22. Seminar Radijske Komunikacije

Šum v radijskih komunikacijah

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Peta mednarodna konferenca Solvay uglednih fizikov in kemikov na temo „Elektroni in fotoni“ (oktober 1927). Najuglednejša udeleženca Albert Einstein in Niels Bohr se nista razumela:

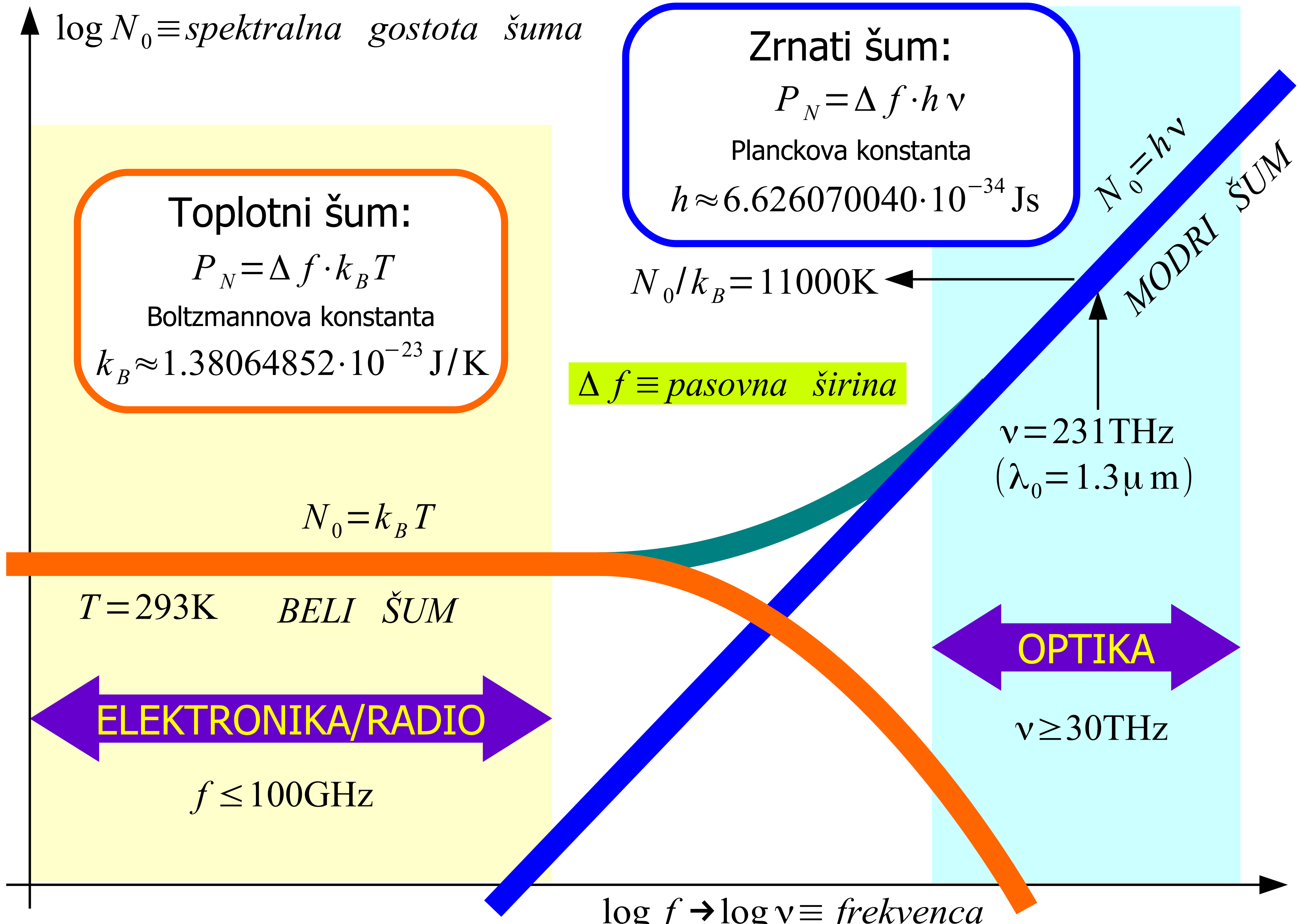
Albert Einstein: „God does not play dice!“
(Bog ne kocka! Torej ne dopušča naključnosti v naravnih zakonih.)

Niels Bohr: „Einstein, stop telling God what to do!“ (Einsten, nehaj učiti Boga, kaj naj počne!)

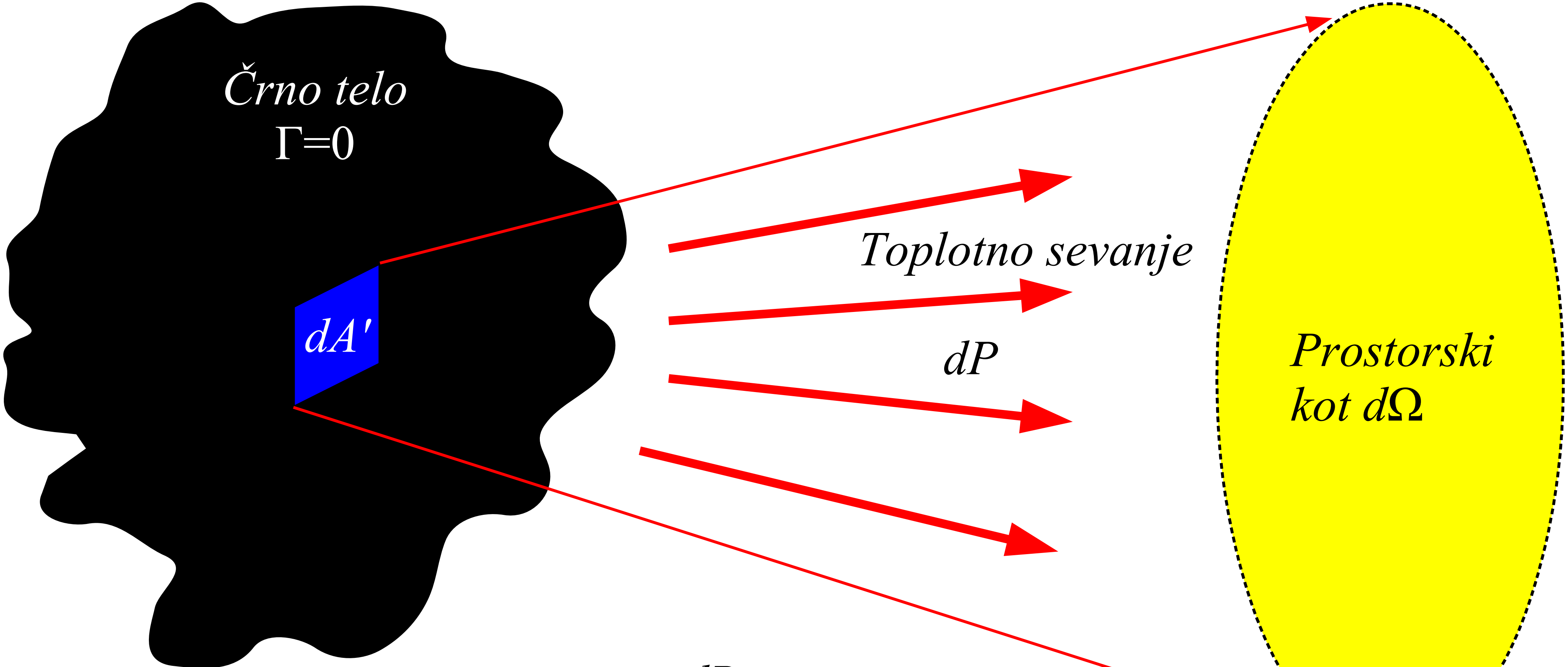
V telekomunikacijah naključnost imenujemo šum. Šum omejuje domet vsake zveze.

Šum je makroskopski opis kvantnih pojavov!

1 - Nesporazum slavnih znanstvenikov



2 - Spektralna gostota šuma



Spektralna svetlost $B_f(f, T) = \dfrac{dP}{df\, dA'\, d\Omega}$

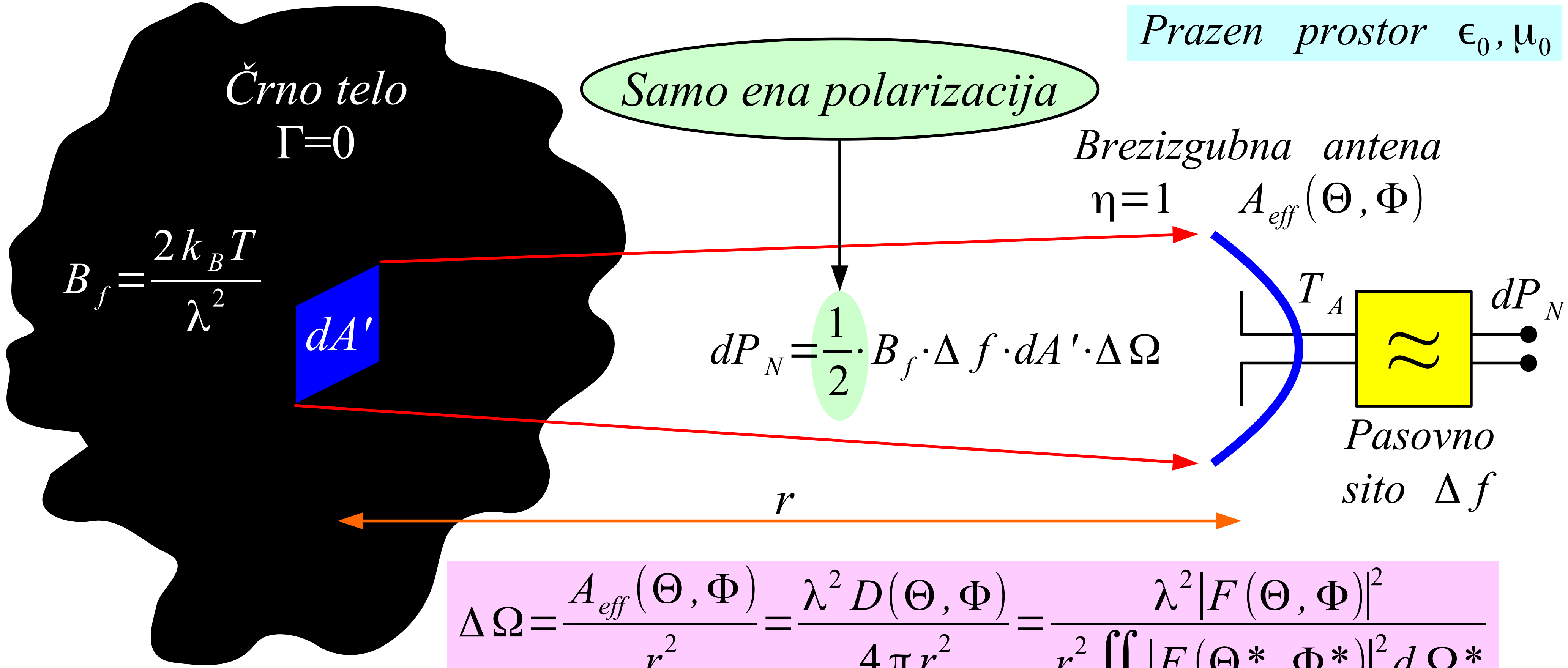
Planckov zakon $B_f(f, T) = \dfrac{2hf^3}{c_0^2} \cdot \dfrac{1}{e^{\frac{hf}{k_B T}} - 1}$

Prazen prostor ϵ_0, μ_0

$c_0 = 299792458\,\mathrm{m/s} \approx 3 \cdot 10^8\,\mathrm{m/s}$

Radio $hf \ll k_B T$ → *Rayleigh − Jeansov približek* $B_f(f, T) \approx \dfrac{2 k_B T f^2}{c_0^2} = \dfrac{2 k_B T}{\lambda^2}$

3 – Toplotno sevanje črnega telesa



$$\Delta\Omega = \frac{A_{eff}(\Theta,\Phi)}{r^2} = \frac{\lambda^2 D(\Theta,\Phi)}{4\pi r^2} = \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

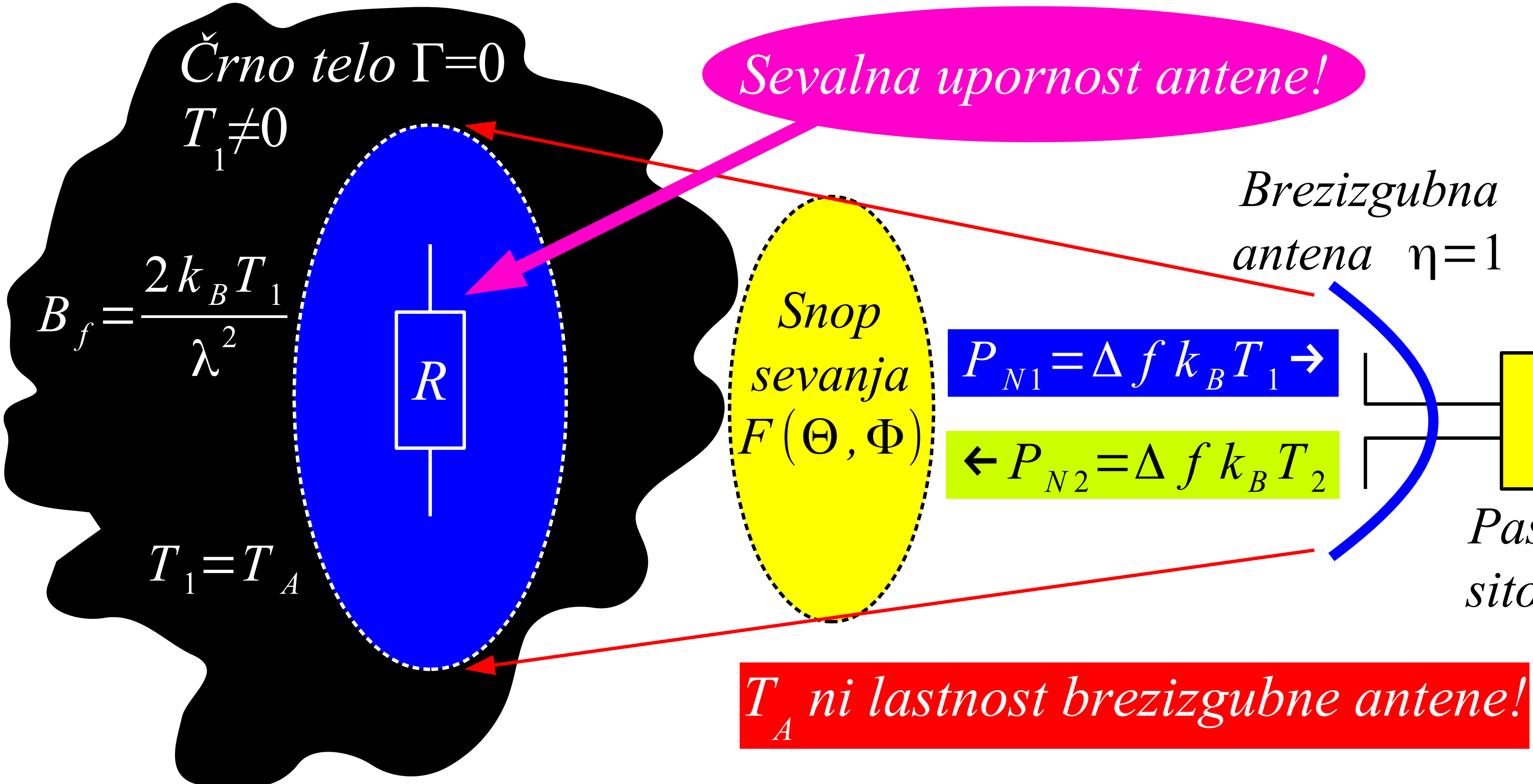
$$dA' = r^2 d\Omega$$

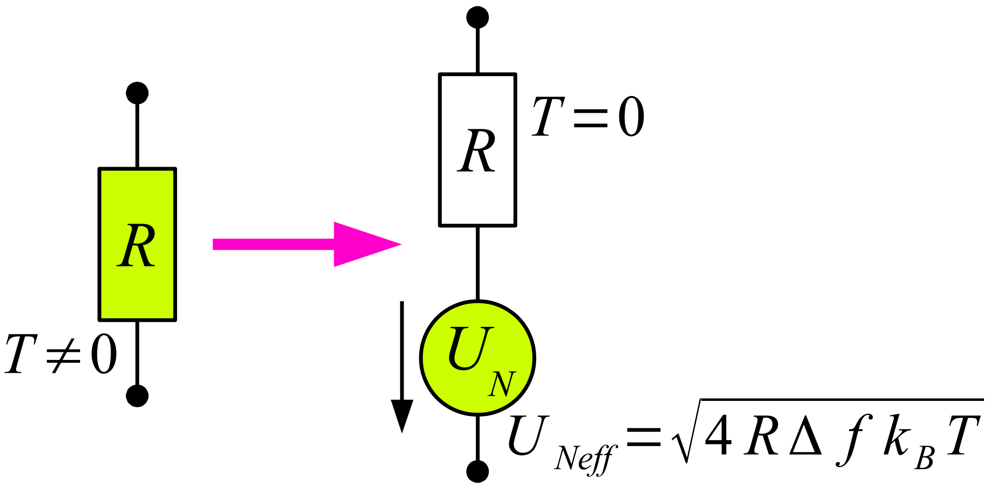
$$P_N = \iint_{A'} \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta\Omega = \iint_{4\pi} \frac{1}{2} \cdot \frac{2 k_B T(\Theta,\Phi)}{\lambda^2} \cdot \Delta f \cdot r^2 d\Omega \cdot \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

