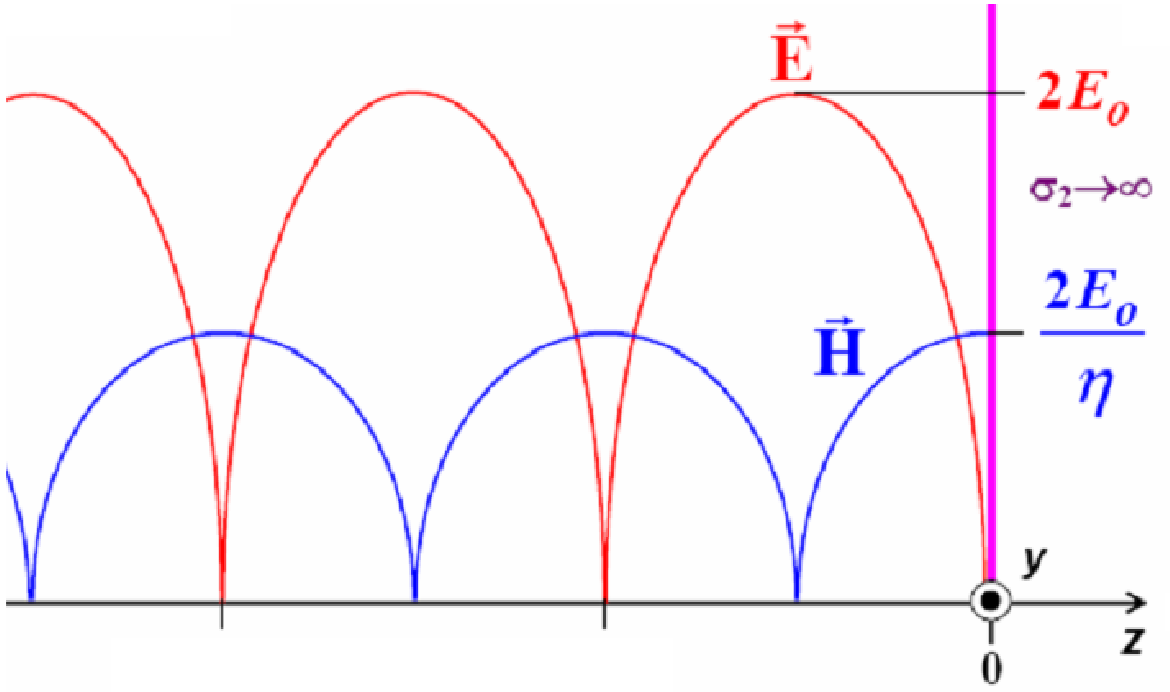
Valovi in odbojnost



Vpad in odboj planega vala proti oviri.
Stojni val električnega in magnetnega polja v prostoru pred kovinsko steno.

Mobitel d.d., izobraževanje

9. 10. 2009, predavanje 23

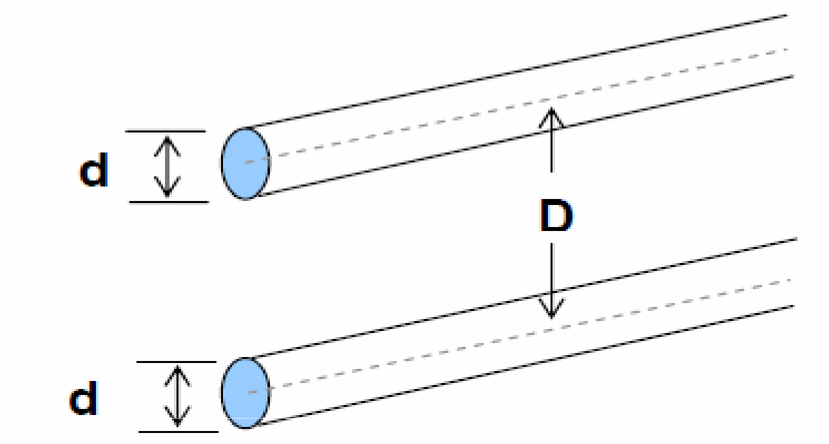
Prof. dr. Jožko Budin

Vsebina

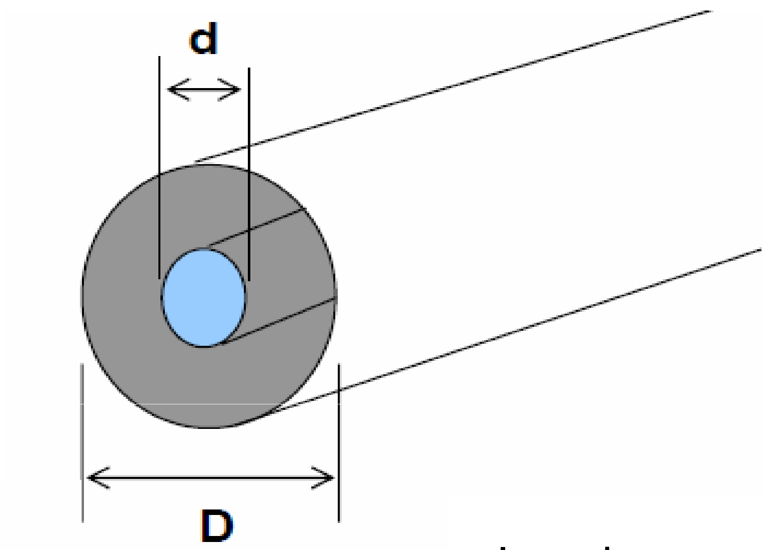
1. Vpad, odboj in prenos vala
2. Prenosni medij s porazdeljenimi elementi
3. Karakteristična impedanca Z_k in prenosna konstanta γ
4. Potujoči in stojni valovi napetosti in toka
5. Odbojnost (Γ) in neubranost (SWR)
6. Kazalčni diagram napetosti in toka
7. Potek amplitude in faze napetosti in toka
8. Potek odbojnosti in impedance
9. Impedanca kratko staknjene in odprte linije
10. Prehodni pojav pri vklopu generatorja na linijo

Prenosni vodi

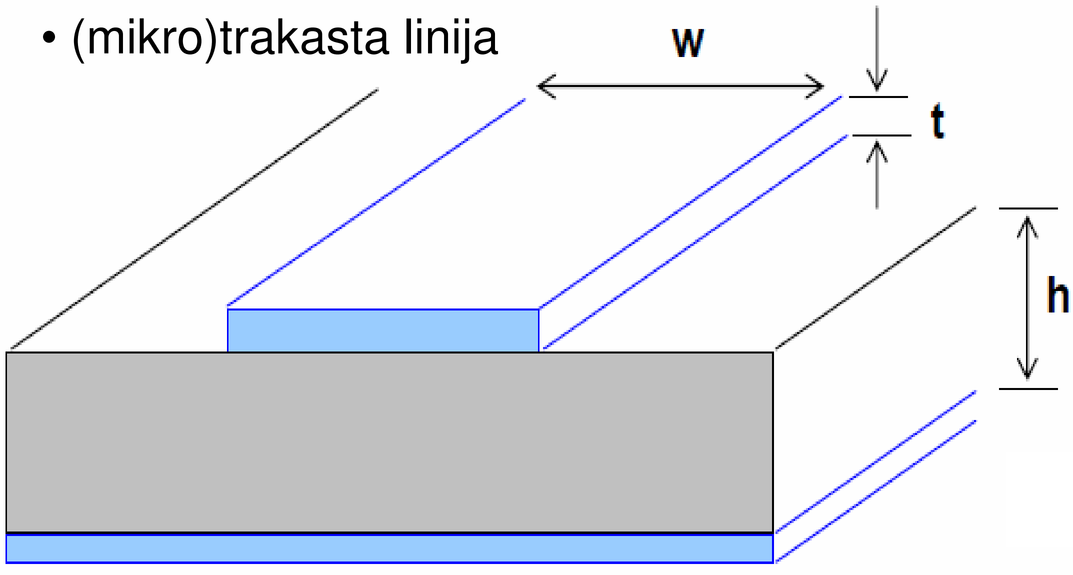
• dvovod



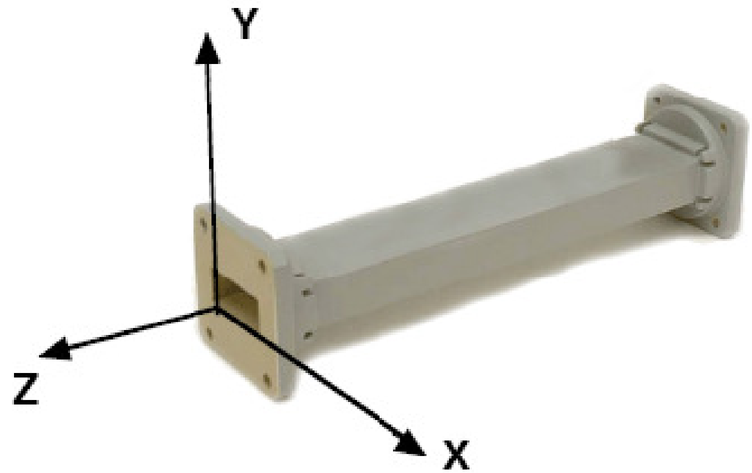
• koaksialni vod



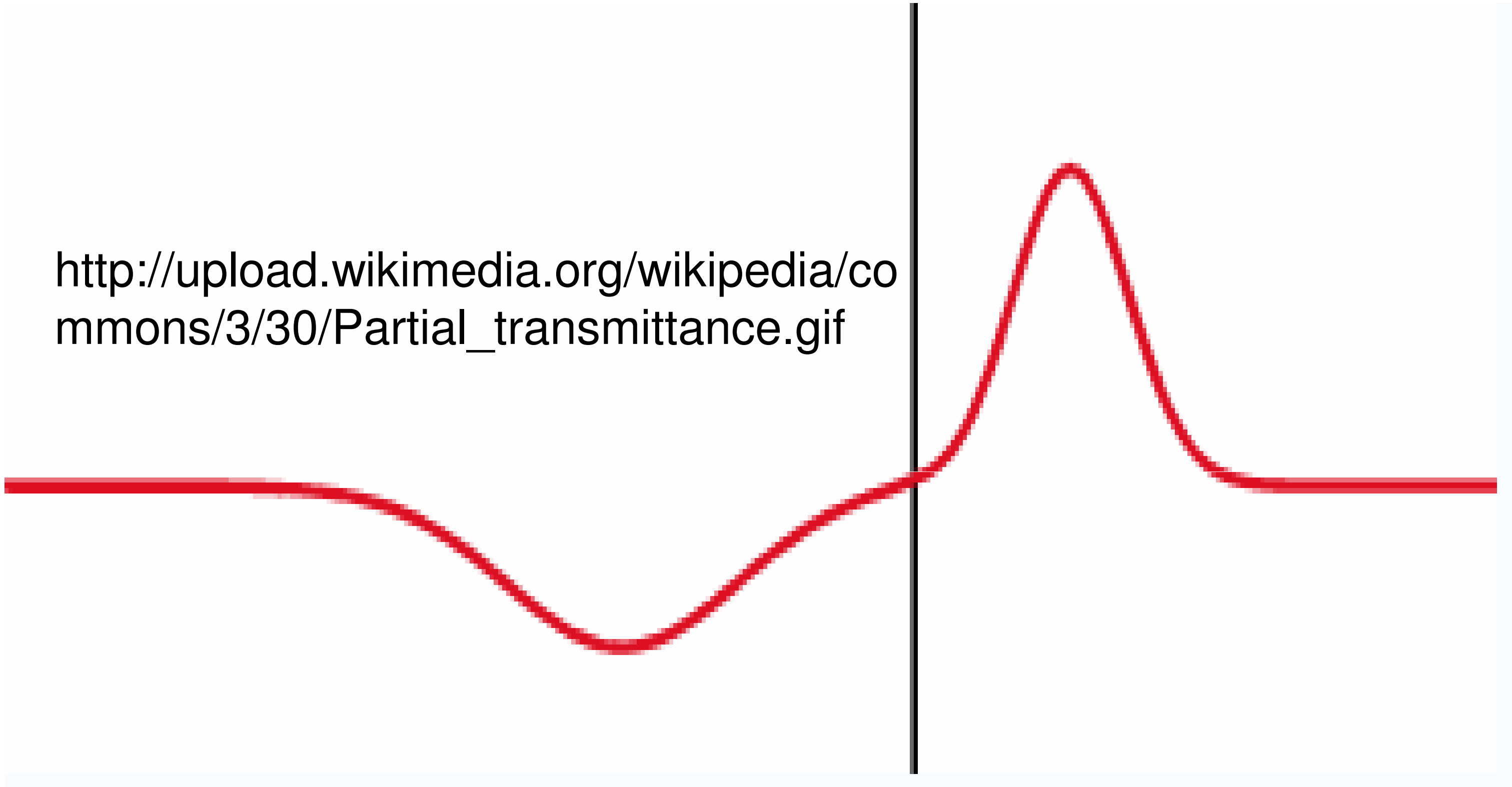
• (mikro)trakasta linija



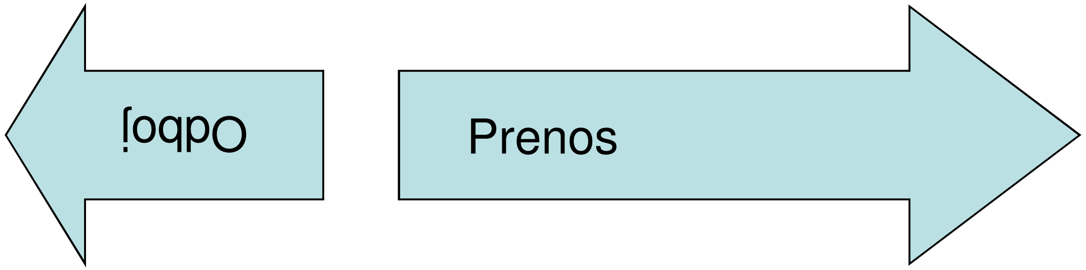
• valovod



Animacija odboja in prenosa



http://upload.wikimedia.org/wikipedia/commons/3/30/Partial_transmittance.gif



Vpad, odboj, prenos elmg. vala

- Mejni pogoj za električno in magnetno polje in gostoto moči na mestu diskontinuitete:

vpad $\mathbf{E}^{vp}$, $\mathbf{H}^{vp}$

$|E^{vp}|^2/2Z_0$

prenos $(1+\Gamma)\mathbf{E}^{vp}$, $(1-\Gamma)\mathbf{H}^{vp}$

$(1-|\Gamma^2|)\,|E^{vp}|^2/2Z_0$

odboj $\Gamma\mathbf{E}^{vp}$, $-\Gamma H^{vp}$

$|\Gamma|^2|E^{vp}|^2/2Z_0$

Vsota električnega polja vpadnega in odbitega vala je enaka električnemu polju prevedenega vala.

