. GENERAL BACKGROUND OF THE PROBLEM

A. The Hooge Equation and the Hooge Parameter

1) Collision-Limited Devices: We first turn to the Hooge equation itself. When a constant voltage $V$ is applied to a semiconductor resistor of resistance $R$, a fluctuating current $I(t)$ is developed. This can only come about because the resistance $R(t)$ of the device fluctuates. Since

$$ I(t) = V/R(t) = \text{const.} $$

$$ \frac{dI}{dt} = - \frac{dR}{dt} \frac{S(f)}{R^2} = \frac{S(f)}{R^2}. \quad (1) $$

If $R$ and $\delta R$ are independent of current, $S(f)/R^2$ will be independent of current also. This is true for generation-recombination (g-r) spectra caused by traps; they give Lorentzian spectra of the form $\text{const}/(1 + \omega^2 \tau^2)$. It is therefore also true for $1/f$ spectra caused by a superposition of Lorentzian spectra, as in McWhorter's theory of $1/f$ noise (Section IV). But, as we shall see, the possibility must also be left open that there are true $1/f$ spectra, not caused by such a superposition.

Irrespective of the cause of the $1/f$ noise, $S(f)/R^2$ may be written as

$$ \frac{S(f)}{R^2} = \frac{\text{const}}{f} \quad (1a) $$

and it may be implied that the noise is caused by resistance fluctuations. Clarke and Voss [4], [5] showed the presence of such resistance fluctuations in a beautiful experiment.

The question is now what other parameters enter into the constant introduced by $(1a)$. Hooge suggested that for a rectangular semiconductor the missing parameter was the number $N$ of carriers of the sample and wrote the empirical formula, now known as the Hooge equation,

$$ \frac{S(f)}{R^2} = \frac{S(f)}{R^2} = \frac{\alpha_H}{N}. \quad (2) $$

This equation neither proves anything nor predicts anything, but merely gives an operational definition of the Hooge parameter $\alpha_H$. It is always valid, but is only useful if one can extract useful information out of the value of $\alpha_H$.

Since a rectangular semiconductor bar of length $L$ and cross-sectional area $A$ has a resistance $R = L^2(\mu N)$, where $\mu$ is the carrier mobility, $N$ follows from $R$, and hence $\alpha_H$ from $(2)$. When one does this for a number of different semiconductor resistors of comparable length $L$, one can characterize the noisiness of the various materials by the parameter $\alpha_H$. Hooge [1] found in that manner that for many semiconductor samples $\alpha_H$ had a value of about $2 \times 10^{-3}$, nearly independent of the material. Hanafi et al. [6] found for ten $\text{Hg}_1-x_{-} \text{Cd}_x \text{Te}$ resistor bars with different doping and (or) different values of $x$ (but all made by similar techniques), that $\alpha_H$ had an average value of $5 \times 10^{-3}$ with a spread of less than a factor 2. The near constancy of $\alpha_H$ suggests that this $1/f$ noise is due to a fundamental mechanism of unknown origin; this is useful information that will be found to be valid in other situations as well.

Later it was found [7], [8] that $\alpha_H$ could be considerably smaller than $2 \times 10^{-3}$ for sufficiently short resistors ($L < 100 \mu m$) whereas the Hooge value of $2 \times 10^{-3}$ was obtained for sufficiently long devices ($L < 500 \mu m$). A systematic experimental study of the dependence of $\alpha_H$ upon the device length $L$, which has not been made so far, would be very helpful.

Hooge gave no proof of $(2)$, but it is easily seen that an equation like $(2)$, with constant $\alpha_H$, could be expected if the $1/f$ noise is generated by $N$ independent carriers. For in that case both $S(f)$ and $S(f)$ would be proportional to $N$ so that $S(f)/n^2$ would be inversely proportional to $N$. This would be fundamental noise.

Since the resistance $R$ is inversely proportional to the product $\mu N$, where $\mu$ is the carrier mobility, there can be fluctuations $\delta \mu$ in $\mu$ and (or) $\delta N$ in $N$, so that, since $\delta \mu$ and $\delta N$ are independent

$$ \frac{\delta R}{R} = \frac{\delta \mu}{\mu} \frac{\delta N}{N}. $$

or

$$ \frac{S(f)}{R^2} = \frac{S(f)}{R^2} = \frac{\alpha_H}{N}. \quad (3) $$

If the fluctuation in $\mu$ predominates

$$ \frac{S(f)}{R^2} = \frac{S(f)}{R^2} = \frac{\alpha_H}{N^2}. \quad (3a) $$

and the noise is called mobility fluctuation $1/f$ noise, whereas

$$ \frac{S(f)}{R^2} = \frac{S(f)}{R^2} = \frac{\alpha_H}{N^2}. \quad (3b) $$

if the fluctuations in $N$ predominate; the noise is then called number fluctuation $1/f$ noise. In principle either relationship can occur, but in practice mobility fluctuation $1/f$ noise predominates in many cases. We come back to that problem in Sections IV and V.

Do $(3a)$ and $(3b)$ result in the Hooge equation? In order that this be the case, $S(f)$ and $S(f)$ must vary as $1/f$ over a wide frequency range and in addition $S(f)/\mu^2$ and (or) $S(f)/N^2$ must vary as $1/N$. We come back to the spectral dependencies in Section IV, but wish to point out here that the latter is the case if $S(f)$ is proportional to $N$.

We shall now show that $S(f)/\mu^2$ always varies as $1/N$. In addition, if each electron, in and by itself, produces $1/f$ noise, the full Hooge equation $(2)$ results.

The proof is simple, as Hooge [9] and van der Ziel et al. [10] have demonstrated. We introduce the short-term mobility $\mu_t$ of the individual carriers. If $N$ does not fluctuate, and the $\mu_t$'s are independent

$$ \mu_N = \sum_{i=1}^{N} \mu_i, \quad \mu = \frac{1}{N} \sum_{i=1}^{N} \mu_i, \quad \mu = \mu_t. \quad (4) $$

$$ \delta \mu = \frac{1}{N} \sum_{i=1}^{N} \delta \mu_i. $$

$$ S(f) = \frac{1}{N^2} \sum_{i=1}^{N} S(f) = \frac{1}{N} S(f) $$

$$ S(f) = \frac{1}{N} S(f) $$

so that $S(f)/\mu^2$ varies as $1/N$. In addition $S(f)/\mu_t^2$ is independent of $N$ and was postulated to have a $1/f$ dependence.
We may then write

$$S_{\nu}(f) = \frac{\alpha_H}{f} \frac{S_{\nu}(t)}{(t)^2} = \frac{\alpha_H}{fN}$$  \hspace{1cm} (4b)$$

We thus see that for mobility fluctuations the Hooge equation is always valid and that $\alpha_H$ is defined as the relative mobility $1/f$ spectrum $S_{\nu}(t)$ of a single electron. This would then be fundamental $1/f$ noise.

Because of the Einstein relation $eD = kT\mu$, mobility fluctuations correspond to fluctuations in the diffusion constant $D$. Consequently

$$\frac{\delta D}{D} = \frac{\delta \mu}{\mu} \text{ or } \frac{S_{\nu}(t)}{(t)^2} = \frac{S_{\nu}(f)}{f^2}.$$  \hspace{1cm} (5)

A Hooge-type equation may therefore also be expected for solid-state devices governed by diffusion processes, such as occur in p$^{-}$-n and n$^{+}$-p junction diodes, p$^{-}$-n-p and n$^{+}$-p-n BJTs, and Schottky-barrier diodes operating in the diffusion mode. Corrections may be needed for degenerate systems.

Since FETs are bias-dependent semiconductor resistors, they should show $1/f$ noise. For devices operating at near-zero drain bias the device is a uniform semiconductor resistor, but for larger bias the resistor becomes nonuniform due to channel pinch-off. For such nonuniform resistors one must replace the Hooge equation by its differential form holding for each section $\Delta x$ at $x$

$$S_{\nu}(x, f) = \frac{\alpha_H}{f N(x)} \chi^2(x)$$  \hspace{1cm} (6)

where $N(x)$ is the carrier density per unit length and $\chi(x)$ the current at $x$. It is thus possible to treat the Hooge equation as the spectrum of a distributed noise source $H(x, f)$. By evaluating the contributions of individual sections $\Delta x$ to the spectrum $S_{\nu}(f)$ of the total current $I$, one can express $S_{\nu}(f)$ in terms of $\alpha_H$ and other measurable device parameters, so that $\alpha_H$ can be determined for all these devices and the relative noisiness of the various noise mechanisms can be established. The methods for solving such distributed noise problems are discussed in Section III. They work so long as $\Delta x$ is larger than the free path length of the carriers.

There is one other further noise problem that requires attention. In relatively long $n^{+}$-p diodes the current carriers disappear by recombination. In that case the lifetime $\tau$ of the individual carriers fluctuates. It is shown in Section III that for $C = 1/\tau$ (C is independent of $x$)

$$S_{\nu}(x, f) = \frac{\alpha_H}{f N(x)} \chi^2(x)$$  \hspace{1cm} (7)

so that this problem can be incorporated into the general schematic.

A related problem is the noise due to fluctuations in the contact recombination velocity $s_{\text{on}}$ at an ohmic contact ($s_{\text{on}} = 10^{7}$ cm/s). To that end consider a planar $n^{+}$-p diode with a length $w_x$ of the p-region ($w_x \ll L_x$, short diode) where $L_x = (D_x \tau_s)^{1/2}$ is the diffusion length of the electrons in the p-region. Then $I_x = \varepsilon_{\text{on}} N_x(x)$, and, in analogy with (7)

$$S_{\nu}(x, f) = \frac{\alpha_H}{f N_{\text{on}}} s_{\text{on}}^2$$  \hspace{1cm} (7a)

where $N_{\text{on}} = 1/2 \left[N(0) + N(w_x)\right] w_x$ is the effective number of minority carriers in the base region (see below).

A similar effect can occur in the surface recombination velocity $s_{\text{on}}$ of a junction space-charge region or in the surface recombination velocity in the base region of a BJT. Here $s$ is usually much smaller than $s_{\text{on}} = 6 \times 10^3$ cm/s in a contact on n-type silicon, and $s < 1$ cm/s for a well-passivated surface on n-type silicon.

Many papers have been written about samples of non-rectangular geometry. For references see Hooge et al.

2) Collision-Free Devices: Up to here we discussed only semiconductor devices that were collision limited, so that $\alpha_H$ was determined by collision processes. We now turn to devices in which collisions either do not exist, as in vacuum tubes and in Schottky-barrier diodes operating in the thermionic mode, or to devices in which collision processes are not the determining factor, as in long p-i-n diodes.

In that case a Hooge type equation of the form [12], [13]

$$S_{\nu}(f) = \frac{\alpha_H e}{f r N}$$  \hspace{1cm} (8)

describes the $1/f$ noise. Here $\alpha_H$ may have a different magnitude than in collision-dominated devices, but $N$ again is the number of carriers in the system. This is, e.g., the case for vacuum tubes like space-charge-limited vacuum diodes, triodes, and pentodes, or saturated vacuum photodiodes, and secondary emission multiplication stages, etc. It holds for any system in which the $N$ carriers generate $1/f$ noise independently (fundamental $1/f$ noise).

Since the current flow is by injection, $I/e$ is the number of carriers injected per second and $N = h\nu e$ is the number of carriers present in the sample. Consequently, if we substitute for $N$,

$$S_{\nu}(f) = \frac{\alpha_H e}{f r}$$  \hspace{1cm} (8a)

where $\tau$ is the carrier transit time. For an electron traveling between two parallel electrodes at a distance $d_{\nu}$, with negligible charge between them

$$\tau = \frac{2d_{\nu}}{V_2 + V_1}$$  \hspace{1cm} (8b)

where $V_2$ and $V_1$ are the carrier velocities at the electrodes 2 and 1, respectively. For space-charge-limited current flow between two parallel electrodes of distance $d_1$

$$\tau = \frac{3(d_1 - d_2)}{V_1}$$  \hspace{1cm} (8c)

where $d_2$ is the distance between the potential minimum and cathode, $v_1 = (2e/m)^{1/2}(V + V_a)^{1/2}$ is the velocity with which the electrons arrive at the anode, $V_a$ is the anode potential, and $V_m$ the depth of the potential minimum in front of the cathode. For a long p-i-n diode $\tau$ is the time constant associated with the generation and recombination of one hole-electron pair. In each case $\alpha_H$ can be determined from $S_{\nu}(f)$ if $\tau$ is known.

Not all noises in collision-free devices satisfy (8a). Whether or not they do, must be determined by comparing the measured value of $\alpha_H$ with the theoretical values predicted in the next section.

B. Handel's Quantum Equations [2], [3]

1) Collision-Free Devices: We first start with a semiclassical consideration of collision-free devices. Since the only
physical process present in such devices is acceleration, the observed 1/f noise must be associated with this acceleration. Now an accelerated electron generates low-frequency Bremsstrahlung; since its energy spectrum is independent of the quantum energy \( \epsilon \) for small \( \epsilon \), and the number spectrum is found by dividing the energy spectrum by \( \hbar \nu \), it is obvious that this number spectrum varies as 1/f. The near-field interaction of an electron with its own Bremsstrahlung will therefore give current 1/f noise in the external circuit that is described by the Hooge parameter \( \alpha_H \). The effect is semiclassical; to evaluate \( \alpha_H \) one needs wave mechanics.

Handel uses a somewhat different model. He splits the electron wave function into a large unperturbed part and a small part that is perturbed by the Bremsstrahlung emission. In the calculation the two parts beat with each other and so give 1/f noise back. Handel thus finds the following [14]:

\[
\alpha_H = \frac{4a \Delta \nu^2}{3\pi c^2} S_f = \frac{4a_0 \Delta \nu^2 e\nu}{3\pi c^2 \nu f} \tag{9}
\]

The first part is known as the Handel equation. Here \( c \) is the velocity of light, \( \Delta \nu \) the vectorial change in velocity along the electron path, and \( a \) the fine structure constant. For motion between two parallel electrodes of distance \( d_{12} \) with terminal velocities \( v_1 \) and \( v_2 \), \( \Delta \nu = v_2 - v_1 \), and \( a = 2d_L(v_2 + v_1) \), as mentioned before. The main objection to this approach is against the beat process. For single electrons, in MKS units, where \( \mu_0 = 4\pi \times 10^{-7} \) H/m

\[
\alpha = \frac{a_0}{4} = \frac{\mu e^2}{2h} = \frac{1}{137} \quad S_f = \frac{4a_0 \Delta \nu^2 e\nu}{3\pi c^2 \nu f} \tag{9a}
\]

But in some cases the current flows in charge conglomerates \( q \). Since they are accelerated as a unit, they produce Bremsstrahlung as a unit, and hence generate 1/f current noise as a unit; consequently \( e^2 \) must be replaced by \( q^2 \), or

\[
\alpha = \alpha_0 \frac{q^2}{e^2} \quad S_f = \frac{4a_0 q^2 \Delta \nu^2 e\nu}{3\pi c^2 \nu f} \tag{9b}
\]

As a first example we consider a space-charge-limited vacuum diode. Here the shot noise is space-charge-suppressed by a factor \( 1/f^2 \) (=0.10 for normal operation) so that \( S_f(f) \) may be written

\[
S_f(f) = 2e(f^2 + 2(e\nu^2)h = 2qf \tag{10}
\]

corresponding to shot noise of charges \( q \), so that the effective charge is \( q = e\nu^2 \). Hence the 1/f noise may be written

\[
S_f(f) = \frac{4a_0 q^2 \Delta \nu^2 e\nu}{3\pi c^2 \nu f} \tag{10a}
\]

In vacuum diodes with oxide-coated cathodes the noise is masked by 1/f noise generated in the cathode coating, so that (10a) is not verifiable.

In vacuum photodiodes \( \Gamma^4 = 1 \) (no space-charge suppression), \( \Delta \nu \) is much larger than in the previous case, and hence \( S_f(f) \) becomes

\[
S_f(f) = \frac{4a_0 \Delta \nu^2 e\nu}{3\pi c^2 \nu f} \tag{11}
\]

where \( a = 2d_L(v_2 + v_1) \) and \( \Delta \nu = (v_2 - v_1) \). If \( V \) is the anode voltage and \( v_1 = 0 \), \( S_f(f) \) varies as \( I V^2 \); this should be measurable, unless masked by classical 1/f noise due to fluctuations in the electron affinity of the photocathode (see Section V).