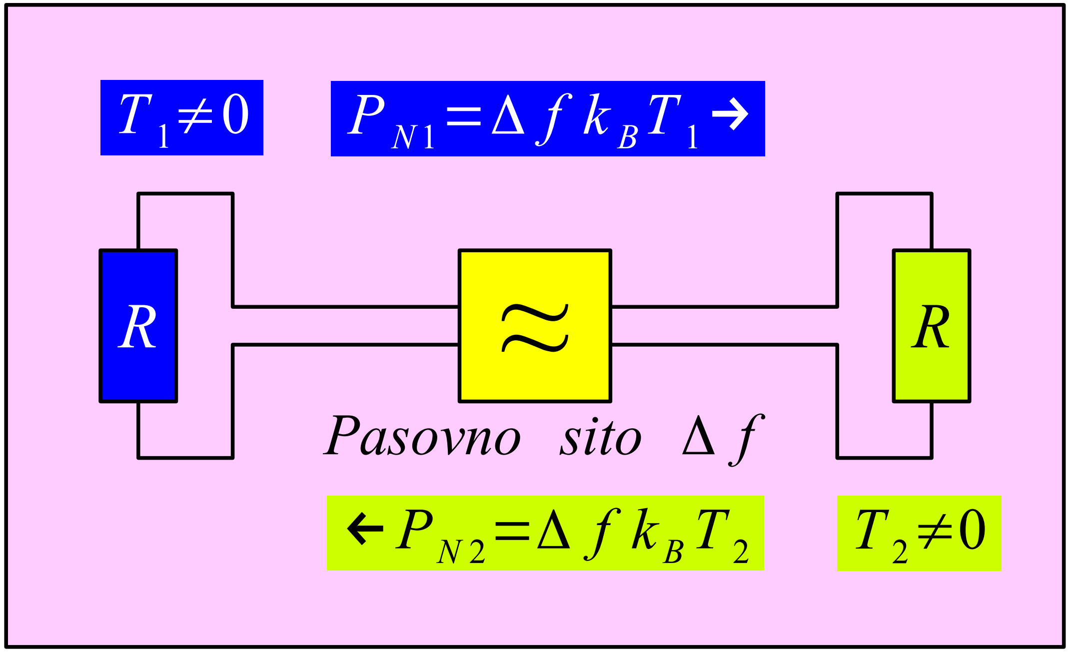
$$P_N = \Delta f\, k_B \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega} = \Delta f\, k_B T_A$$

$$T_A = \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

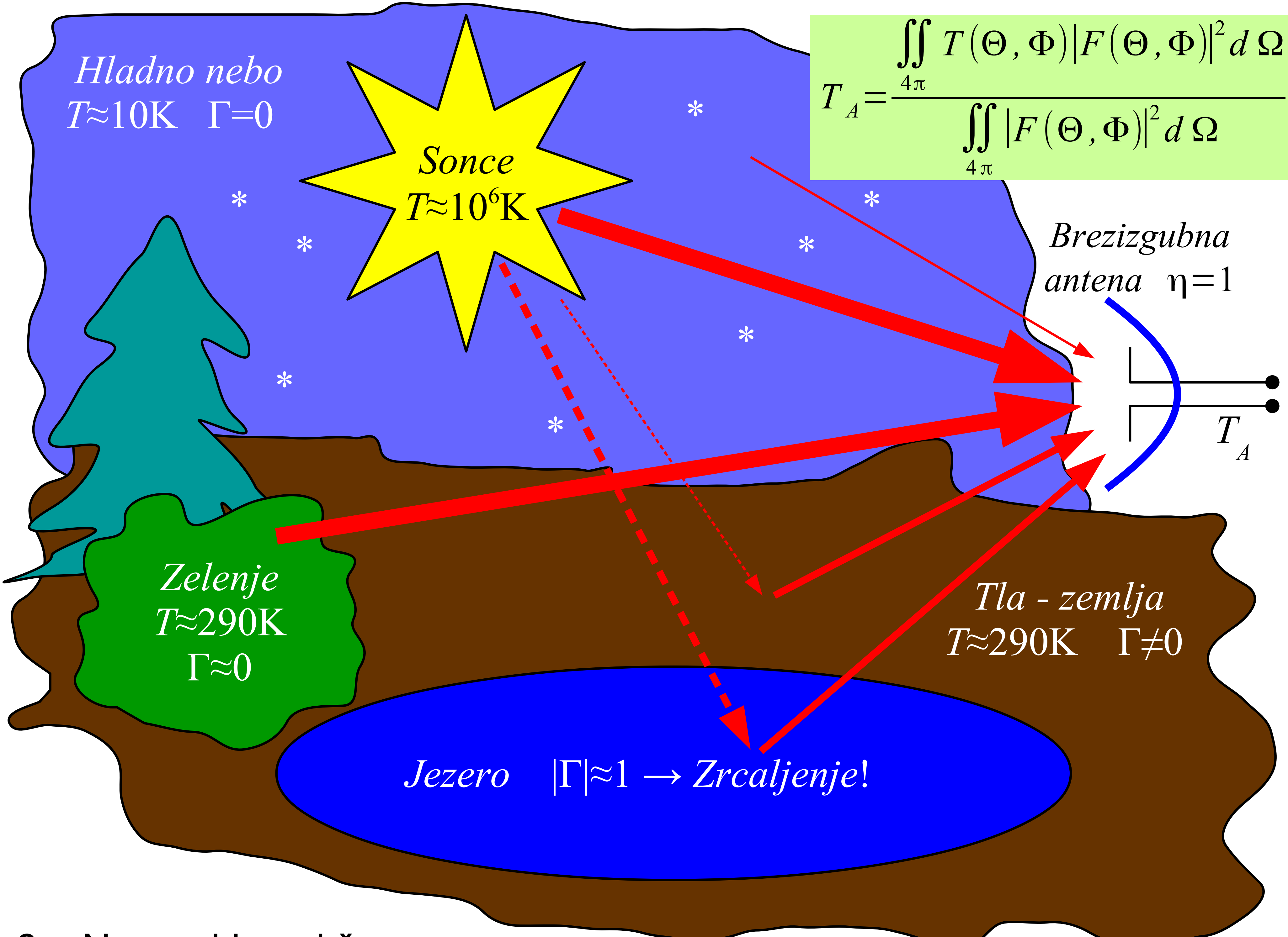
4 – Sprejeta moč toplotnega šuma



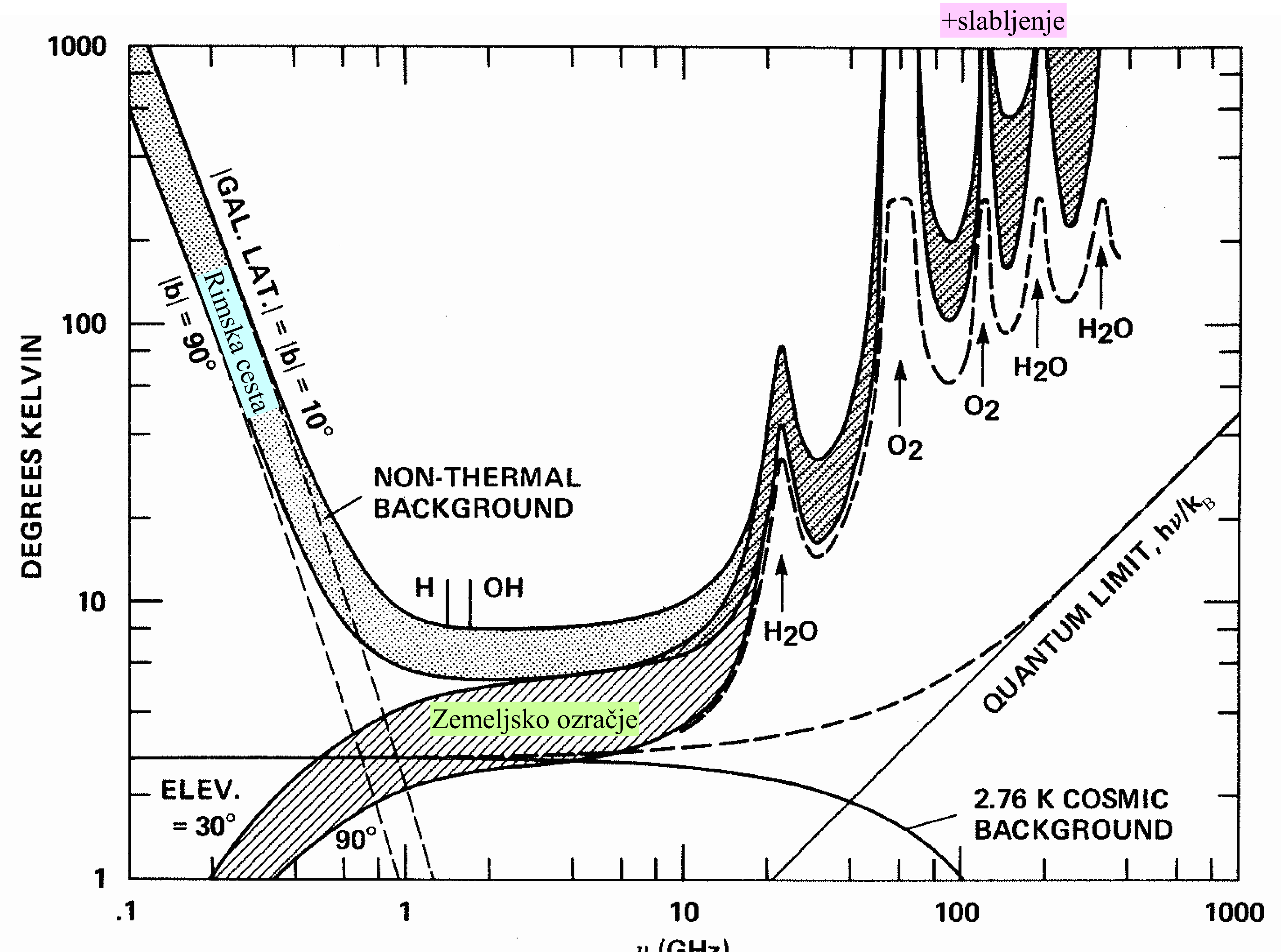




5 – Toplotno ravnovesje

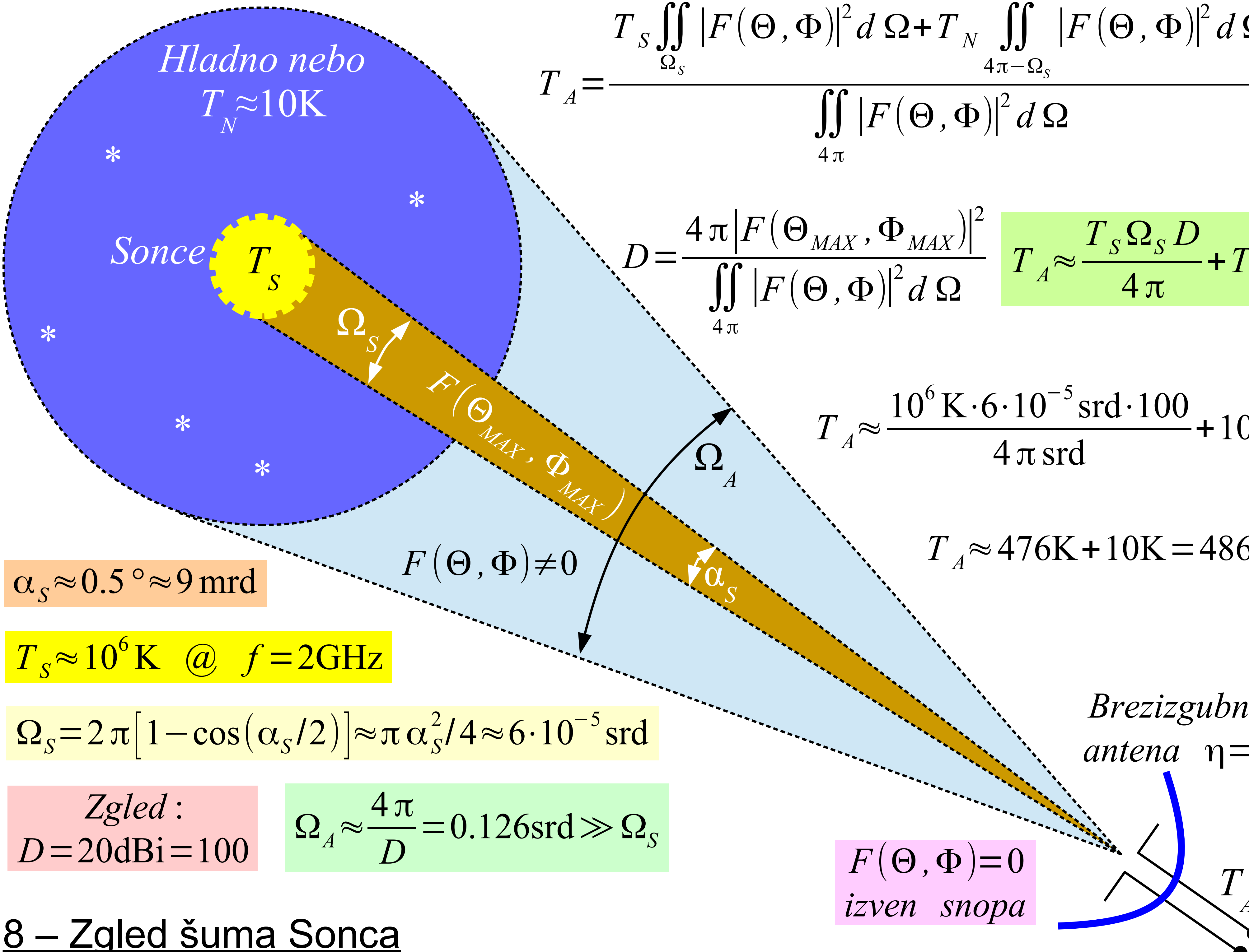


6 – Naravni izvori šuma



7 – Naravni šum neba

8 – Zgled šuma Sonca



$$T_A = \frac{T_S \iint\limits_{\Omega_S} |F(\Theta,\Phi)|^2 d\Omega + T_N \iint\limits_{4\pi-\Omega_S} |F(\Theta,\Phi)|^2 d\Omega}{\iint\limits_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$D = \frac{4\pi |F(\Theta_{MAX},\Phi_{MAX})|^2}{\iint\limits_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$T_A \approx \frac{T_S \Omega_S D}{4\pi} + T_N$$

$$T_A \approx \frac{10^6\,\text{K} \cdot 6 \cdot 10^{-5}\,\text{srd} \cdot 100}{4\pi\,\text{srd}} + 10\text{K}$$

$$T_A \approx 476\text{K} + 10\text{K} = 486\text{K}$$

$\alpha_S \approx 0.5° \approx 9\,\text{mrd}$

$T_S \approx 10^6\,\text{K} \quad @ \quad f = 2\text{GHz}$

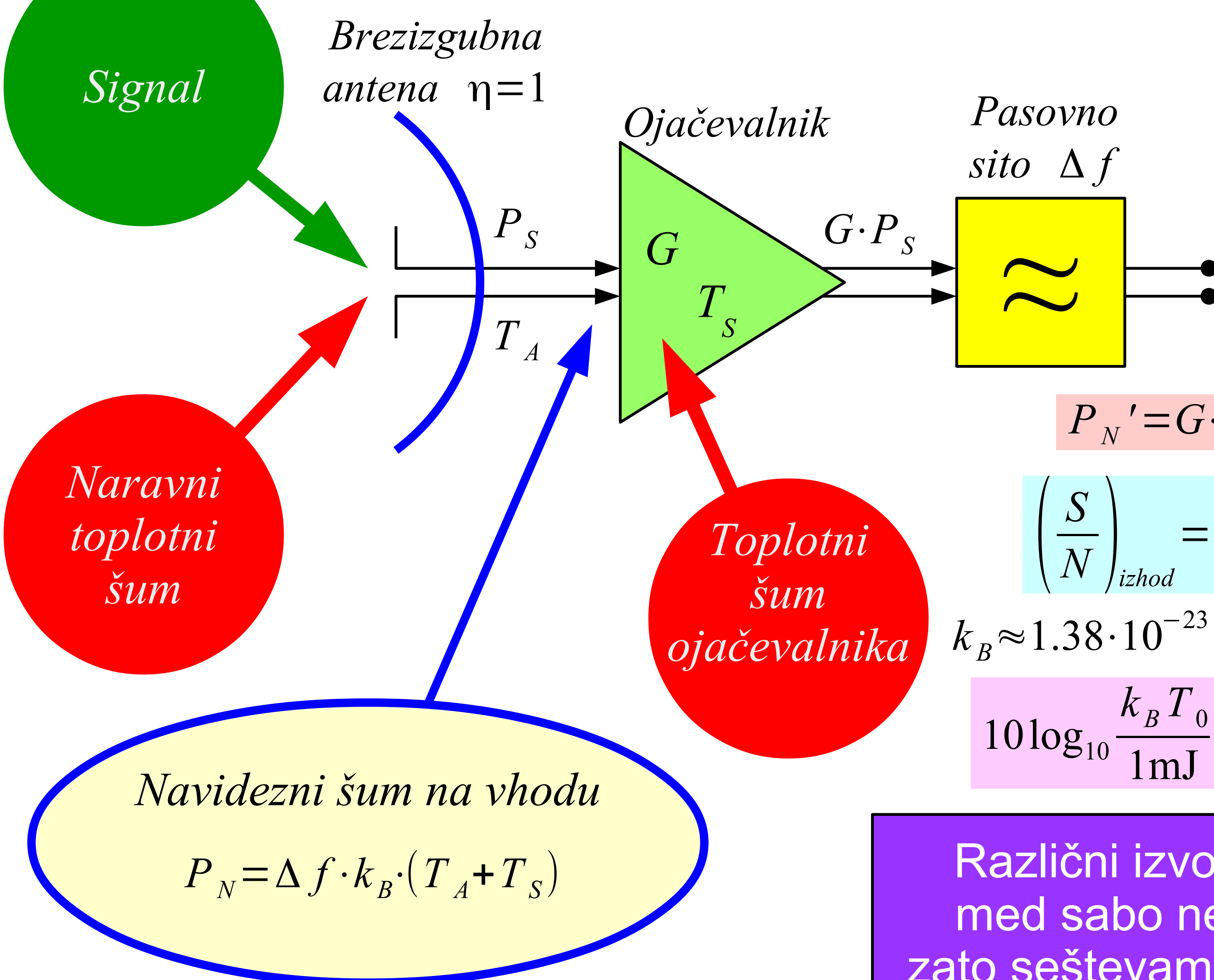
$\Omega_S = 2\pi[1-\cos(\alpha_S/2)] \approx \pi\alpha_S^2/4 \approx 6 \cdot 10^{-5}\,\text{srd}$

Zgled:
$D = 20\text{dBi} = 100$

$\Omega_A \approx \frac{4\pi}{D} = 0.126\text{srd} \gg \Omega_S$

$F(\Theta,\Phi) = 0$
izven snopa

$T_S \equiv$ *temperatura ojačevalnika preračunana na vhod !*



$$P_S' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S'}{P_N'}$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot (T_A + T_S)$$

$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_S)}$$

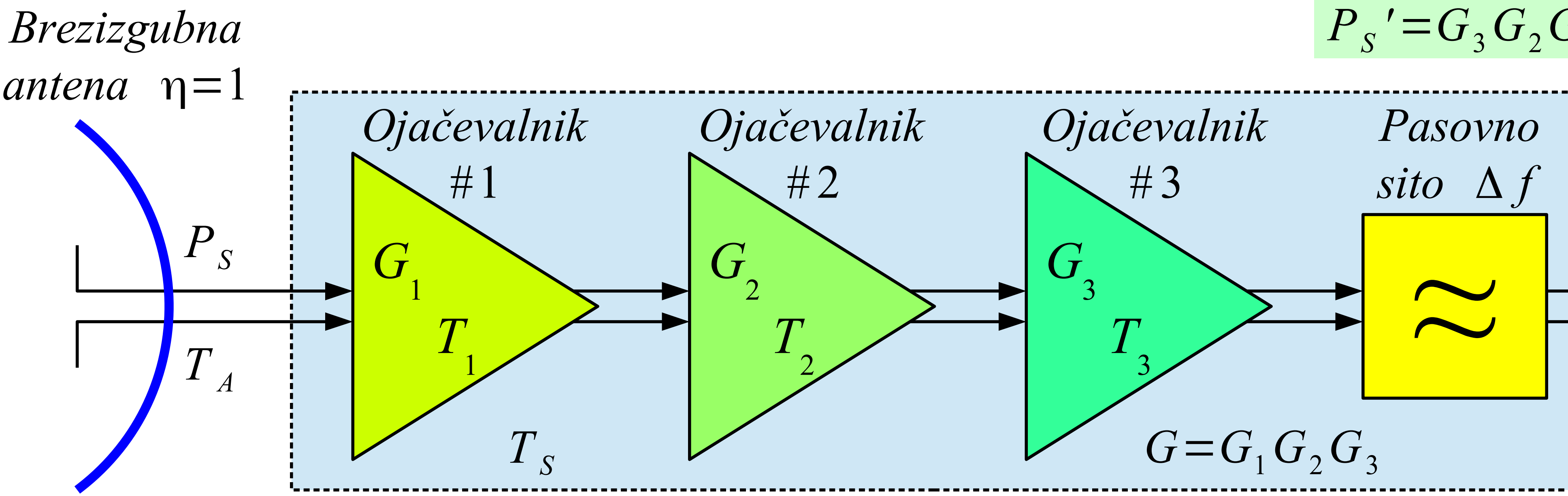
$$k_B \approx 1.38 \cdot 10^{-23}\ \mathrm{J/K} \qquad T_0 = 290\mathrm{K}$$

$$10\log_{10} \frac{k_B T_0}{1\mathrm{mJ}} \approx -174\ \mathrm{dBm/Hz}$$

Različni izvori šuma so med sabo nekorelirani, zato seštevamo moči šuma oziroma temperature šuma!