Razlika moči vpadnega in odbitega vala je enaka moči prevedenega vala

Mejni pogoj na koncu linije - odboj

Nastanek odbitega vala na koncu linije je posledica zveznega prehoda napetosti na koncu linije. Breme impedance Z_b vsiljuje razmerje med napetostjo in tokom $U/I = Z_b$. Vpadni val, ki ima razmerje $U^{vp}/I^{vp} = Z_k$, tega pogoja ne zadovoljuje. Zato se mu pridruži odbiti val, ki je sorazmeren vpadnemu valu $U^{odb} = \Gamma\ U^{vp}$ oz. $I^{odb} = -\ \Gamma\ I^{vp}$ tako, da napetost in tok prehajata zvezno:

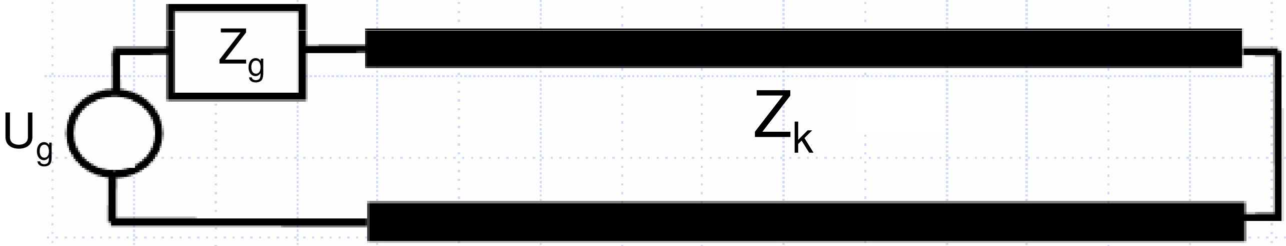
$$U/I = U^{vp}(1 + \Gamma)/I^{vp}(1 - \Gamma) = Z_k(1 + \Gamma)/(1 - \Gamma) = Z_b$$

Tri značilne zaključitve linije

Prilagojena linija

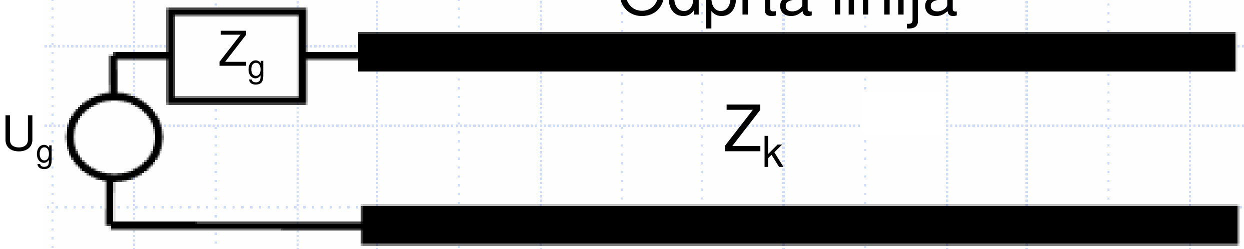
$$\Gamma = \frac{Z_k - Z_k}{Z_k + Z_k} = 0$$

Kratkostaknjena linija



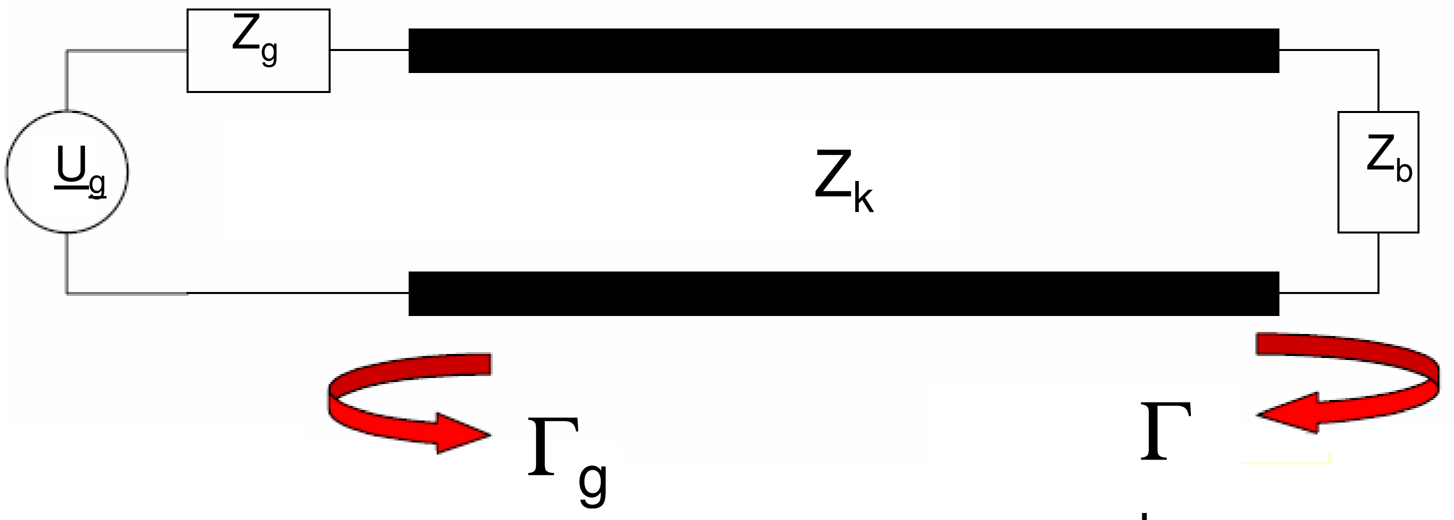
$$\Gamma = \frac{0 - Z_k}{0 + Z_k} = -1$$

Odprta linija



$$\Gamma = \frac{\infty - Z_k}{\infty + Z_k} = 1$$

Odboj na generatorju in odboj na



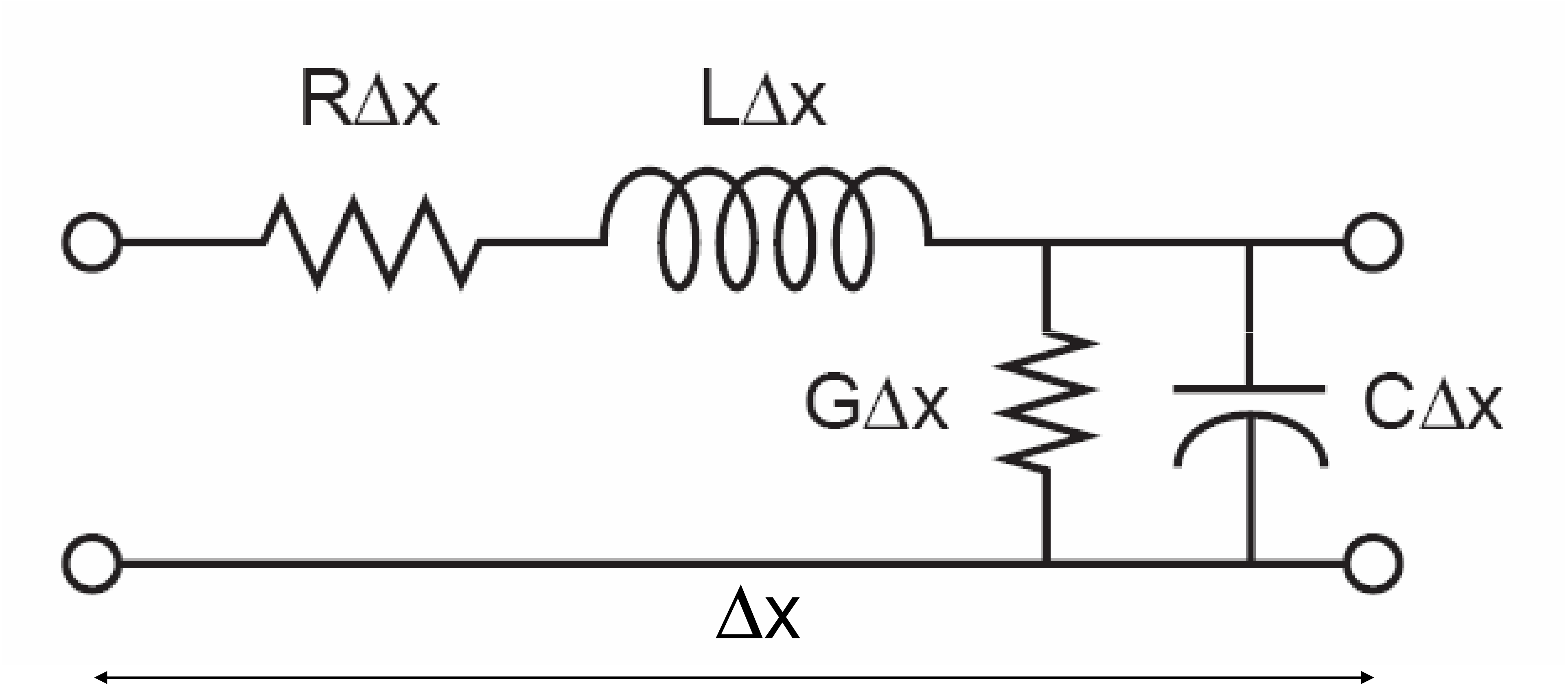
Vpadni val se odbije od bremena, odbiti val se odbije od generatorja. Oslabitev prenosa moči nastaja v enaki meri zaradi neprilagojenosti na obeh koncih linije.

$$\Gamma = \frac{Z_b - Z_k}{Z_b + Z_k} \qquad T_b = 1 + \Gamma_b = \frac{2Z_b}{Z_b + Z_k}$$

Impedanca, odboj in prenos so odvisni od frekvence

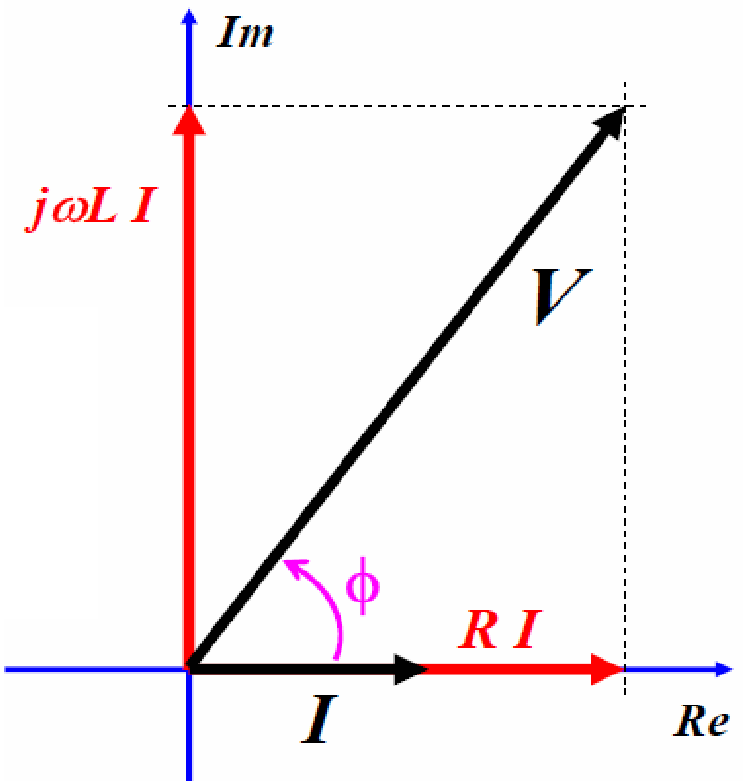
Porazdeljeni elementi

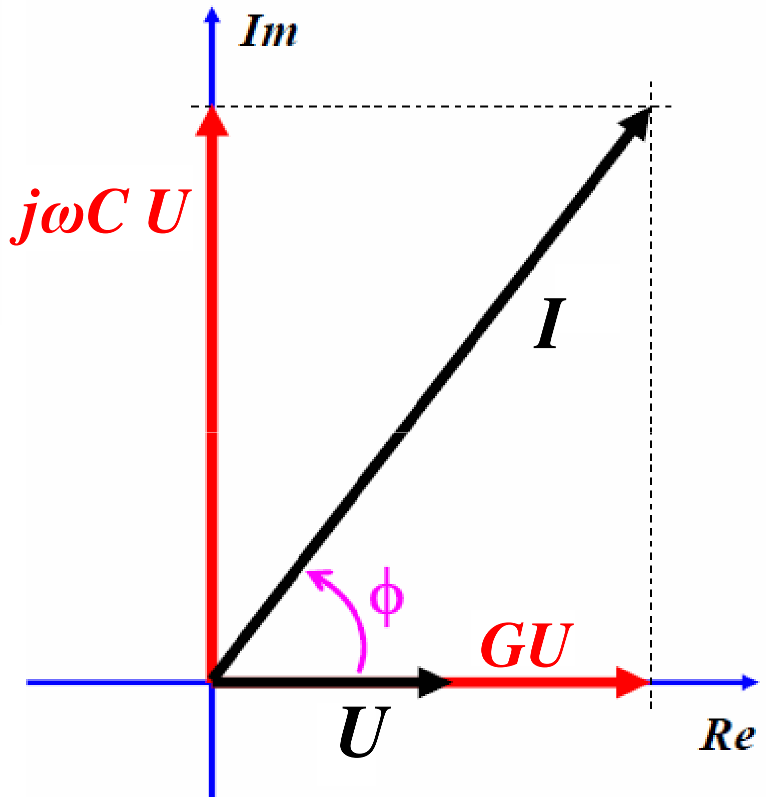
• Vzdolžni R, vzdolžni L; Prečni G, prečni C



Metamateriali: vzdolžni C in prečni L !!

Kompleksorji





$$V = Z\,I = R\,I + j\omega L\,I$$

$$I = GU + j\omega C\,U$$

Valovna enačba napetosti in toka

$$\frac{\mathrm{d}I}{\mathrm{d}z} = -(j\omega C + G)V$$

Diferencialna enačba napetosti:

$$\frac{\mathrm{d}^2 V}{\mathrm{d}z^2} = -(j\omega L + R)\frac{\mathrm{d}I}{\mathrm{d}z} = (j\omega L + R)(j\omega C + G)V$$

$$\frac{\mathrm{d}^2 I}{\mathrm{d}z^2} = -(j\omega C + G)\frac{\mathrm{d}V}{\mathrm{d}z} = (j\omega C + G)(j\omega L + R)I$$

Diferencialna enačba toka

$$\frac{\mathrm{d}V}{\mathrm{d}z} = -(j\omega L + R)I$$

Napetostni in tokovni potujoči valovi na liniji z izgubami

- Linija z izgubami $\gamma = \alpha + j\,\beta$
- Vpadni val se širi v smeri osi z, odbiti val v smeri osi -z

$$V(z) = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z}$$

Stojni val — Vpadni val — Odbiti val

$$I(z) = \frac{1}{Z_0}\left(V^{+}e^{-\gamma z} - V^{-}e^{\gamma z}\right)$$

Stojni val — Vpadni val — Odbiti val

Odbiti val toka je v protifazi z odbitim valom napetosti

Konstanta širjenja

- Linija z izgubami, γ brez zanemaritev:

$$\gamma = \sqrt{(j\omega L + R)(j\omega C + G)}$$

$$= \sqrt{j\omega L \; j\omega C \left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{1 + \frac{R}{j\omega L} + \frac{G}{j\omega C} - \frac{RG}{\omega^2 LC}}$$

Konstanta slabljenja in fazna konstanta

- Linija z izgubami

$$\gamma = \sqrt{(j\omega L + R)(j\omega C + G)} = \alpha + j\beta$$

Konstanta slabljenja α (realni del) in fazna konstanta β (imaginarni del) sta:

$$\alpha = \frac{1}{\sqrt{2}}\sqrt{RG - \omega^2 LC + [(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]^{1/2}}$$

$$\beta = \frac{1}{\sqrt{2}}\sqrt{-RG - \omega^2 LC + [(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]^{1/2}}$$

Karakteristična impedanca in fazna hitrost

- Linija z zanemarljivimi izgubami, R << ωL, G << ωC

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{L}{C}}$$

$$v_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{LC}}$$

Konstanta skabljenja in fazna konstanta

- Linija z majhnimi izgubami, $RG \ll \omega^2 LC$

$$\gamma \approx j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right]$$

$$= \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \qquad \beta = \omega\sqrt{LC}$$