As a second example we take a secondary emission multiplier stage [15]-[17]. Let \( I_p \) be the primary current of the multiplier and \( S \) the secondary multiplication factor, then the output current \( I = S I_p \) so that the current consists of charge conglomerates of charge \( q = 6e \). Hence

\[
S_f(f) = \frac{4a_0 q^2 \Delta \nu^2 e\nu}{3\pi c^2 \nu f} \tag{12}
\]

when \( r \) is the transit time between the secondary emission electrode (dynode) and the collecting electrode (anode). In secondary emission pentodes this noise is usually masked by the 1/f noise, \( S_f(f) \), of the primary current. The latter can be suppressed satisfactorily by appropriate cathode feedback [12], [13], [15], [17]; in that case \( S_f(f) \) becomes measurable. Again, \( S_f(f) \) varies as \( I V^2 \), where \( V \) is the potential difference between anode and dynode, and this can be verified (see Section V).

In vacuum pentodes cathode 1/f noise is distributed between screen grid and anode, whereas partition 1/f noise flows from screen grid to anode. The latter is not space-charge suppressed, whereas the former is. Nevertheless, the partition 1/f noise is masked by cathode 1/f noise, unless the latter is sufficiently reduced by a feedback resistor \( R \) in the cathode lead. In that case the partition 1/f noise becomes measurable [12], [13], [18] and its possible quantum character can be investigated (Section V).

We have here discussed the predictions made by Handel's quantum 1/f noise theory for various vacuum tubes. By comparing the calculated spectra with the experimental data we may then be able to either refute or verify these predictions. The experimental data are independent of Handel's equations (9)-(12), and so can serve as independent checks of those equations.

2) Collision-Dominated Devices: We now turn to quantum 1/f noise in semiconductor devices. Here the devices are collision-dominated and (9) must be appropriately modified: in (9) \( \Delta \nu^2 \) must now be averaged over all collisions and replaced by \( \Delta \nu^2 \). Bremsstrahlung 1/f noise is still considered the initiating process, however. Since the carriers are single electrons or single holes, they always have a charge \( \pm e \); hence the fine structure constant \( a \) always has the value \( a_0 = 1/(137) \). Consequently, for a single scattering process, (9) may be written as

\[
\alpha_H = \frac{4a_0 \Delta \nu^2}{3\pi c^2} = 3.09 \times 10^{-3} \Delta \nu^2/c^2 \tag{13}
\]

where the averaging must be performed in \( k \)-space over all scattering angles \( \theta \) and over the electron-velocity distribution [2], [3], [19].

There are different scattering processes possible, each having its associated mobility \( \mu \), and Hooge parameter \( \alpha_H \). We then have according to Kousik and van Vliet [20]

\[
\frac{1}{\mu} = \sum \frac{1}{\mu_i} \left( \frac{\delta \mu}{\delta \mu_i} \right) = \sum \frac{\delta \mu_i}{\mu_i} \tag{13a}
\]
Introducing
\[ \alpha_H = \frac{S_H(f)}{\mu^2} \text{ (fN)} \]
and multiplying by \( \mu^2 \) yields
\[ \alpha_H = \sum \alpha_{Hn} \left( \frac{\mu}{\mu_0} \right)^2 \cdot \rho \]

We now have the following semiclassical processes:

**a) Normal Collision Processes (Acoustical Phonon Scattering, Optical Phonon Scattering, Impurity Scattering):**
Calculating the \( \alpha_{Hn} \) and \( \mu_0 \) for each of these processes, Kousik and van Vliet found \( \alpha_{Hn} = 3.3 \times 10^{-9} \) for n-type Si at \( T = 300 \) K (mostly acoustical phonon scattering) and \( \alpha_H = 1.6 \times 10^{-8} \) for n-type GaAs at \( T = 300 \) K (mostly optical phonon scattering).

Approximate values can be found by assuming elastic scattering [19]. In that case, the change in velocity \( \Delta v = 2v \sin \theta \) has a mean square value
\[ \Delta v^2 = 4v^2 \left( \frac{\sin^2 \theta}{3} \right) = \frac{2v^2}{m^*} \]
for a Maxwellian velocity distribution, or
\[ \alpha_H = \frac{4\alpha_0}{3\pi} \frac{6kT}{m^*} \cdot \frac{3}{1/m_1^* + 1/m_2^* + 1/m_3^*} = 0.241m. \]

The approximation is often not very reliable, as the examples show; the values for \( \alpha_{Hn} \) as obtained by the Kousik-van Vliet method [2], [3], [19], [20] are much more accurate. Combining both considerations yields an estimated value \( \alpha_{Hn} \approx (2-3) \alpha_{Hn} \) or \( \alpha_H \approx (6-10) \times 10^{-9} \).

**b) Umklapp Processes [2], [3], [19], [20]:** In an Umklapp process an electron can give up a momentum \( h/a \) to the lattice or accept a momentum \( h/a \) from the lattice, while being scattered into the next Brillouin zone; here a is the lattice spacing. Hence \( \Delta v = h(m^*a) \) and
\[ \alpha_{Hu} = \frac{4\alpha_0}{3\pi} \left( \frac{h}{m^*a} \right)^2 \cdot \exp \left( \frac{-\theta_0/4T}{2} \right) + \frac{\Delta v^2}{\mathcal{N}_{norm}} \left[ 1 - \exp \left( \frac{-\theta_0/4T}{2} \right) \right] \]

since \( \mu^{-1} = \mu_{t}^{-1} + \mu_{norm}^{-1} \). The last term involving \( \Delta v^2 \mathcal{N}_{norm} \) is usually negligible.

The first term of (15b) corresponds to the van der Ziel-Handel [19] heuristic form for Umklapp 1/f noise and the derivation gives it a firmer theoretical basis. It should hold for weakly doped n-type Si, but should be absent for weakly p-type Si because it has no intervalleys. It could also hold for degenerate materials, narrow-bandgap materials, and for strongly inverted MOSFET channels.

**c) Coherent State 1/f Noise [22]:** In (13) \( \Delta v^2 \mathcal{N}_{norm} \) is much smaller than unity and hence \( \alpha_{Hu} \approx 3.09 \times 10^{-10} \). It should therefore be clear that \( \alpha_{Hn} \) values as large as \( (2-5) \times 10^{-9} \) cannot be explained by Handel’s equation. Only in Umklapp processes involving carriers with very low effective masses \( m^* \) (as in Hg, Cd, Te) can \( \Delta v^2 \mathcal{N}_{norm} \) be so large that there is a small relativistic correction [24] to (13). Hence the large observed values of \( \alpha_{Hn} \) for long semiconductor resistors cannot be explained in this manner.

Handel has proposed [22], [23] a different fundamental mechanism, called “coherent state” quantum 1/f noise. The term “coherent state” is a wave-mechanical term and the process itself is difficult to explain, and we are not making inverted MOSFET channels [19]. In that case, Kousik et al. find \( \mu_{norm} \approx \exp (-\theta_0/4T) \), where \( \theta_0 \) is the Debye temperature.
an attempt here. The result, however, is very simple; Handel predicted

$$\alpha_H = \frac{2\alpha_0}{\pi} = 4.6 \times 10^{-3}$$

(16)

in close agreement with some experimental values $(2-5) \times 10^{-3}$. Whether or not this is a coincidence remains to be seen, but at least it explains how Hooge’s result might indicate a fundamental process.

Whereas Hooge’s result holds for long resistors, it has also been found that $\alpha_H$ can be of the order of $10^{-8}-10^{-9}$ for short devices [25], [26], [19] like short FETs and BJTs. Hence there should be a transition from ‘high’ 1/f noise $(2 \times 10^{-8})$ to ‘low’ 1/f noise $(10^{-9}-10^{-8})$ when going to shorter lengths. Handel [23] has proposed a formula for this transition, but it can only be tested when reliable experimental data have been obtained.

3) Ballistic Devices: We next return to a collision-free device: the n-type Schottky barrier diode operating in the thermionic mode. The electrons with a forward velocity $v_1 > v_0$ can pass the potential barrier and contribute to the forward current. Here

$$v_0 = [e(V_{diff} - V)/m^*]^{1/2}$$

(17)

where $(V_{diff} - V)$ is the barrier height and $m^*$ the effective mass. The electrons passing the space-charge region are decelerated and hence produce 1/f noise. Trippe gave a computer solution of the problem [27] whereas Luo et al. [28] gave a solution in closed form.

To outline their approach we write the second part of (9) in differential form, put $dV = v_1 - v_0$, and $\tau = 2dl(v_1 + v_2)$, where $v_1$ is the initial velocity (at $x = 0$) and $v_2$ the final velocity at the barrier (at $x = d$), and $d$ the width of the barrier. This yields

$$dS(f) = \frac{4\alpha_0}{\pi} \frac{(v_1 - v_0)^3 \epsilon v d_l}{c^2} \frac{\epsilon v d_l}{2\pi f} (v_1 + v_2).$$

(18)

Here $dS_A = \epsilon v n A$ is the differential current, $A$ the cross section of the device, and $dn$ the number of electrons arriving with an initial velocity between $v_1$ and $v_1 + dv_1$

$$dn = N_0 \left[\frac{m^*}{2\pi kT}\right]^{1/2} \exp\left(-\frac{m^* v_1^2}{2kT}\right) dv_1$$

(18a)

where $N_0$ is the donor concentration. Moreover

$$v_2^2 = v_1^2 - 2e(V_{diff} - V)/m^*$$

(18b)

$$d = \frac{2\epsilon v_0}{\epsilon v d_0}(V_{diff} - V)^{1/2}$$

(18c)

where $\epsilon v_0$ and $\epsilon$ are the MKS conversion factor and the relative dielectric constant, respectively. Introducing the limits $v_0$ and $\infty$, Luo et al. [28] found

$$S(f) = \frac{4\alpha_0}{\pi} \frac{e^2}{\epsilon v d_0} \frac{(V_{diff} - V)}{m^*} \left[\frac{N_0}{\epsilon v d_0}\right]^{1/2}$$

$$\left[1 - \frac{\pi kT}{4e(v_0)} (V_{diff} - V)^{1/2}\right].$$

(19)

4) $\Delta V^2$ Described in Terms of an Energy $E$: We [29]-[32] finally discuss a set of processes in which $\Delta V^2$ can be described in terms of an energy $E$ such that $\Delta V^2 = 2E m^*$, where $m^*$ is the effective mass.

In that case $\alpha_H$ may be written

$$\alpha_H = \frac{4\alpha_0}{\pi} \frac{2E}{3\pi m^* e^2 T^2}$$

(20)

The question is to find the energy $E$. We give several examples.

a) Fluctuation in Carrier Injection Across Junction Barriers [29]-[31]: If there are no collisions, $E = e(V_{diff} - V)$ where $(V_{diff} - V)$ is the barrier height in electron-volts (the references have an additional term $3kT/2$ that should be removed). If there are collisions, the energy $E$ is lost in steps and $v^2$ must be replaced by $\Delta V^2$, so that (20) might still be valid if $\Delta V^2 = \Sigma \Delta E/m^* = 2E/m^*$. It is doubtful that this will be the case.

b) Recombination of Electrodes in the Junction Space-Charge Region: This is a two-step process, involving the subsequent capture of an electron (mass $m_e^*$) and a hole (mass $m_h^*$). Van der Ziel and Handel [29], [30] find for the collision-free case

$$\alpha_H = \frac{4\alpha_0}{\pi} \frac{2E}{3\pi \left(m_e^* + m_h^*\right)^{1/2}}$$

(20a)

if the capture of an electron and a hole are independent events. This may need correction if the two events are correlated.

c) Recombination of Electrons in the p-region of an n+ - p Diode: According to van der Ziel [31] $E = 3/2kT$ because the captured electron arrives with an average kinetic energy $3kT$. But it might also be argued that an “activation energy” $E_a$ should be added with $E_a = E_{phonon}$ for traps at midband and $E_a = E_{phonon}$ for direct band-to-band transitions; here $E_{phonon}$ is the band gap.

d) Recombination at Surfaces and at Contacts [32]: This problem is similar to case c). Van der Ziel et al. [32] added a term $E_S = E_{phonon}$ for contact recombination. This should be replaced by $\Delta E_{phonon} = E_{phonon}$ where $E_{phonon}$ is the Fermi level, since the electron drops from the bottom of the conduction band in the semiconductor to the Fermi level in the metal at the contact.

None of the processes a)-d) have been observed so far [33]. It seems that this problem requires further scrutiny, especially the presence of the activation energy $E_a$ (c), d) and the degree of correlation between subsequent electron and hole captures b).

5) Summary: We have now discussed most of the predictions made by Handel’s quantum 1/f noise theory. We shall see in Section V whether these predictions can be refuted or verified by experiment.

The spectra $S(f)$ of the collision-free devices are all proportional to the current $I$. That is a direct consequence of the Hooge equation and comes about because the number $N$ of carriers in the system is proportional to $I$. We shall see in Section III that the same is true for diffusion-dominated junction devices like n+ - p and n+ - p-n and p+ - n-BJT’s, all at strong forward bias, and we shall also see that it comes about because the minority carrier density $N(x)$ per unit length at $x$ is proportional to the current $I(x)$ at $x$.

Nevertheless, there are many cases on record where the 1/f noise spectra of junction devices vary as $1/f$ with $\gamma > 1$. It should be clear that in such cases the noise cannot be described by the Hooge equation. It is also found that MOS-FETs show a bias dependence $S(f)$ different from what is predicted in Section III-A; apparently the Hooge equation...
is not valid in these cases either. We shall see that in those cases traps in the surface oxide are responsible (Section IV).

Another feature that can be explained by traps is that the 1/f noise of many devices may vary strongly from unit to unit and from batch to batch. It comes about because the 1/f noise is proportional to the trap density and will therefore vary if that density varies.

On the other hand, the diffusion or mobility 1/f noise in BJTs or FETs under comparable conditions (i.e., for comparable interaction processes) all have the same value of \( \alpha_T \). This is not a consequence of Handel's quantum 1/f noise theory, but comes about because comparable devices are subjected to identical diffusion or mobility fluctuation processes, even from a classical point of view. As a consequence the \( \alpha_T \)'s should be identical.

Since the trapping 1/f noises in BJTs have usually a different current dependence than the quantum 1/f noise, it is often possible to discriminate between the two types of processes.

III. USING HOOGES'S 1/f EQUATION AS A LANGEVIN NOISE SOURCE

We saw how for a uniform semiconductor resistor \( R \) of length \( L \) and cross-sectional area \( A \) the parameter \( \alpha_T \) could be directly evaluated with the help of definition (2). Things are less simple for nonuniform devices such as JFETs and MOSFETs at arbitrary drain bias, junction diodes, \( p^+ - n - p \) and \( n^+ - p - n \) BJTs, and Schottky-barrier diodes. In nearly all these cases a generalized Langevin approach can be used, in which Hooge's equation (2), in a distributed form, is used to give an expression for the cross-correlation spectrum of the Langevin (distributed) noise source \( H(x, \theta) \). The approach is in itself not new, but is here applied to the various devices mentioned before, so that the analogy between the various applications becomes obvious. It then also becomes clear why in some cases a modified approach must be used.

For a section \( dx \) at \( x \) of a nonuniform device (2) may be written

\[
S_n(x, t) = \frac{I_n(x) \alpha_T}{f_N(x) dx}
\]

where \( N(x) \) is the carrier density for unit length at \( x \) and \( \alpha_T \) is assumed to be independent of \( x \). Usually \( f \) is independent of \( x \), but in long \( n^+ - p \) diodes ("long" means that the length \( w_p \) of the \( p \)-region is large in comparison with the electron diffusion length \( L_e = (D_e \tau_e)^{1/2} \)), \( f(x) \) depends on \( x \). We shall see that this requires a modification in the method of approach.

Consequently the cross-correlation spectrum of the distributed Langevin noise source is

\[
S_n(x, x', t) = \alpha_T \frac{f_N(x) f_N(x') dx}{f_N(x) dx} \delta(x' - x).
\]  

One can now write down the Langevin equation of the system, linearize it, integrate with respect to \( x \) over the device length \( L \), apply the boundary conditions at \( x = 0 \) and \( x = L \), and express the resulting external current fluctuation \( \delta(I, \theta) \) in terms of the integral of \( \delta(H, \theta) \) with respect to \( x \). One then transforms to spectra, carries out the integration, and obtains \( S_n(\theta) \).