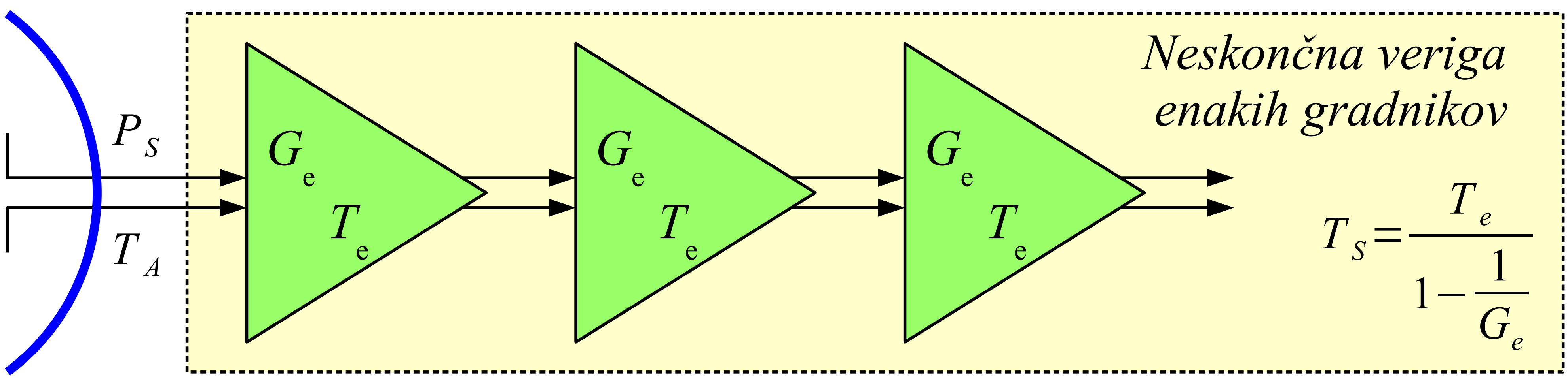
9 – Razmerje signal/šum sprejemnika

$$P_S' = G_3 G_2 G_1 P_S$$



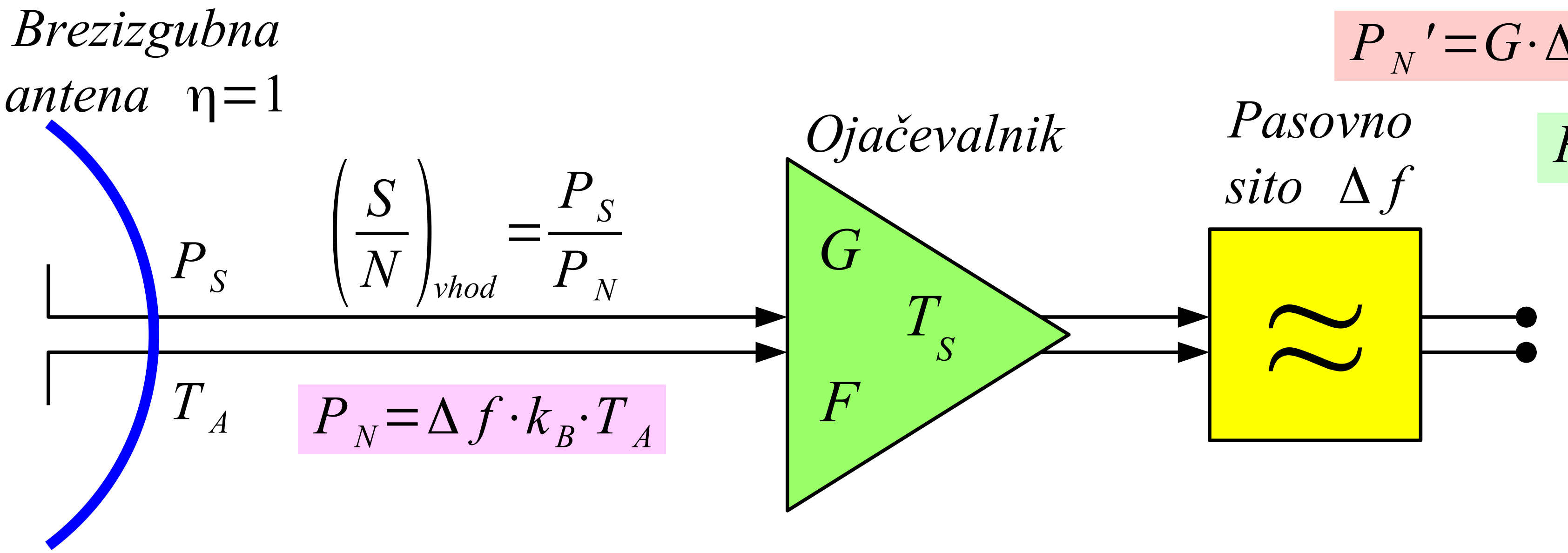
$$P_N' = \Delta f\, k_B \left[G_3 G_2 G_1 (T_A + T_1) + G_3 G_2 T_2 + G_3 T_3 \right]$$

$$P_N' = G_3 G_2 G_1 \Delta f\, k_B (T_A + T_S) \quad \rightarrow \quad T_S = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$



$$T_S = \frac{T_e}{1 - \frac{1}{G_e}}$$

10 – Šumna temperatura verige



$$\left(\frac{S}{N}\right)_{vhod}=\frac{P_S}{P_N}\qquad P_N=\Delta f\cdot k_B\cdot T_A$$

$$P_N{}'=G\cdot\Delta f\cdot k_B\cdot\left(T_A+T_S\right)\qquad P_S{}'=G\cdot P_S\qquad \left(\frac{S}{N}\right)_{izhod}=\frac{P_S{}'}{P_N{}'}$$

Nesmiselna definicija šumnega števila:

$$F=\frac{\left(\frac{S}{N}\right)_{vhod}}{\left(\frac{S}{N}\right)_{izhod}}=\frac{\dfrac{P_S}{\Delta f\,k_B T_A}}{\dfrac{G\,P_S}{G\,\Delta f\,k_B\left(T_A+T_S\right)}}=\frac{T_A+T_S}{T_A}=1+\frac{T_S}{T_A}$$

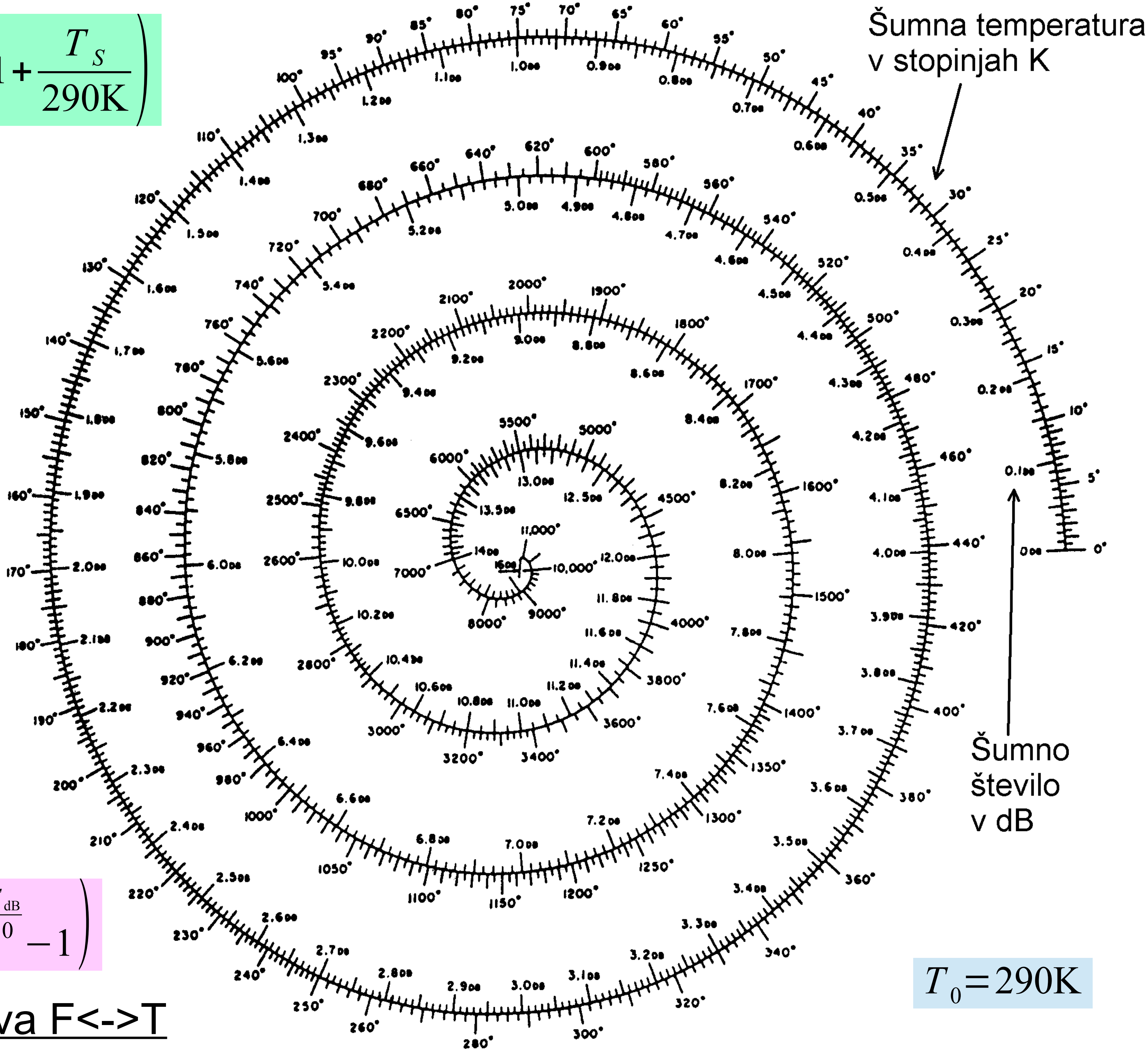
Lastnost ojačevalnika ne more biti funkcija T_A!

Smiselna definicija $\quad F=1+\frac{T_S}{T_0}\quad @\quad T_0=290\mathrm{K}\quad\leftrightarrow\quad T_S=T_0\left(F-1\right)$

Logaritemske enote $\quad F_{\mathrm{dB}}=10\log_{10}F=10\log_{10}\left(1+\frac{T_S}{T_0}\right)\quad\leftrightarrow\quad T_S=T_0\left(10^{\frac{F_{\mathrm{dB}}}{10}}-1\right)$

11 – Šumno število ojačevalnika

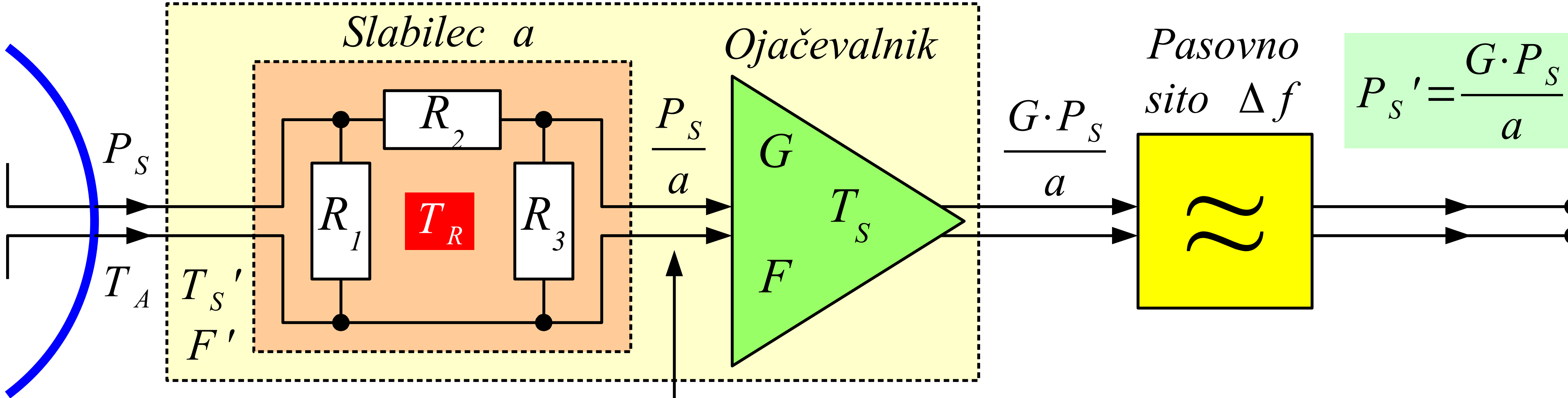
$$F_{\mathrm{dB}} = 10\log_{10}\left(1 + \frac{T_S}{290\mathrm{K}}\right)$$



$$T_S = 290\mathrm{K}\left(10^{\frac{F_{\mathrm{dB}}}{10}} - 1\right)$$

$$T_0 = 290\mathrm{K}$$

12 – Povezava F<->T



Brezizgubna antena $\eta = 1$

$$\frac{T_A}{a} + T_R\left(1 - \frac{1}{a}\right)$$

$$P_N' = G\cdot\Delta f\cdot k_B\cdot\left[\frac{T_A}{a} + T_R\left(1-\frac{1}{a}\right) + T_S\right]$$

$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S'}{P_N'} = \frac{P_S}{\Delta f\cdot k_B\cdot\left[T_A + T_R(a-1) + aT_S\right]}$$

$$T_S' = T_R(a-1) + aT_S$$

$$F' = 1 + \frac{T_S'}{T_0} = 1 + \frac{T_R}{T_0}(a-1) + a\frac{T_S}{T_0}$$

Pogost primer $T_R \approx T_0 = 290\mathrm{K}$

$$F' \approx a + a\frac{T_S}{T_0} = a\left(1 + \frac{T_S}{T_0}\right) = a\cdot F$$

$$F_{dB}' \approx a_{dB} + F_{dB}$$

Primeri slabilcev $T_R \approx T_0 = 290\mathrm{K}$

$F' \approx a\cdot F$ *oziroma* $F_{dB}' \approx a_{dB} + F_{dB}$

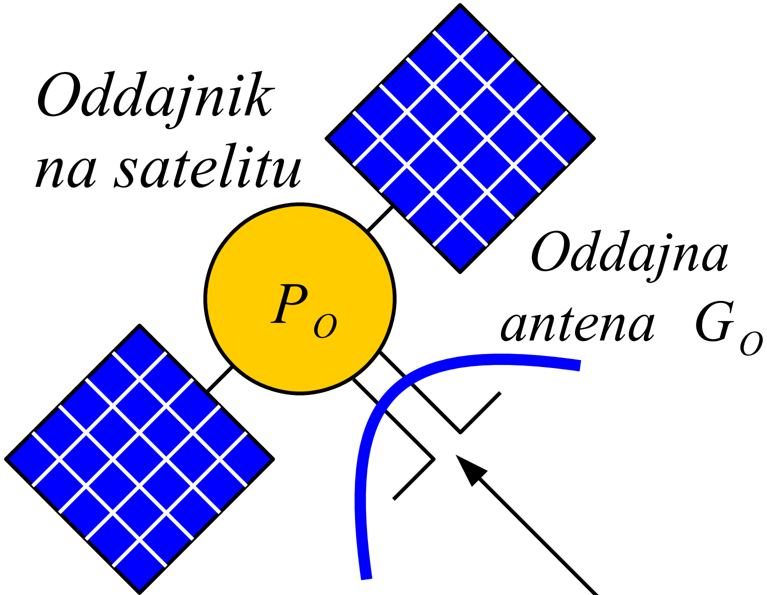
(1) *izgubna antena* $a_{dB} = -10\log_{10}\eta$

(2) *prenosni vod z izgubami* a_{dB}

(3) *pasovno sito s slabljenjem* a_{dB}

(4) *slabljenje pasivnega mešalnika* a_{dB}

13 – Šum slabilca



Zveza v praznem prostoru $P_S = P_O \cdot G_O \cdot G_S \cdot \left(\frac{\lambda}{4\pi r}\right)^2$

Oddajnik | Sistem | Sprejemnik

$$\left(\frac{S}{N}\right)_{izhod} = P_O \cdot G_O \cdot \frac{1}{\Delta f \cdot k_B} \cdot \left(\frac{\lambda}{4\pi r}\right)^2 \cdot \frac{G_S}{(T_A + T_S)}$$

r

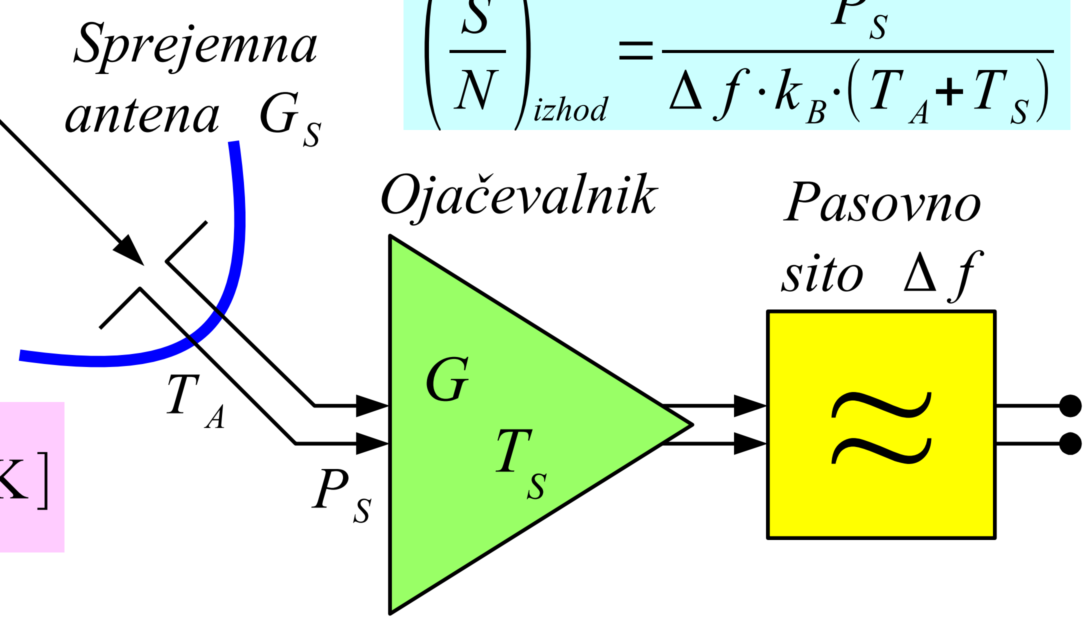
Sprejemna postaja

$$(G/T) = \frac{G_S}{(T_A + T_S)} \quad [\mathrm{K}^{-1}]$$

$$(G/T)_{\mathrm{dB/K}} = 10 \log_{10} \frac{G_S \cdot 1\mathrm{K}}{(T_A + T_S)} \quad [\mathrm{dB/K}]$$

$$(G/T)_{\mathrm{dB/K}} = G_{S\,\mathrm{dB}} - 10 \log_{10} \frac{T_A + T_S}{1\mathrm{K}} \quad [\mathrm{dB/K}]$$