Valovni pojavi na prenosnih linijah

1. Slabljenje signala na izgubnih elementih

   Konstanta slabljenja α dB/m

2. Odboj signala na snovnih ali geometrijskih

   diskontinuitetah linije

   Odbojnost Γ, prenosnost T

3. Sprememba spektra signala zaradi nelinearne

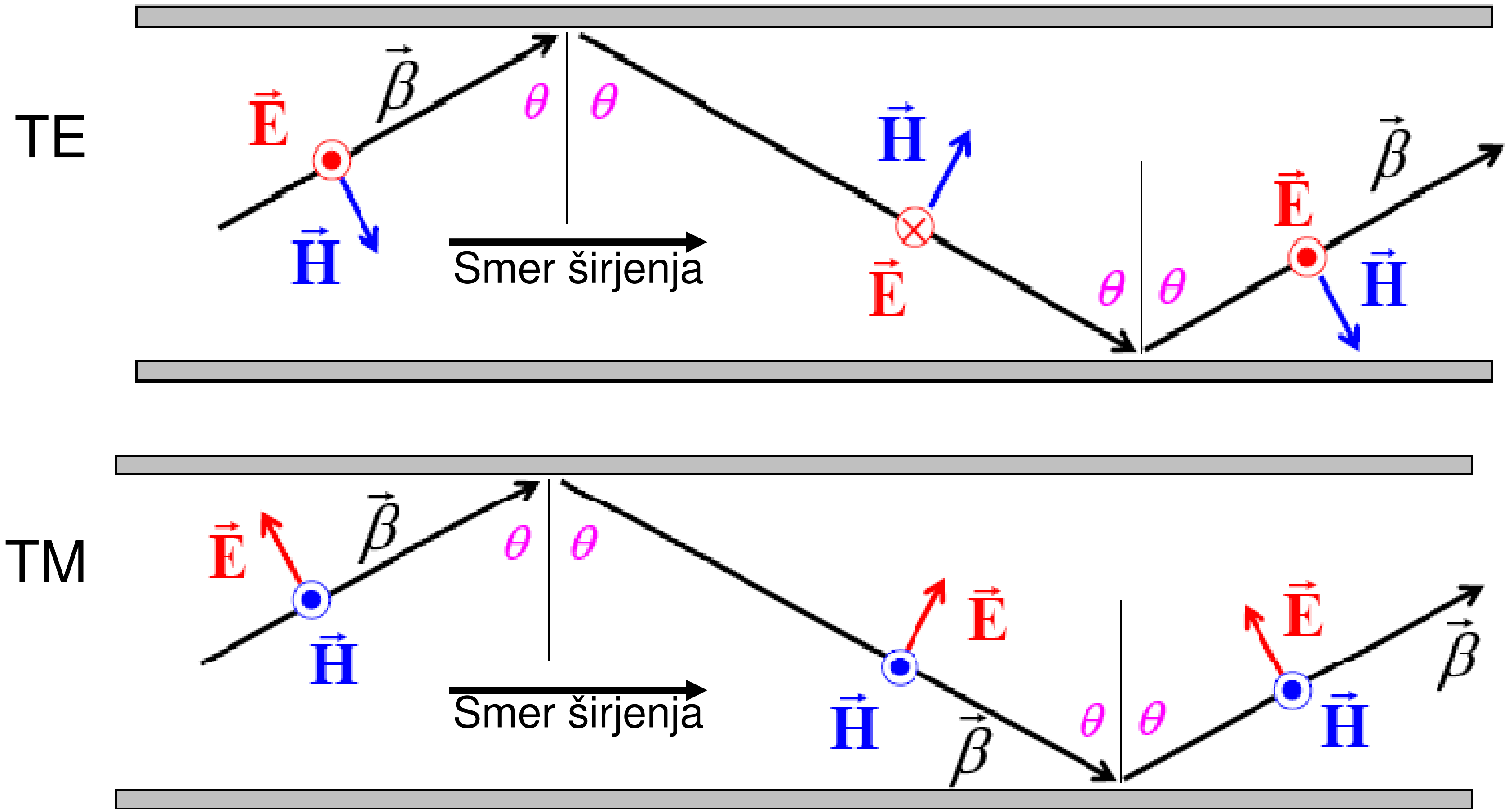
   odvisnosti fazne konstante β s frekvenco

   Razpršitev (disperzija) skupinske hitrosti

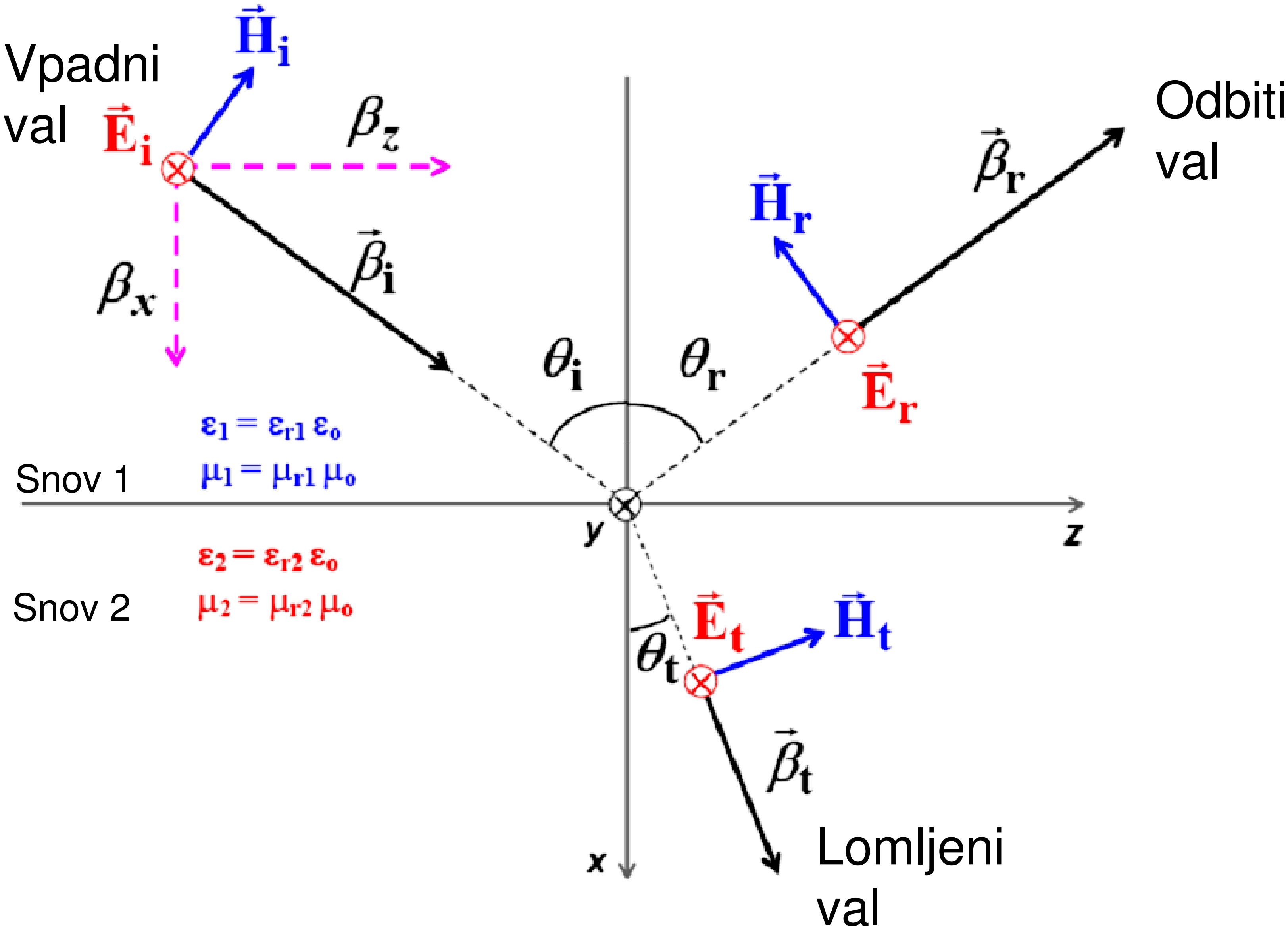
Polje TE in TM v ploščnem valovodu

TE: Električno polje vzporedno s planparalelnimi ploščami, nima komponente v smeri širjenja

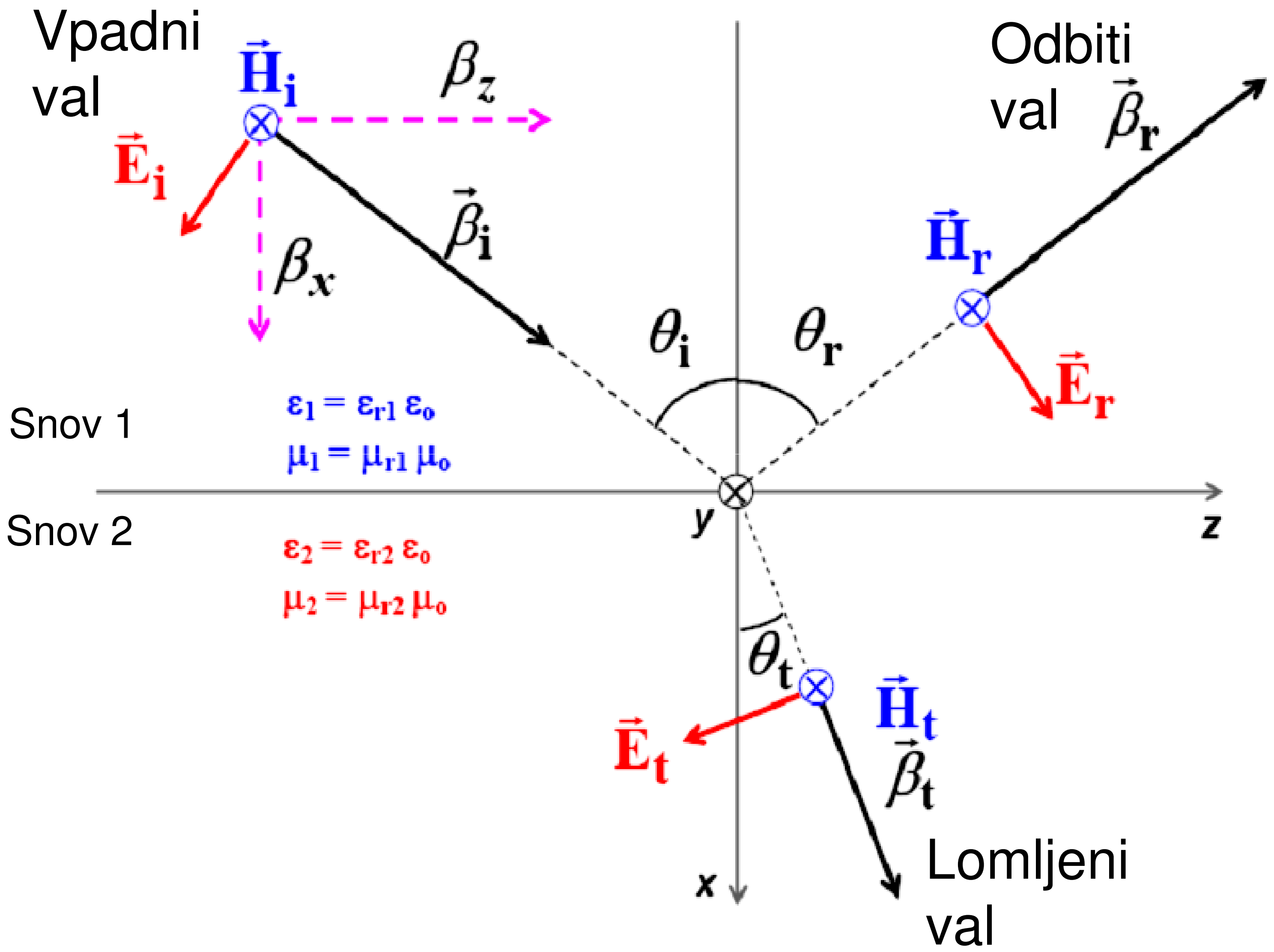
TM: Magnetno polje vzporedno s planparalelnimi ploščami, nima komponente v smeri širjenja