We give several examples in the following sections.

A. FET (MOSFET and JFET)

The Langevin equation is

\[
I_d = g(V) \frac{dV}{dx} + H(x, t)
\]

where \( I_d = I_{th} + \Delta I_d(t) \) is the current in the channel, \( V = V_g + \Delta V(x, t) \) the voltage distribution along the channel, and \( H(x, t) \) the random source function. Substituting for \( I_d \) and \( V \) and neglecting second-order terms yields

\[
I_{th} = g(V) \frac{dV}{dx}
\]

\[
\Delta I_d(t) = \frac{d}{dV_d} (g(V) \Delta V(x, t)) + H(x, t) dx.
\]  

We now h.f. short-circuit the drain to the source, so that \( \Delta V(0, t) = \Delta V(L, t) = 0 \) for all \( t \), integrate with respect to \( x \) for constant \( t \), and divide by \( L \); this yields

\[
S_n(t) = \frac{1}{L^2} \int_0^L \int_0^L S_n(x, x', t) dx \ dx'.
\]

This holds for both thermal noise \([34]\) and 1/f noise \([35]\). In the latter case

\[
S_n(x, x', t) = \frac{f_N(x) f_N(x') dx}{f_N(x) dx} \delta(x' - x)
\]

or

\[
S_n(t) = \alpha_T \frac{e^{\theta t} P_d}{E_t^2}
\]

for \( V_d < V_{ds} \). This result was already obtained by Klaassen \([35]\). If \( \alpha_T \) is independent of \( V_d \), the result holds for arbitrary \( V_d \) as long as the device is not saturated; if \( \alpha_T \) depends on \( V_d \), a suitable average \( \alpha_T \) must be taken. For small \( V_d \), (23) is always correct because the device acts as a uniform resistor.

B. Short \( p^+ - n \) Diode (\( w_n \ll \) Hole Diffusion Length \( L_p \))

The Langevin equation may be written

\[
I_p = -eD_p \frac{dP}{dx} + H(x, \theta).
\]

Substituting \( I_p = I_{po} + \Delta I_p(t) \) and \( P = P_0(x) + \Delta P(x, t) \) yields

\[
I_{po} = -eD_p \frac{dP_0}{dx}
\]

\[
\Delta I_p(t) dx = -eD_p \Delta P(x, t) + H(x, \theta) dx.
\]  

If the device is h.f. short-circuited \( \Delta P(0, t) = \Delta P(w_n, t) = 0 \). Carrying out the integration yields

\[
S_{\Delta P}(t) = \frac{1}{w_n} \int_0^{w_n} \int_0^L H(x, \theta) dx \ dx
\]

\[
S_p(t) = \frac{1}{w_n} \int_0^{w_n} \int_0^L S_n(x', t) \ dx \ dx'.
\]

in complete analogy with the FET case. But, according to Hooge we have

\[
S_{th}(x', \theta) = \alpha_T \frac{e^{\theta t} P_d}{E_t^2} \delta(x' - x).
\]
One might argue whether \( P_0(x') \) should be replaced by the excess hole concentration \( P_0(x') \). We believe that this should not be done, because one cannot distinguish between "normal" holes and "excess" holes. Moreover, experiments seem to favor \( P_0(x') \) rather than \( P_0(x) \) (see Section V).

Carrying out the integrations yields, since \( I_{0D} = -eD_{0D} \frac{dP_d}{dx} \),

\[
S_p(f) = \frac{e^2}{2\tau_{Dp}} \ln \left[ \frac{P_d(0)}{P_d(\infty)} \right]
\]

where \( \tau_{Dp} = \tau_{pD} / 2D_p \) is the diffusion time for holes through the n-region. This is the well-known Kleinpenning-van der Ziel result [36]-[38]. Similar equations hold for \( n^+\)-p, \( n^+\)-p-n, and \( p^+\)-n-p devices with slightly different boundary conditions. We come back to that in Section III-D. However, the method breaks down for long diodes \( (w_n > l_p) \), as is shown in Section III-D. It is interesting to note that \( S_p(f) \) is approximately proportional to \( I_{0D} \).

The \( 1/f \) noise investigated in this model is diffusion \( 1/f \) noise. If the current flow is by generation-recombination in the junction space-charge region, a different approach is needed [29], [30].

C. Diffusion \( 1/f \) Noise in \( n^- \)-Type Schottky-Barrier Diodes

The Langevin equation is now, if \( x = 0 \) at the metal electrode,

\[
I = e\mu + eD_{0D} \frac{dN}{dx} + H(x, t)
\]

where \( \mu \) and \( D \) are the mobility and the diffusion constant, respectively, \( N(x) \) the carrier density for unit length, \( F = -d\psi / dx \) the field strength, \( \psi(x) \) the potential at \( x \), and \( H(x, t) \) the random source function.

Multiplying both sides by the integrating factor \( \exp \left( -e\psi(x)/kT \right) dx \), putting \( \tau_{Dp} = eD \), and substituting for \( F(x) = -d\psi / dx \) yields

\[
I \exp \left( -e\psi(x)/kT \right) dx = D, d[\{N(x) \exp \left( -e\psi(x)/kT \right) \}]
\]

If the device is now shortcircuited for h.f., and we put

\[
I = I_0 + \Delta I(t) \quad \psi(x) = \psi_0 + \Delta \psi(x, t)
\]

then \( \Delta N(0, t) = \Delta N(d, t) = 0 \) and \( \Delta \psi(0, t) = \Delta \psi(d, t) = 0 \) for all \( t \). Integrating with respect to \( x \) between the limits 0 and \( d \), eliminating second-order terms, and equating the dc terms and the noise yields

\[
I_0 = eD \int_0^d \left\{-N_0(0) + N_0(d) \exp \left( -e(V_{dd} - V_0) / kT \right) \right\} \exp \left( -e\psi_0 / kT \right) \exp \left( -e\psi_0 / kT \right) dx
\]

where

\[
I_1 = \int_0^d \exp \left( -e\psi_0 / kT \right) dx
\]

so that

\[
S(f) = \frac{1}{\tau_{Dn}} \int_0^d \int_0^d S_p(x, x', f) \exp \left( -e\psi_0 / kT \right) \exp \left( -e\psi_0 / kT \right) dx \exp \left( -e\psi_0 / kT \right) dx'.
\]

This corresponds to the previous cases, except for the weighting factors

\[
\exp \left( -e\psi_0 / kT \right) \quad \exp \left( -e\psi_0 / kT \right)
\]

According to the Hooge equation we have

\[
S_p(x, x', f) = \frac{e^2}{2nD_{0D}} \frac{dN}{dx_0} \Delta (x' - x).
\]

Luo et al. [28] evaluated the integral and found

\[
S(f) = \frac{2}{3} \frac{e^2}{kT} \frac{\alpha_f N_d V_{dd}}{I_0}.
\]

Note that this noise spectrum again varies approximately as \( I_0 \). Van der Ziel applied this method to evaluate shot noise in Schottky-barrier diodes using the appropriate shot-noise source for \( S_d(x, x', f) \) [39].

D. Transfer Function Method for Diode \( 1/f \) Noise (Transmission-Line Method)

We consider an \( n^-\)-p diode having an arbitrary length \( w_p \) of the p-region. We further assume that the device electrodes are h.f. short-circuited; the noise current in the external circuit then equals the noise current at the junction (\( x = 0 \)). However, since the \( 1/f \) noise is a distributed noise source, the noise generated at \( x \) must propagate to the junction at \( x = 0 \). If \( x \) is comparable to \( w_p \), this propagation results in an attenuation. As a consequence, the calculation for the diode noise must be redone by another method. We do this first for the diffusion \( 1/f \) noise sources.

Let \( I_{0D}(x) \) be the electron current in the section \( \Delta x \) at \( x \) in the p-region and \( \Delta I_{0D}(x, t) \) its fluctuation. If \( \alpha_{Hooge} \) is the diffusion Hooge parameter then

\[
S_{\Delta ID}(x, t) = \frac{I_{0D}(x) \alpha_{Hooge}}{f[N(x)\Delta x]}
\]

where \( N(x) \) is the carrier density for unit length at \( x \). The current generator \( \Delta I_{0D}(x, t) \) is connected in parallel to \( Ax \).

We now apply the transmission-line method (see [40], [41]). Since \( (\Delta x / eD_n) \) is the "equivalent resistance" of the section \( \Delta x \), as "seen" by \( \Delta I_{0D}(x, t) \), the fluctuating carrier density \( \Delta N(x, t) \) in the section \( \Delta x \) is

\[
\delta \Delta N(x, t) = \delta I_{0D}(x, t) \frac{\Delta x}{eD_n}
\]

This, in turn, corresponds to a current generator \( \delta \Delta I_{0D}(x, t) \) at the junction, where (see Appendix II)

\[
\delta \Delta I_{0D}(x, t) = \frac{\Delta \Delta I_{0D}(x, t)}{Z_{20}} \frac{\sin \gamma_0 W - \sin \gamma_0 W}{\sin \gamma_0 W - \sin \gamma_0 W}
\]

\[
= \frac{\Delta \Delta I_{0D}(x, t)}{Z_{20}} \frac{\sin \gamma_0 W - \sin \gamma_0 W}{\sin \gamma_0 W - \sin \gamma_0 W}
\]

Here \( Z_{20} = L_/eD_n \) is the "characteristic impedance" and \( \gamma_0 = 1 / L_n \) the "propagation constant" of the "equivalent transmission line" describing the diffusion. Moreover, \( l_n \)
$D_n$ is the electron diffusion length, $D_e$ the electron diffusion constant, and $\tau_n$ the electron lifetime, all in the p-region. Hence

$$\Delta S_n(x, f) = S_m(x, f) \frac{\cosh \gamma_n(w_p - x)}{\sinh \gamma_n w_p}$$

By integrating with respect to $x$ between the limits 0 and $w_p$ we obtain $S_n(f)$.

We first consider a short diode ($w_p << L_p$). Then $\cosh \gamma_n(w_p - x) = 1$, $\sinh \gamma_n w_p = \gamma_n w_p$, and $S_n(x) = I_d$, so that

$$S_n(f) = \frac{1}{w_p^2} \int_0^{w_p} \frac{\alpha_{l,d}(x) dx}{f(x)}$$

This is identical with the Langevin approach [Section III-B] and that is as expected, because in short diodes there is no attenuation.

We next consider a long diode ($w_p >> L_p$). Then the upper limit of integration may be replaced by $\infty$, whereas

$$\cosh \gamma_n w_p = \frac{\exp (-\gamma_n w_p)}{\exp (-\gamma_n w_p)}$$

Hence

$$S_n(f) = \frac{1}{w_p^2} \int_0^{\infty} \frac{\alpha_{l,d}(x) \exp (-2\gamma_n x) dx}{f(x)}$$

Since these integrals have a built-in attenuation factor $\exp (-2\gamma_n x)$, they cannot be derived by the Langevin method. Substituting for $\alpha_{l,d}(x)$ yields, if $u = \exp (-\gamma_n x)$

$$S_n(f) = \frac{1}{w_p^2} \int_0^1 \frac{\alpha_{l,d}(x) \exp (-2\gamma_n x) dx}{f(x)}$$

where $I_d = e(D_e/\tau_n)^{1/2} N_p$ is the back saturation current, $N_p$ the equilibrium concentration of electrons in the p-region, $I_d = I_{d,0}$, $a = \exp (eV/kT) - 1 = N'(0)/N_p$, and

$$\alpha_{l,d}(x) = \frac{eI_d}{f_{r_n}}$$

This was already derived by van der Ziel et al. [32]. Kleinpenning [37] omitted the attenuation factor $\exp (-2\gamma_n x)$. Carrying out the integration one then obtains the same equation but with a different factor $\alpha_{l,d}(x)$

$$S_n(f) = \frac{1}{w_p^2} \int_0^1 \frac{au}{u + 1} du = -\frac{1}{a} \ln (1 + a).$$

It is the merit of the transmission-line method that it introduces the attenuation factor $\exp (-2\gamma_n x)$ automatically.

If we replace $N(x)$ by $N'(x)$ in the expressions for $S_n(f)$, we must replace $(au + 1)$ by $au$; $\alpha_{l,d}(x)$ then follows from

$$\alpha_{l,d}(x) = \pm \int_0^1 u^2 du = \pm \frac{1}{3}$$

(plus sign for forward bias, minus sign for back bias [32], [41]). It must be decided by experiment which of these expressions for $\alpha_{l,d}(x)$ is valid.

E. Recombination 1/1 Noise in a Long n+-p Diode [31], [41]

We now turn to recombination 1/1 noise for the p-region of a long n+-p diode ($w_p/L_p >> 1$). If $N'(x)$ is the excess carrier density for unit length at $x$ and $\Delta N'(x) = N'(x)\Delta x$ is the number of excess carriers in the section $\Delta x$. Consequently, the recombination current $\Delta I_n(x)$ disappearing in the section $\Delta x$ at $x$ is

$$\Delta I_n(x) = \Delta N'(x) e/\tau_n$$

where $\tau_n$ is the electron lifetime in that section.

Because the electron capture cross section of the traps in the p-region fluctuates, the lifetime $\tau_n$ fluctuates and hence $\Delta n(x) = 1/\tau_n$ will fluctuate. The current fluctuation disappearing in the section $\Delta x$ at $x$ is therefore

$$\delta \Delta I_n(x, t) = \Delta N'(x) \frac{\delta N_n(x, t)}{C_n}$$

where, in analogy with Hooge's equation (2)

$$S_{con}(x, f) = \frac{\alpha_{ho} e^2}{f N'(x)}$$

or

$$S_{con}(x, f) = \frac{\alpha_{ho} e^2}{f N'(x)}$$

where $\Delta N'(x) = \Delta N'(x) + \Delta N_p$ and $\Delta N_p$ is the number of equilibrium minority carriers in the section $\Delta x$. We shall prove these relationships in a moment.

Since $N(x) = \Delta N/\Delta x$, we have in case a)

$$\Delta S_n(f) = \Delta I_n(x) \frac{\alpha_{ho}}{f \Delta N}$$

But according to the transmission-line model, the fluctuation current disappearing in the section $\Delta x$ at $x$ corresponds to a current fluctuation $\delta \Delta I_n(x, t)$ at the junction, where (see Appendix II)

$$\delta \Delta I_n(x, t) = \delta \Delta I_n(x, t) \exp (-\gamma_n x)$$

so that, if $u = \exp (-\gamma_n x)$

$$S_n(f) = \int_0^\infty S_n(x, f) dx = \alpha_{ho} e f_{r_n}$$

where $\alpha_{l,d}(x)$ has the same meaning as before. For case b) we find instead $\alpha_{l,d}(x) = 1/3$ (plus sign for forward bias, minus sign for back bias). The total noise is therefore

$$S_n(f) = \alpha_{ho} e f_{r_n} (x)$$

By measuring $S_n(f)$ we can only determine the sum $\alpha_{ho} e f_{r_n}$. 

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According to (40) \( \alpha_m \) is very small \( (=0.5 \times 10^{-9}) \) for electrons in p-type silicon. We now prove case b). We write (33) as

\[
\Delta I_d(x) = e \Delta N'(x) C_m(x) = e \sum_{i=1}^{\Delta N'(x)} C_m(x) = \delta C_m(x) \tag{36}
\]

or, by taking averages on both sides,

\[
\overline{C}_m = \overline{C}_m \tag{36a}
\]

for all \( i \)'s, since all \( C_m \)'s fluctuate independently.

Next we consider fluctuations. Since \( \Delta N'(x) \) does not fluctuate

\[
\delta \Delta I_d(x, t) = e \sum_{i=1}^{\Delta N'(x)} \delta C_m(x, t) \tag{36b}
\]

or

\[
\Delta S_d(x, t) = e^2 \sum_{i=1}^{\Delta N'(x)} S_{cm}(t) = e^2 \Delta N'(x) S_{cm}(t) \tag{36c}
\]

so that

\[
\frac{S_{ad}(x, f)}{\Delta \overline{I}_d(x)} = \frac{S_{cm}(f)}{(\overline{C}_m)^2 \Delta N'(x)} = \frac{\alpha_n}{\overline{C}_m} \tag{36d}
\]

where \( \overline{C}_m = \overline{C} \) and \( S_{cm}(f)/\overline{C}_m^2 \) is independent of \( \Delta N'(x) \) and hence equal to \( \alpha_n/\overline{C}_m^2 \).