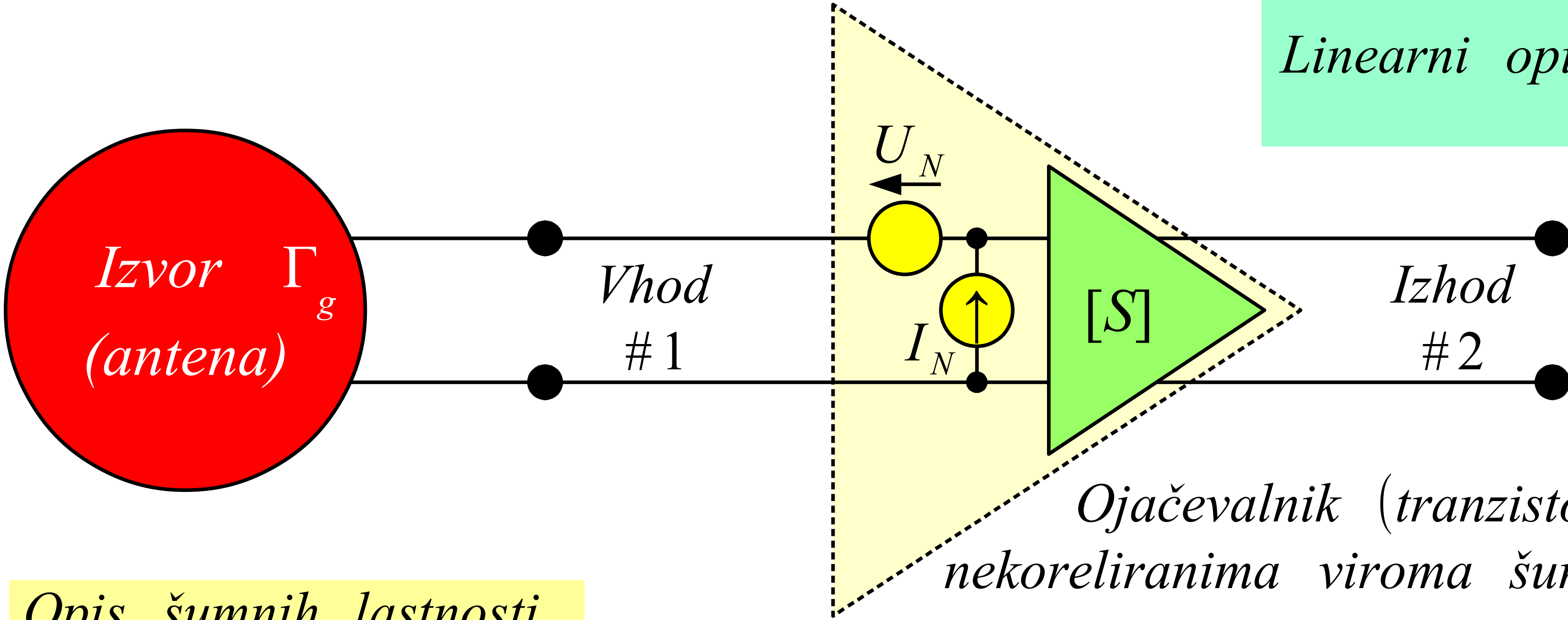
$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_S)}$$



Zemeljska sprejemna postaja

14 – Razmerje G/T

Vrsta ojačevalnika	Ojačanje G [dB]	Temperatura šuma T_S [K]	Šumno število F_{dB} [dB]
Vakuumska cev z mrežicami (trioda, pentoda)	10↔20	1600↔9000	8↔15
Vakuumska cev s hitrostno modulacijo (klistron, TWT)	20↔50	3000↔30000	10↔20
Parametrični ojačevalnik (sobna temperatura)	10↔15	75↔300	1↔3
Si BJT, JFET ali MOSFET (sobna temperatura)	10↔20	75↔300	1↔3
GaAs FET ali HEMT (sobna temperatura)	10↔15	20↔120	0.3↔1.5
GaAs FET ali HEMT (hlajen 77K tekoči dušik)	10↔15	7↔35	0.1↔0.5
Si ali GaAs MMIC ojačevalnik	10↔25	170↔1600	2↔8
Operacijski ojačevalnik	40↔100	10^4↔10^9	16↔66



Ojačevalnik (tranzistor) z nekoreliranima viroma šuma U_N, I_N

Linearni opis $[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

Opis šumnih lastnosti
$U_N, I_N \rightarrow F_{MIN}, \Gamma_O, r_N$

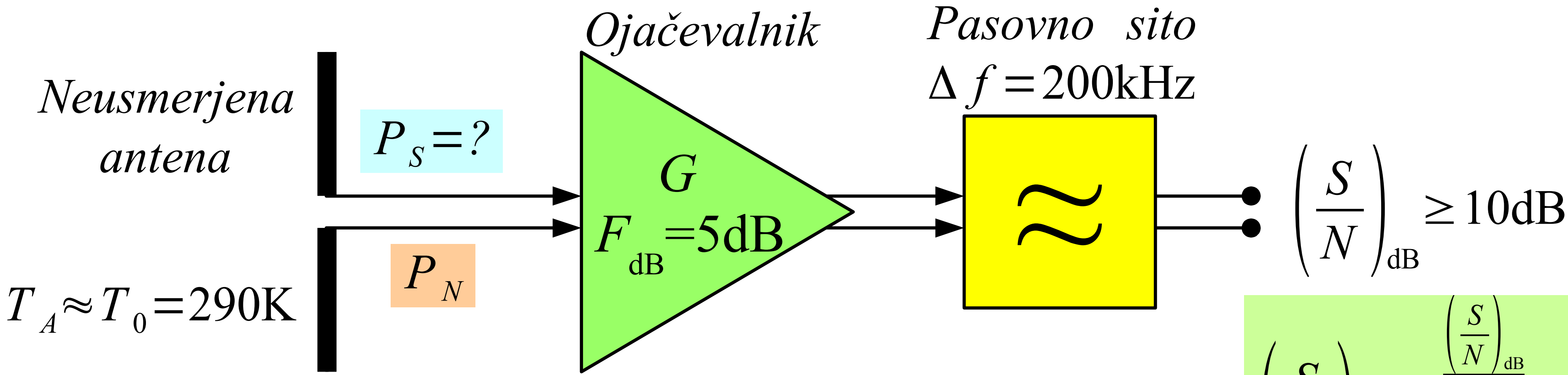
$$F = F_{MIN} + 4\frac{R_N}{Z_K}\cdot\frac{|\Gamma_g-\Gamma_O|^2}{(1-|\Gamma_g|^2)\cdot|1+\Gamma_O|^2} = F_{MIN} + 4r_N\cdot\frac{|\Gamma_g-\Gamma_O|^2}{(1-|\Gamma_g|^2)\cdot|1+\Gamma_O|^2}$$

$F_{MIN} \equiv$ najnižje šumno število pri $\Gamma_g = \Gamma_O$ v linearnih enotah (ne v dB!)

$\Gamma_O \equiv$ optimalna odbojnost izvora za F_{MIN} (nima povezave z matriko $[S]$!)

$r_N = \frac{R_N}{Z_K} \equiv$ normirana šumna upornost (običajno $Z_K = 50\,\Omega$)

16 – Šumni parametri tranzistorja



$$\left(\frac{S}{N}\right) = 10^{\frac{\left(\frac{S}{N}\right)_{\mathrm{dB}}}{10}} \geq 10$$

$$T_S = T_0 \cdot \left(10^{\frac{F_{\mathrm{dB}}}{10}} - 1\right) = 290\mathrm{K} \cdot (3.162 - 1) = 627\mathrm{K}$$

$$k_B \approx 1.38 \cdot 10^{-23}\,\mathrm{J/K}$$

$$P_N = \Delta f \cdot k_B \cdot (T_A + T_S) = 200\mathrm{kHz} \cdot 1.38 \cdot 10^{-23}\,\mathrm{J/K} \cdot (290\mathrm{K} + 627\mathrm{K}) = 2.531 \cdot 10^{-15}\,\mathrm{W}$$

$$P_S = P_N \cdot \left(\frac{S}{N}\right) = 2.531 \cdot 10^{-15}\,\mathrm{W} \cdot 10 = 2.531 \cdot 10^{-14}\,\mathrm{W}$$

$$P_{S\,\mathrm{dBm}} = 10 \log_{10} \frac{P_S}{1\mathrm{mW}} = -106\mathrm{dBm}$$

Poenostavljen izračun izključno v primeru $T_A \approx T_0 = 290\mathrm{K}$

$$P_{S\,\mathrm{dBm}} \approx (S/N)_{\mathrm{dB}} + (\Delta f)_{\mathrm{dB \cdot Hz}} + (k_B T_0)_{\mathrm{dBm/Hz}} + F_{\mathrm{dB}}$$

$$(k_B T_0)_{\mathrm{dBm/Hz}} = 10 \log_{10} \frac{k_B T_0}{1\mathrm{mJ}} \approx -174\,\mathrm{dBm/Hz}$$

$$(\Delta f)_{\mathrm{dB \cdot Hz}} = 10 \log_{10} \left(\frac{\Delta f}{1\mathrm{Hz}}\right) = 53\mathrm{dB \cdot Hz}$$

$$P_{S\,\mathrm{dBm}} \approx 10\mathrm{dB} + 53\mathrm{dB \cdot Hz} - 174\mathrm{dBm/Hz} + 5\mathrm{dB} = -106\mathrm{dBm}$$

17 – Občutljivost GSM telefona

Dva različna sprejemnika #1 in #2:

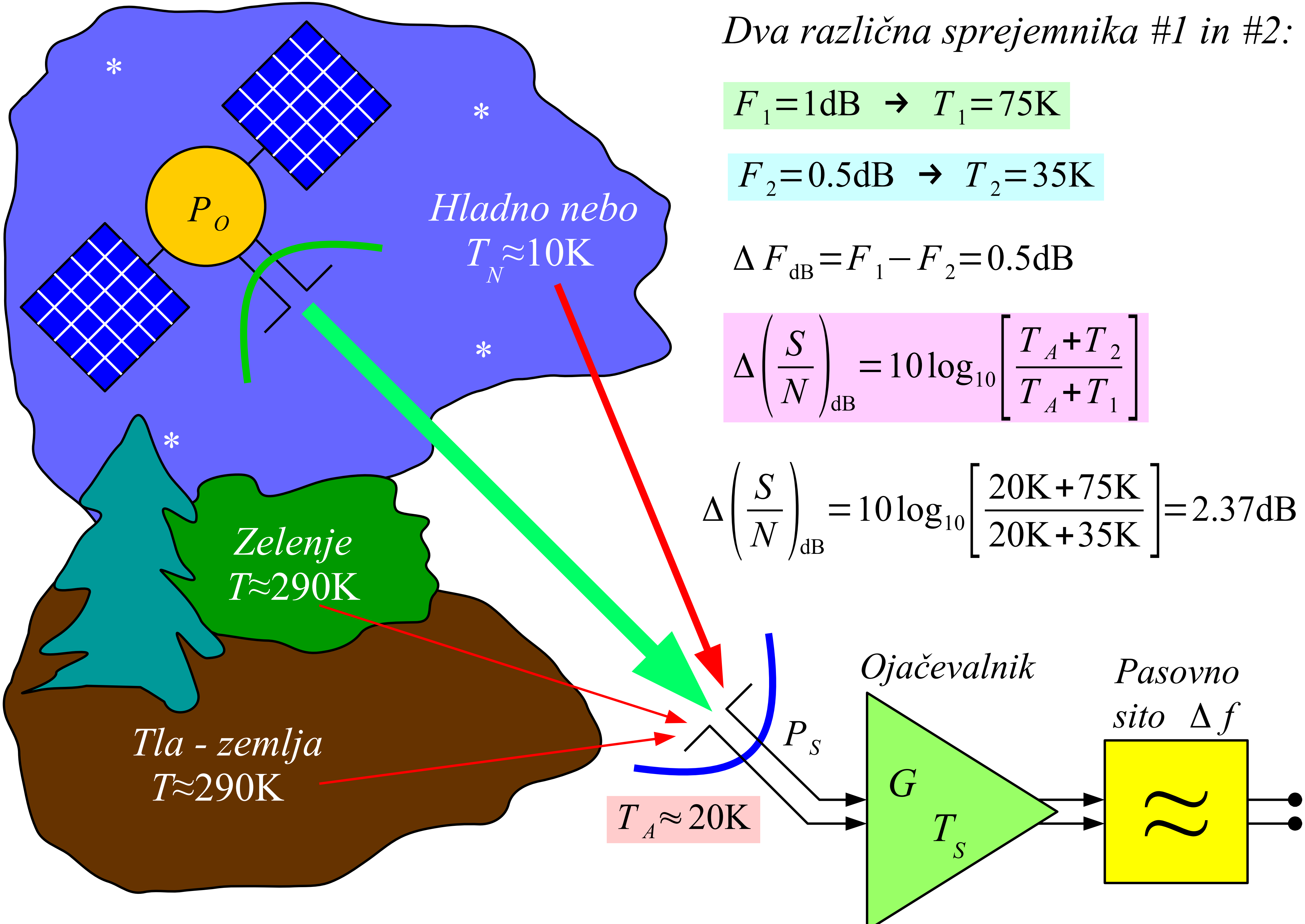
$$F_1 = 1\text{dB} \rightarrow T_1 = 75\text{K}$$

$$F_2 = 0.5\text{dB} \rightarrow T_2 = 35\text{K}$$

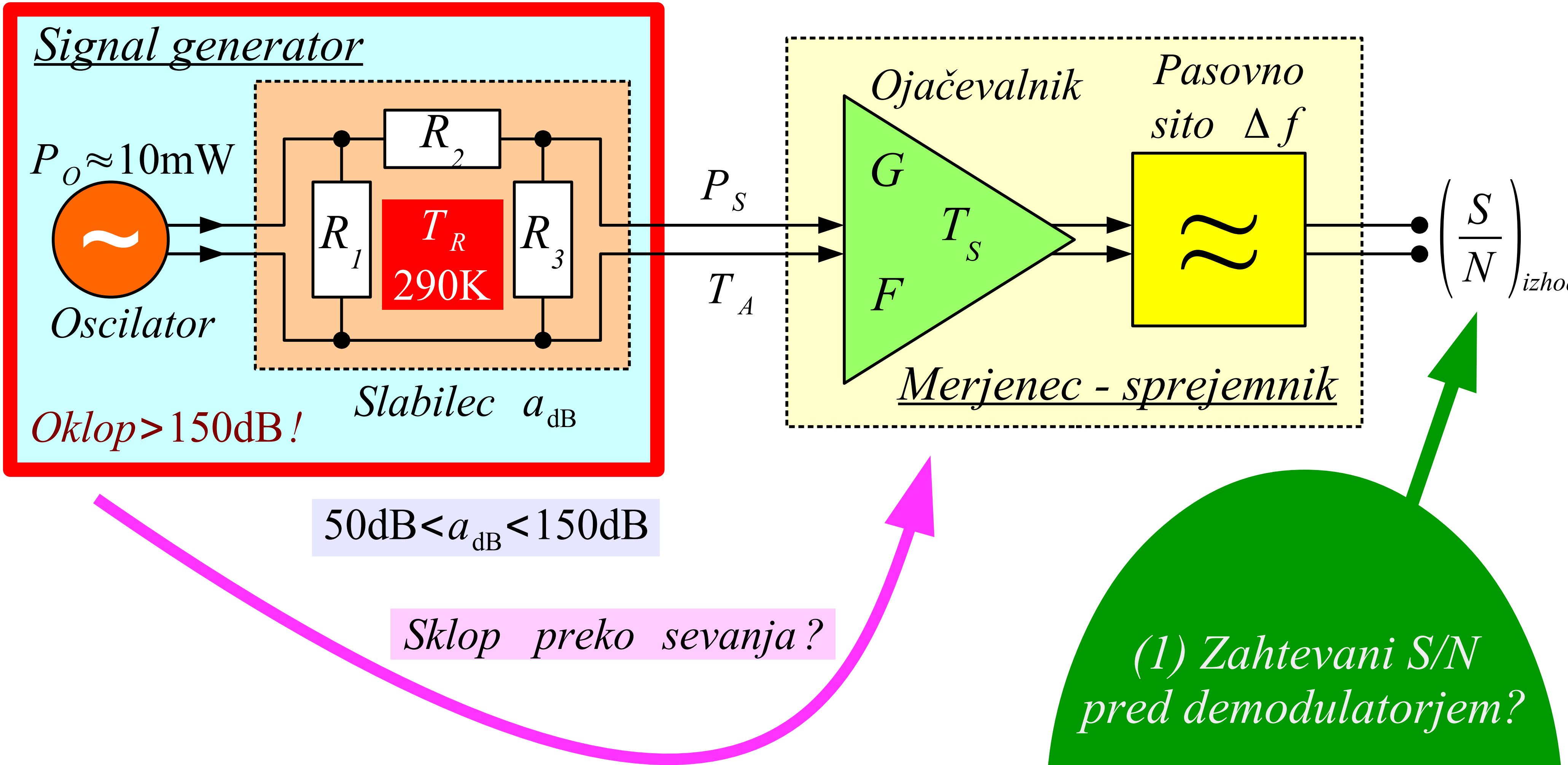
$$\Delta F_{\text{dB}} = F_1 - F_2 = 0.5\text{dB}$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{T_A + T_2}{T_A + T_1}\right]$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{20\text{K}+75\text{K}}{20\text{K}+35\text{K}}\right] = 2.37\text{dB}$$



18 – Spremembi F in S/N



(1) Zahtevani S/N pred demodulatorjem?

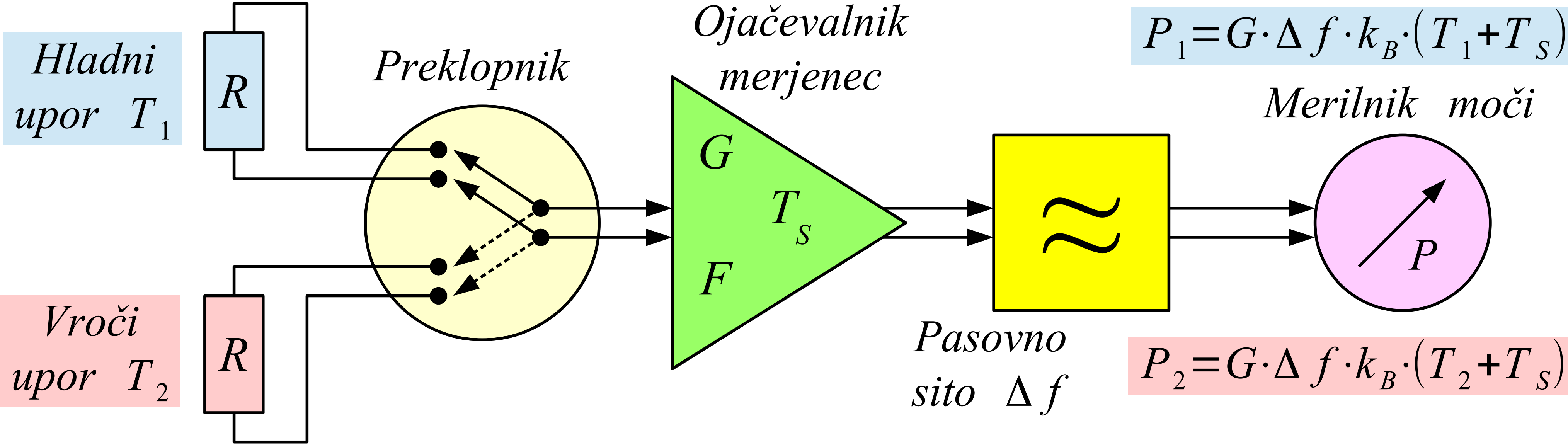
(2) Zahtevani S/N za demodulatorjem?

(3) Zahtevani BER?

Dodatni zahtevi za merilni izvor (signal generator) za merjenje občutljivosti radijskih/mikrovalovnih sprejemnikov:

(1) Oklop>150dB

(2) $T_R = T_A = T_0 = 290K$



V razmerju Y se neznanke $G \cdot \Delta f \cdot k_B$ natančno krajšajo!