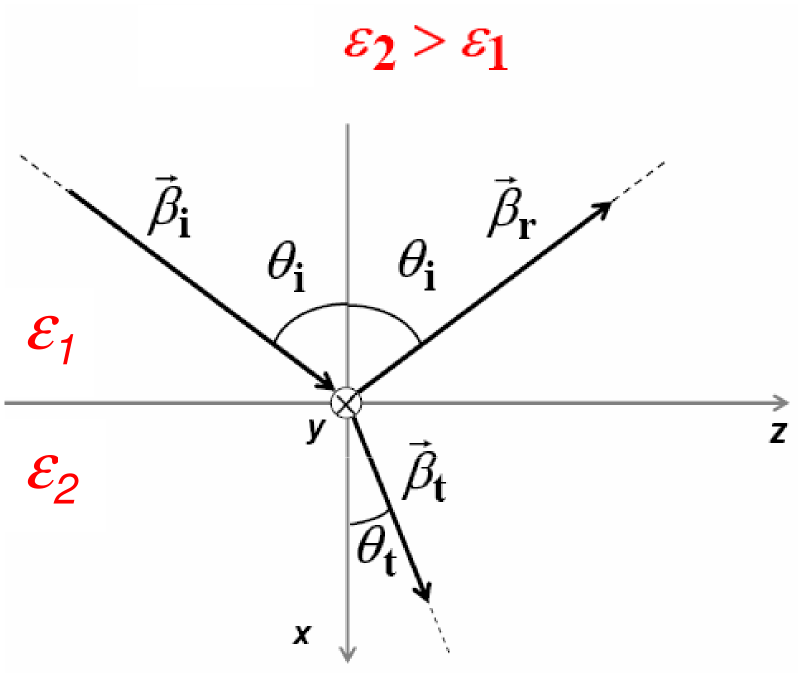
Poševni lom TE vala

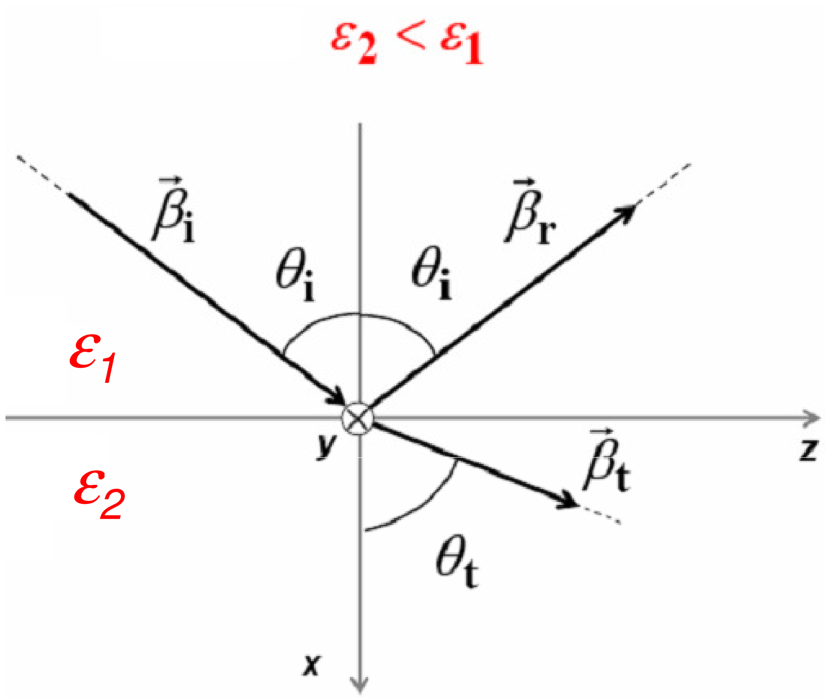


Poševni lom TM vala



Snellov lomni zakon





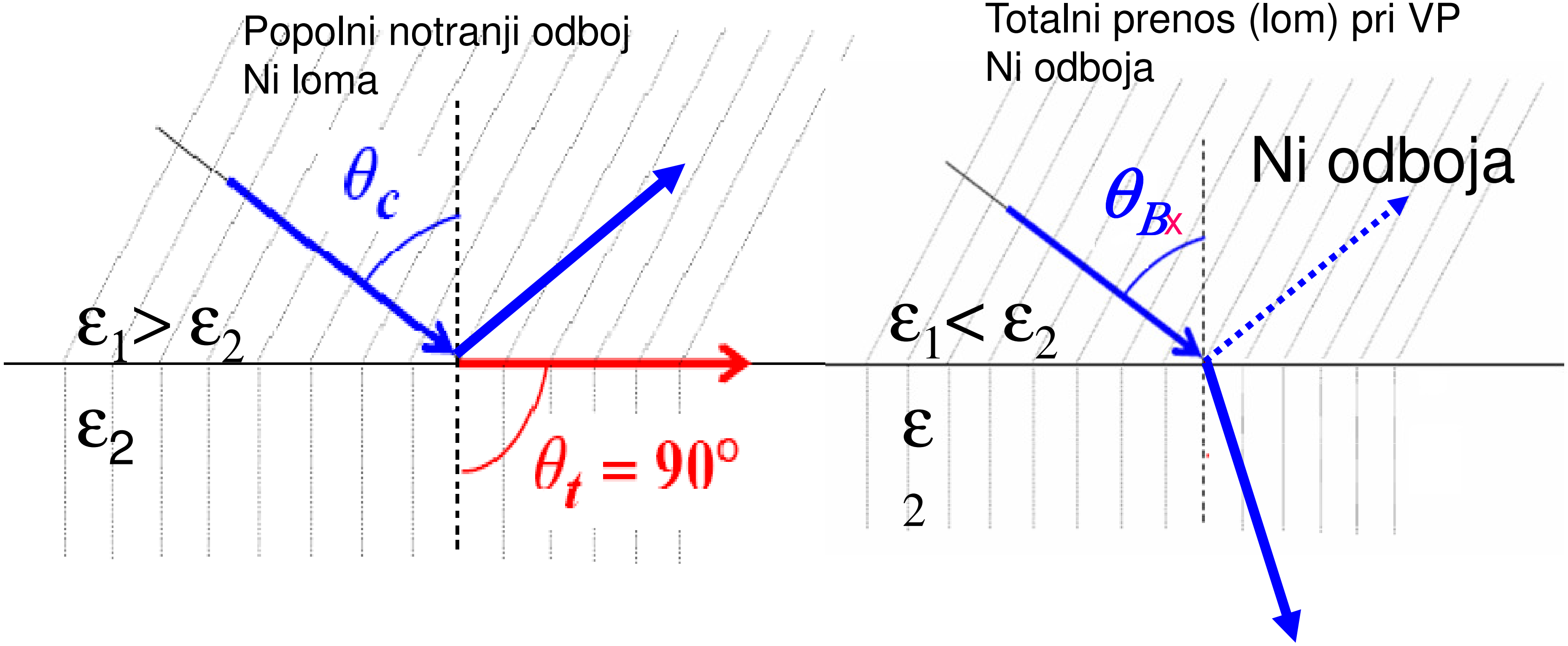
$$\sqrt{\varepsilon_1}\sin\theta_i = \sqrt{\varepsilon_2}\sin\theta_t$$

$$\varepsilon_2 > \varepsilon_1 \;\Rightarrow\; \theta_t < \theta_i$$

$$\sqrt{\varepsilon_1}\sin\theta_i = \sqrt{\varepsilon_2}\sin\theta_t$$

$$\varepsilon_2 < \varepsilon_1 \;\Rightarrow\; \theta_t > \theta_i$$

Totalni odboj in totalni prenos



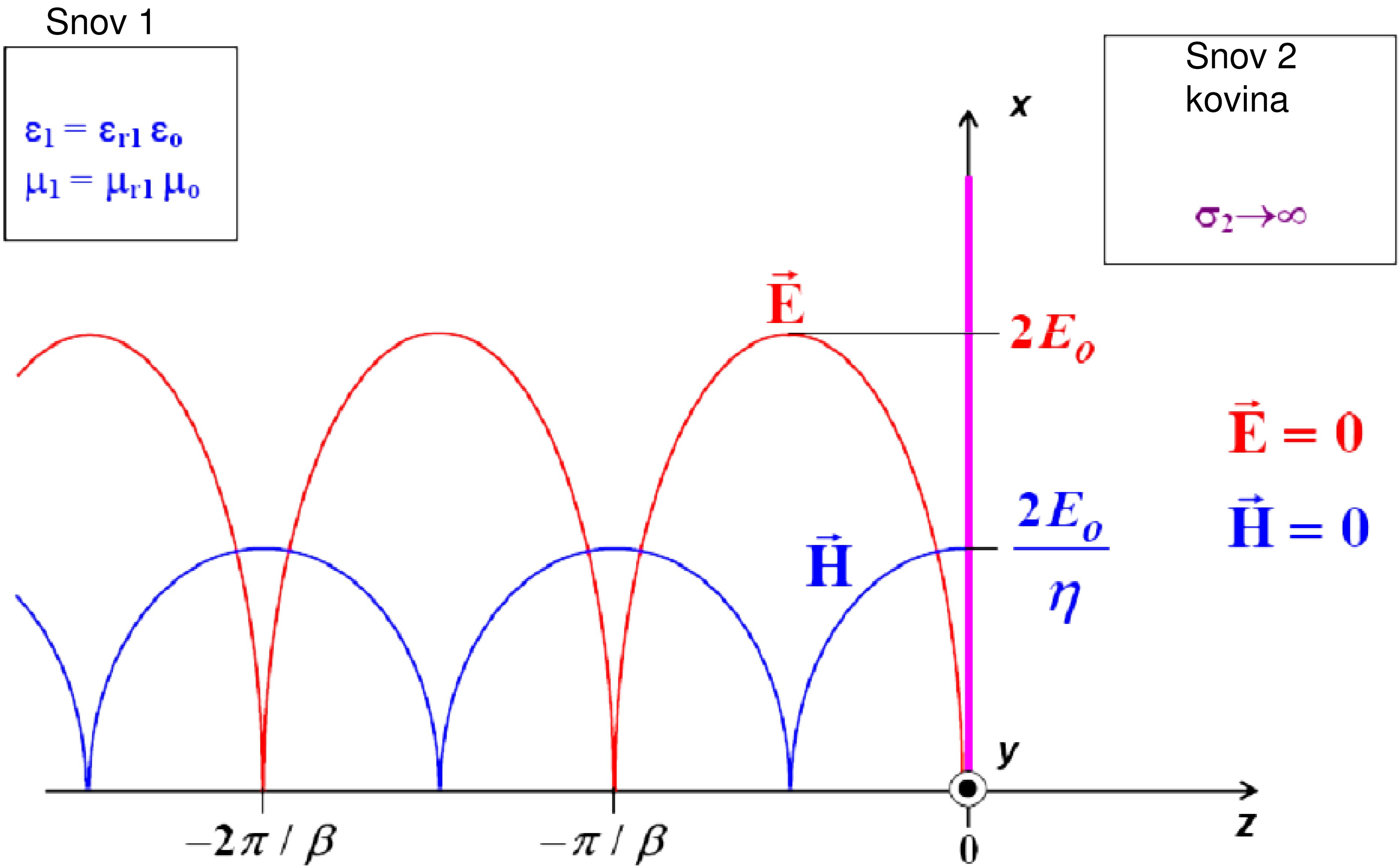
Vpadni kot totalnega odboja:

$$\theta_i = \theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

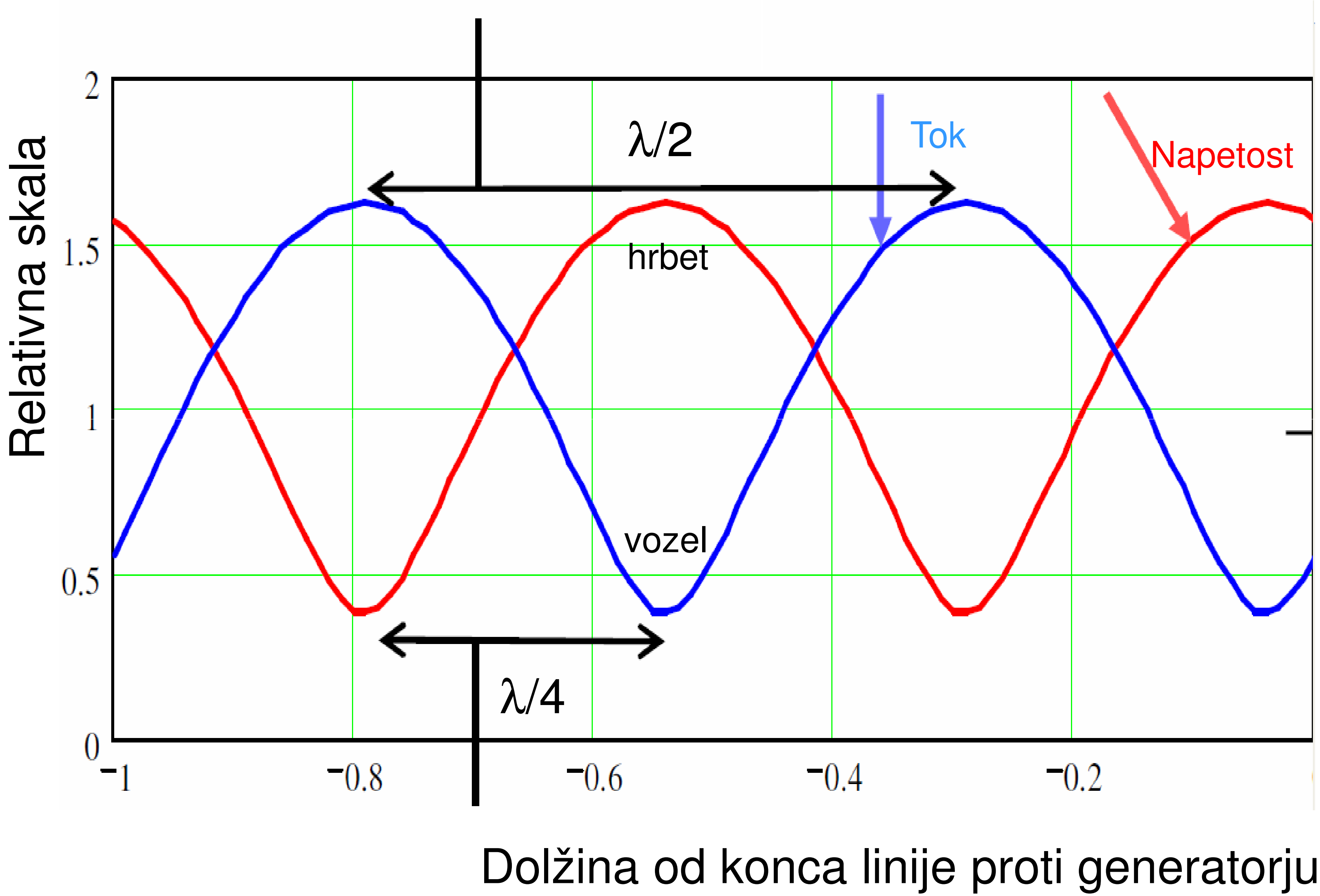
Brewstrov kot totalnega prenosa pri vertikalni polarizaciji polja:

$$\theta_i = \theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

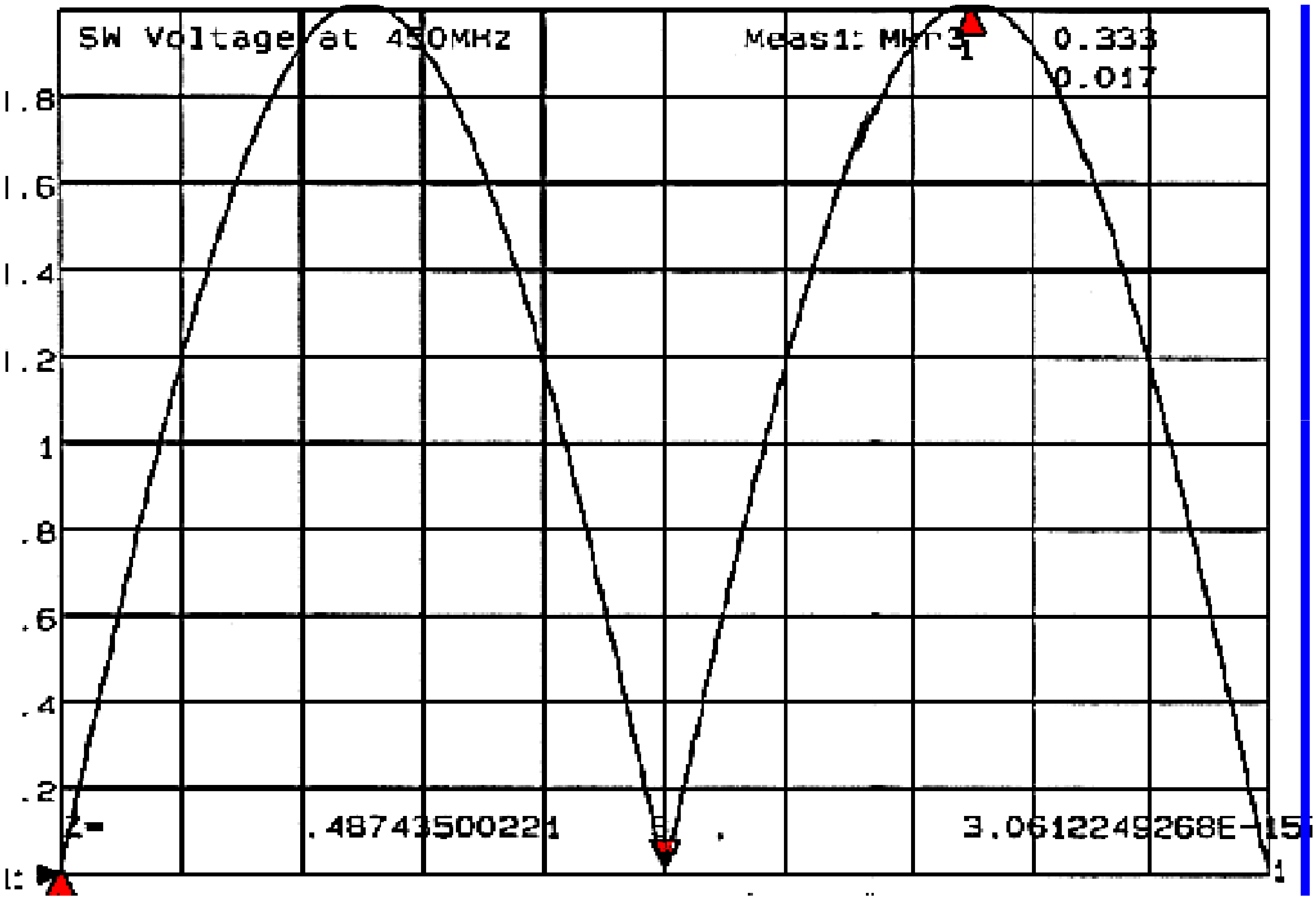
Vpad in odboj planega vala na kovino
Stojni val električnega in magnetnega polja