Case b) assumes that only the lifetimes of the individual excess minority carriers fluctuate. Since one cannot distinguish between equilibrium minority carriers and excess minority carriers, it is more likely that the lifetime of each minority carrier fluctuates. But that corresponds to replacing \( \Delta N'(x) \) by \( \Delta N(x) \); we then obtain case a).

F. Recombination 1/f Noise in the Base Region of a BJT

This problem can also be solved by the transmission-line model. Consider an n−p−n BJT with a base-length \( w_b \). Let the equilibrium electron concentration and the excess electron concentration \( N'(w_b) \) at \( x = w_b \) be small in comparison with the excess electron concentration \( N'(0) \) at \( x = 0 \). Then

\[
N'(x) = N'(0) (1 - x/w_b) \tag{37}
\]

\[
\Delta I_d(x) = \frac{eN'(0)\Delta x}{r_n} \tag{37a}
\]

\[
I_{bp} = \int_0^{w_b} \Delta I_d(x) \, dx = \frac{eN'(0)}{r_n} \int_0^{w_b} \left(1 - \frac{x}{w_b}\right) \, dx = \frac{eN'(0)}{2w_b} \tag{37b}
\]

whereas the noise in the section \( \Delta x \) has a spectrum (see (34))

\[
S_{ad}(x, f) = \alpha_n e^2 \frac{N'(0)}{r_n^2} \left(1 - \frac{x}{w_b}\right) \Delta x \tag{37c}
\]

According to the transmission-line theory the transfer function is (Appendix II)

\[
\frac{\sinh \gamma_d(w_b - x)}{\sinh \gamma_d w_b} = \left(1 - \frac{x}{w_b}\right) \tag{37d}
\]

for small \( \gamma_d w_b \). Consequently

\[
S_{ad}(f) = \int_0^{w_b} S_{ad}(x, f) \, dx = \alpha_n e^2 \frac{N'(0)}{r_n^2} \int_0^{w_b} \left(1 - \frac{x}{w_b}\right)^3 \, dx
\]

\[
= \frac{1}{4} \alpha_n e^2 \frac{N'(0)}{r_n^2} \tag{37e}
\]

Substituting for \( I_{bp} \) yields

\[
S_{ad}(f) = \alpha_n e^2 \frac{I_{bp}}{2r_n}. \tag{37f}
\]

This result has not been published before. According to the end of Section II-B4

\[
\alpha_n = \frac{4e^2}{3\pi} \left[ \frac{kT}{m_e^2} \right] \tag{37g}
\]

Since \( kT/e = 25 \) mV, \( \alpha_n = 0.5 \times 10^{-9} \) for electrons in p-type silicon.

The competing 1/f noise mechanism in n−p−n BJTs is due to hole injection from the base into the emitter followed by diffusion toward the emitter contact. The base current \( I_{bp} \) is associated with this process. Hence by analogy with (25a)

\[
S_{ad}(f) = \alpha_n e^2 \frac{I_{bp}}{2r_n} \tag{37h}
\]

where \( \alpha_n = (6-10) \times 10^{-9} \) (see Section VI), \( I_{bp} \gg I_{bp} \), and the diffusion time \( \tau_{dp} = \frac{w_b^2}{4D_p} \gg \tau_r \). In the base region \( \tau_{dn} = \frac{w_b^2}{2D_n} \) and \( \tau_{dp} = \frac{w_b^2}{2D_p} \ll \tau_r \) (for n−p−n BJTs with a large \( \beta \)), whereas \( \tau_{dp} \) and \( \tau_{dn} \) are comparable. Consequently, \( S_{ad}(f) \gg S_{ad}(f) \), so that the recombination effect in the base is generally a negligible source of 1/f noise.

G. Fluctuations in the Contact Recombination Velocity at the Contact to the p-Region of a Short n−p Diode

If a short n−p diode has a length of \( w_p \) of the p-region and the ohmic contact to the p-region has a contact recombination velocity \( s_{cn} \) for electrons then the electron current is given by

\[
I_{ro} = \frac{eD_n}{w_p} \left[ N'(0) - N'(w_p) \right] = e s_{cn} N'(w_p) \tag{42}
\]

where \( N'(0) \) and \( N'(w_p) \) are the excess electron concentrations at the junction \( x = 0 \) at the contact \( x = w_p \), respectively. Solving for \( N'(w_p) \) yields \( N'(w_p) = \frac{eS_{cn}}{w_p} + D_n/w_p \), and

\[
I_{ro} = \frac{eN'(0)}{w_p} \left[ \frac{S_{cn}}{S_{cn}} + \frac{D_n}{D_n+w_p} \right] \tag{43a}
\]

where \( s_{cn} = 6 \times 10^4 \) cm/s for electrons in Si, and \( s_{cp} = 4 \times 10^6 \) cm/s for holes in Si.
If \( s_{cn} \) fluctuates, \( I_{na} \) fluctuates and hence

\[
\delta I_{na} = eN'(0) \frac{D_n}{w_p} \frac{D_n/w_p}{D_n/w_p} \frac{s_{cn}}{s_{cn}}. 
\]

(43a)

For an n\textsuperscript{+}-p Si diode with \( w_p = 1 \mu m \) we have \( D_n/w_p \ll s_{cn} \). Hence in first approximation we may replace \( s_{cn} + D_n/w_p \) by \( s_{cn} \) so that

\[
I_{na} = eN'(0) \frac{D_n}{w_p} 
\]

and

\[
\delta I_{na} = I_{na} \left( \frac{D_n}{w_p s_{cn}} \right) \cdot \frac{\delta s_{cn}}{s_{cn}}. 
\]

(43b)

Consequently, according to (7a)

\[
S_d(f) = \alpha_{f\!d} f_{\!d n} \left[ \frac{D_n}{w_p s_{cn}} \right]^2 
\]

\[
= \alpha_{f\!d} f_{\!d n} \left[ \frac{D_n}{w_p s_{cn}} \right]^2 \frac{\alpha_{en}}{N_{eff}} 
\]

(44a)

\[
S_n(f) = \alpha_{n\!d} f_{\!d n} \ln \left[ \frac{N(0)}{N(w_p)} \right]. 
\]

(44b)

where \( N_{eff} = \frac{1}{2} N'(0) w_p \) and \( \alpha_{en} \) is given by (40). Substituting for \( N_{eff} \) yields, if \( \tau_{dn} = w_p^2/2 D_n \)

\[
S_d(f) = \alpha_{f\!d} f_{\!d n} \left[ \frac{D_n}{w_p s_{cn}} \right]^2. 
\]

This result has not been published before.

For electron diffusion 1/f noise in the p-region (see (25a))

\[
S_n(f) = \alpha_{n\!d} f_{\!d n} \ln \left[ \frac{N(0)}{N(w_p)} \right]. 
\]

(44b)

Here \( \alpha_{f\!d} \) and \( \alpha_{n\!d} \) can have comparable values and \( \ln [N(0)/N(w_p)] = 3-4 \) (see next section) so that (44a) is smaller than (44b) by a factor

\[
\left( \frac{s_{cn}w_p}{D_n} \right)^2 = \left( \frac{6 \times 10^8 \times 10^{-8}}{35} \right) = 300
\]

for \( w_p = 10^{-4} \) cm. Therefore (44a) is negligible in comparison with (44b).

H. Evaluation of \( [N(0)/N(w_p)] \) and Its Application to Transistor Noise

According to the previous section we have for a short n\textsuperscript{+}-p diode

\[
\frac{N(0)}{N(w_p)} = 1 + \frac{s_{cn}w_p}{D_n} 
\]

(45a)

for large forward bias, \( \exp (eV/kT) \gg 1 \) and \( N'(0) \gg N_p \) and \( N(w_p) \gg N_p \). In that case (45) may be written [31]

\[
\frac{N(0)}{N(w_p)} = 1 + \frac{s_{cn}w_p}{D_n} 
\]

(45a)

or

\[
S_n(f) = \alpha_{n\!d} f_{\!d n} \ln \left[ 1 + \frac{s_{cn}w_p}{D_n} \right]. 
\]

(45b)

where \( \tau_{dn} = w_p/2 D_n \). Here \( S_n(f) \) is the 1/f diffusion noise (25a).

This is easily applied to an n\textsuperscript{+}-p-n transistor. Here the base current \( I_b \) is normally due to hole injection from the base into the emitter and diffusion through the emitter region toward the emitter contact. Then

\[
P(0)/P(w_p) = 1 + \frac{s_{en}w_e}{D_e} 
\]

(45c)

\[
S_e(\tau) = \alpha_{en} w_e \ln \left[ 1 + \frac{s_{en}w_e}{D_e} \right] 
\]

(45d)

where \( \tau_{en} = w_e/2 D_e \) and \( w_e \) is the length of the emitter region.

For the collector current \( I_c \) of an n\textsuperscript{+}-p transistor the current flow is due to electron diffusion through the base region. It is usually assumed [31] that the electrons leave the base region with the limiting velocity \( v_e = 10^7 \) cm/s; more exactly, the velocity in question is the electron velocity in the collector space-charge region. We must then replace \( s_{en} \) by \( v_e \), \( P \) by \( N_p \), \( p \) by \( n \), \( l_c \) by \( I_b \), and \( w_l \) by \( w_p \), so that

\[
\frac{N(0)}{N(w_p)} = 1 + \frac{v_{en}w_{en}}{D_v} 
\]

(46)

\[
S_n(\tau) = \alpha_{en} \frac{v_{en}}{2 \tau_{en}} \ln \left[ 1 + \frac{v_{en}w_{en}}{D_v} \right]. 
\]

(46a)

In short n\textsuperscript{+}-p Hg, Cs, Cd, Te photodiodes one needs to known \( N(0)/N(w_p) \) for zero near-zero bias or for back bias. Equation (42) must then be rewritten as

\[
I_{en} = eD_p \left[ \frac{N(0) - N(w_p)}{w_p} \right] = eN_p [N(w_p) - N_p] 
\]

where \( N_p = N(0) \exp (-eV/kT) = A/\gamma N_p \) is the equilibrium hole concentration for unit length. Here \( A \) is the cross-sectional area of the diode, \( \gamma \), the intrinsic carrier concentration, and \( N_p \) the acceptor concentration in the p-region. Then by substituting for \( N_p \) and solving for \( N(w_p) \)

\[
S_n(\tau) = \alpha_{en} \frac{I_{en}}{2 \tau_{en}} \ln \left[ 1 + \frac{N(0)}{N(w_p)} \right]. 
\]

(46c)

\[
\frac{N(0)}{N(w_p)} = 1 + \left( \frac{v_{en}w_{en}}{D_v} \right) \exp (-eV/kT) 
\]

(46d)

where \( V \) is the applied bias, \( I_{en} = I_{en} \exp (eV/kT) - 1 \)

\[
I_b = eN_p s_{en} + D_n/w_p. 
\]

(46c)

For large negative bias \( I_{en} = -I_b \) and \( N(0)/N(w_p) = \exp (-e|V|/kT) \).

IV. NONFUNDAMENTAL 1/f NOISE SOURCES

Nonfundamental noise sources are noise sources that involve carrier trapping by and carrier detrapping from traps. These traps may be in a conducting channel, in a space-charge region, or in a surface oxide, and may cause Lorentzian or 1/f type spectra. They are called "nonfundamental," since the magnitude of their spectra is proportional to the trap density; the noise effect can thus be strongly reduced by eliminating most of the traps. On the other hand, the various scattering 1/f noise sources in collision-dominated devices and the Bremsstrahlung 1/f noise in collision-free devices are essential to the operation of the device, and hence should be called "fundamental."

A. The McWhorter Theory

The earliest theory of flicker noise (Schottky, 1926) [42] involved a process governed by a time constant \( \tau \). The Lan-
The gevin equation of the process
\[ \frac{dX}{dt} + \frac{X}{\tau} = H(t) \] (47)
yields the spectrum
\[ S_n(f) = S_n(0) \frac{N^2}{1 + \omega^2} = 4X^2 \frac{\tau}{1 + \omega^2} \] (47a)
where \( \overline{X} = S_n(0)\tau \). Such a spectrum is called a Lorentzian spectrum. A good example is trapping and detrapping of electrons by surface traps, as in the MOS capacitor of MOSFETs, in the surface oxide on the base of a BJT, on the surface of the space-charge region of a p-n junction, or in the bulk space-charge region of a JFET.

A single trap level of time constant \( \tau \) can be described by (47); hence the number fluctuation spectrum is in analogy with (47a)
\[ S_n(f) = 4\Delta N^2 \cdot \frac{\tau}{1 + \omega^2} \] (47b)
where \( \Delta N^2 \) is the variance of the fluctuation \( \Delta N \) in \( N \), the number of carriers in the sample. By analogy we have for discrete, multiple-trap levels
\[ S_n(f) = \sum_i \Delta N_i^2 \cdot \frac{\tau_i}{1 + \omega^2} \] (47c)
These spectra are called generation–recombination spectra (g–r for short). Such spectra are often found in Si-JFETs, where they can mask the small amount of 1/f noise present in such devices.

It soon became clear that the spectrum was 1/f rather than Lorentzian, and efforts were made to develop models for 1/f spectra. Gradually the idea emerged that a proper distribution in time constants \( \tau \) might explain the spectra. The idea was first proposed by von Schweidler for the theory of dielectric losses; he could then explain why \( \tan \delta \) is nearly independent of frequency over a wide frequency range (1907) [43]. Gever (1946) applied this theory to his experimental dielectric loss data [44]. Later the idea was applied by du Pree [45] and by van der Ziel [46]. McWhorter [47] applied it to semiconductor devices and made it very popular; for that reason the 1/f model still bears his name.

If one has a time constant distribution \( g(\tau) \, d\tau \), then by analogy with (47a)
\[ S_n(f) = 4\Delta N^2 \int_0^\infty \frac{g(\tau) \, d\tau}{1 + \omega^2} \] (48a)
where
\[ \int_0^\infty g(\tau) \, d\tau = 1 \] (normalization).

In particular the normalized distribution:
\[ g(\tau) \, d\tau = \frac{d\tau}{\ln (\tau_2/\tau_1)} \] for \( \tau_1 < \tau < \tau_2 \)
\[ g(\tau) \, d\tau = 0, \] otherwise (48b)
yields the 1/f spectrum
\[ S_n(f) = \frac{\Delta N^2}{f \ln (\tau_2/\tau_1)} \] for \( 1/\tau_2 < \omega < 1/\tau_1 \) (49)

whereas
\[ S_n(f) = \frac{2\Delta N^2}{\pi f \ln (\tau_2/\tau_1)} \tan^{-1} (\omega \tau_2) \] for \( \omega \tau_2 < 1 \)
\[ S_n(f) = \frac{2\Delta N^2}{\pi f \ln (\tau_2/\tau_1)} \left( \frac{1 - 2}{\pi} \tan^{-1} (\omega \tau_2) \right) \] for \( \omega \tau_2 >> 1 \) (49b)
so that \( S_n(f) \) is constant for \( \omega \tau_2 << 1 \) and \( S_n(f) \) varies as \( 1/f^2 \) for \( \omega \tau_2 >> 1 \). Equation (49a) was observed on CdHgTe samples in which the surface had been cleaned by sputtering in a mercury discharge; apparently the sputtering removed the surface centers with long time constants [48]. Equation (49b) was observed by Suh and van der Ziel [49] in GaAs MESFETs; this is one of the many different types of spectra observed in these devices [50]–[52] and therefore not much emphasis should be placed on a particular one.

In most cases \( \tau_2 \) is so long and \( \tau \) so short that only the 1/f part of the spectrum is observed. How can one then be sure that the 1/f spectrum is really due to a distribution of Lorentzians? By going to samples of very small area (= 1-μm diameter). The effective number of traps in the system then becomes so small that individual Lorentzians become visible.

How can a distribution function of the form (48a) be realized? There are two obvious possibilities: a) excitation from trap levels with activation energies \( E \) (distribution in \( E \)) [43]–[46], and b) tunneling to trap levels inside surface oxide at depth \( z \) (distribution in \( z \)) [47].