$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_S}{T_1 + T_S}$$

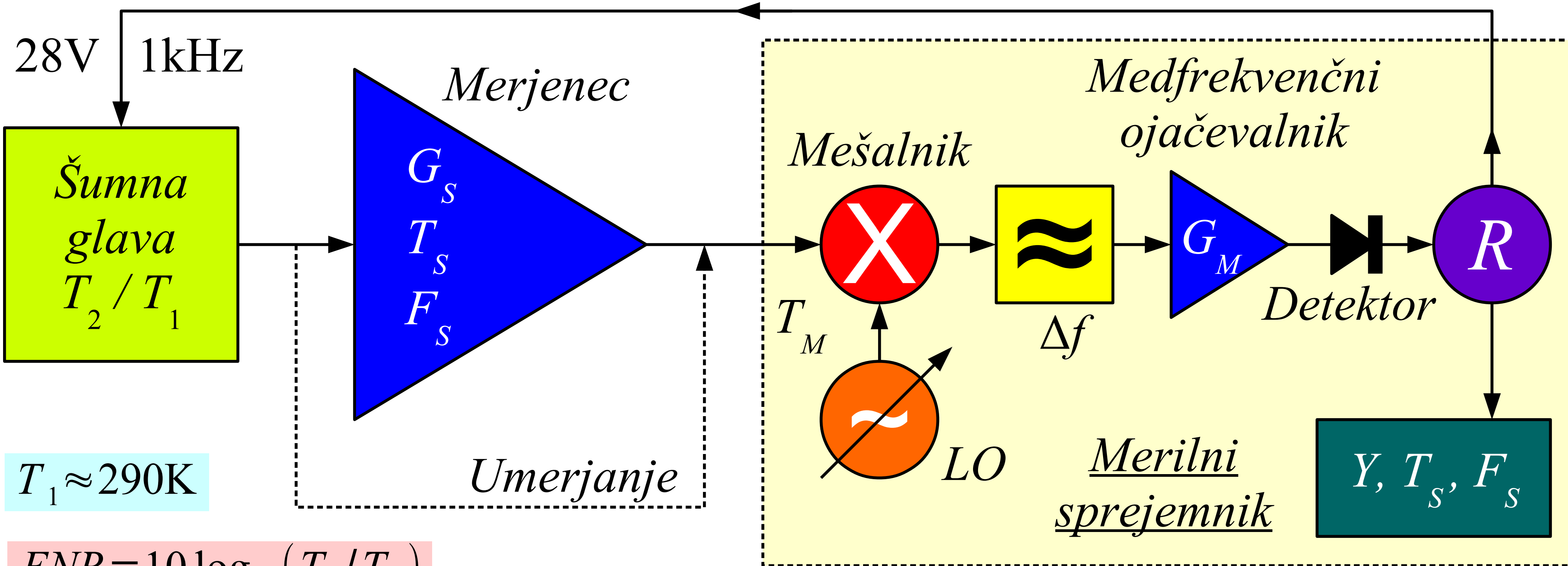
$$T_S = \frac{T_2 - Y \cdot T_1}{Y - 1}$$

$$T_0 = 290\text{K}$$

$$(F_S)_{dB} = 10 \log_{10}\left[1 + \frac{T_2 - Y \cdot T_1}{(Y-1) \cdot T_0}\right]$$

Vrsta upora	Temperatura
Antena v hladno nebo	~20K
R hlajen tekoči dušik	~77K
Antena v absorber	~290K
R sobna temperatura	~290K
Nitka žarnice kot R	~2000K
Ioniziran plin kot R	~10^4K
Plazovni preboj v diodi	~10^6K

<u>20 – Postopek vroče/hladno</u>



$T_1 \approx 290\text{K}$

$ENR = 10\log_{10}(T_2/T_1)$

$G_M \Delta f \equiv nezanesljiv!$

Dve meritvi brez umerjanja:

$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_S + T_M/G_S}{T_1 + T_S + T_M/G_S}$$

$$T_S = \frac{T_2 - Y\cdot T_1}{Y-1} - \frac{T_M}{G_S} \leftarrow \ poznam \ \ G_S$$

$$(F_S)_{dB} = 10\log_{10}\left[1 + \frac{1}{T_0}\cdot\left(\frac{T_2 - Y\cdot T_1}{Y-1} - \frac{T_M}{G_S}\right)\right]$$

Štiri meritve z umerjanjem:

(1) $P_1 = G_M G_S \Delta f k_B (T_1 + T_S + T_M/G_S)$

(2) $P_2 = G_M G_S \Delta f k_B (T_2 + T_S + T_M/G_S)$

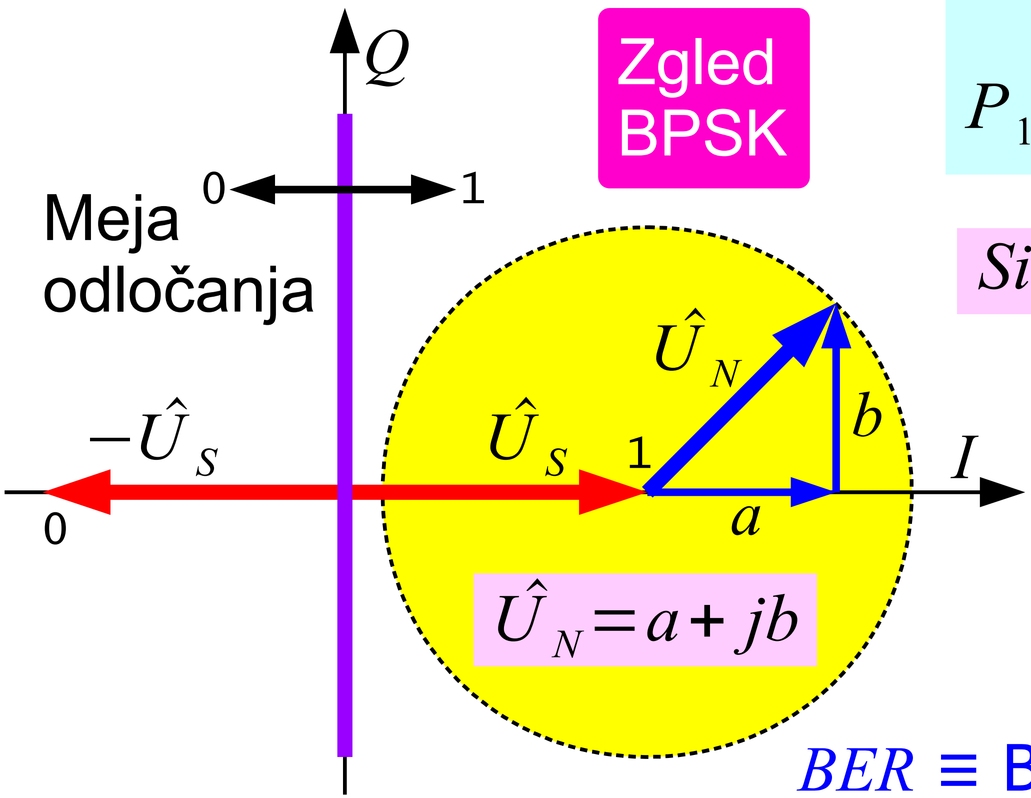
(3) $P_3 = G_M \Delta f k_B (T_1 + T_M)$

(4) $P_4 = G_M \Delta f k_B (T_2 + T_M)$

Rešujem 4 enačbe za 4 neznanke:

$T_S, \ G_S, \ T_M \ in \ (G_M \Delta f k_B)$

21 – Merilnik šumnega števila



$$P_{1\to 0} = \int_{-\infty}^{-|\hat{U}_S|} p(a)\,da$$

$$P_{0\to 1} = \int_{|\hat{U}_S|}^{\infty} p(a)\,da$$

$$Simetrična\ \ meja:\ \ P_{1\to 0} = P_{0\to 1} = BER$$

$$BER = \int_{|\hat{U}_S|}^{\infty} \frac{1}{\sqrt{\pi\langle|\hat{U}_N|^2\rangle}} e^{-\frac{a^2}{\langle|\hat{U}_N|^2\rangle}}\,da$$

BER ≡ Bit-Error Rate

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2}\,du$$

Gaussova porazdelitev gostote verjetnosti sofazne a in kvadraturne jb komponente šuma

$$p(a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{a^2}{2\sigma^2}}$$

$$p(b) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{b^2}{2\sigma^2}}$$

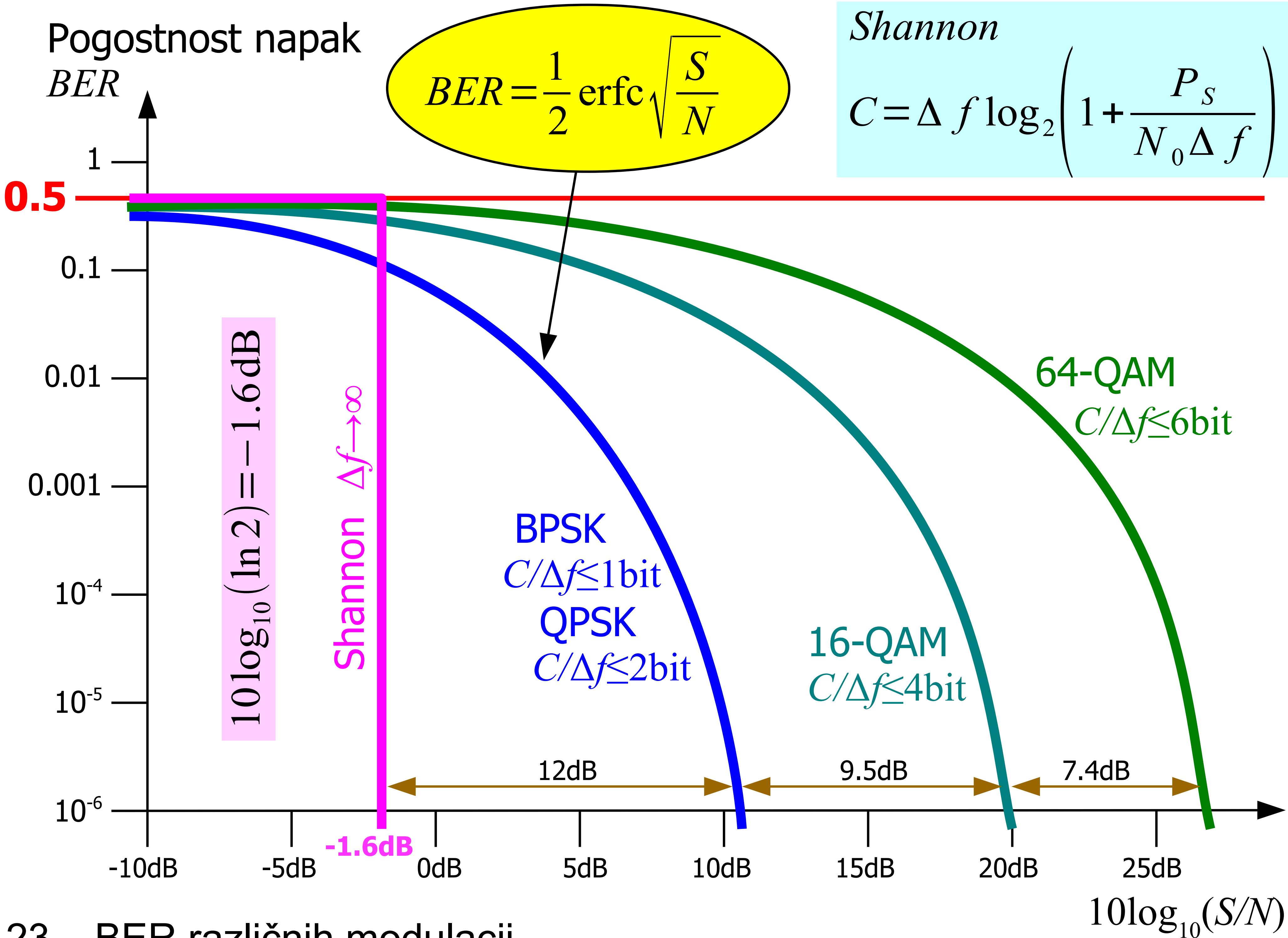
$$BER = \frac{1}{2}\,\mathrm{erfc}\left(\frac{|\hat{U}_S|}{\sqrt{\langle|\hat{U}_N|^2\rangle}}\right)$$

$$P_S = \alpha|\hat{U}_S|^2 \qquad P_N = \alpha\langle|\hat{U}_N|^2\rangle$$

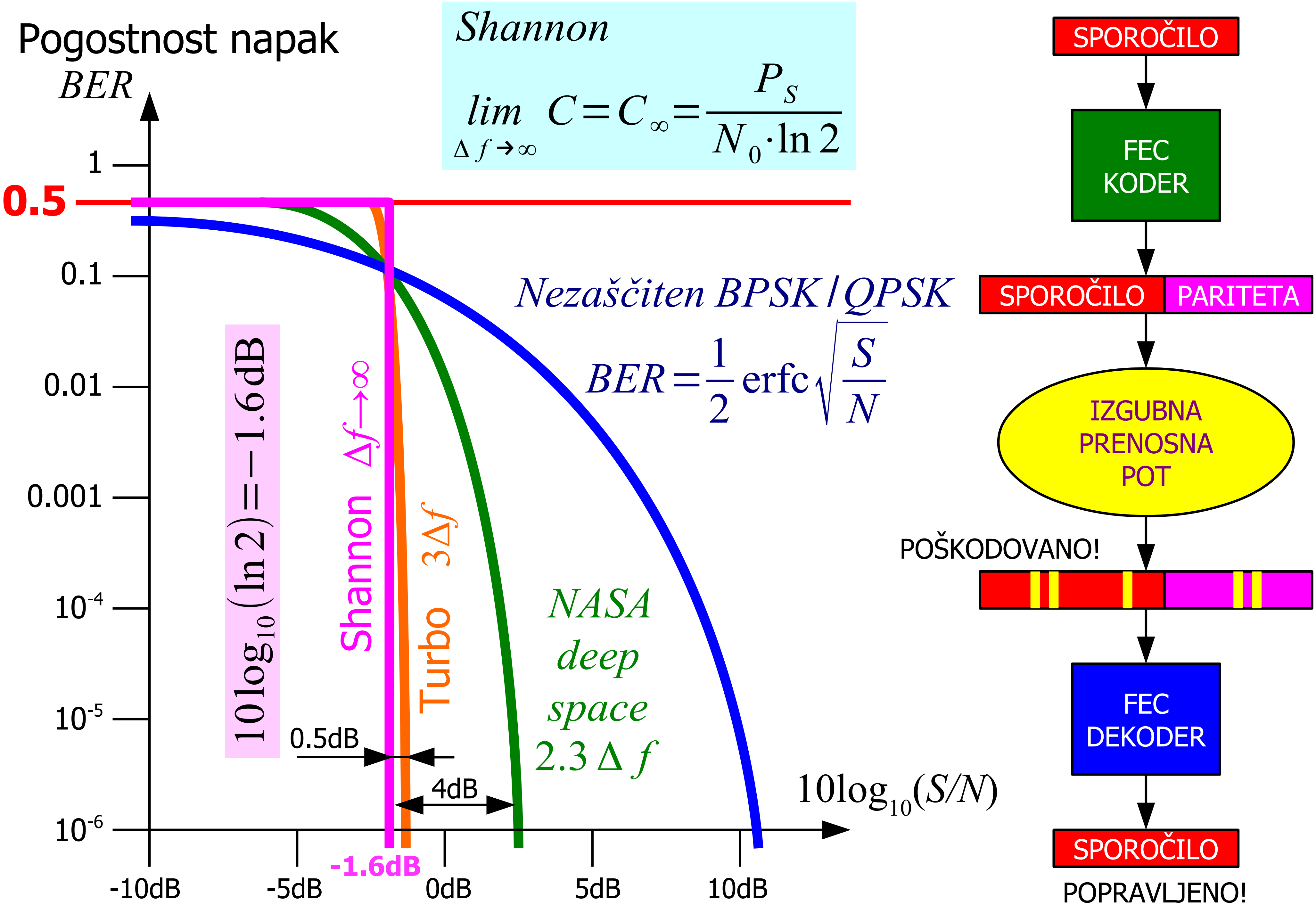
$$\langle|\hat{U}_N|^2\rangle = \langle a^2\rangle + \langle b^2\rangle = 2\sigma^2$$