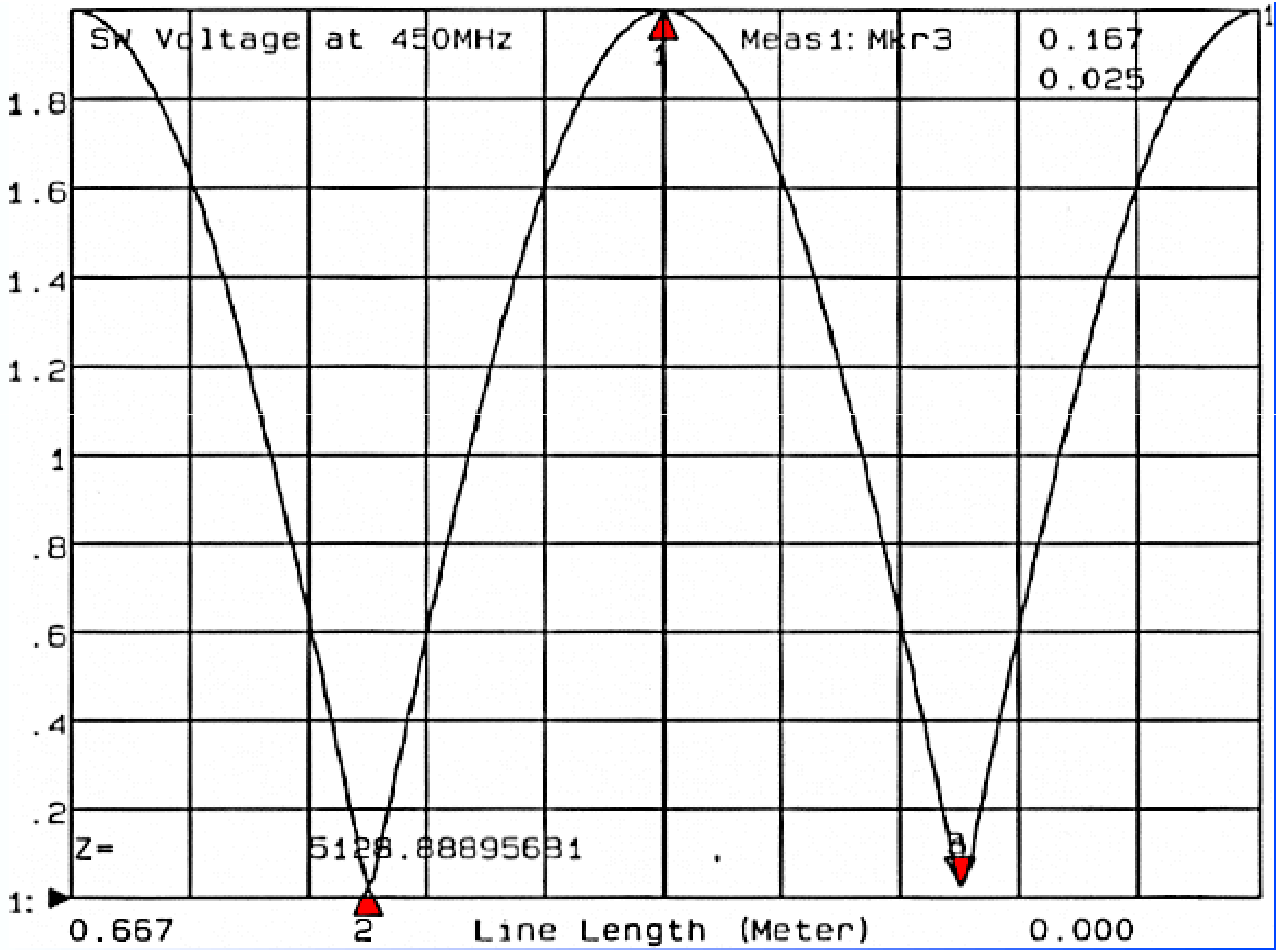
Stojni val napetosti in toka



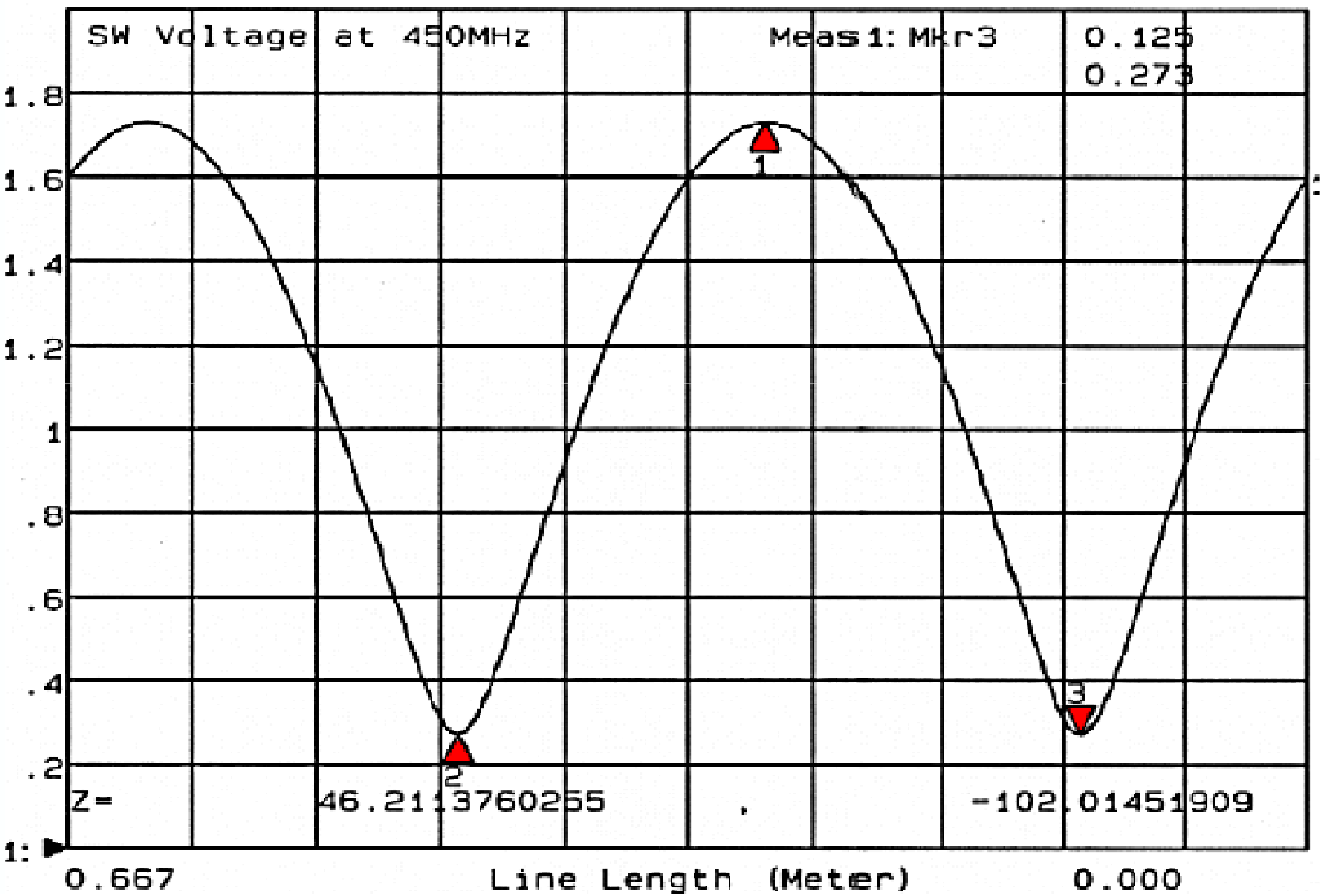
Popolni stojni val



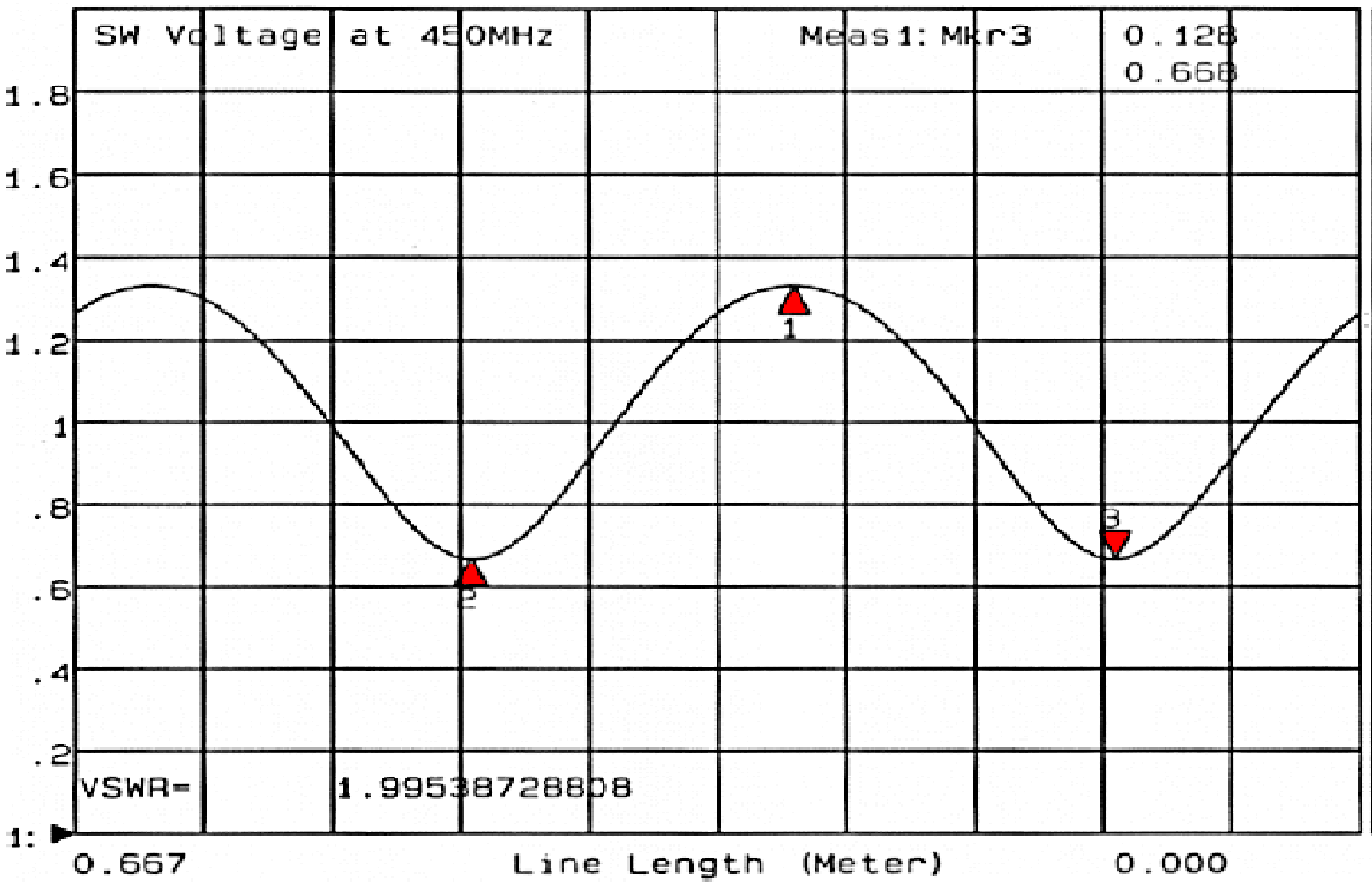
Popolni stojni val toka



Nepopolni stojni val



Nepopolni stojni val



Preglednica enačb za napetost in tok

Linija brez izgub:

$$V(\mathrm{d}) = V^+ e^{j\beta \mathrm{d}} \left(1 + \Gamma_R e^{-2j\beta \mathrm{d}}\right)$$

$$I(\mathrm{d}) = \frac{V^+ e^{j\beta \mathrm{d}}}{Z_0} \left(1 - \Gamma_R e^{-2j\beta \mathrm{d}}\right)$$

$$\Gamma(\mathrm{d}) = \Gamma_R e^{-2j\beta \mathrm{d}}$$

$$V(\mathrm{d}) = V^+ e^{j\beta \mathrm{d}} (1 + \Gamma(\mathrm{d}))$$

$$I(\mathrm{d}) = \frac{V^+ e^{j\beta \mathrm{d}}}{Z_0} (1 - \Gamma(\mathrm{d}))$$

Linija z izgubami:

$$V(\mathrm{d}) = V^+ e^{\gamma \mathrm{d}} \left(1 + \Gamma_R e^{-2\gamma \mathrm{d}}\right)$$

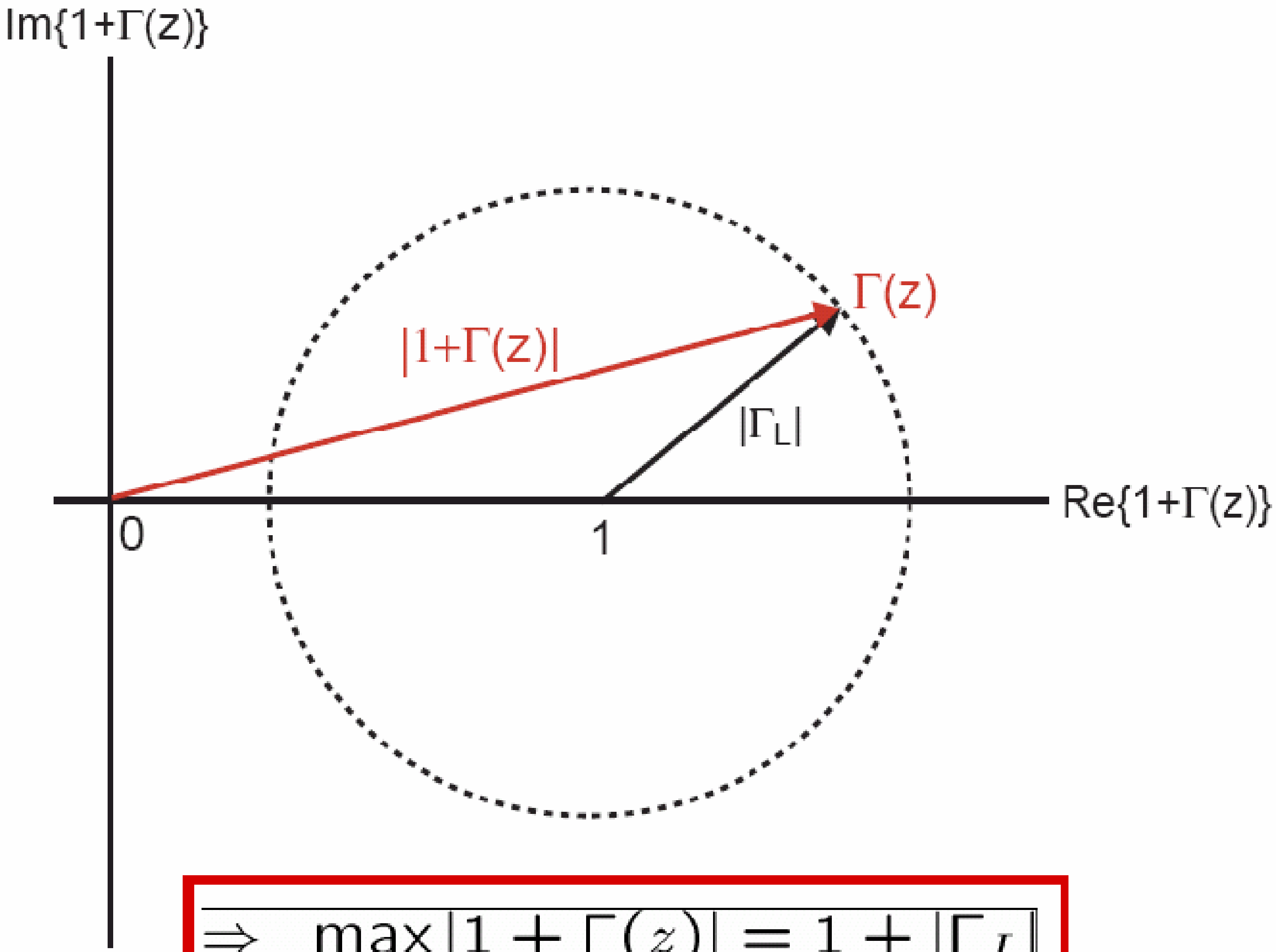
$$I(\mathrm{d}) = \frac{V^+ e^{\gamma \mathrm{d}}}{Z_0} \left(1 - \Gamma_R e^{-2\gamma \mathrm{d}}\right)$$

$$\Gamma(\mathrm{d}) = \Gamma_R e^{-2\gamma \mathrm{d}}$$

$$V(\mathrm{d}) = V^+ e^{\gamma \mathrm{d}} (1 + \Gamma(\mathrm{d}))$$

$$I(\mathrm{d}) = \frac{V^+ e^{\gamma \mathrm{d}}}{Z_0} (1 - \Gamma(\mathrm{d}))$$