Case a): Since the time constant \( \tau \) depends exponentially on \( E \),
\[ \tau = \tau_0 \exp \left( \frac{E}{kT} \right) \]
\[ \tau_1 = \tau_0 \exp \left( \frac{E_1}{kT} \right) \]
\[ \tau_2 = \tau_0 \exp \left( \frac{E_2}{kT} \right) \] (50)
and the distribution function
\[ g(E) \, dE = \frac{dE}{E_{22} - E_{11}} \] for \( E_{11} < E < E_{22} \) (50a)
and zero otherwise. This is possible for migrating ions with a distribution in activation energies, but is impossible for a distribution of trap levels in the energy gap of a semiconductor.

The reason is simple. All traps a few \( kT \) above the Fermi level are empty and all traps a few \( kT \) below the Fermi level are filled. The spectrum thus consists of two somewhat smeared-out Lorentzians, one due to transitions from the Fermi level to the conduction band and the other due to transitions from the Fermi level to the valence band. Consequently bulk number fluctuations in semiconductors cannot give 1/f noise.

Case b): Since the process is due to tunneling,
\[ \tau = \tau_1 \exp (\gamma z) \]
\[ \tau_1 = \tau_1 \exp (\gamma z_0) \]
\[ g(\tau) \, d\tau = \frac{dz}{z_0} \] for \( 0 < z < z_0 \) (50b)
and zero otherwise. Here \( \gamma \) is the tunneling parameter (≡ 10^9 cm^-1) and \( z_0 \) is the average distance between traps.
VAN DER ZIEL: 1/\nu NOISE IN ELECTRONIC DEVICES

Jindal and van der Ziel [53] have proposed a McWhorter model for the interaction of electrons and acoustical phonons. While a 1/\nu spectrum results, it seems unlikely that it can extend to sufficiently low frequencies. The Kousik-van Vliet model, based on Handel's theory, is a more likely candidate, as we shall see in Section V.

B. 1/\nu Noise in MOSFETs [54]-[58]

In MOSFETs electrons tunnel from traps in the oxide, at a distance \( z \) from the interface, to the conducting channel and vice versa. As a result, the number of trapped electrons \( \Delta N_\nu \) in a volume element \( \Delta x \Delta y \Delta z \) in the oxide fluctuates with a mean square value

\[
\langle \delta N_\nu^2 \rangle = \Delta N_\nu(E) \Delta E \Delta x \Delta y \Delta z \tau(1 - \tau)
\]

where \( \tau \) is the Fermi function and \( \Delta N_\nu(E) \) the number of traps per unit volume with an energy between \( E \) and \( \Delta E \). Since \( \tau = \tau_0 \exp(\gamma z) \) is the time constant of a trap at \( z \), where \( \gamma = 10^9 \text{cm} \) is the tunneling parameter of the traps, one has for the spectrum

\[
S_{\delta N_\nu}(f) = 4N_\nu(E) \Delta E \Delta x \Delta y \Delta z \tau(1 - \tau)
\]

and hence by integrating with respect to the trap energy \( E \), the distance \( z \), and \( \gamma \), one obtains [56], [57]

\[
S_{\delta N_\nu}(x, f) = \frac{N_\nu(E) \Delta z \Delta x \Delta y}{\gamma} \left( \frac{\delta N}{\delta N_\nu} \right)^2
\]

where

\[
\frac{\delta N}{\delta N_\nu} = \frac{C_n}{C_d + C_{ox} + C_{ox} + C_n}.
\]

Here \( C_n = e^2 N/kT \) is the channel charge capacitance per unit area, \( C_d \) the depletion capacitance per unit area, \( C_{ox} \) the surface state capacitance per unit area; \( C_{ox} \) the oxide capacitance per unit area, and \( N \) the electron density in the channel per unit area. Furthermore

\[
S_{\delta N_\nu}(x, f) = \frac{P_\nu}{\Delta N_\nu} S_{\delta N_\nu}(x, f)
\]

where \( [\Delta S_{\delta N_\nu}(x, f)]^{1/2} \) is the noise current generator in parallel to the section \( \Delta z \) at \( z \); \( S_{\delta N_\nu}(f) \) is the spectrum of the current fluctuation in the drain and \( L \) is the device length.

At weak inversion [56], \( C_n \ll (C_d + C_{ox} + C_{ox}) \), \( N_\nu(E) \) is practically independent of \( V \), and \( \nu_\nu \) and hence \( S_{\delta N_\nu}(f) \) is the same as the spectrum of the noise current generator in parallel to the section \( \Delta z \) at \( z \); \( S_{\delta N_\nu}(f) \) is the spectrum of the current fluctuation in the drain and \( L \) is the device length.

At weak inversion [56], \( C_n \ll (C_d + C_{ox} + C_{ox}) \), \( N_\nu(E) \) is practically independent of \( V \), and \( \nu_\nu \) and hence \( S_{\delta N_\nu}(f) \) is the same as the spectrum of the noise current generator in parallel to the section \( \Delta z \) at \( z \); \( S_{\delta N_\nu}(f) \) is the spectrum of the current fluctuation in the drain and \( L \) is the device length.

is independent of \( V \) and \( V_d \), the equivalent noise resistance \( R_{\nu e}(f) \) is independent of \( V \) and \( V_d \), and hence \( I_{\nu e} \). Measurements indicate that \( R_{\nu e}(f) \) can have turn-over frequencies as low as 1000 Hz at weak inversion [58]. At stronger inversion, \( R_{\nu e}(f) \) increases with increasing saturation current, so that the weak-inversion amplifier is more useful.

At strong inversion, where \( C_n \gg (C_d + C_{ox} + C_{ox}) \), \( N_\nu(E) \) depends on \( V_g \) and \( V_d \), because the position of the Fermi level depends on \( N(x) \). Reimbold neglected this effect [57], [58], but Klaassen gave an approximate solution [36] by assuming that \( N_\nu(E) \) was proportional to the carrier density \( N(x) \) per unit length at \( x \). One can then introduce a Hooge parameter \( \alpha_H \) by the definition [31]

\[
\alpha_H = \frac{N_\nu(E) \gamma N(x)}{\nu_\nu}.
\]

This parameter is independent of bias and hence yields

\[
S_{\delta N_\nu}(f) = \alpha_H \frac{P_\nu}{\nu_\nu}, \quad \text{for} \quad V \leq V_d.
\]

It is now possible to evaluate \( \alpha_H \) as a function of bias, and so determine whether \( \alpha_H \) is independent of bias. In many instances it is [25] but this is not self-evident and needs experimental proof in each case.

We now evaluate the quantum limits for silicon MOSFETs and calculate \( \alpha_H \) and \( \alpha_{n, p} \). Since \( \theta_0 = 645 \text{K}, a = 5.43 \times 10^{-8} \text{cm} \) (there are three kinds of holes), we obtain \( \alpha_{n, p} = 2.1 \times 10^{-1} \) and \( \alpha_{n, p} = 4.2 \times 10^{-1} \), if they exist. The lowest measured values for \( n \)- and \( p \)-channel MOSFETs are: \( \alpha_{n, p} = 1.0 \times 10^{-8} \) and \( \alpha_{n, p} = (3-9) \times 10^{-9} \).

We thus conclude that the surface 1/\nu noise always masks the quantum 1/\nu noise in n-channel MOSFETs, whereas in p-channel MOSFETs the quantum 1/\nu noise may just be observable in the best units. We come back to this problem in Section V.

In MOSFETs with ion-implanted channels, \( S_{\delta N_\nu}(f) \), measured as a function of the drain voltage \( V_d \), has a maximum [60] well before saturation. It comes about because at a given point \( x \) in the channel the potential energy \( \psi(x) \) has a minimum away from the surface. As a consequence, the electrons must climb a potential barrier before they can reach the surface and interact with oxide traps [62]. The 1/\nu noise is therefore reduced, and this becomes more pronounced near saturation.

We can now understand why most MOSFETs have surface 1/\nu noise and most bulk semiconductor resistors have volume 1/\nu noise. According to (51b) and (53), \( \Delta S_{\delta N_\nu}(f) \) is proportional to the surface area, whereas in resistors \( N \), and hence \( S_{\delta N_\nu}(f) \), are proportional to the device volume. Devices with a small surface-to-volume ratio have therefore bulk 1/\nu noise whereas devices with a large surface-to-volume ratio have surface 1/\nu noise, unless the surface is well-passivated.

We finally mention an MOS capacitor experiment by Amberiadis [63]. The channel was p-type and the gate voltage \( V_g \) was raised from below flat-band to strong inversion. When \( V_g \) passes the flat-band situation, the holes are repelled from the surface, the interaction of holes with traps in the surface oxide diminishes and hence the surface 1/\nu noise due to holes in gradually eliminated. There was some 1/\nu noise left; that can be attributed to hole mobility 1/\nu noise.

When an appreciable inversion sets in, the surface part of the channel becomes n-type. The electrons now react with the oxide traps and produce surface 1/\nu noise due to elec-
trons; as a consequence $S(f)$ increases by more than one order of magnitude. The merit of this experiment is that it shows how surface 1/f noise can be turned off and on. The trapping of carriers in the surface oxide can also give rise to surface potential fluctuations, and hence to mobility 1/f noise. This problem is studied at the University of Rochester (private communication).

C. Trapping Noise in p-n Junctions and BJTs

We saw that in the case of collision 1/f noise in BJTs the spectra $S_\text{mob}(f)$ and $S_\text{recomb}(f)$ were proportional to $I_\text{c}$ and $I_\text{f}$, respectively. While this occurs in very-low-noise devices, the collision 1/f noise is usually masked by trapping noise. Earlier theories (Fonger (1956) [64], Hsu et al. (1970) [65]) described this noise by fluctuations $\delta s$ in the surface recombination velocity $s$ of the surfaces. Van der Ziel [66] used this model to reconcile seemingly conflicting data. The noise phenomena can be distinguished from the collision 1/f noises with the help of their current dependence (as $I_\text{c}$, with $\gamma = 1$ or 2).

Fonger's model can be understood from the McWhorter model as follows: The recombination at the surface goes via fast surface states, but these surface states are modulated by the trapping or detrapping of carriers in the surface oxide. The fluctuation $\delta s$ in $s$ should be proportional to the fluctuation $\delta N_\text{tr}$ in the number of trapped electrons in the oxide and hence $S_\text{mob}(f)$ should be proportional to $S_\text{tr}(f)$, but $S_\text{tr}(f)$ is proportional to $N_\text{tr}$. Since $s$ is also proportional to $N_\text{tr}$ we obtain

$$S_s(f) = Cs/f$$

which is Fonger's starting point.

We now apply this to the base $I_\text{b}$ of a BJT. We write

$$I_\text{b} = I_\text{b}^\text{eff} + I_{\text{b1}} + I_{\text{b2}}$$

where $I_\text{b}^\text{eff}$ is due to carrier injection from the base into the emitter, diffusion through the emitter, and recombination at the emitter contact; whereas $I_{\text{b1}}$ and $I_{\text{b2}}$ are due to recombination at the base surface and at the surface of emitter-base space-charge region, respectively. In modern devices $I_{\text{b1}}$ predominates, but nevertheless most of the noise often comes from $I_{\text{b1}}$ and $I_{\text{b2}}$.

We first consider the noise of $I_{\text{b1}}$. Since

$$\frac{\delta I_{\text{b1}}}{I_{\text{b1}}} = \frac{\delta s}{s} \quad \text{or} \quad \frac{S_{\text{b1}}(f)}{I_{\text{b1}}^2} = \frac{S_s(f)}{s^2} = \frac{C}{s^2}$$

so that $S_{\text{b1}}$ varies as $I_{\text{b1}}^2$; if $I_\text{b} \gg I_{\text{b1}}$, $S_{\text{b1}}(f)$ varies as $I_\text{b}^2$.

Next we turn to $I_{\text{b2}}$. Here the recombination occurs in a well-defined part of the space-charge region, characterized by the coordinate $x_1$; at this point, the surface electron and hole densities are comparable. Let for an applied voltage $V$ the potential at $x$ change by $V(x) = V_1$ for a symmetric junction, and $V_1 = 3/4V$ for a strongly asymmetric junction, and let $p(x)$ and $p_1$ be the hole concentrations for the applied voltages $V$ and 0, respectively. Then $I_{\text{b2}}$ may be written

$$I_{\text{b2}} = \exp (x_1) A_{\text{eff}} = \exp \left( \frac{eV_1}{kT} \right) A_{\text{eff}}$$

where $A_{\text{eff}}$ is the effective recombination area. If $s$ now fluctuates

$$\delta I_{\text{b2}} = eA_{\text{eff}} \exp \left( \frac{eV_1}{kT} \right) \delta s$$

$$S_{\text{b2}}(f) = (eA_{\text{eff}}^2 \exp \left( \frac{2eV_1}{kT} \right) S_s(f).$$

Hence $S_{\text{b2}}(f)$ varies as $I_{\text{b2}}^2$, and if $I_{\text{b2}}$ predominates, $S_{\text{b2}}(f)$ varies as $I_{\text{b2}}^2 V_1/V$. Here $I = I_\text{b} \exp (eV/kT)$, where $m = 1$ if diffusion predominates and $m = V_1/V$, if recombination at the surface of the emitter-base space-charge region predominates. In the second case $S(f)$ varies as $f_2$, whereas in the first case ($f \gg f_2$), $S(f)$ varies as $f^{1/2}$. This explains Hsu et al. data [65].

We now discuss a 1/f noise mechanism observed by Trippp [67] in small p+−n+ tunnel diodes. There was a large amount of 1/f noise in the excess current regime at forward bias and at large back bias. In both cases $S(f)/f^2$ was constant and the values in the two regimes were nearly equal. The 1/f spectrum had some small g−r "bumps," and the h.f. noise was suppressed shot noise, as expected for a two-stage process (van Vliet [68]).

The current flow in each case is caused by tunneling via intermediate states in the space-charge region followed by a transition to the valence or conduction band. The large amount of 1/f noise is probably caused by 1/f modulation of the transition probabilities due to the fluctuating occupancy of traps near the intermediate states; the deviation from 1/f noise may indicate that relatively few traps are involved. According to this interpretation the noise mechanism represents nonfundamental noise.

D. Location of 1/f Noise Sources in BJTs

We finally discuss a method of locating 1/f noise sources in transistors [69]. There are three noise sources, $i_{\text{bc}}$ between base and collector, $i_{\text{be}}$ between base and emitter, and $i_{\text{ec}}$ between emitter and collector; they come from independent mechanisms and should therefore be uncorrelated. They have spectra $S_{\text{b2}}(f)$, $S_{\text{b1}}(f)$, and $S_{\text{b2}}(f)$, respectively. There are a base series resistance $r_{\text{bn}}$ in series with the base (Fig. 1). It should be borne in mind that $i_{\text{bc}}$ is not connected to one of the two endpoints of $r_{\text{bn}}$, but somewhat in between.

$$\text{Fig. 1. Equivalent noise circuit of BJT with feedback.}$$

Evaluating the 1/f noise in the base and the collector yields

$$S_{\text{b1}}(f) = S_{\text{b1}}(f) + S_{\text{b2}}(f)$$

$$S_{\text{b2}}(f) = S_{\text{b2}}(f) \left( g_{\text{m}}(r_{\text{bn}}) + f_{\text{bc}} \right)^2$$

$$S_{\text{b2}}(f) (1 + g_{\text{m}}(r_{\text{bn}})^2 + S_{\text{b2}}(f).$$

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Inserting a large resistor $R_e (R_e \gg r_{eb})$ in the emitter lead yields the feedback spectra

$$S_4(f) = S_{in}(f) + S_{jc}(f) + S_{ic}(f) \left( \frac{g_m R_e}{1 + g_m R_e} \right)^2$$  

(60b)

$$S_{ic}(f) = S_{in}(f) \left( \frac{g_m R_e}{1 + g_m R_e} \right)^2$$  

$$+ S_{jc}(f) + S_{ic}(f) \left( \frac{1 + g_m R_e}{1 + g_m R_e} \right)^2.$$

(60c)

The four equations (60a)-(60c) may often enable one to locate the three noise sources and identify them by their location and their current dependence. The h.f. noise is

$$[S_{ie}(f)]_{bd} = 2eI_c + 4kT r_{nb} g_m,$$

so that $r_{nb}$ can be evaluated from h.f. noise data.