$$BER = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{P_S}{P_N}}\right)$$

22 – Izračun pogostnosti napak BER

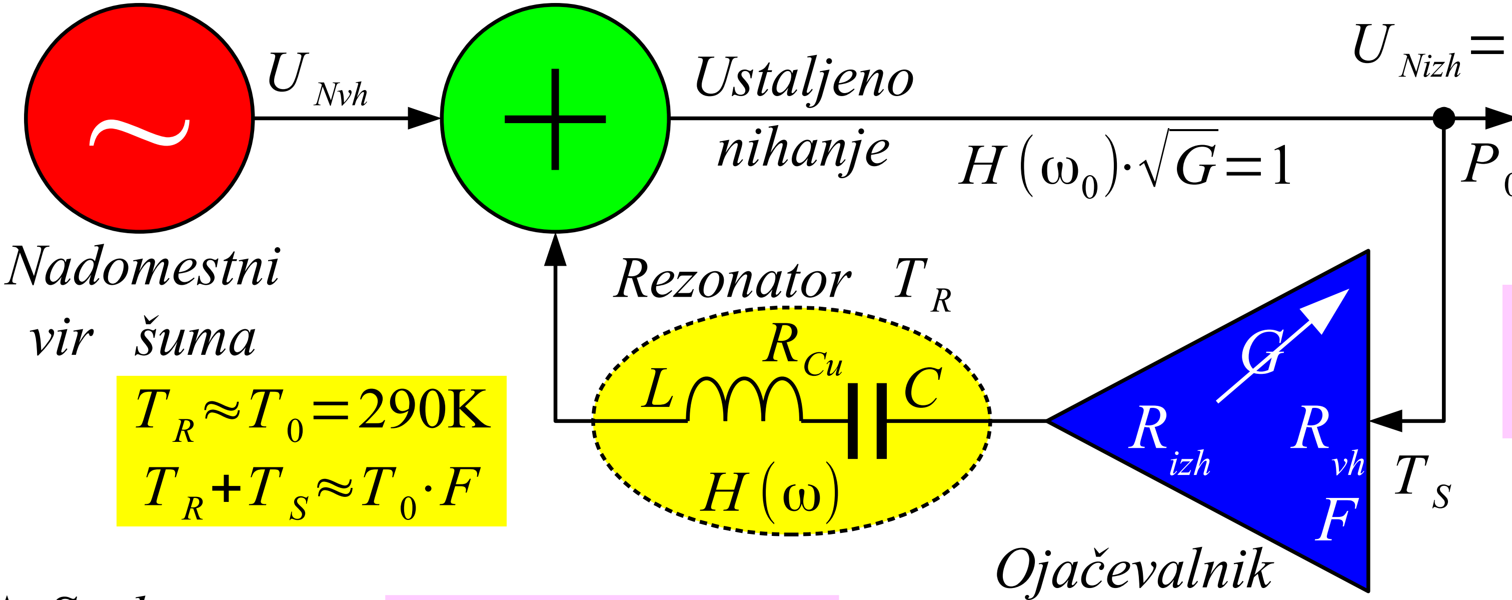


23 – BER različnih modulacij



24 – Vnaprejšnje popravljanje napak

FEC ≡ Forward Error Correction

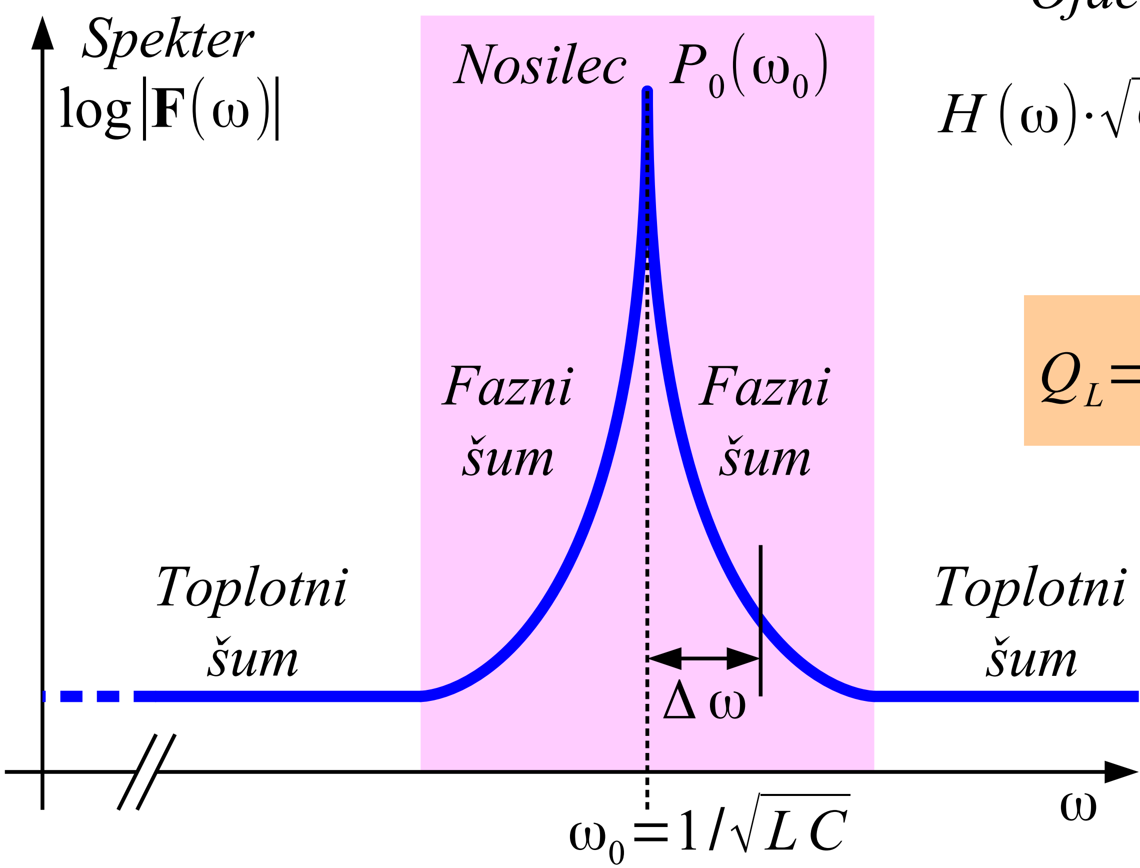


$$U_{Nizh} = U_{Nvh} + H(\omega)\cdot\sqrt{G}\cdot U_{Nizh}$$

Velja pri $P_{Nizh} \ll P_0$

$$U_{Nizh} = \frac{U_{Nvh}}{1 - H(\omega)\cdot\sqrt{G}}$$

$$\Sigma R = R_{izh} + R_{Cu} + R_{vh}$$



$$H(\omega)\cdot\sqrt{G} = \frac{\Sigma R}{\Sigma R + j\omega L + \dfrac{1}{j\omega C}} \approx \frac{1}{1 + j2Q_L\dfrac{\Delta\omega}{\omega_0}}$$

$$Q_L = \frac{\omega_0 L}{\Sigma R}$$

$$U_{Nizh} \approx U_{Nvh}\cdot\left(1 + \frac{\omega_0}{j2Q_L\Delta\omega}\right)$$

$$P_{Nizh} \approx P_{Nvh}\cdot\left[1 + \left(\frac{\omega_0}{2Q_L\Delta\omega}\right)^2\right]$$

Amplitudni in fazni šum

$$P_{Nizh} \approx P_{Nvh}\cdot\left[1 + \left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]$$

25 – Fazni šum oscilatorja

26 – Leesonova enačba

$P_0 \equiv moč \;\; nosilca$

$$\frac{dP_{Nvh}}{df} = N_0 = k_B (T_R + T_S) \approx k_B T_0 F$$

Normirana spektralna gostota faznega šuma $\log L(\Delta f)$ $[\text{dBc/Hz}]$

$$L(\Delta f) = \frac{1}{P_0}\cdot\frac{dP_\varphi}{df} = \frac{1}{2}\cdot\left[1 + \left(\frac{f_0}{2 Q_L \Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1 + \frac{f_C}{|\Delta f|}\right) \quad [\text{Hz}^{-1}]$$

Velja pri $L(\Delta f)\cdot\Delta f \ll 1$

Samo fazni šum

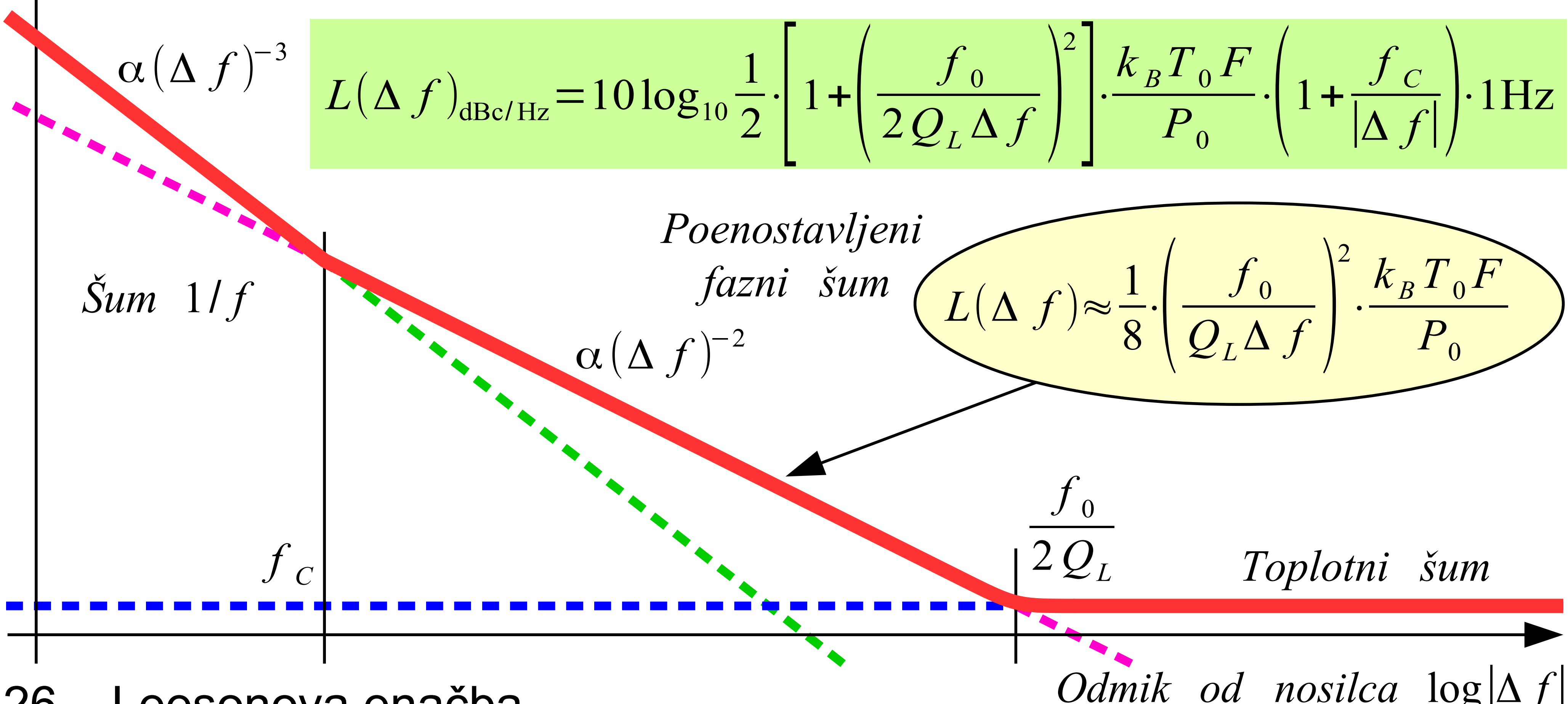
Nasičenje odstrani amplitudni šum

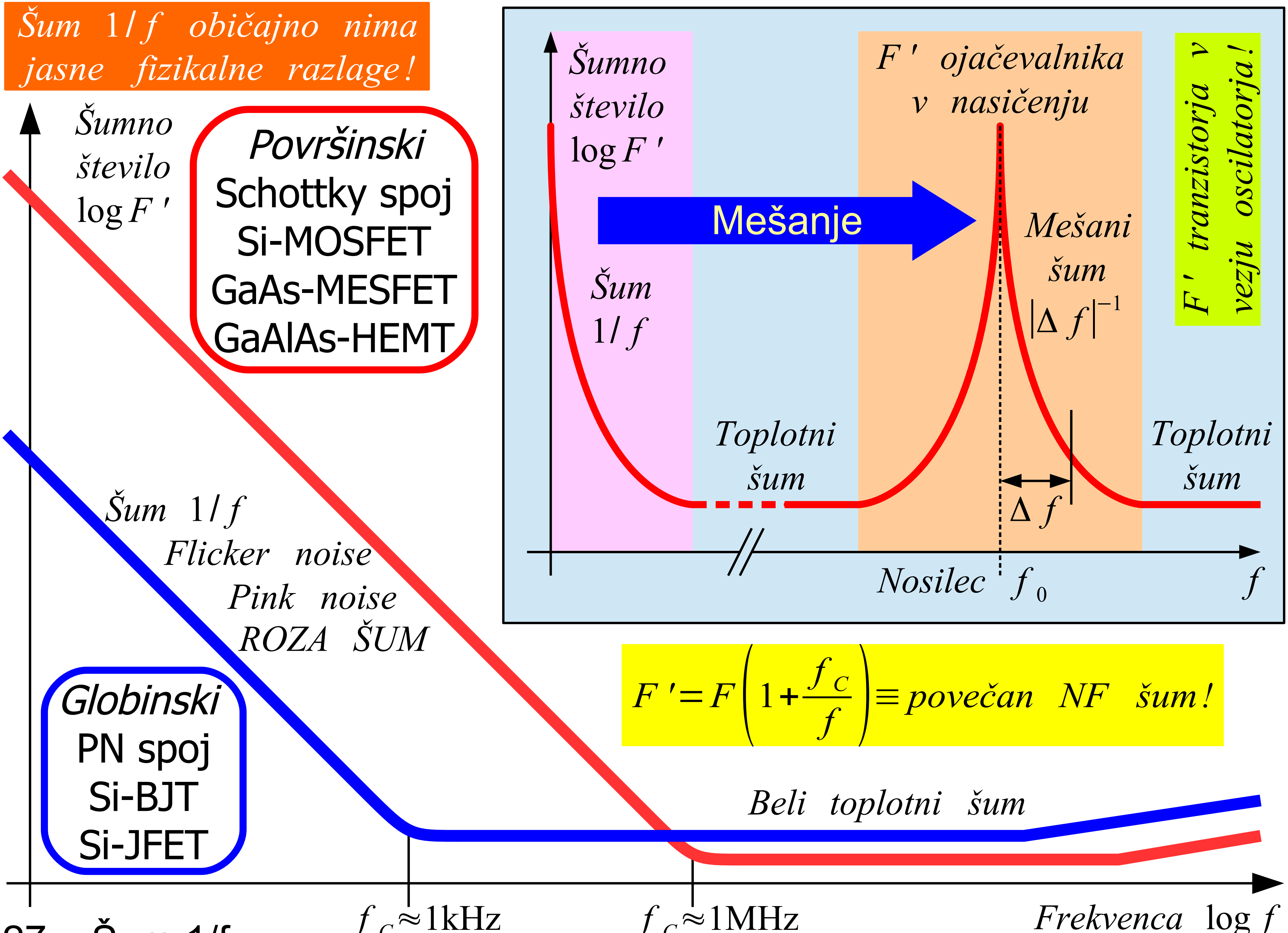
Šum 1/f

$$L(\Delta f)_{\text{dBc/Hz}} = 10\log_{10}\frac{1}{2}\cdot\left[1 + \left(\frac{f_0}{2 Q_L \Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1 + \frac{f_C}{|\Delta f|}\right)\cdot 1\text{Hz}$$

Poenostavljeni fazni šum

$$L(\Delta f) \approx \frac{1}{8}\cdot\left(\frac{f_0}{Q_L \Delta f}\right)^2\cdot\frac{k_B T_0 F}{P_0}$$




Šum 1/ f običajno nima jasne fizikalne razlage!
Šumno število log F '
Površinski
Schottky spoj
Si-MOSFET
GaAs-MESFET
GaAlAs-HEMT
Šum 1/ f
Flicker noise
Pink noise
ROZA ŠUM
Globinski
PN spoj
Si-BJT
Si-JFET
Beli toplotni šum
f C ≈1kHz
f C ≈1MHz
Frekvenca log f
Šumno število log F '
F ' ojačevalnika v nasičenju
F ' tranzistorja v vezju oscilatorja!
Mešanje
Mešani šum
|Δ f|^−1
Šum 1/ f
Toplotni šum
Toplotni šum
Δ f
Nosilec f 0
f
F ' = F (1 + f C / f) ≡ povečan NF šum!

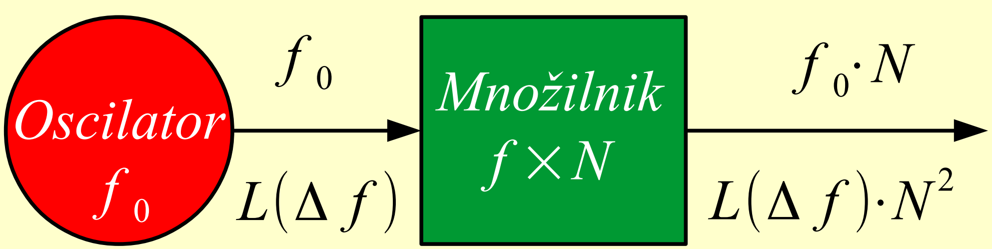
27 – Šum 1/f

Kvaliteta obremenjenega rezonatorja Q_L je ključnega pomena za fazni šum!

$$L(\Delta f)=\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)$$

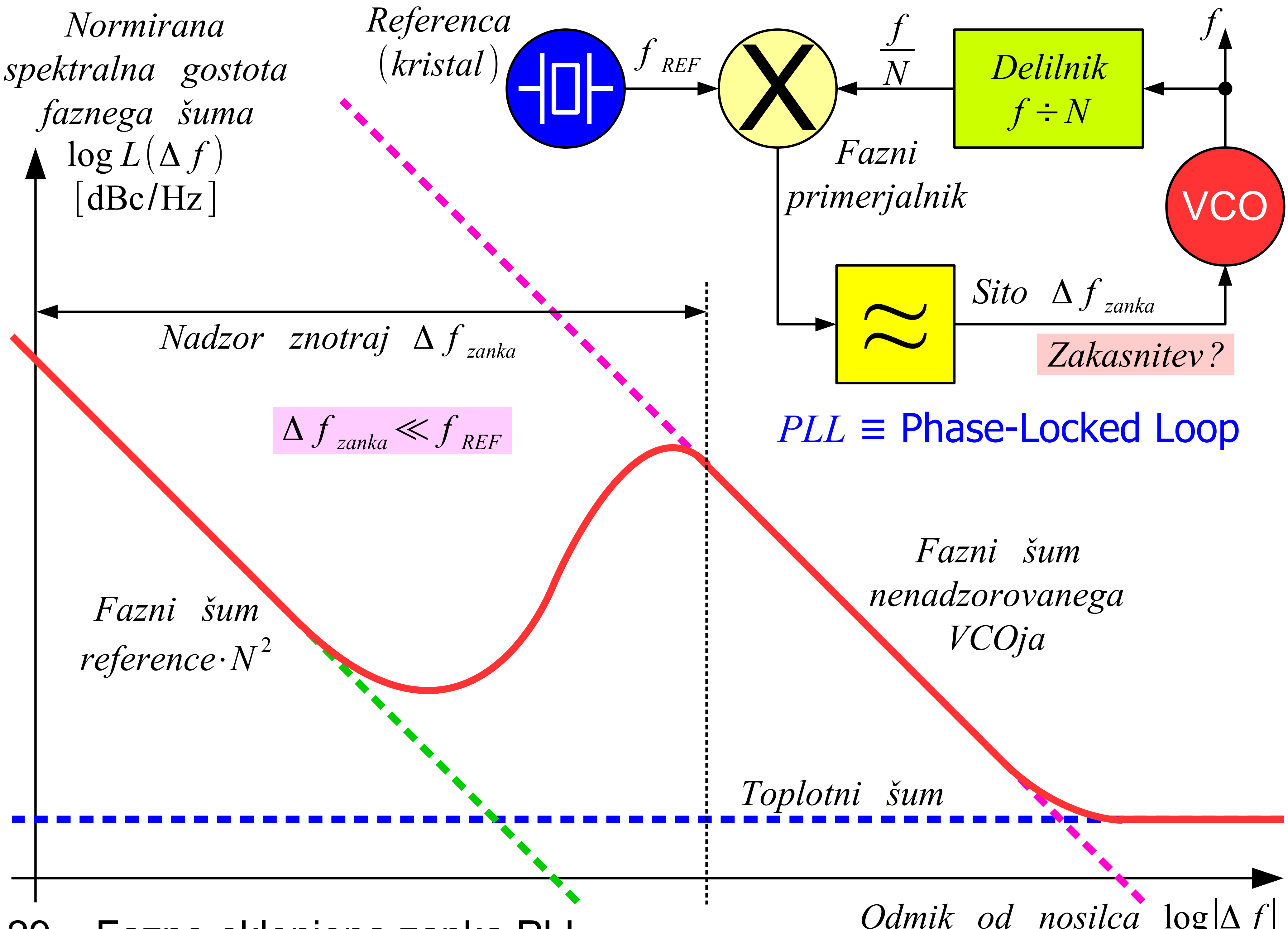
Frekvenčno nastavljivi oscilatorji	Q_L
RC VCO	~1
Cev BWO	~1
Varikap LC VCO	10↔30
YIG ($Y_3Fe_5O_{12}$) oscilator	300↔1000

Oscilatorji fiksne frekvence	Q_L
RC multivibrator	~1
LC nihajni krog	30↔100
Votlinski rezonator	1000↔3000
Keramični dielektrični rezonator	1000↔3000
AT kremenov kristal (osnovna rezonanca)	3000↔10000
AT kremenov kristal (tretji/peti overton)	10000↔30000
Elektro-optični zakasnilni vod ($)	$\sim 10^5$
Safirjev dielektrični rezonator ($$$)	$\sim 3.10^5$
Rdeč HeNe LASER	$\sim 10^8$



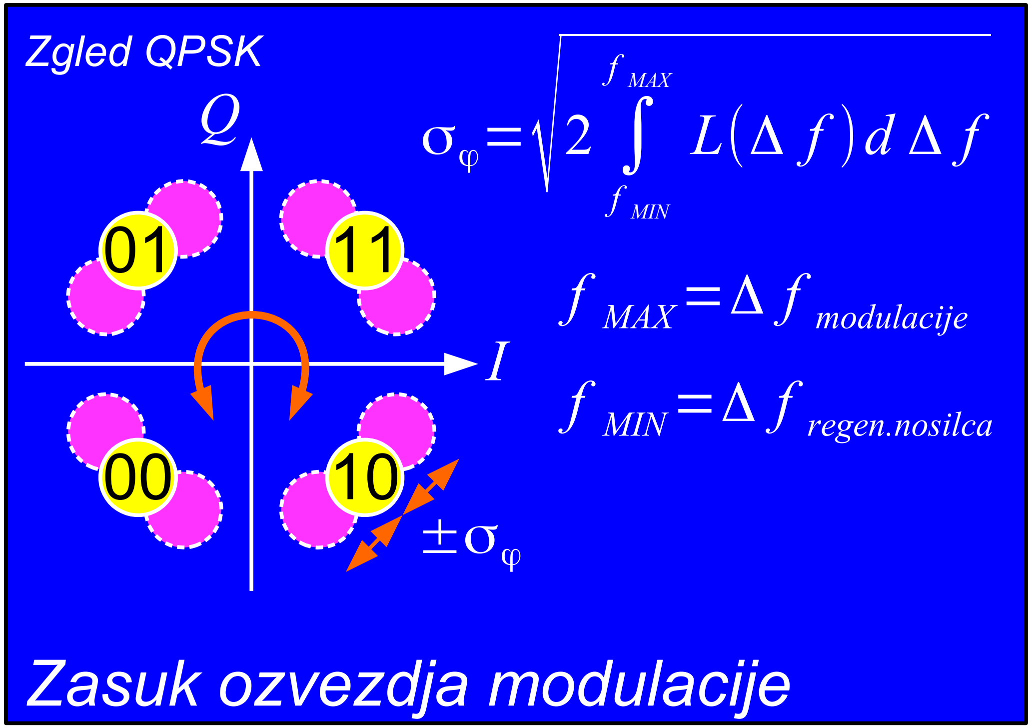
Fazni šum se množi s kvadratom množenja frekvence!