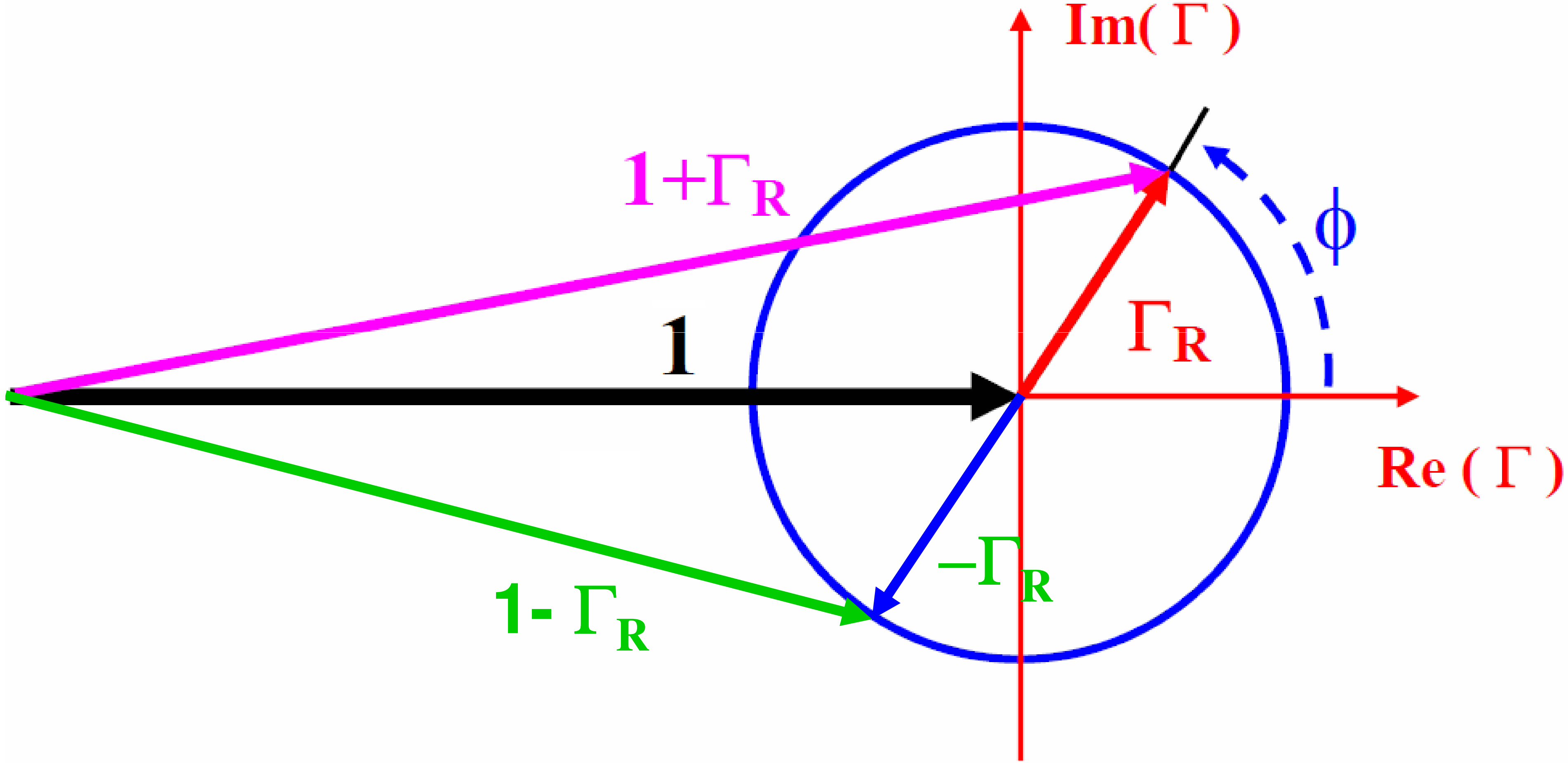
Kazalčni prikaz stojnega vala napetosti



$$\Rightarrow \quad \max|1+\Gamma(z)| = 1 + |\Gamma_L|$$

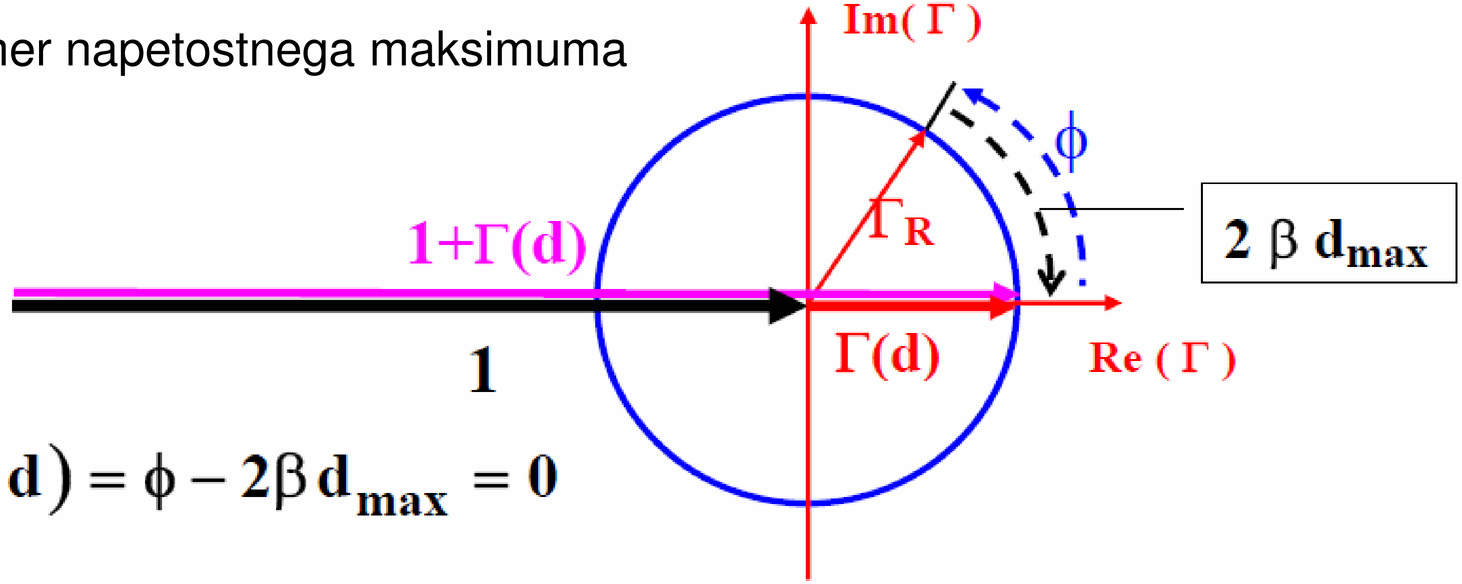
$$\Rightarrow \quad \min|1+\Gamma(z)| = 1 - |\Gamma_L|$$

Kazalci odbojnosti, napetosti in toka



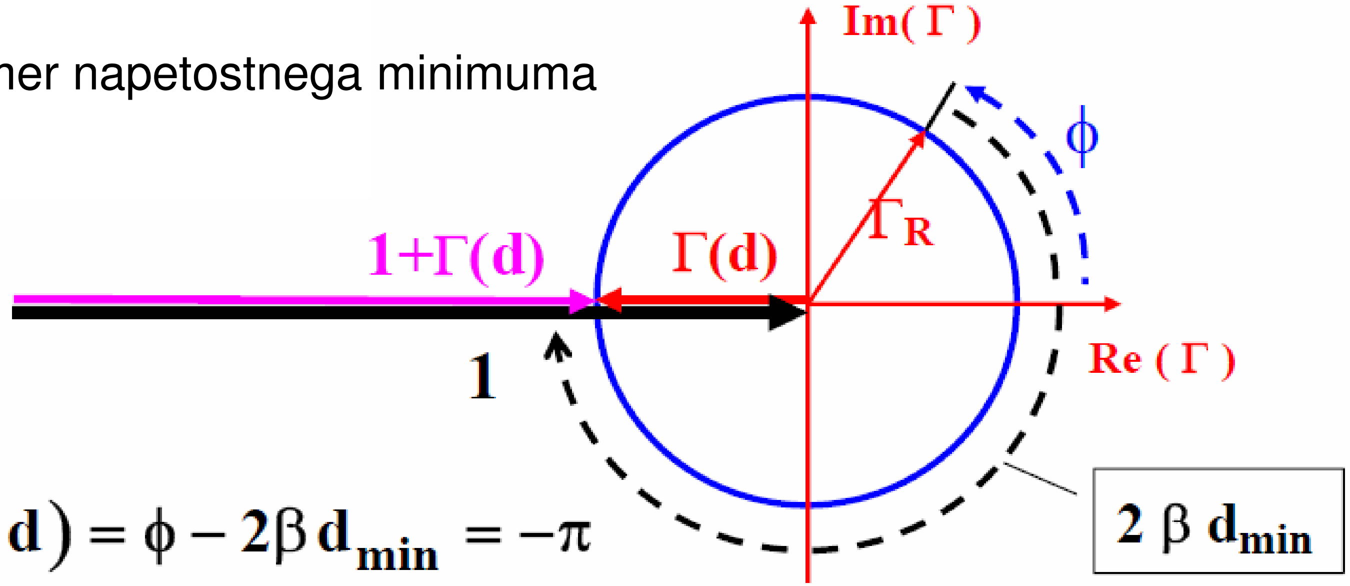
Primera maksimuma in minimuma napetosti

1. Primer napetostnega maksimuma



$$\angle\Gamma(d) = \phi - 2\beta d_{max} = 0$$

2. Primer napetostnega minimuma



$$\angle\Gamma(d) = \phi - 2\beta d_{min} = -\pi$$

Napetost in tok

Linija brez izgub

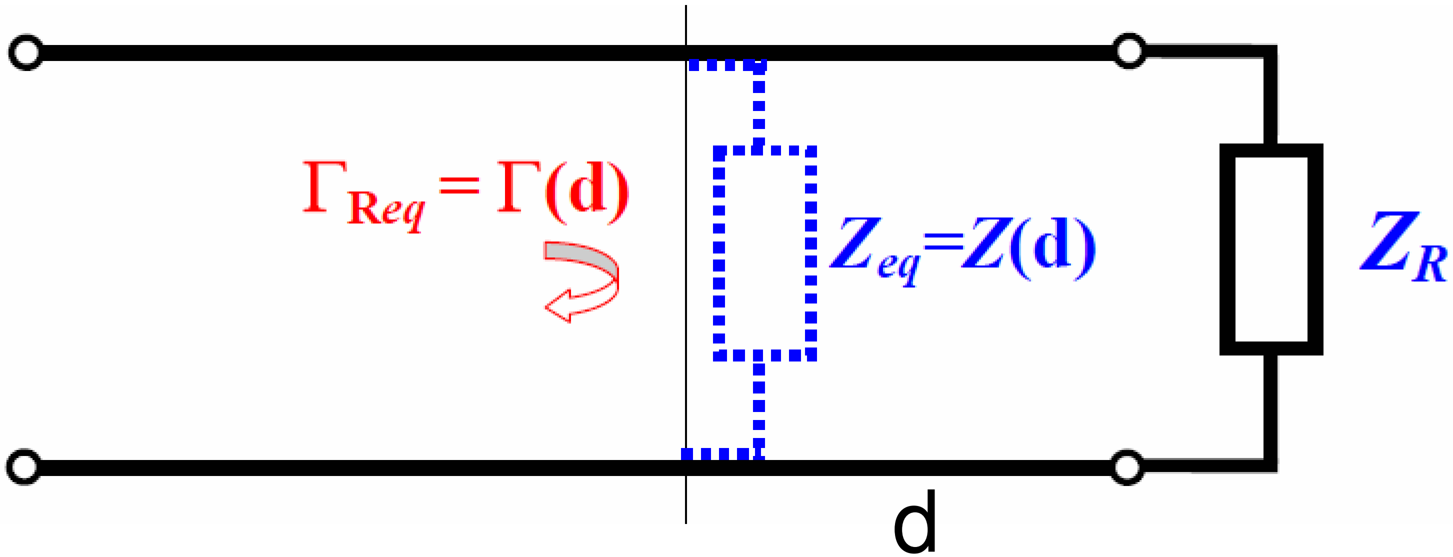
$$\begin{cases} |V(d)| = |V^+| \cdot |(1+\Gamma(d))| \\ |I(d)| = \left|\dfrac{V^+}{Z_0}\right| \cdot |(1-\Gamma(d))| \end{cases}$$

Linija z izgubami

$$\begin{cases} |V(d)| = |V^+ e^{\alpha d}| \cdot |(1+\Gamma(d))| \\ |I(d)| = \left|\dfrac{V^+ e^{\alpha d}}{Z_0}\right| \cdot |(1-\Gamma(d))| \end{cases}$$

Odbojnost in impedanca linije

$$Z(\mathrm{d}) = \frac{V(\mathrm{d})}{I(\mathrm{d})} = Z_0 \frac{1+\Gamma(\mathrm{d})}{1-\Gamma(\mathrm{d})}$$



Razmerje stojnega vala

- Razmerje stojnega vala, VSWR, SWR, S, σ
- Valovitost
- Neubranost

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma_R|}{1-|\Gamma_R|}$$

$$\Gamma_R = 0 \qquad \Rightarrow \qquad VSWR = 1$$

Absolutna odbojnost in valovitost

$$\left|\Gamma_R\right| = 1 \qquad \Rightarrow \qquad VSWR \to \infty$$

$$\left|\Gamma_R\right| = \frac{VSWR - 1}{VSWR + 1}$$

Vhodna impedanca obremenjene linije

- Linija brez izgub $\gamma = j\beta$; dolžina linije d

$$Z(\mathrm{d}) = Z_0 \frac{1+\Gamma_R e^{-2j\beta \mathrm{d}}}{1-\Gamma_R e^{-2j\beta \mathrm{d}}} = Z_0 \frac{Z_R + jZ_0 \tan(\beta\, \mathrm{d})}{jZ_R \tan(\beta\, \mathrm{d}) + Z_0}$$