V. EXPERIMENTS

We saw in the previous sections how in collision-dominated devices the Hooge parameter $\alpha_H$ could be evaluated. Comparing the experimental data with the theoretical predictions, we apply the following rules:

1) If $(\alpha_H)_{expt} > (\alpha_H)_{theory}$, the process in question is masked by another noise source.

2) If $(\alpha_H)_{expt} = (\alpha_H)_{theory}$, theory and experiment agree.

3) If $(\alpha_H)_{expt} < (\alpha_H)_{theory}$, the process in question is not present.

In collision-free processes similar rules apply to $S_i(f)$.

In BJTs and junction diodes, especially at forward bias, $S_i(f)/I = \text{constant}$ for the fundamental collision processes, whereas $S_i(f)$ varies as $I^\gamma$ for $\gamma > 1$ for surface 1/f noise processes. In that case one should measure $S_i(f)$ as a function of current, find the low-current regime for which $\gamma = 1$, and then apply the above rules to determine whether $\alpha_H$ agrees with one of the fundamental collision processes.

If surface 1/f noise predominates, $\alpha_H$ and $S_i(f)$ vary from unit to unit and from batch to batch. If a particular fundamental collision process predominates, $\alpha_H$ has a characteristic value. In the same way, in collision-free processes $S_i(f)$ is described by a formula without adjusting parameters that should be the same from unit to unit.

In many applications the optimum noise performance is obtained not by minimizing $\alpha_H$, but rather by minimizing $\alpha_H \tau$, where $\tau$ is the time constant of the carriers or of the system.

A. Collision-Free Processes

1) Vacuum Pentodes: We first consider partition 1/f noise in a vacuum pentode. To that end we first apply (8) and the first part of (9) to vacuum diodes and write for the quantum 1/f noise

$$S_q(f) = \frac{4a_0 \kappa_4}{3\pi} \sqrt{\frac{\rho}{I_{eb}}} \Gamma^4$$

(61)

where $I_e$ is the anode current, $\kappa_4$ the velocity with which the electrons reach the anode, and $N_e = I_e \tau_m/e$ is the number of electrons between potential minimum and anode; here $\tau_m = \tau_{ma} + \tau_{sa}$ and $\tau_{ma}$ and $\tau_{sa}$ are the transit times between potential minimum and control grid and between control grid and anode, respectively. The effect cannot be observed since it is masked by classical emission 1/f noise caused by classical fluctuations of the cathode emission.

The situation is more favorable for a vacuum pentode. Here we have not only cathode 1/f noise, distributed between the screen grid $g_s$ and the anode $a$, but also partition 1/f noise flowing from screen grid to anode [12]. The cathode 1/f noise components flowing in the screen grid and the anode leads are fully and positively correlated, whereas the partition 1/f noise components in both leads are fully and negatively correlated.

If $S_p(f)$ is the partition 1/f noise spectrum, we may thus write (see [14])

$$S_p(f) = \frac{2a_0 \kappa_4}{3\pi} \sqrt{\frac{\rho}{I_{eb}}} \Gamma^4 + S_q(f)$$

(61a)

$$S_p(f) = \frac{2a_0 \kappa_4}{3\pi} \sqrt{\frac{\rho}{I_{eb}}} \Gamma^4 + S_q(f)$$

(61b)

$$S_{bh}(f) = \frac{4a_0 \kappa_4}{3\pi} \sqrt{\frac{\rho}{I_{eb}}} \Gamma^4 - S_p(f)$$

(61c)

where $N_s = I_g \tau_m/e$ and $N_a = I_e \tau_g/e$; $\tau_m$ and $\tau_a$ are the transit times from the potential minimum to the screen grid and from the screen grid to the anode, respectively, whereas $I_g = I_e + I_s$.

To evaluate $S_p(f)$, we observe that for white shot noise and white partition noise we have by analogy

$$S_{bh}(f) = 2a_0 \kappa_4 \Gamma^4 - 2a_0 \kappa_4 \Gamma^4.$$

(61d)

But this is zero for $\Gamma^2 = 1$; that means that the total white fluctuations in $I_e$ and $I_s$ are independent in saturated pentodes. This should also be valid for 1/f noise; hence one would expect $S_p(f) = 0$ for $\Gamma^2 = 1$ in (61c), so that

$$S_p(f) = \frac{4a_0 \kappa_4}{3\pi} \sqrt{\frac{\rho}{I_{eb}}} \left[ g_s N_s \right]^{1/2}.$$

(62)

In all pentodes without feedback $S_p(f)$ is masked by classical cathode emission 1/f noise. But by inserting a large resistor $R_e$ in the cathode lead, all linear cathode current noises are reduced by the feedback factor $(1 + g_m R_e)$, whereas the partition noise fluctuations are not affected; here $g_m$ is the cathode transconductance. In that manner $S_p(f)$ can be accurately measured, a discrimination between 1/f noise and quantum partition 1/f noise can be made.

When one does so, one usually finds agreement between theory and experiment within the limit of accuracy ($\pm 30$ percent) [12], [70]. This is also true for Schwantes’s 1960 data [18], [70].

Of particular interest is the dependence of $S_p(10)$ on the anode voltage $V_a$. This can be measured much more accurately, since only $V_a$ has to be varied. If $V_a$ was kept constant and only $V_a$ was varied, van der Ziel et al. [69] found for a 6AU6 tube

$$\frac{S_p(10) V_a}{S_p(10) V_a} = \frac{454 V}{134 V} \exp^{-1.74}$$

(62a)

whereas the theoretical ratio was 1.75, in excellent agreement [70]. This effect comes mainly from the $v_e$ term in (62), for (454)/(134) = 3.44. On the other hand, in the anode current of a tube without feedback, $S_p(10)$ was independent of $V_a$ [71]. This is easily understood, for that case $S_p(f)$ is due
to emission fluctuations generated at the cathode, far away from the screen grid-anode region.

What is missing is a classical model of partition 1/f noise. Such a model might be constructed as follows. Consider the screen grid and look in a direction parallel to the grid and perpendicular to the grid wires. Noise processes at or near the cathode might now give rise to a fluctuating electron velocity component at the screen grid in that direction and this might result in partition 1/f noise. Such a model might perhaps explain the anomalous features of the partition 1/f noise in 6C6E pentodes [70].

Some further investigation seems therefore warranted.

2) Secondary Emission Pentodes EFP60: According to (12), the spectrum of secondary emission 1/f noise is due to the flow of carriers from the dynode \( d \) to the anode \( a \), so that

\[
S_a(f) = \frac{4\pi e^2}{3\pi} \left( \frac{V_a}{V_d} \right)^2 \frac{e f}{\alpha d a}
\]

where \( i_a \) is the anode current, \( \delta \) the secondary emission factor, \( \tau_{da} \) the transit time of the secondary electrons from the screen grid to the anode, and \( d_{da} \) the path length of the secondary electrons between dynode and anode.

The effect was discovered by Schwantes in 1958 [15], and van der Ziel [16] gave a quantitative quantum interpretation with the help of (63). Fang [17] measured two EFP60 tubes and showed that \( S_a(10)/(V_a - V_d)^{3/2} \) at constant \( f \) and \( \alpha d_{da} \) at constant \( V_a - V_d \) were independent of bias [see Table 1]. This verifies the \( \delta \) term in (63), which indicates a fine structure constant \( \delta/\sqrt{137} \), and the \( V_a^2 \) term in (63), which implies that acceleration (or Bremsstrahlung) is initiating the secondary emission 1/f noise process. From the averages of Table 1 one can evaluate \( S_a(10)/(V_a - V_d)^{3/2} \) for device 1 and \( 0.85 \times 10^{-20} \) A Hz\(^{-1}\) V\(^{-3/2}\) for device 2. Fig. 2 shows \( S_a(10)/(V_a - V_d)^{3/2} \) as a function of \( V_a - V_d \).

Taking the average over devices 1 and 2 yields \( 1.03 \times 10^{-20} \) A Hz\(^{-1}\) V\(^{-3/2}\) and this, in turn, gives \( d_{da} = 0.56 \) cm, thus verifying the earlier estimate of \( d_{da} \).

This shows the consistency of the data and indicates that (62) gives a complete description of the secondary emission 1/f noise phenomenon. This is no coincidence but represents an established fact.

Apparently contradicting this result is a report by Schwantes et al. [72], according to which \( S_a(f) \) for photomultipliers was white down to 1 Hz. However, since this white noise was not further analyzed, it is not certain that it corresponds to amplified shot and secondary emission.

### Table 1

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<tr>
<th>EFP 60</th>
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<td>( f = 10 ) Hz ( (V_a - V_d) )</td>
<td>( \delta = 3.6 )</td>
<td>( \delta = 3.1 )</td>
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<tr>
<td>( i_a = 0.00 ) mA</td>
<td>( S_a(f)/(V_a - V_d)^{3/2} )</td>
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<tbody>
<tr>
<td>( f = 10 ) Hz ( V_a - V_d = 125 ) (V)</td>
<td>( L_a = 8.0 ) mA</td>
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<tr>
<td>( V_a )</td>
<td>( S_a(f)/(V_a - V_d)^{3/2} )</td>
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noise; there might have been excess white noise present. For that reason a more detailed analysis was carried out by Fang et al. (see next section) [73].

The first stage of a photomultiplier is a vacuum photodiode. By tying all the dynodes together and measuring the noise between this "anode" and the photocathode, one can study the white noise and the 1/f noise of this photodiode.

By connecting all the electrodes beyond d2 to d2 one has a configuration consisting of a vacuum photodiode and a single secondary-emission stage in series. One can then determine the 1/f noise and the white noise added by the multiplier stage.

3) Vacuum Photodiodes: By analogy with (63)

\[ S_\text{sh}(f) = \frac{4e_0}{3\pi} \left( \frac{2e}{c} \right)^2 \left( \frac{e_0}{\epsilon_0} \right) n^2, \]

\[ S_\text{sh}(\infty) = 2e I_0 n, \]  \hspace{1cm} (64)

where \( d_{ca} \) is the cathode-anode distance and

\[ V_a = \frac{(2e/m)^{1/2}V_a^{1/2}}{d_{ca}^{1/2}} \]

and \( n > 1 \) takes into account that the photoelectrons are emitted in bunches. There are several reasons for this, but we will not go into further detail. \( S_\text{sh}(\infty) \) is shot noise of charge conglomerates.

The experimental data agree well with (64); at large \( V_a \) \( S_{\text{sh}}(f) \) is proportional to \( I_0 \) and to \( V_a^{2} \); Fig. 3 shows the latter dependence. The shot-noise data gave \( n = 1.5 \). Both shot noise and the white noise hampered the measurements, and limited their accuracy, but nevertheless the data seem to agree with (64). More work is needed.

Classically, one might expect 1/f noise due to fluctuations in the work function \( \chi \). It is easily shown that the leads to a spectrum \( S_{\text{sh}}(f) = \text{const.} V_a^{1/2} \). This has a voltage and a current dependence different from (64). This noise source is present at low \( V_a \) (see Fig. 3) [74].

4) Schottky-Barrier Diodes: Early measurements were reported by Hsu [74], [75]. In order to avoid surface 1/f noise, the metal contact was protected by a guard ring. Kleinpenning gave the essential theory in terms of mobility 1/f noise (or diffusion 1/f noise) in 1979 [76] but made a few mathematical errors. For example, his method of approach could not incorporate the integrating factor needed to solve the Langevin equation correctly. Luo [28] has corrected these errors and obtained (29) and (19) for the collision-dominated and for the collision-free models, respectively. Hsu's measurements agreed approximately with (19) but there was one difficulty.

The collision-free model required that there were no collisions in the whole space-charge region. But when they compared the free path length of the electrons with the length \( d \) of the space-charge region, they found that the electrons made about \( 7-10 \) collisions in that region. Any agreement with (19) thus seemed fortuitous.

The dilemma can be resolved by introducing the image-force model [28]. Here the potential energy has a maximum at \( x = x_m \) for the bulk space-charge region the current flow is by diffusion, but at \( x = x_m \) the current flow can also be considered as being due to thermionic emission (TE-D model). This yields a current

\[ I = eN_d \left( \frac{V_d}{V_a} \right)^{1/2} \exp \left( -\frac{V_d}{kT} \right) \]

\[ V_a = \left( \frac{kT}{2\pi m^*} \right)^{1/2} \]

\[ V_d = \frac{kT}{\epsilon_0} \left( \frac{V_d}{V_a} \right)^{1/2}. \]  \hspace{1cm} (65a)

For \( V_d \gg V_a \) the characteristic is said to be diffusion-limited and for \( V_d \gg V_a \) it is said to be thermionic-limited (TE-model); the latter does not imply absence of collisions in the space-charge region.

Luo [28] therefore combined the diffusion model with the image-force model and obtained the general equation (TE-D model)

\[ S_{\text{sh}}(f) = \frac{e^2 \alpha d}{3kT} \left( \frac{V_d}{V_a} \right)^{1/2} \exp \left( -\frac{2N_d(V_d - V_a) + V_d}{\epsilon_0} \right). \]  \hspace{1cm} (66)

This leads to the diffusion model for \( V_d \gg V_a \) and hence
to (29), and for \( v_d \gg v \), to the approximate TE model with

\[
S(f) = \frac{e^2 \alpha_{TE} |N_d(V_{dd} - V)|^{1/2}}{3f \sqrt{ee \pi kT M^*}}
\]  

(66a)

for \( v_d \gg v \), Kleinpenning already discussed the TE-D model [76]. For corrected expressions see Luo et al. [28].

Luo found that (66a) could be fitted to Hsu’s measurements by taking \( \alpha_{HT} = 1.8 \times 10^{-8} \) (Fig. 4). According to Kou-

Pawlikiewicz [77] found some n-channel Si-JETS (Silicon) with very low g-r noise. He was now able to measure \( \alpha_n \), for the 1/f noise in the temperature range 225 K < \( T < 400 \) K, but below 225 K the 1/f noise was masked by g-r noise. Paw-
likiewicz’s data fitted with the formula

\[
\alpha_{HTn} = 6.2 \times 10^{-8} \exp \left( -\frac{322.5}{T} \right)
\]  

(67b)

with an accuracy better than 20 percent (Fig. 5).

---

**Fig. 4.** \( I_{eq}(f) \) versus \( f \) for Pt-n-Si in Schottky-barrier diodes at 20 Hz. Upper broken line: diffusion model. Lower broken line: ballistic model. Full line: TE model involving image effect; circles: Hsu’s data.

**Fig. 5.** \( \alpha_{HTn} \) versus \( T \) for Si n-channel JFET. Comparison of the measured \( \alpha_{HTn} \) with (67b). Also shown is \( \alpha_T \) for the interval-

valley process.

The accurate determination of \( \alpha_{HTn} \) requires knowledge of the temperature dependence \( \mu(T) \) of the mobility \( \mu \). This can be obtained as follows. The output conductance \( g_{od}(T) \) at zero drain bias can be accurately measured. Since \( g_{od}(T) \) is proportional to the mobility \( \mu(T) \)

\[
\frac{g_{od}(T)}{g_{od}(300)} = \frac{\mu(T)}{\mu(300)}
\]  

(68)

Putting \( \mu(300) = 1400 \text{ cm}^2/\text{V} \cdot \text{s} \), as taken from Sze’s book [78], yields a functional dependence of \( \mu(T) \) independent of any theory.

However, there remained one difficulty in interpretation. Since the n-channel in each case was rather weakly doped, one would expect the true Umklapp process to have so small a probability that it would be unobservable. On the other hand, intervalley scattering should have a much larger probability, combined with an \( \exp(-\theta_D/2T) \) temperature dependence. However, as Fig. 5 shows, the theoretical expression for \( \alpha_{HTn} \) lies about a factor 3 above (67a), so that the intervalley scattering model must be discarded also.

What is left is the combined effect of intervalley scattering followed by Umklapp into another intervalley. According to van Vliet [79] the combination should be treated as a single process, described by \( \alpha_T = \alpha_{HTn} \mu(n) \mu(p) \), where \( \mu(n) \mu(p) = \exp(-\theta_D/4T) \). This solves all discrepancies. It is now expected that JFETs or MOSFETs made on different interfaces (100, 110, 111) might show different noise behavior. For example, it could be that the combined process could be forbidden for some interfaces. This problem, which is of fundamental interest, requires further study.