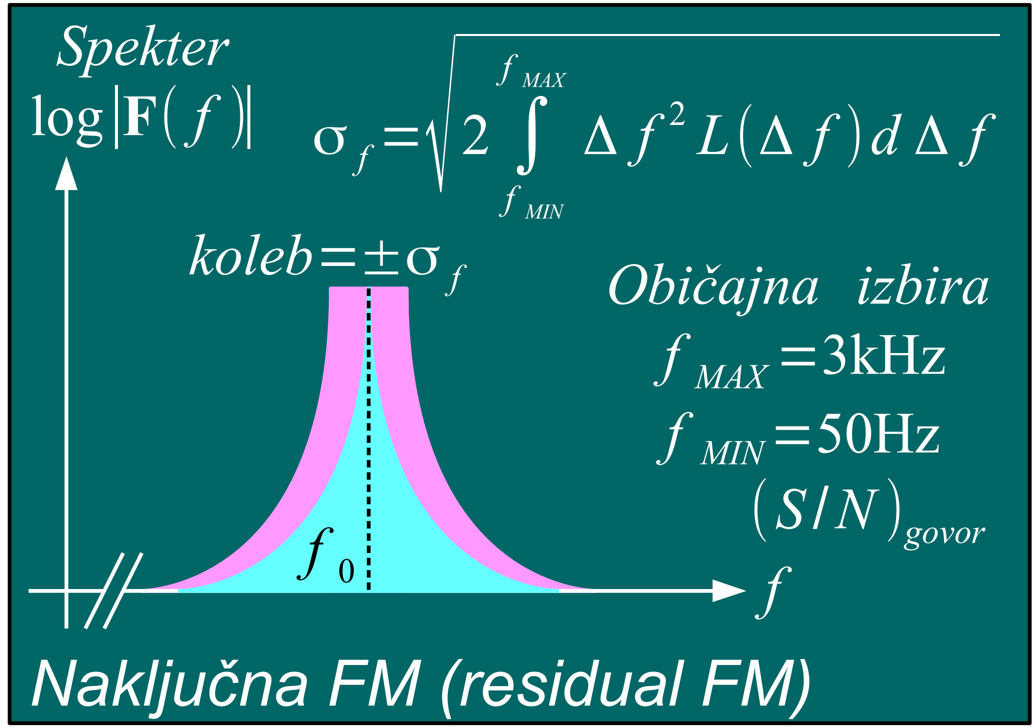
28 – Kvaliteta rezonatorja

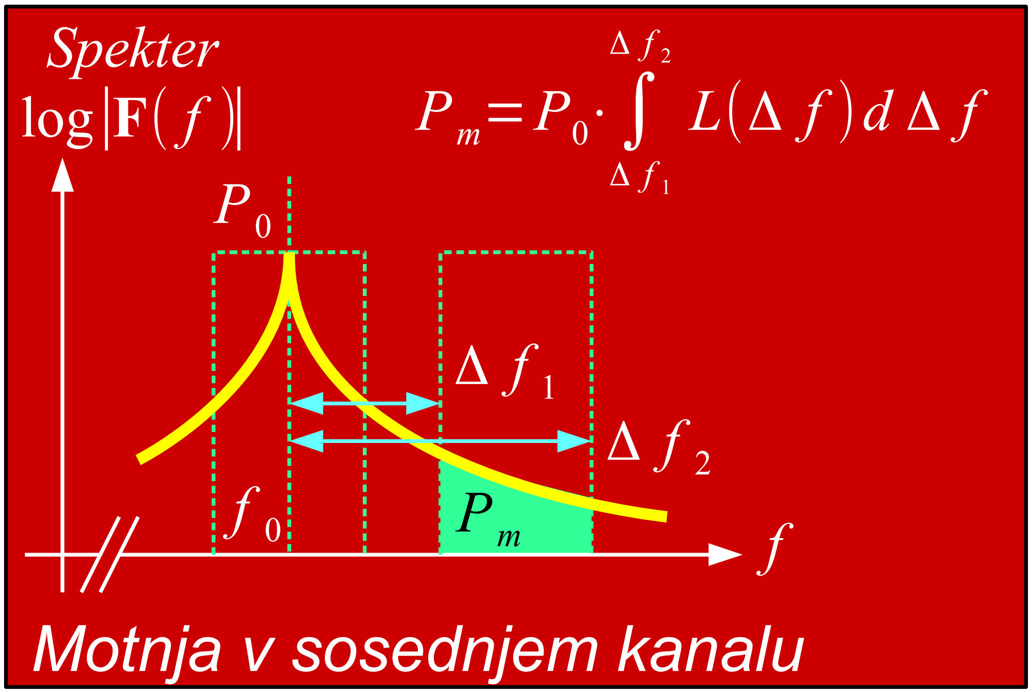


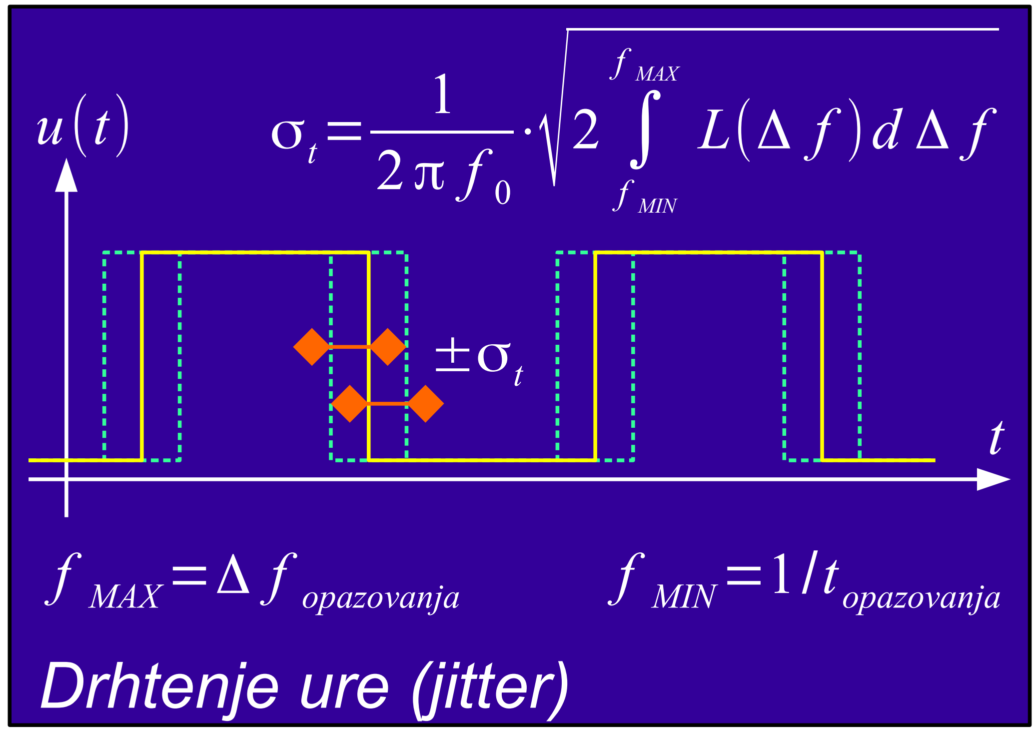
29 – Fazno-sklenjena zanka PLL

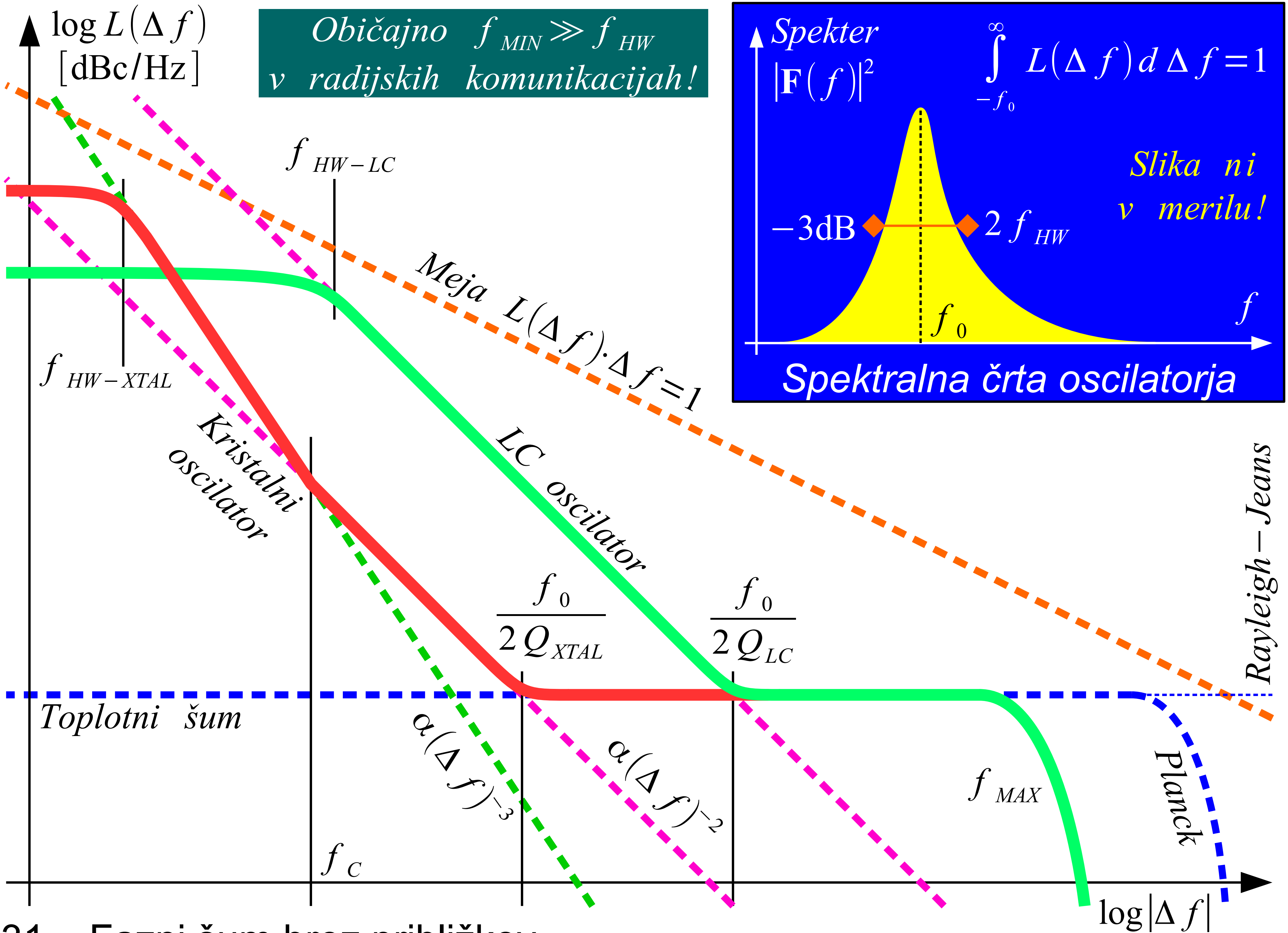
30 – Posledice faznega šuma



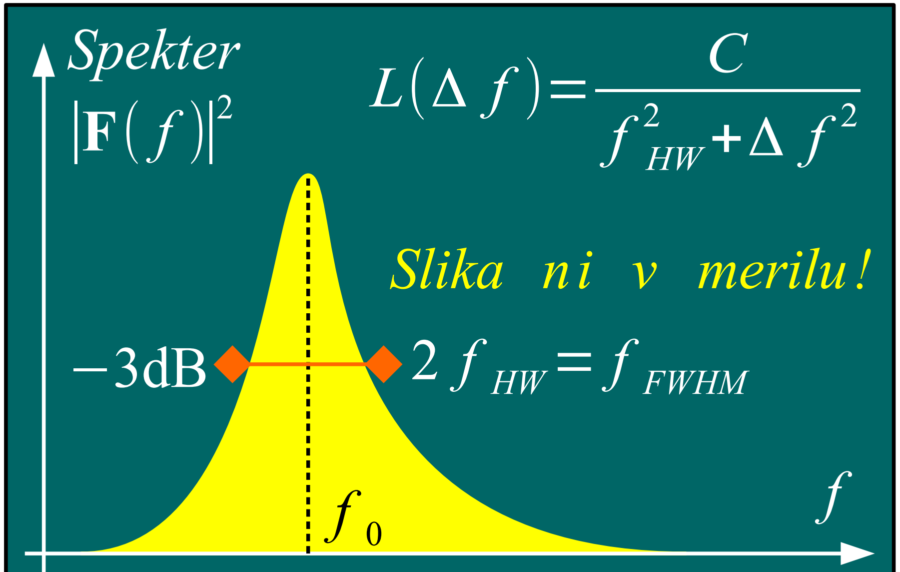








31 – Fazni šum brez približkov



Lorentzova spektralna črta

Prispevek toplotnega šuma je zanemarljiv f_{MAX} vezja oziroma Planckov zakon

Šum $1/f$ LC oscilatorja je zanemarljiv

$$L(\Delta f)=\frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{1}{f_{HW}^2+\Delta f^2}\cdot\frac{k_B T_0 F}{P_0}$$

Lorentzova črta v Leesonovi enačbi

$$\int_{-f_0}^{\infty} L(\Delta f)\,d\Delta f=1\approx\int_{-\infty}^{\infty} L(\Delta f)\,d\Delta f=\frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{k_B T_0 F}{P_0}\int_{-\infty}^{\infty}\frac{1}{f_{HW}^2+\Delta f^2}\,d\Delta f=$$

$$=\frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{k_B T_0 F}{P_0}\cdot\left[\frac{1}{f_{HW}}\cdot\arctan\frac{\Delta f}{f_{HW}}\right]_{\Delta f=-\infty}^{\Delta f=\infty}=\frac{k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{\pi}{f_{HW}}$$

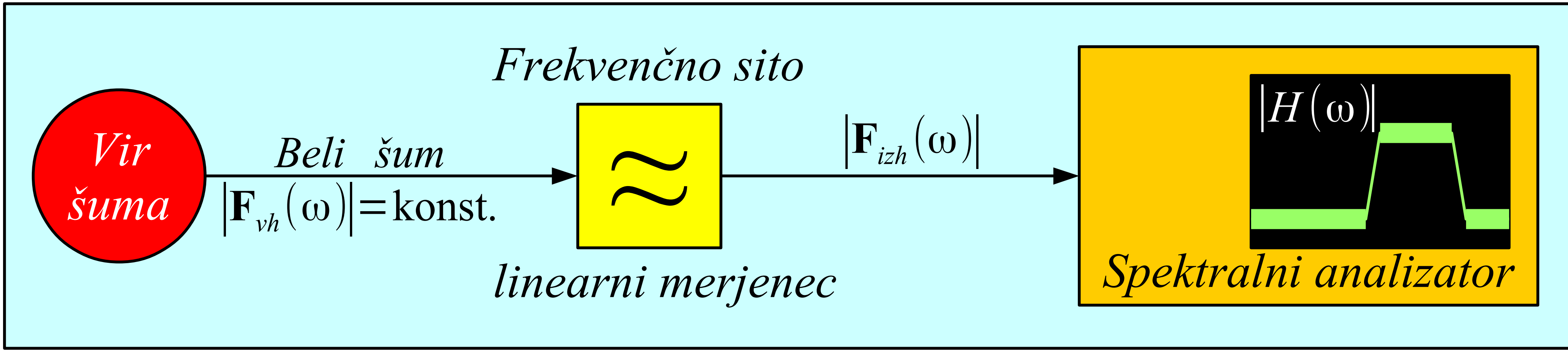
$$f_{HW}=\frac{\pi k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2$$

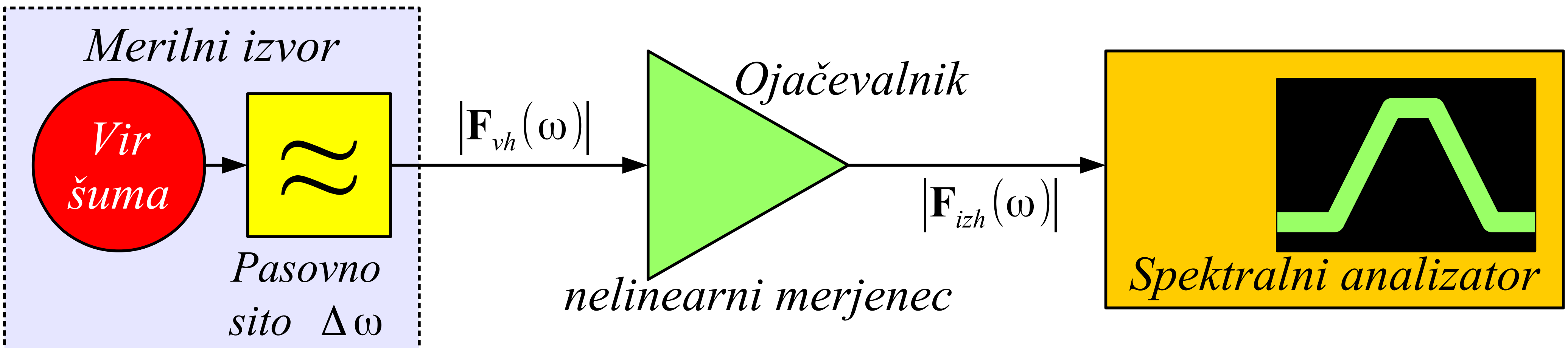
Zgled $f_0=3\text{GHz}$ $Q_L=10$
$P_0=0.1\text{mW}$ $F=10\text{dB}$
$f_{HW}=14\text{Hz}$ $f_{FWHM}=28\text{Hz}$

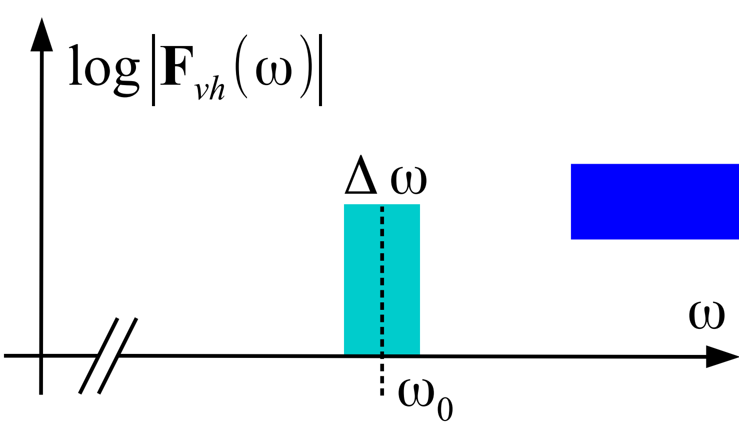
$$C=\frac{k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2=\frac{f_{HW}}{\pi}$$