- Linija z izgubami, $\gamma = \alpha + j\beta$; dolžina linije d

$$Z(\mathrm{d}) = Z_0 \frac{1+\Gamma_R e^{-2\gamma \mathrm{d}}}{1-\Gamma_R e^{-2\gamma \mathrm{d}}} = Z_0 \frac{Z_R + Z_0 \tanh(\gamma\, \mathrm{d})}{Z_R \tanh(\gamma\, \mathrm{d}) + Z_0}$$

d ... dolžina linije

Prenašana moč, moč potujočih valov

$$\langle P(\mathrm{d},t)\rangle = \frac{1}{2}\mathrm{Re}\{V(\mathrm{d})\,I^*(\mathrm{d})\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{V^+ e^{j\beta \mathrm{d}}\left(1+\Gamma_R e^{-j2\beta \mathrm{d}}\right) \times\right.$$

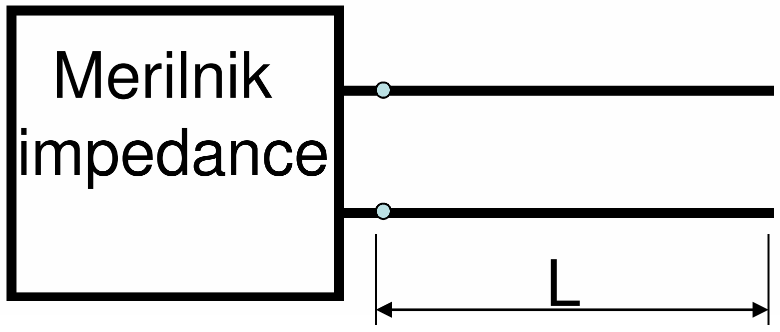
$$\left.\times\ \frac{1}{Z_0}(V^+)^* e^{-j\beta \mathrm{d}}\left(1-\Gamma_R e^{-j2\beta \mathrm{d}}\right)^*\right\}$$

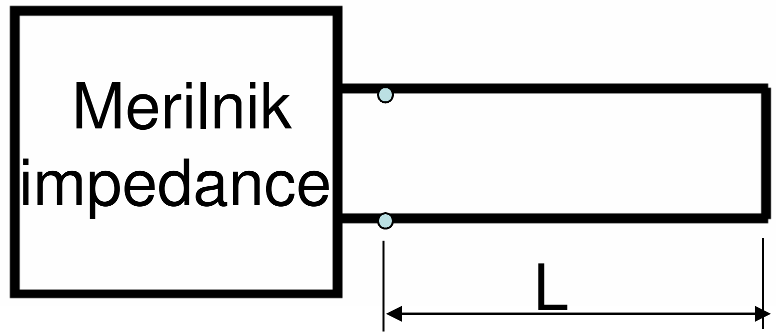
Moč vpadnega vala: Moč odbitega vala;

$$= \frac{1}{2Z_0}\left|V^+\right|^2 - \frac{1}{2Z_0}\left|V^+\right|^2\left|\Gamma_R\right|^2$$

Prenašana moč je enaka razliki moči vpadnega in odbitega vala

Meritev Z_k in v/c





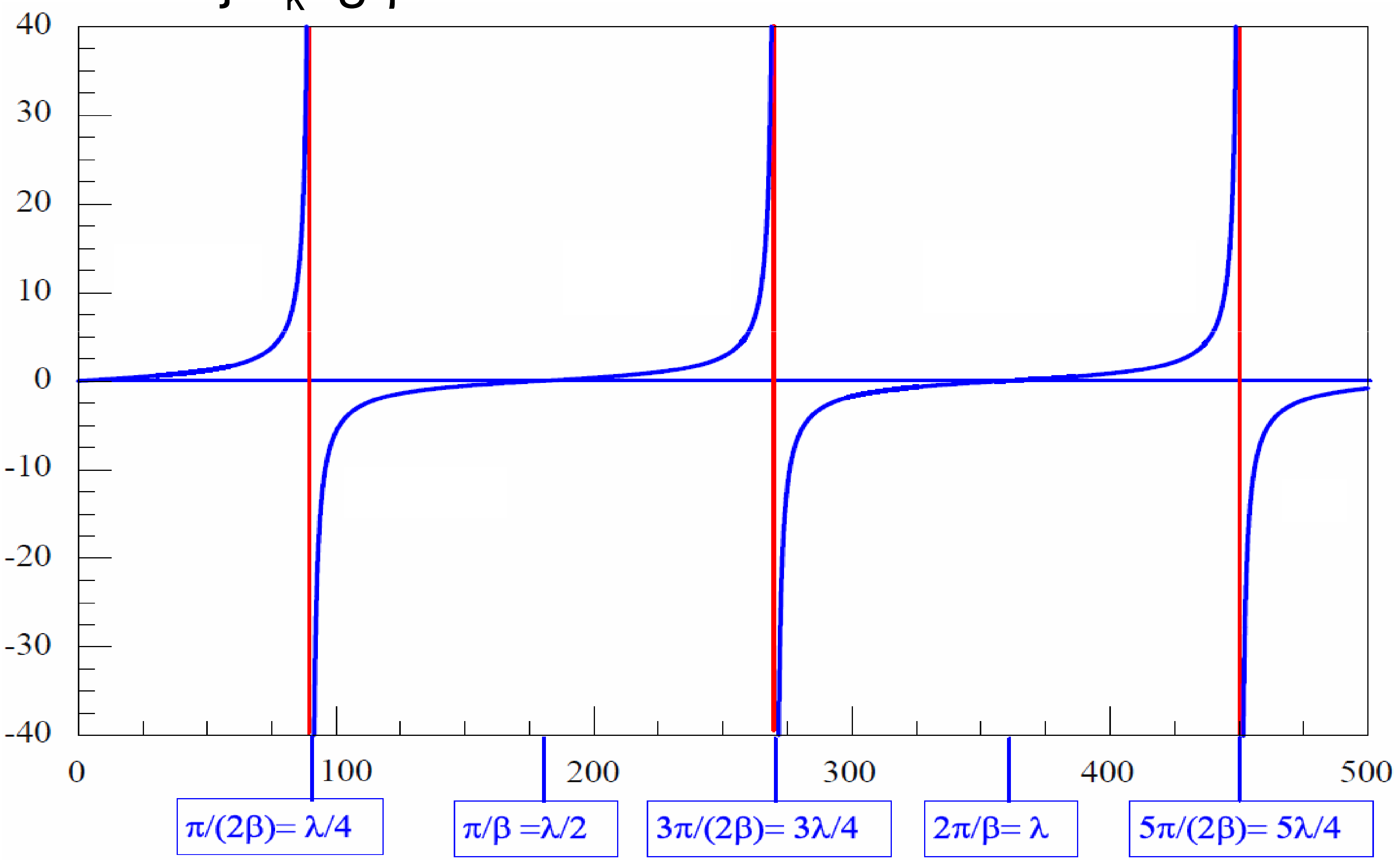
$$Z_k = \sqrt{Z_{kratkostaknjeni} \times Z_{odprti}}$$