The devices studied by Duh and by Pawlikiewicz had very different structures, but showed the same value of \( \alpha_n \). This is as expected, since they are subjected to identical noise processes. Nevertheless, measurements on a few more samples of the same type as well as on a few samples of different types might be useful.

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**Picture:** The page contains graphs showing the relationship between current and frequency for different types of diodes, and a theoretical model for calculating the noise in these devices. The text is mathematical and technical, discussing various models and experiments related to noise in electronic devices.
Pawlikiewicz [80] and Birbas [81] measured l.f. noise in p-channel JFETs. They found at first that the l.f. noise varied as $1/f^2$, as expected for a g-$r$ process with a time constant of many seconds. Apparently there was no $1/f$ noise observable.

What kind of $1/f$ noise could be expected? There should, of course, be normal collision $1/f$ noise for holes, with $\alpha_{hp} = (6-10) \times 10^{-5}$. But since there are no intervalleys, and the doping of the channel is low, one would not expect noise that is describable by (67a).

As mentioned in Section II (67a) yields $\alpha_{hp} = 4.2 \times 10^{-7}$. As before, (23) yields for $V_g < V_{ds}$

$$S(f) = \alpha_{hp} \frac{e\mu I_g V_g}{f^2}.$$

(69)

Substituting for the device parameters and for $\alpha_{hp}$ yielded a curve lying above the measured curve $[S(f)]_{meas}$ (Fig. 6).

![Fig. 6. $S(f)_{meas}$ versus frequency in Si p-channel JFETs. (a) Theoretical spectrum for $\alpha_{hp} = 4.2 \times 10^{-7}$ (see (67b)). (b) Theoretical spectrum for $\alpha_{hp} = 10^{-8}$ (phonon collision model). In cases (a) and (b) the h.f. thermal noise was added.](image)

According to rule 3 at the beginning of Section V, this means that Umklapp $1/f$ noise is absent, as predicted in the previous paragraph. Normal collision $1/f$ noise, however, should have $\alpha_{hp} = (6-10) \times 10^{-5}$. Substituting $\alpha_{hp} = 10^{-5}$ into (69) yielded a curve that was usually located below the measured curve $[S(f)]_{meas}$, indicating that normal collision $1/f$ noise was now masked by g-$r$ noise in those units. Only the lowest noise units showed some $1/f$ noise; then $[S(f)]_{meas}$ and $[S(f)]_{theo}$ coincided at the higher frequencies, indicating that normal collision $1/f$ noise was compatible with those data (Fig. 6).

The measurements should be continued to indicate whether Umklapp $1/f$ noise is always missing in p-channel JFETs. Moreover, a search should be made for more devices of $(-V_g)$ and independent of the device length $L$, as expected for Umklapp $1/f$ noise. It is therefore quite likely that Duh actually observed Umklapp $1/f$ noise. The direct proof would be to measure the temperature dependence of $\alpha_{hp}$. Unfortunately, the devices were lost before tests could be made; a search for similar low-noise devices is in progress.

The noise mechanism of $1/f$ noise in GaAs MOSFETs and MODFETs is most likely due to deep traps in the space-charge regions of the devices. If one turns the data into an equivalent $\alpha_{tv}$, one finds $\alpha_{tv} = (2-10) \times 10^{-5}$ [26]. Because the band structure has higher valleys (process d), (intervalley + Umklapp) is possible. With $m^*_v = 0.067 m, \theta_0 = 350$ K, $a = 5.65 \times 10^{-8}$ cm, (67a) then yields $\alpha_{tv} = 7.1 \times 10^{-6}$.

This verifies again that the Umklapp limit, if any, has not yet been reached. It would be very worthwhile to continue to search for it, especially since the best units are only a factor 3 away from it [82]. Both the presence or the absence of this limit would be important.

2) Collision $1/f$ Noise in BJTs: The first cases of possible quantum $1/f$ noise in BJTs were published by Kilmer et al.

![Fig. 7. $\alpha_{hp}$ versus $V_g$ for p-channel MOSFETs at channel length $L = 7.5$ and 17.5 $\mu$m. $V_g = -4.0$ V, $f = 100$ Hz.](image)
[83], [84] and by Zhu et al. [26]. They based their identification on the fact that \( S(f) \) was independent of \( I \) and that \( \alpha_H \) has values close to the value for Umklapp 1/f noise. For example, in GE 82-185 p^+ -n-p transistors \( \alpha_{H_{\text{meas}}} \) was measured from \( S_L(f) \). Kilmer found \( \alpha_{H_{\text{meas}}} = 9 \times 10^{-5} \) and Zhu found \( \alpha_{H_{\text{meas}}} = 6 \times 10^{-5} \), both at 300 K, whereas the Umklapp value for \( \alpha_H \) was 2.1 \times 10^{-5}. This agreement is perhaps acceptable, but it is not always so. For microwave n^+ -n-p transistors, Zhu found from the measured spectrum \( S_L(f) \) that \( \alpha_{H_{\text{meas}}} = 1.1 \times 10^{-6} \), whereas the Umklapp value was 4.2 \times 10^{-7}, both at 300 K. We now believe these devices represent trapping noise with \( S(f)/f \) = constant.

Later, cases were discovered where \( S(f)/f \) was constant but \( \alpha_H \) did not have a fundamental value; apparently there can be trapping noise where \( S(f) \) is proportional to \( f \). The clue is that \( \alpha_{H_{\text{meas}}} \) is not constant but varies from unit to unit. Pawlikiewicz et al. [85] found GE 82-185 transistors in which \( S_L(f) \) yielded \( \alpha_{H_{\text{meas}}} = 3.5 \times 10^{-5} \), in very reasonable agreement with Kousik's value \( \alpha_{H_{\text{meas}}} = 3.3 \times 10^{-5} \), calculated for normal collision 1/f noise. To obtain this value he assumed \( w_L = 0.5 \mu m \) and \( D_m = 15 \mu m^2/\mu s \); this estimate of \( \alpha_{H_{\text{meas}}} \) may be off by 30 to 50 percent. Such low-noise units are hard to find; their \( \alpha_H \) values became measurable after extending the spectral observation down to 1 Hz.

Zhu and Kilmer found no measurable collector 1/f noise down to 20 Hz and Zhu therefore concluded that in the collector \( \alpha_{H_{\text{meas}}} = 5 \times 10^{-9} \) for a GE 82-185 p^+ -n-p BJT, whereas \( \alpha_{H_{\text{meas}}} = 1.6 \times 10^{-9} \) for a microwave n^+ -p-n BJT. At first that did not cause much difficulty, but after the normal collision 1/f noise was identified, it became questionable whether these inequalities even excluded that process too.

To find that out, Pawlikiewicz et al. [85] and Fang [86] extended the spectral measurement down to 1 Hz and looked for units in which the collector 1/f component was clearly identifiable and not perturbed by amplified base noise. Pawlikiewicz found \( \alpha_{H_{\text{meas}}} = 5.3 \times 10^{-9} \) and \( 6.5 \times 10^{-9} \) for a GE 85-182 p^+ -n-p transistor at \( T = 300 \) K at low collector currents, (0.1 and 0.2 mA, respectively) in very reasonable agreement with the estimated theoretical value (6-10) \times 10^{-9}. Fang [86] found \( \alpha_{H_{\text{meas}}} = 5 \times 10^{-9} \) for an experimental n^+ -p-n transistor at 300 K and \( I_L = 2 \) mA, in reasonable agreement with the theoretical value \( \alpha_{H_{\text{meas}}} = 3.3 \times 10^{-9} \). This thus seems clear that the collector 1/f component shows normal collision 1/f noise, just as the base 1/f component in low-noise BJTs does. There was one discrepancy, however, in that Pawlikiewicz found \( \alpha_{H_{\text{meas}}} = 1.4 \times 10^{-9} \) at \( I_L = 2 \) mA. This requires further study.

There is indication that surface 1/f noise may depend on the interface. It seems that (111) surfaces give more noise than the (100) and (110) surfaces [87]. This requires further study. One of the reasons is rather trivial: Many units have very large values at \( \alpha_{H_{\text{meas}}} \) (e.g., 250 \Omega instead of 10 \Omega at \( I_L = 1 \) mA). As a consequence, the component \( I_{B_{\text{HP}}} \) gives a large contribution \( S_B(f) \) \((g_{nB_{\text{HP}}} f_0^3) \) to \( S_L(f) \); since \((g_{nB_{\text{HP}}} f_0^3) \approx 100 \), the amplified base noise predominates by far over the other \( S_L(f) \) components. It should be clear that such devices are unsuitable for collision 1/f noise studies.

The final conclusion is that Si-BJTs always have normal collision 1/f noise but that Umklapp 1/f noise, and inter-valley 1/f noise do not show up in the external circuit. This "selection rule" requires a theoretical foundation. Also, it should be investigated whether such a selection rule applies to all interfaces or only to some of them.

3) Collision 1/f Noise in Diodes: In silicon n^+ -p diodes at relatively low forward bias the current flow is by recombination at the surface of the space-charge region. This is usually caused by 1/f modulation of surface recombination due to the fluctuating occupancy of oxide traps. This yields a current dependence of \( S(f) \) of the form \( f^\gamma \) with \( \gamma = 1.5 \) or 2. The possibility of \( \gamma = 1 \) needs further study, since it might indicate the presence of another fundamental noise process (see Section V-C).

For GaAs laser diodes at relatively low currents \( S(f)/f \) is often constant; this might indicate the presence of a fundamental noise source [88]. Since the device is heavily doped (\( \approx 10^{17}/\text{cm}^3 \)), the diffusion constant \( D_m \) for electrons may be as small as 25 cm^2/s. Consequently, since the carrier lifetime \( \tau_s \) is of the order of \( 2 \times 10^{-9} \) s, the diffusion length \( L_m = (D_m \tau_s)^{1/2} = 2.2 \times 10^{-4} \) cm. Since the doped regions have a comparable length, the diode is neither long (\( w/L_m \gg 1 \)), nor short (\( w/L_m \ll 1 \)). This case needs further study.

\( Hg_{1-x}Cd_x \) Te n^+ -p diodes with \( x = 0.30 \) give \( \alpha_H = 5.3 \times 10^{-10} \) at 273 K at back bias, whereas the Umklapp theory gives \( \alpha_H = 4.9 \times 10^{-3} \) in excellent agreement [89]. Apparently the Umklapp process was responsible. The theoretical value follows from Zhu et al. tables [90], whereas the experimental value was obtained from \( S(f) \) with the help of (32) and (32a). This implies a long diode (\( w/L_m \gg 1 \)) and current flow by diffusion; it furthermore indicates that all carriers contribute to the noise (for proof, see Section V-C). The lifetime \( \tau_s \) follows from the Honeywell tables (\( \tau_s = 1.2 \times 10^{-7} \) s) [91].

C. Coherent State 1/f Noise

We saw that for long resistors the Hooge parameter \( \alpha_H \) was \( 2 \times 10^{-3} \). We shall see that there are other cases (long \( Hg_{1-x}Cd_x \) Te n^+ -p diodes) where comparable values for \( \alpha_H \) are found. This makes it plausible that the same principles are involved.

Handel's coherent state theory gives \( \alpha_H = 4.6 \times 10^{-3} \); unfortunately, it does not give guidelines for deciding when the theory applies. It is the aim of this section to provide clarification.

1) Hooge's experiments [3]: The data speak for themselves, but there is additional information. According to Hooge and Vandamme \( \alpha_H \) depends on doping in the following manner [92]:

\[
\alpha_H = 2 \times 10^{-3} \left( \frac{\mu}{\mu_{\text{limit}}} \right)^2
\]

where

\[
\frac{1}{\mu} = \frac{1}{\mu_{\text{limit}}} + \frac{1}{\mu_{\text{imp}}}
\]

(70a)

where \( \mu_{\text{limit}} \) and \( \mu_{\text{imp}} \) are the mobilities due to phonon and to impurity scattering, respectively. Applying the Kousik-van Vliet-Bosman approach [20], if \( \mu_{\text{limit}} \) and \( \mu_{\text{imp}} \) both fluctuate

\[
\alpha_H = \alpha_{\text{limit}} \left( \frac{\mu}{\mu_{\text{limit}}} \right)^2 + \alpha_{\text{imp}} \left( \frac{\mu}{\mu_{\text{imp}}} \right)^2
\]

(70b)

This must apply both for normal scattering and for the Hooge-type process.

As an example take normal collisions in n-type silicon. Here \( \alpha_{\text{limit}} = 3.3 \times 10^{-5} \) and \( \alpha_{\text{imp}} \) is not negligible at high
doping; as a consequence the first term will predominate at low doping and the second term at high doping. Next take the Hooge process; here \( \alpha_{\text{Hooge}} = 2 \times 10^{-2} \) and \( \alpha_{\text{temp}} \) must be much smaller, for otherwise (70) would not hold. There is thus an enhancement in \( \alpha_{\text{Hooge}} \) by a factor \( 10^6 \). Of course, one cannot be quite sure that we started from the normal collision process; but if we had started from the Umklapp process the enhancement factor would still be \( 10^6 \), so our conclusion about the enhancement factor changes little. This problem requires further study.

In the second place \( \alpha_{\text{t}} \) may be field-dependent. According to Bosman et al. [7], [8] for n- and p-type silicon resistors

\[
\alpha_{H} = \frac{\alpha_{H}(0)}{1 + (V_f V_c)^2} \tag{71}
\]

where \( \alpha_{H}(0) \) is the low-field value and \( \mu F \) corresponds to the velocity of sound. This suggests that the effect may be due to phonon emission [93]. Bosman used planar geometry; Kleinpenning [94], who used a more complicated point-contact geometry, did not observe the effect.

2) Long Hg, Cd, Te n+-p Diodes with \( x = 0.30 \): Wu et al. measured long n+-p diodes with a nonplanar geometry [89]. Assuming a planar approximation, and a diffusion-recombination type current flow one obtains

\[
S_j(f) = \frac{\alpha_{\text{end}}(0) e^{-f(a)}}{\alpha_f (a)} \tag{72}
\]

Here \( \alpha_{\text{end}}(0) \) is the diffusion (Hooge) parameter, \( \alpha_f \) the lifetime, and

\[
f(a) = f_0(a) = \frac{1}{3} \cdot \frac{1}{2a} + \frac{1}{a^2} - \frac{1}{a} \cdot \ln (1 + a), \quad \text{with } a = \exp \left(\frac{eV/kT}{2}\right) - 1 \tag{72a}
\]

if all minority carriers contribute to the noise and \( f_0(a) = f_0(3/2) \). Only the excess minority carriers contribute. They plotted \( S_j(f)S_j(f) \) versus \( eV/kT \) used (72a) and found a horizontal line, indicating that all minority carriers contribute equally to the noise (Fig. 8).

They then measured the diode admittance and determined the lifetime. According to the diffusion theory [95]

\[
V \omega = g_0 + jfb_0 = \alpha_{\text{end}}(0) + j\alpha_{\text{end}}(0) \tag{73}
\]

where \( \gamma = 1/2 \). This was quite well satisfied for devices operating near zero bias but farther away from zero bias the value of \( \gamma \) was 0.7-0.9, indicating that the lifetime measurement was not reliable in that case. This is probably due to the nonplanar geometry. Otherwise (73) seemed to be valid.

Evaluating \( \alpha_{\text{end}}(0) \) from \( S_j(f)S_j(f) \) at \( T = 193 \) K they found that \( \alpha_{\text{end}}(0) \) was nearly independent of bias (Fig. 8). Using the value of \( \alpha_f \) determined by the above method, they found that \( \alpha_{\text{end}}(0) = 3 \times 10^{-3} \) near zero bias and that \( \alpha_f \) agreed with the Honeywell lifetime tables [91]. Farther away from zero bias, the \textit{measured} values of \( \alpha_f \) were larger than the \textit{theoretical} values obtained from the lifetime tables. As a consequence, \( \alpha_{\text{end}}(0) \) was now larger than \( 5 \times 10^{-3} \), and this was due to the error in the measured values of \( \alpha_f \) (Fig. 9). Finally, they used \( \alpha_{\text{end}}(0) = 4.6 \times 10^{-13} \), evaluated \( \alpha_f \) from the measured values of \( \alpha_{\text{end}}(0) \) and obtained good agreement with the lifetime tables. They therefore concluded that \( \alpha_{\text{end}}(0) \) had values close to \( 4.6 \times 10^{-3} \), as expected for coherent state noise, and that \( \alpha_f = (1-3) \times 10^{-7} \) at \( T = 193 \) K.