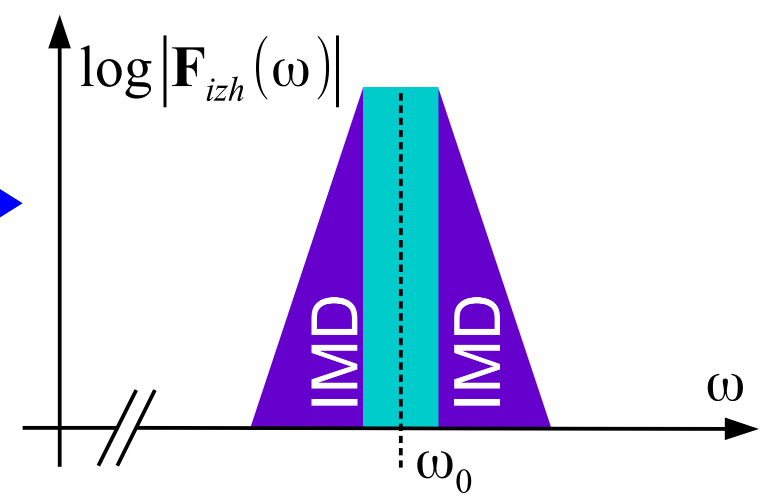
$$L(\Delta f)=\frac{f_{HW}/\pi}{f_{HW}^2+\Delta f^2}$$

32 – Širina Lorentzove spektralne črte

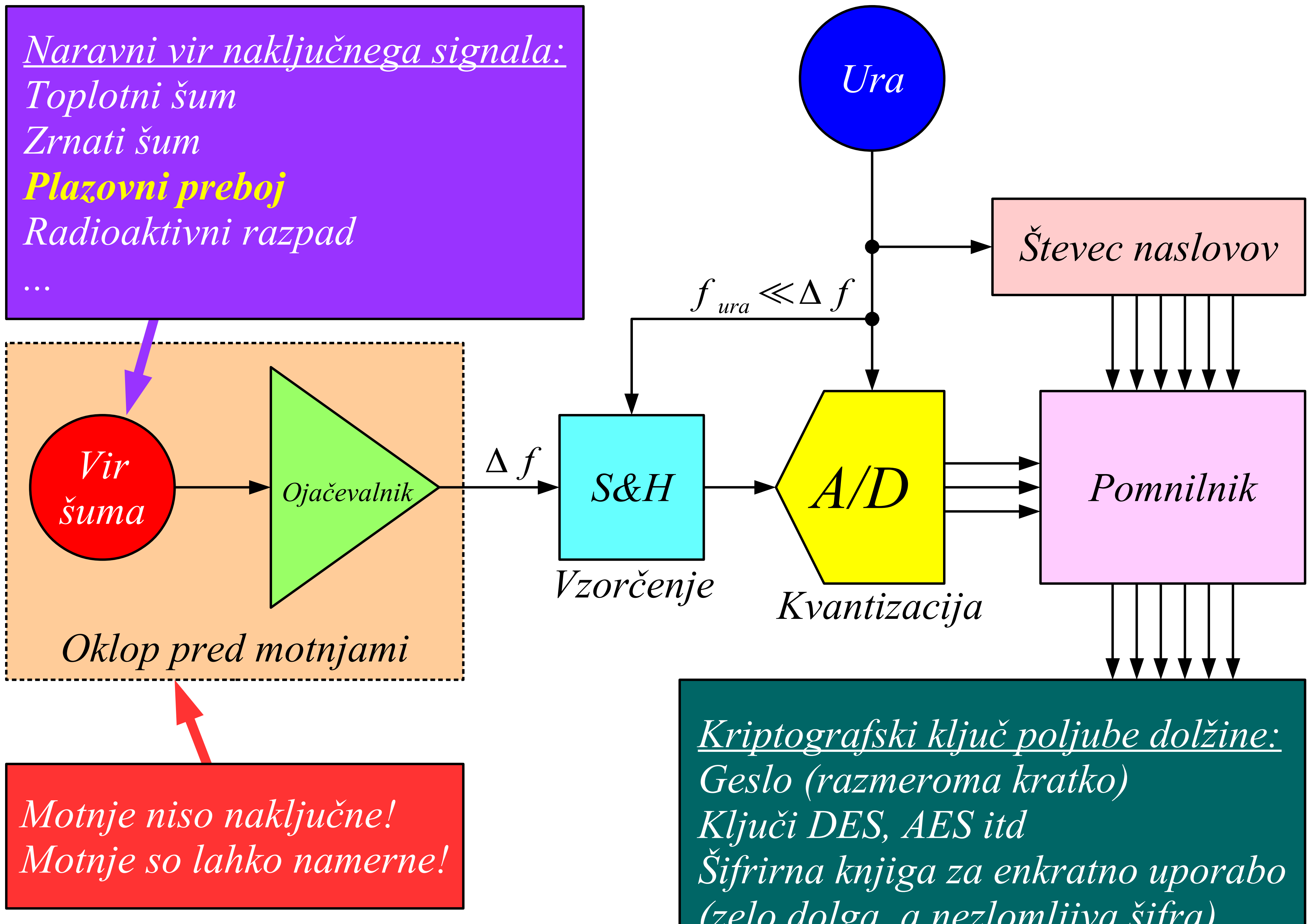




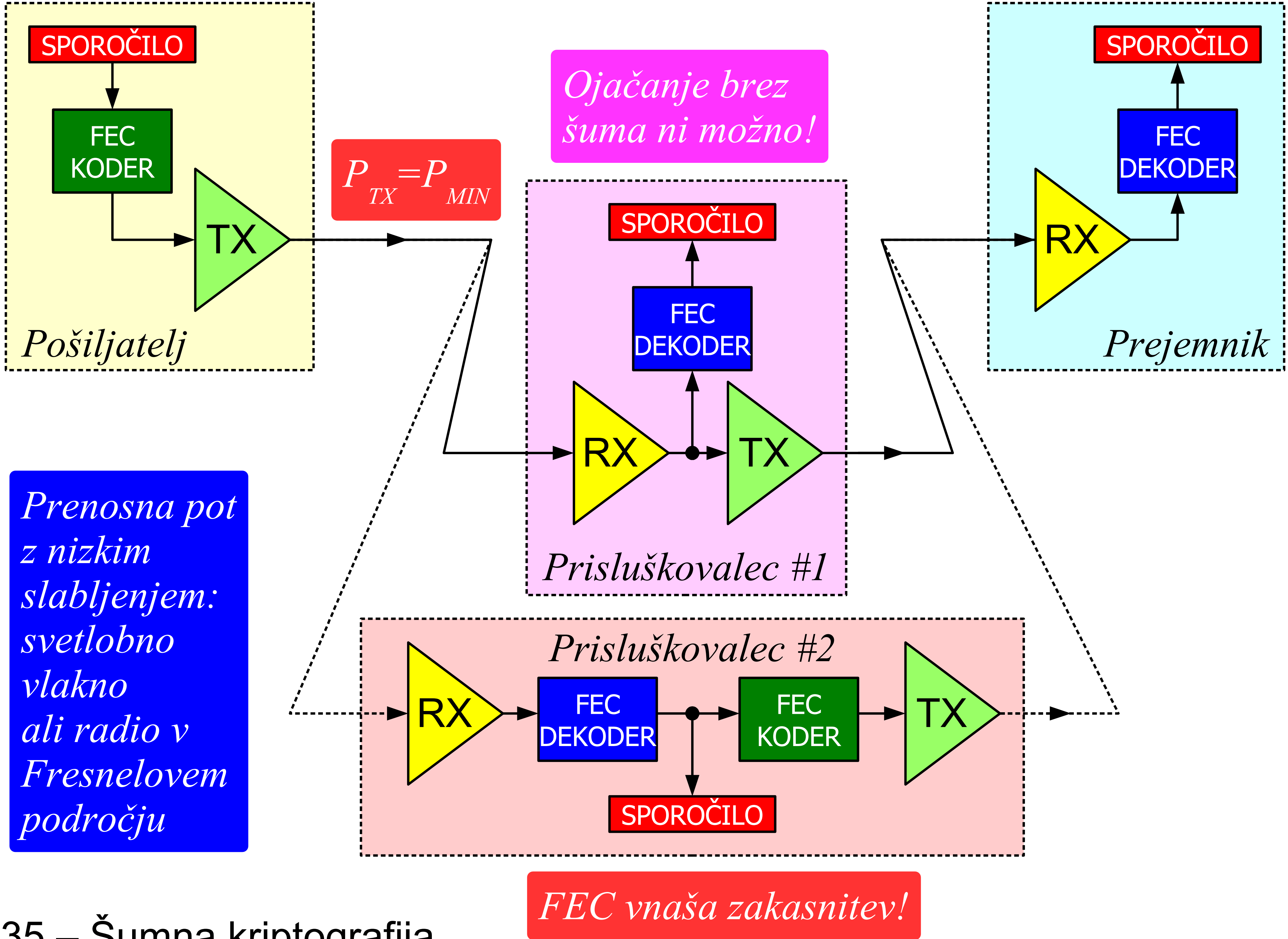




33 – Šum kot merilni signal

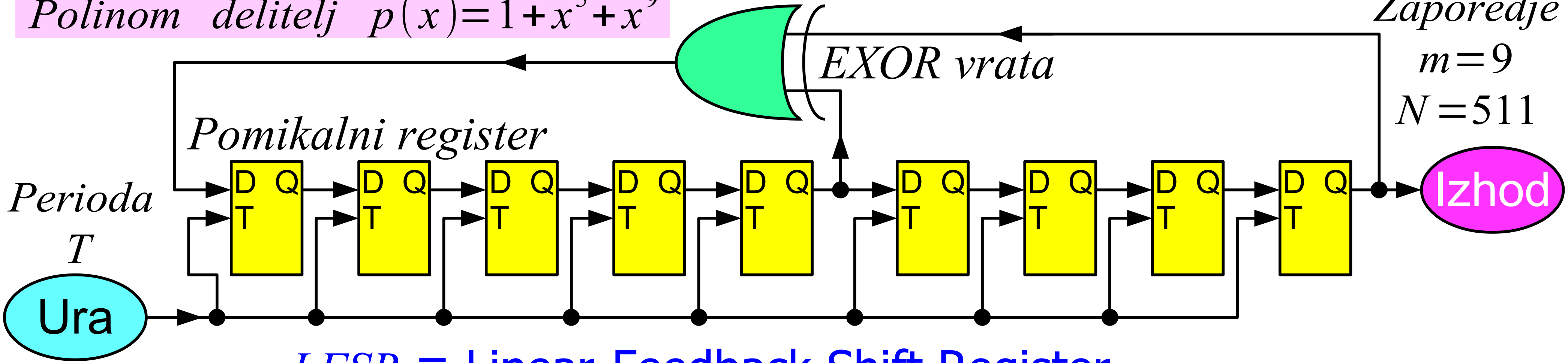


34 – Vir kriptografskega ključa



35 – Šumna kriptografija

Polinom delitelj $p(x)=1+x^5+x^9$



LFSR ≡ Linear-Feedback Shift Register

Nerazcepni polinom $p(x)=1+x^l+x^m$ → *zaporedje dolžine max* $N=2^m-1$

2^{m-1} enic in $2^{m-1}-1$ ničel razporejenih v skupine

1X m enic, m-1 ničel

1X m-2 enic in ničel

2X m-3 enic in ničel

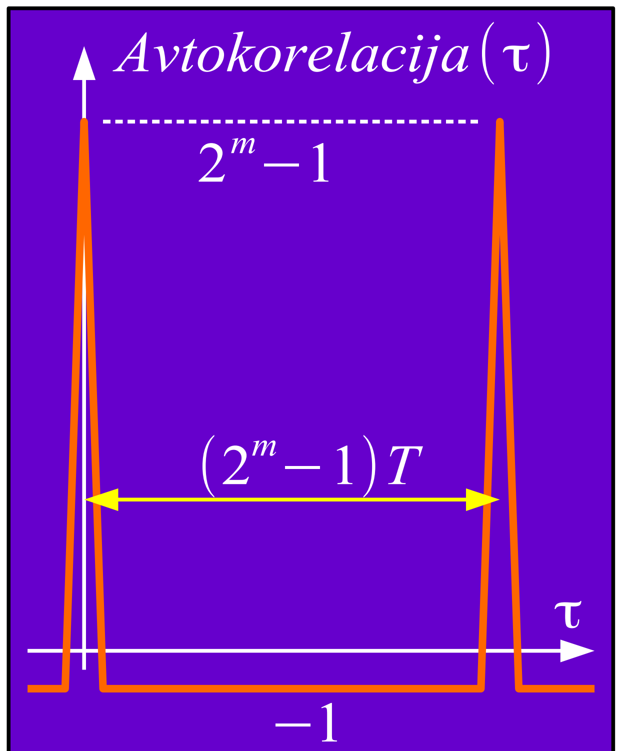
4X m-4 enic in ničel

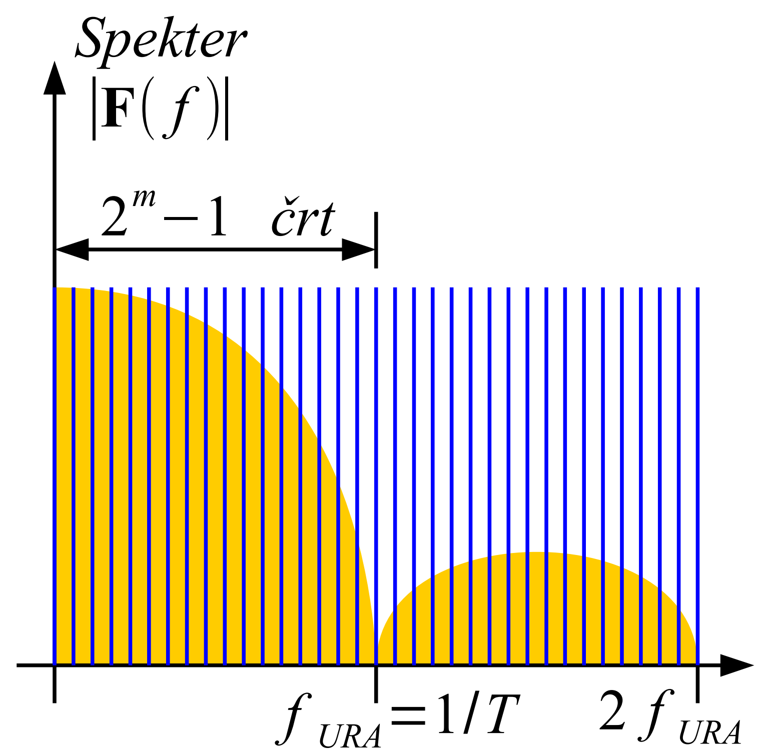
....

2^{m-5} skupin 111 in 000

2^{m-4} skupin 11 in 00

2^{m-3} posamičnih 1 in 0





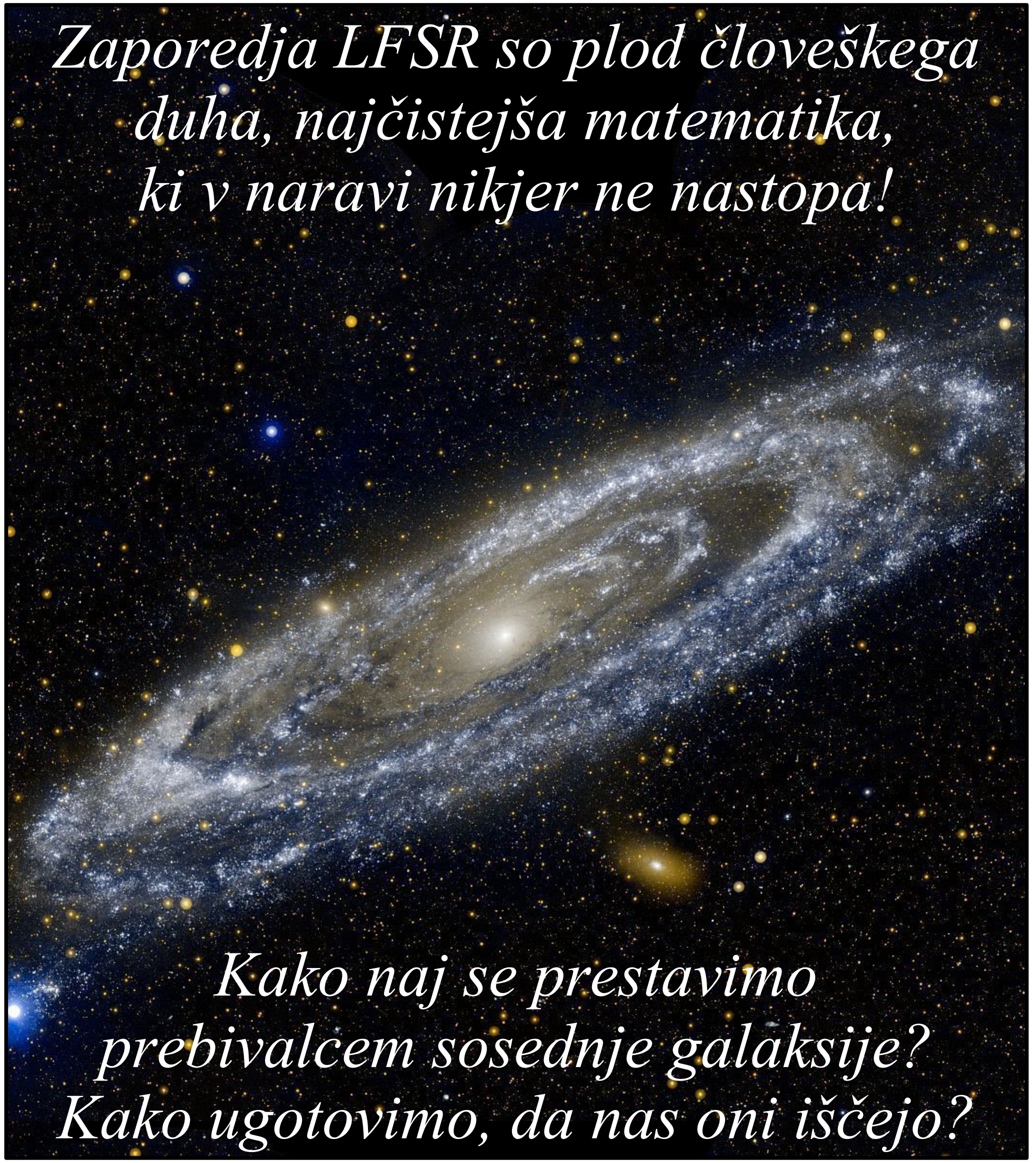
Sliši in vidi se kot beli šum!

36 – Psevdonaključna zaporedja LFSR

Avtokorelacija ima dve vrednosti z enim samim izrazitim vrhom:
- zaporedja za sinhronizacijsko glavo podatkovnih okvirjev
- razširitvena zaporedja v CDMA
- natančen prenos časa v radionavigaciji (GPS, GLONASS)

Psevdonaključna zaporedja LFSR nimajo kriptološke vrednosti: algoritem Berlekamp-Massey 1969

Brezhiben spekter enako velikih, enakomerno razmaknjenih črt ter preprosto proizvajanje/preverjanje:
- preizkusni podatki za vse vrste zvez v telekomunikacijah
- skrambliranje (randomization) podatkov kot linijsko kodiranje



Razmerje vršna moč / povprečje:

$$LFSR: \frac{P_{MAX}}{\langle P \rangle} \approx 1 \qquad Šum: \frac{P_{MAX}}{\langle P \rangle} \rightarrow \infty$$

37 – Uporaba psevdonaključnih zaporedij