$$v_f = \frac{2\pi f L}{c} \frac{1}{\cot^{-1}\sqrt{\frac{-Z}{Z}}}$$

skrajševalni faktor

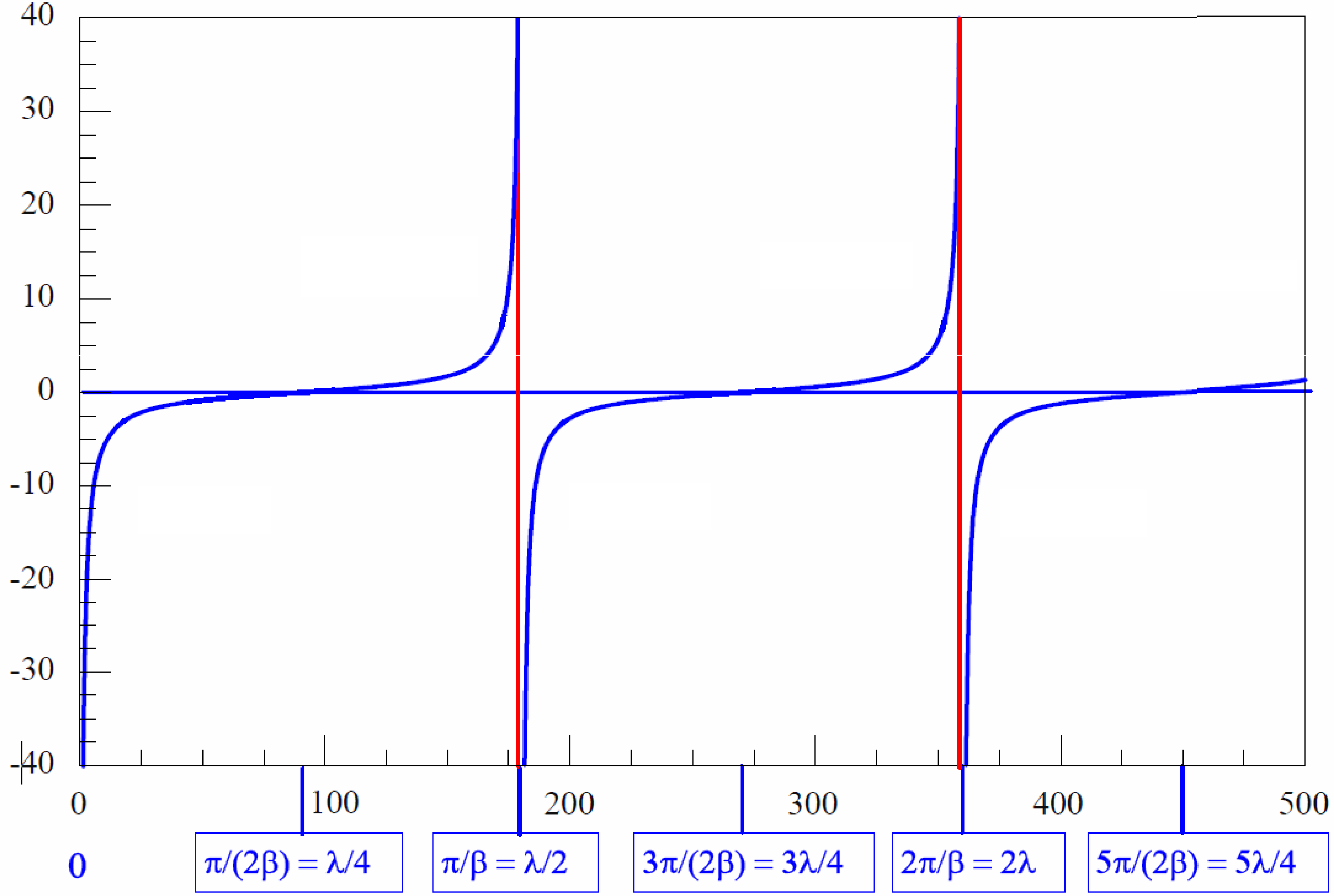
Reaktanca kratkostaknjene linije

$Z = j\, Z_k\, \mathrm{tg}\, \beta L$



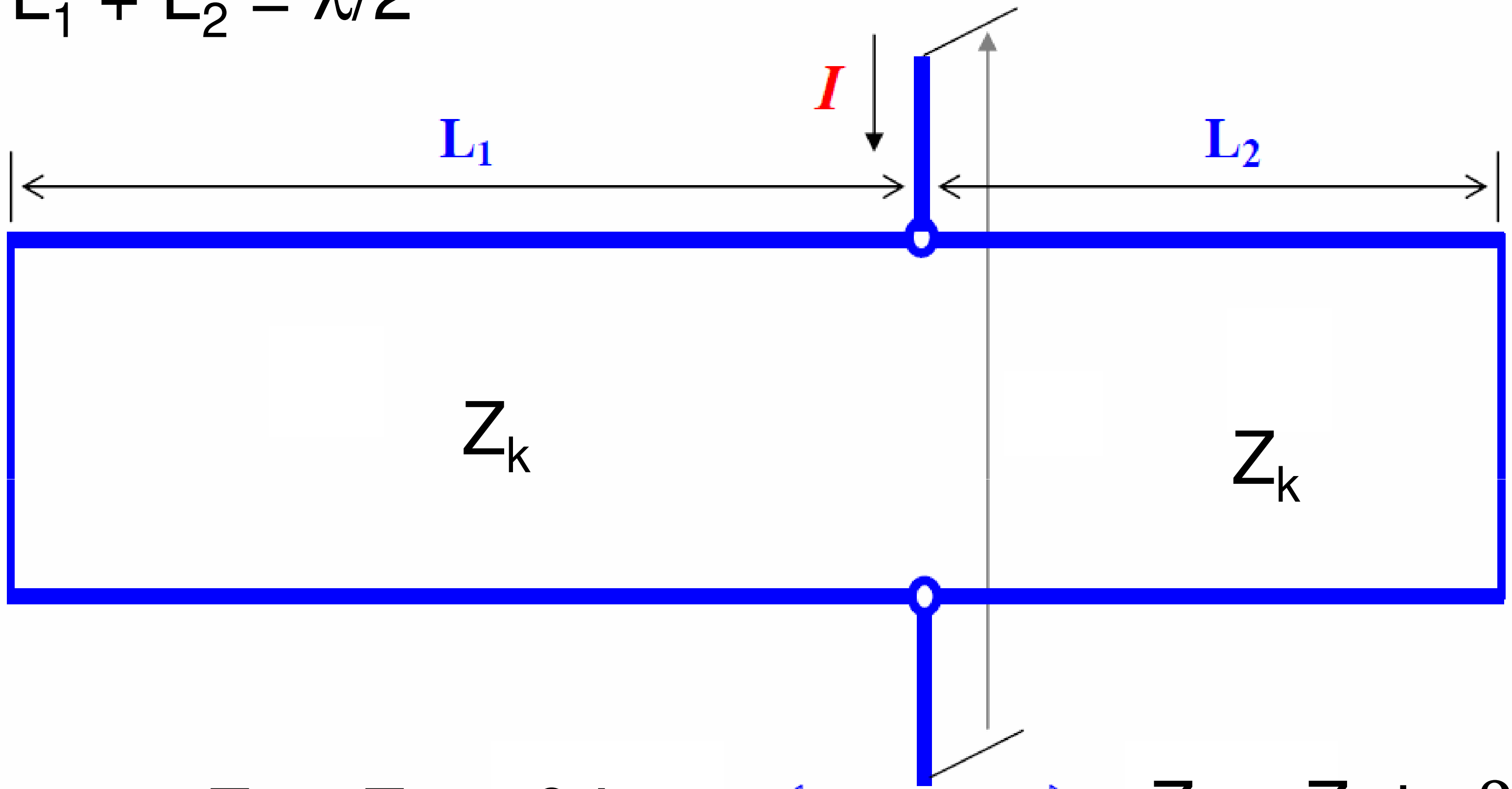
Reaktanca odprte linije

$Z = -j Z_k \operatorname{ctg} \beta L$



Linijski resonator

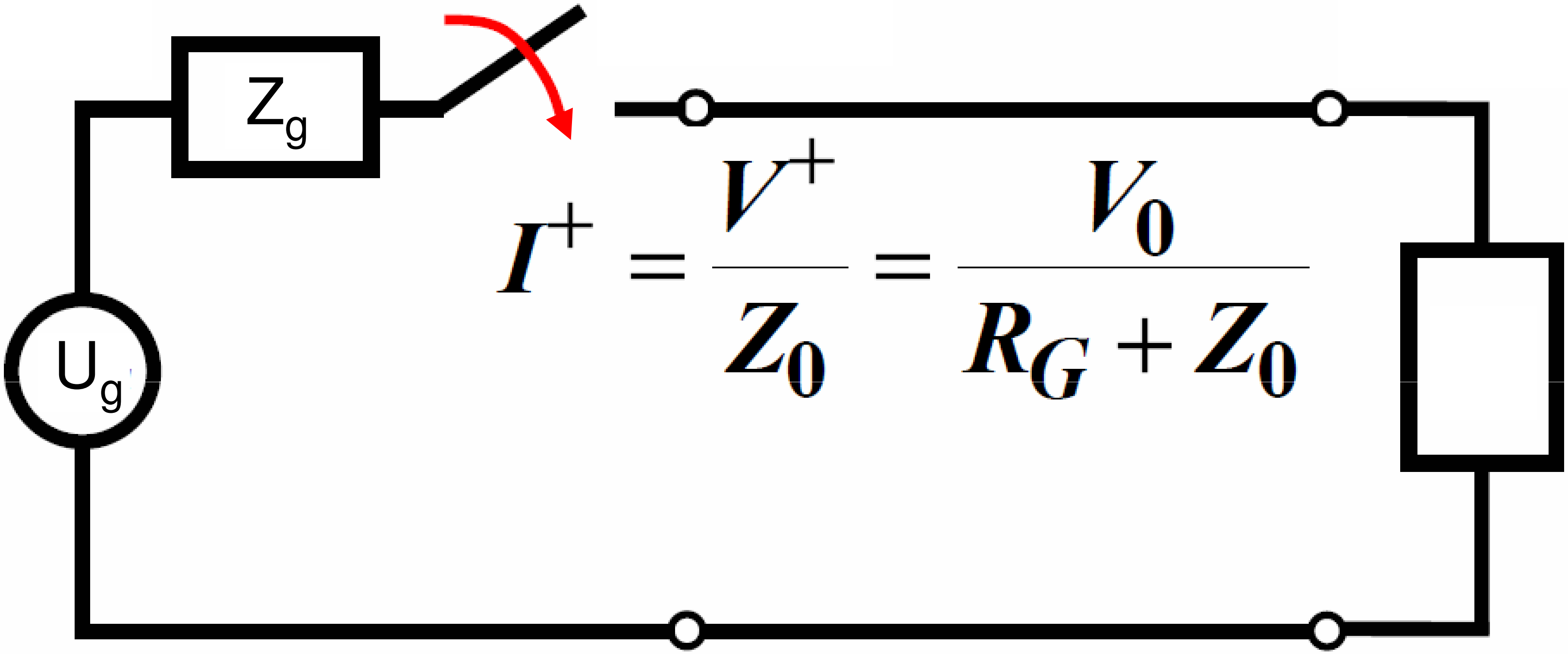
$L_1 + L_2 = \lambda/2$



$Z_1 = Z_k \operatorname{tg} \beta L_1$ $Z_2 = Z_k \operatorname{tg} \beta L_2$

Impedanci Z_1 in Z_2 sta po velikosti enaki in po znaku nasprotni. Nastaja vzporedna resonanca pri poljubnih dolžinah L_1 in L_2 pri pogoju, da znaša skupna dolžina pol valovne dolžine ($\lambda/2$).

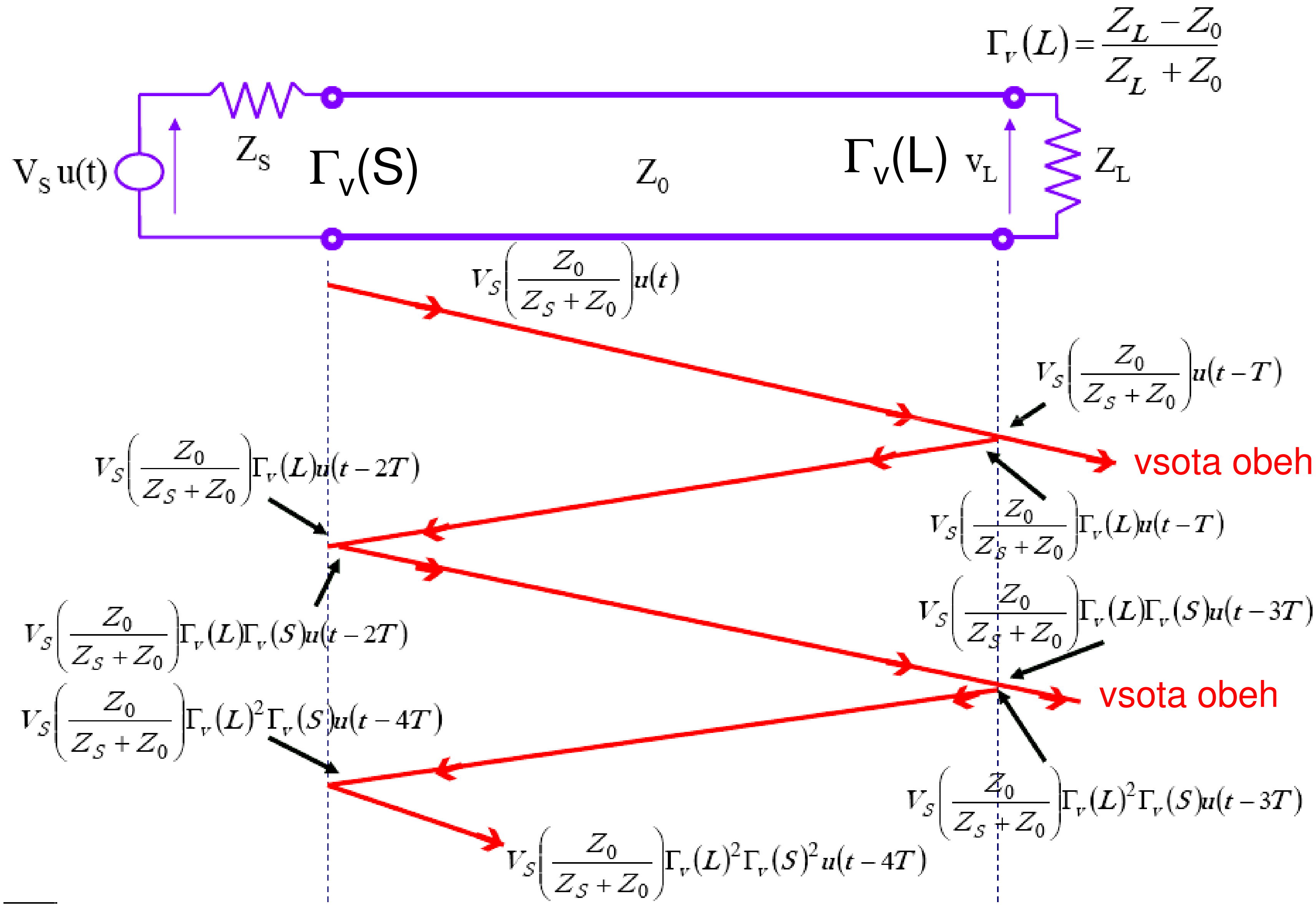
Vklop generatorja na linijo



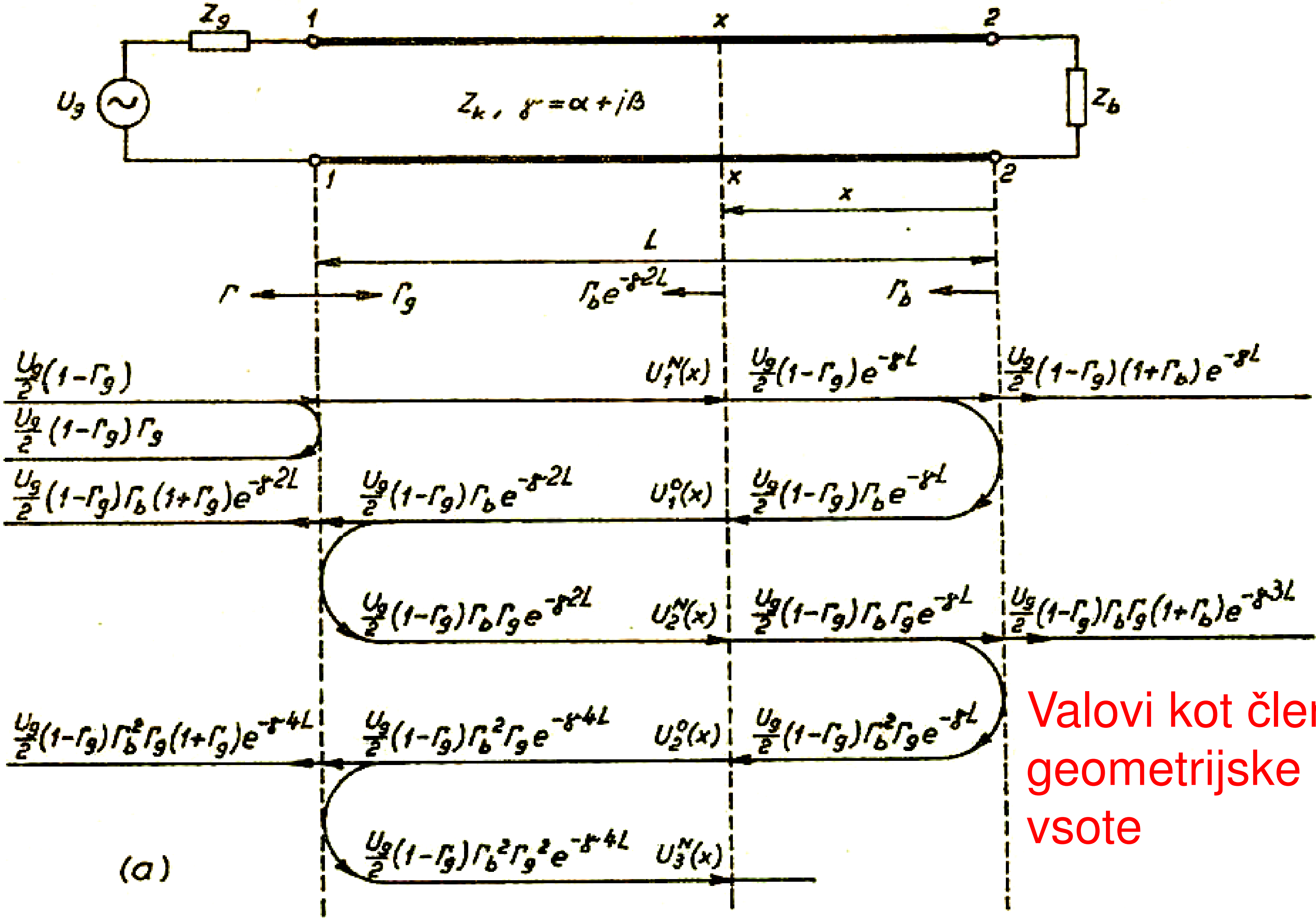
$$I^+ = \frac{V^+}{Z_0} = \frac{V_0}{R_G + Z_0}$$

$$V^+ = I^+ Z_0 = V_G \frac{Z_0}{R_G + Z_0}$$

Razvoj valov po vklopu



Valovni pojav na homogeni liniji



Valovi kot členi geometrijske vsote

Potujoči val in stojni val

$$U^N(x) = \frac{U_g}{2}(1-\Gamma_g)\frac{e^{-\gamma(L-x)}}{1-\Gamma_g\Gamma_b e^{-\gamma 2L}}, \qquad I^N(x) = \frac{U^N(x)}{Z_k},$$

Napredujoči in odbiti val napetosti in toka

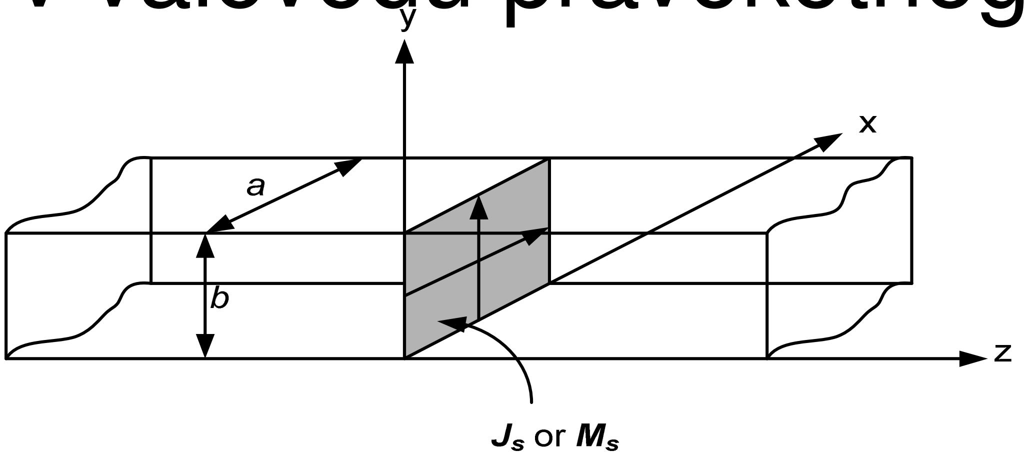
$$U^o(x) = \frac{U_g}{2}(1-\Gamma_g)\frac{e^{-\gamma(L-x)}}{1-\Gamma_g\Gamma_b e^{-\gamma 2L}}, \qquad I^o(x) = -\frac{U^o(x)}{Z_k}.$$

$$U(x) = U^N(x) + U^o(x) = \frac{U_g}{2}(1-\Gamma_g)\frac{e^{-\gamma(L-x)}}{1-\Gamma_g\Gamma_b e^{-\gamma 2L}}(1+\Gamma_b e^{-\gamma 2x}),$$

Stojni val napetosti in toka .

$$I(x) = I^N(x) + I^o(x) = \frac{U_g}{2Z_k}(1-\Gamma_g)\frac{e^{-\gamma(L-x)}}{1-\Gamma_g\Gamma_b e^{-\gamma 2L}}(1-\Gamma_b e^{-\gamma 2x}).$$

Polje v valovodu pravokotnega prereza



$$E_x^{\pm} = \eta \left(\frac{n\pi}{b}\right) A_{mn}^{\pm} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{\mp j\beta z}, \qquad (1)$$

$$E_y^{\pm} = -\eta \left(\frac{m\pi}{a}\right) A_{mn}^{\pm} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{\mp j\beta z}, \qquad (2)$$

$$H_x^{\pm} = \pm \left(\frac{m\pi}{a}\right) A_{mn}^{\pm} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{\mp j\beta z}, \qquad (3)$$

$$H_y^{\pm} = \pm \left(\frac{n\pi}{b}\right) A_{mn}^{\pm} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{\mp j\beta z} \qquad (4)$$

Osnovni rod polja TE_{10}

$$H_z = A_{10} \cos \frac{\pi x}{a} e^{-j\beta z},$$

$$E_y = \frac{-j\omega\mu a}{\pi} A_{10} \sin \frac{\pi x}{a} e^{-j\beta z},$$

$$H_x = \frac{j\beta a}{\pi} A_{10} \sin \frac{\pi x}{a} e^{-j\beta z},$$

$$E_x = E_z = H_y = 0.$$

$$k_c = \pi / a, \beta = \sqrt{k^2 - (\pi / a)^2}$$

Sklep

1. Znanje o elektromagnetnih pojavih na prenosnih linijah je osnovno znanje za delo na mikrovalovih in za razumevanje drugih vodniških in prostorskih valovanj.
2. Interferenca vpadnega in odbitega vala daje osnovno predstavo o valovnih pojavih na liniji. Valovi so primarni pojav, napetost, tok in impedanca pa posledice. Pomembna je odbojnost.
3. Jasno predstavo o valovanju na liniji daje analiza prehodnega pojava, ki vključuje zaporedje vpadnih in odbitih valov.
4. Analiza prostorskih valovanj (primer valovod) je razširitev analize enodimenzionalnih (linijskih) valovanj in uporablja prikazan način obravnave.

Varujmo se velikih valov