**Fig. 8.** \( S_j(f)S_j(f) \) versus \( eV/kT \) for Hg, Cd, Te n+-p diodes with \( x = 0.30 \) at \( T = 193 \) K and \( f = 20 \text{ Hz} \). This indicates that for back bias and near forward bias all minority carriers contribute to the noise and that \( \alpha_{\text{end}}/\alpha_f \) is a constant.

Similar results were obtained for all devices at 193 and 113 K whereas for one unit at 273 K they found \( \alpha_{\text{end}} = 5 \times 10^{-13} \), as expected for the Umklapp process. No transition to \( \alpha_{\text{end}} = 5 \times 10^{-3} \) was found, however.

Since the coherent state mechanism gives the highest fundamental value of \( \alpha_{\text{end}} \) considerable improvement could be obtained in the noise performance of these diodes if one could learn to operate them in the Umklapp mode, and still further improvement could be made by operating them in the normal collision mode. This would be an interesting consequence of a systematic noise study of HgCdTe diodes.

3) Noise in Long Si p+-i-n Diodes: Fang [96] measured noise in long Si p+-i-n diodes at 410 K. Here the current flow is associated with hole-electron pair recombination (for forward bias) and hole-electron pair generation (for back bias). According to the theory (Section II)

\[
S_j(f) = \alpha_{\text{H}} e^{-f/\alpha_f} \tag{74}
\]

where \( \tau \) is the time constant associated with the generation or recombination of a single hole-electron pair. He plotted \( S_j(f)S_j(f) \) versus \( eV/kT \) at back bias for several diodes and found it to be independent of bias; he could then evaluate...
The values were all very close together and the average value was $3.1 \times 10^9$s. He then evaluated $\tau$ from admittance data and found $\tau = 1.4 \times 10^{-5}$ s, so that $\alpha_H$ had an average value of $4.3 \times 10^{-4}$, in excellent agreement with the coherent state value $\alpha_H = 4.6 \times 10^{-3}$.

A few words should be said about the device admittance. The equivalent circuit consists of an $R - C$ parallel connection, with a current-dependent series resistance $R_C$. The current modulation of $R_C$ gives rise to a negative resistance $-R_{pp} = I_d/\tau$ in series with $R_C$ plus a parallel combination $(R_{pp} - L)$ where $L/R_{pp} = \tau$. Finally there is a capacitance $C_1$ from the bottom of the $R - C$ parallel connection to the bottom of the $R_{pp} - L$ parallel connection [95], [96]. This equivalent circuit is quite different from that of the diffusion type diode, which showed no resonances and antiresonances in $g(\omega)$ and $b(\omega)$. Having obtained the circuit elements from $Y(\omega)$ one puts $\tau = CR$.

In p-n junctions and BJTs one often finds modulation-type 1/f noise due to the modulation of recombination centers by the fluctuating occupancy of oxide traps (Section IV). Let it have a Hooge parameter $\alpha_{tip}$. The recombination centers themselves should also show Handel-type quantum 1/f noise. If it has a Hooge parameter $\alpha_H$, then $\alpha_{tip} = \alpha_H + \alpha_{tip}$. If there are no coherent state effects, $\alpha_{tip}$ is very small and $\alpha_{tip}$ predominates by far. If the coherent effects are fully developed $\alpha_H$ is raised to $4.6 \times 10^{-3}$ and it predominates by far, whereas $\alpha_{tip}$ is unaffected. This picture is fully equivalent to the one presented in Section IV-C1.

As was already mentioned, the Hooge parameter of semiconductor resistors seems to increase with increasing device length $L$, reaching the coherent state value for large $L$. Birbas, Peng, and Amberiadis [97] are studying p-channel MOSFETs with different channel lengths ($L = 14$ $\mu$m to $L = 190$ $\mu$m), made on the same chip, and biased at low drain bias $(V_D = -0.2$ V) so that the device behaves as a linear resistor. Preliminary data seem to indicate that $\alpha_H$ varies as $L^2$ at intermediate lengths. Much more work is needed, however, to establish beyond doubt that this represents indeed a transition from normal collision 1/f noise to coherent state 1/f noise. Present indications are that this is a new quantum 1/f noise source caused by carrier acceleration between collisions and coherent generation of 1/f noise along the carrier drift path. An elementary theory allows to calculate the time $\tau$ between collisions and to compare with the $\tau$ value deduced from the data. Reasonable agreement has been obtained for n-type MOSFETS.

VI. Conclusions

We have shown how a generalized representation of the noisiness in electronic devices can be given with the help of the measured Hooge parameter $\alpha_H$. In collision-free devices (vacuum pentodes, secondary emission tubes, and the vacuum photodiode part of photomultipliers) $\alpha_H$ is given by fundamental formulas. In collision-dominated devices one observes fundamental noise sources of the normal collision type, the Umklapp type, and the intervalley + Umklapp type scattering processes; pure intervalley type scattering processes have not been found, even in n-type Si at low doping; they cannot occur in p-channel JFETs since they have no intervalls. Normal collision 1/f noise seems to occur in Si-BJTs (n"-p-n and p"-n-p), in p-channel Si JFETs and in Schottky barrier diodes.

Where applicable, Handel's predictions for $\alpha_H$ are usually verified. This is the more remarkable, since there is severe criticism about Handel's derivation. The criticism is not directed against the Bremsstrahlung hypothesis as such, since for collision-free devices it seems to be the only 1/f noise process available. In Appendix I we derive the Handel formula for $\alpha_H$, from semiclassical considerations applied to collision-free devices.

There are several cases of coherent state 1/f noise with $\alpha_H = 5 \times 10^{-3}$ on record, but we have no satisfactory criteria for predicting its occurrence or nonoccurrence in advance. Otherwise, the general features of the generalized representation seem to have been established, even though evidence from larger samples would be helpful.

This project started as an attempt to verify or refute the predictions made by Handel's quantum 1/f noise theories; more particularly his theory of the Hooge parameter $\alpha_H$. This is now practically complete, except for some more work on vacuum photodiodes, on BJTs and on ballistic devices. We see from Section V, that Handel's result, if properly applied to the device under test, agrees with our measurements in nearly all cases. Both the experimental numbers for the various $\alpha_H$ values and their agreement with Handel's predictions represent scientific information that should not be ignored.

Our project cannot check the validity or invalidity of Handel's derivation of his predictions for $\alpha_H$. This is the domain of the theoreticians. They have every right to criticize the derivation and replace it by a better one. In the latter case, they should see to it that their prediction for $\alpha_H$ agrees with Handel's prediction for $\alpha_H$ when the latter has been verified experimentally. Up to now this has not been done. It is difficult for some scientists to understand how a theory that is in their opinion incorrect can give correct predictions. It must be emphasized that only experiment can decide whether a conclusion is correct or incorrect. In our situation experiments decided that the predictions were right, and I see no way to avoid this conclusion.

Since the accuracy of the measurements is +30 percent, correction factors close to unity cannot be detected.

Appendix I

Semiclassical Derivation for Handel's Expression of the Hooge Parameter of Collision-Free Devices

According to Hooge's equation the noise spectrum of collision-free devices is

$$S(f) = \frac{\alpha_H \beta}{f} \frac{N_{eh}}{c}$$

(A1)

where $L$ is the current, $N_{eh} = I_l e$ is the effective number of carriers in the system, $\tau$ the appropriate transit time of the electrons, and $\alpha_H$ is the Hooge parameter; (A1) defines $\alpha_H$. According to Handel

$$\alpha_H = \frac{4 \alpha \Delta V^2}{3 \pi \alpha^2}$$

(A2)

where $\alpha = \alpha_0 (q/e)^2$ and $\alpha_0 = 2 e \pi^2 \hbar c = 1/137$. Here $\alpha_0$ is the fine structure constant for electrons, and $\alpha$ the fine structure constant for a charge conglomerate $q$; moreover, $\Delta V$ is the change in velocity along the electron path.

We now derive (A2) for $q = e$ or $\alpha = \alpha_0$. As is well known,
the Bremsstrahlung power emitted by a single electron is
\[ P(t) = \frac{2e^2}{3c^3} a^2, \quad \text{for } 0 < t < \tau \] (A3)
and zero otherwise; here \( \tau \) is the transit time (\( = 10^{-9} \) s) and \( a = \frac{dv}{dt} \) is the acceleration of the electron.

We now find the spectrum associated with linear pulses
\[ P(t) = \left( \frac{2e^2}{3c^3} \right)^{1/2} a, \quad \text{for } 0 < t < \tau. \] (A3a)
If we make the Fourier transform \( F(j\omega) \) for \( P(t) \), then for \( \omega \tau < 1 \),
\[ F(j\omega) \approx F(0) \left( \frac{2e^2}{3c^3} \right)^{1/2} \Delta v \] (A3b)
where \( \Delta v = v(d) - v(0) \) is the change in velocity along the electron path and \( d \) is the length of that path. Since \( \tau = 10^{-9} \) s, \( \omega \tau < 1 \) means that \( F(j\omega) \) is white up to about 50 MHz.

Since \( \lambda = l/e = N_{e/d} \tau \) is the rate at which pulses occur per second, we have for the \( \nu P \) (Carson's theorem)
\[ S_{\nu P}(f) = 2F(0)^2\lambda = \frac{4e^2 \Delta v^2}{3c^3} \lambda. \] (A3c)
But we are not interested in the spectrum \( S_{\nu P}(f) \), but rather in the quantum spectrum associated with the pulses \( P \). This spectrum corresponds to the rate of quantum emission and is related to \( S_{\nu P}(f) \) by
\[ S_q(f) = \frac{S_{\nu P}(f)}{hf} = \frac{4e^2 \Delta v^2}{3c^3} \frac{\lambda}{hf} \] (A4)
where \( hf \) is the quantum energy; \( S_q(f) \) thus has a \( 1/f \) spectrum.

So far the theory is straightforward and (A4) is a rigorous consequence of (A3). We must now transform from the quantum emission rate spectrum to the current spectrum \( S_{\nu P}(f) \). To that end we observe that the charge transferred by a single electron pulse is (Ramo's theorem)
\[ Q(t) = \frac{e}{d} \int_0^d V(t) \, dt = e. \] (A5)
The shot noise associated with the current \( I \) is then
\[ [S(I)]_s = 2e^2\lambda = 2eI \] (A5a)
as is well known.

We must now connect (A4) and (A5a) to find \( S_q(f) \). \( S_q(f) \) must be proportional to \( S_{\nu P}(f) \) and must contain the factor \( [S(I)]_s/\lambda = 2e^2 \) (we cannot introduce \( \lambda \) twice!). We thus write
\[ S_q(f) = CS_q(f) \cdot 2e^2 = C \frac{8e^2 \Delta v^2 e^{\lambda \tau}}{ec^3 \lambda \tau} \] (A6)
where \( C \) is a dimensionless proportionality factor that will be determined. Next we rewrite (A1) as
\[ S_q(f) = \alpha_H f^3 \frac{N_{e/d}}{\tau} = \alpha_H \frac{e^{\lambda \tau}}{\tau} \] (A7)
so that comparison with (A6) gives
\[ \alpha_H = C \frac{8e^2 \Delta v^2}{3c^3} = C \frac{4\alpha_D \Delta v^2}{3\pi^2 c^2}. \] (A7a)
For \( C = 1 \) we obtain Handel's result (A2).

We can also argue as follows. Experimentally (A2) was found to be correct within 20-30 percent and hence \( C = 1 \) with an accuracy of 20-30 percent. We are now independent of Handel's derivation of \( \alpha_H \).

Equation (A6) is in itself a heuristic expression, but the factor \( 2Ce^2 \) is so chosen that \( C \) is dimensionless and that \( \alpha_H \) fits with \( \alpha_H \). This requires \( 0.70 < C < 1.30 \).

Equations (A6) and (A7a) are fully equivalent: (A7a) follows from (A6) and by inversion (A6) follows from (A7a).

Our theory is essentially a one-particle theory, since each electron only interacts with its own Bremsstrahlung. With "interaction" we mean that the Bremsstrahlung energy comes from the accelerated electron (energy law).

We now take \( C = 1 \) and write (A6) as
\[ S_q(f) = \frac{S_{\nu P}(f)}{2e^2}. \] (A6a)
This illustrates energy law stated above.

We now show that (A2) is independent of the model. To that end we consider a collision-free system in which the current is carried by charge conglomerates \( q \) that are uniformly accelerated. Taking \( P(t) \) from (3) and replacing \( e^2 \) by \( q^2 \), the average Bremsstrahlung energy emitted per pulse is
\[ E = \frac{P(t)\tau}{\nu} = \frac{2e^2 q^2 \Delta v^2}{3c^3 e^2 c^2} \frac{1}{\tau} \] (A8)
since \( a = \frac{dv}{dt} = \frac{\Delta v}{\tau} \) is the uniform acceleration.

We now consider Handel's equation (A2) and rewrite it as
\[ \alpha_H = 4\alpha_D \frac{q^2 \Delta v^2}{3\pi^2 e^2 c^2} \frac{4}{h}. \] (A9)
(Note that (A8) and (A9) have the same factor between square brackets in common. The assumption that the \( 1/f \) noise in collision-free devices is due to Bremsstrahlung already implies the validity of the two terms \( q/e \) and \( \Delta v/c \) that are most easily verified. A detailed analysis of \( \alpha_H \) only adds a factor \( 4h \).)

Where does the factor come from? Comparing with the preceding we see that the factor \( 4 \) comes from applying Carson's theorem twice; each application adds a factor \( 2 \). In addition, the factor \( 1/h \) comes from the change-over from the energy spectrum to a quantum emission rate spectrum. Any derivation of \( \alpha_H \) that involves these two steps gives the same result.

The expression of \( \alpha_H \) is therefore relatively independent of the detailed electron-photon interaction process.

An exact theory, both for the collision-free and the collision-dominated devices, is urgently needed.

**APPENDIX II**

**TRANSMISSION LINE MODEL FOR 1/f NOISE IN DIFFUSION DOMINATED n+-p or p+-n DIODES AND n+-p-n and p+-n-p BJTs**

We first evaluate the response to the series emf \( e_d \) (Fig. 10(a)). It is equivalent to Fig. 10(b), with
\[ Z_1 = Z_{d0} \tanh \gamma_d \quad Z_{\tau} = Z_{d0} \tanh \gamma_d(x - \tau). \] (A10)
and hence
\[
i_0 = \frac{e_d}{Z_1 + Z_2} = \frac{e_d}{Z_0} \left[ \frac{1}{\tanh \gamma_0 x + \tanh \gamma_0 (w - x)} \right]
\]
\[
= \frac{e_d \cosh \gamma_0 x \cosh \gamma_0 (w - x)}{Z_0 \sinh \gamma_0 w}.
\]  
(A10a)

Try for \(0 < y < x\) the expression:
\[
i(y) = a \cosh \gamma_0 y + b \cosh \gamma_0 (x - y).
\]  
(A11)

For \(y = 0\):
\[
i(0) = i_1 = a + b \cosh \gamma_0 x
\]

and for \(y = x\):
\[
i(x) = -i_0 = a \cosh \gamma_0 x + b.
\]  
(A11a)

For \(x \to 0\) the \(b \cosh \gamma_0 x\) term blows up; hence we must take \(b = 0\). This means
\[
i_1 = i = \frac{e_d \cosh \gamma_0 (w - x)}{Z_0 \sinh \gamma_0 w}
\]
\[
i_2 = \frac{e_d \cosh \gamma_0 x}{Z_0 \sinh \gamma_0 w}.
\]  
(A11b)

We now evaluate the response of the current generator
\[
i_d = i_d \frac{Z_1 Z_2}{Z_1 + Z_2}
\]
\[
i_0 = \frac{e_d}{Z_1} \frac{Z_2}{Z_1 + Z_2} = \frac{i_d \sinh \gamma_0 (w - x) \cosh \gamma_0 x}{\sinh \gamma_0 w}
\]  
(A12a)

\[
i_1 = \frac{i_0}{\cosh \gamma_0 x} = \frac{i_d \sinh \gamma_0 (w - x)}{\sinh \gamma_0 w}.
\]
\[
i_2 = \frac{i_0}{\sinh \gamma_0 w}
\]  
(A12b)

so that the two transfer functions for \(e_d\) and for \(i_0\) have now been evaluated.

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