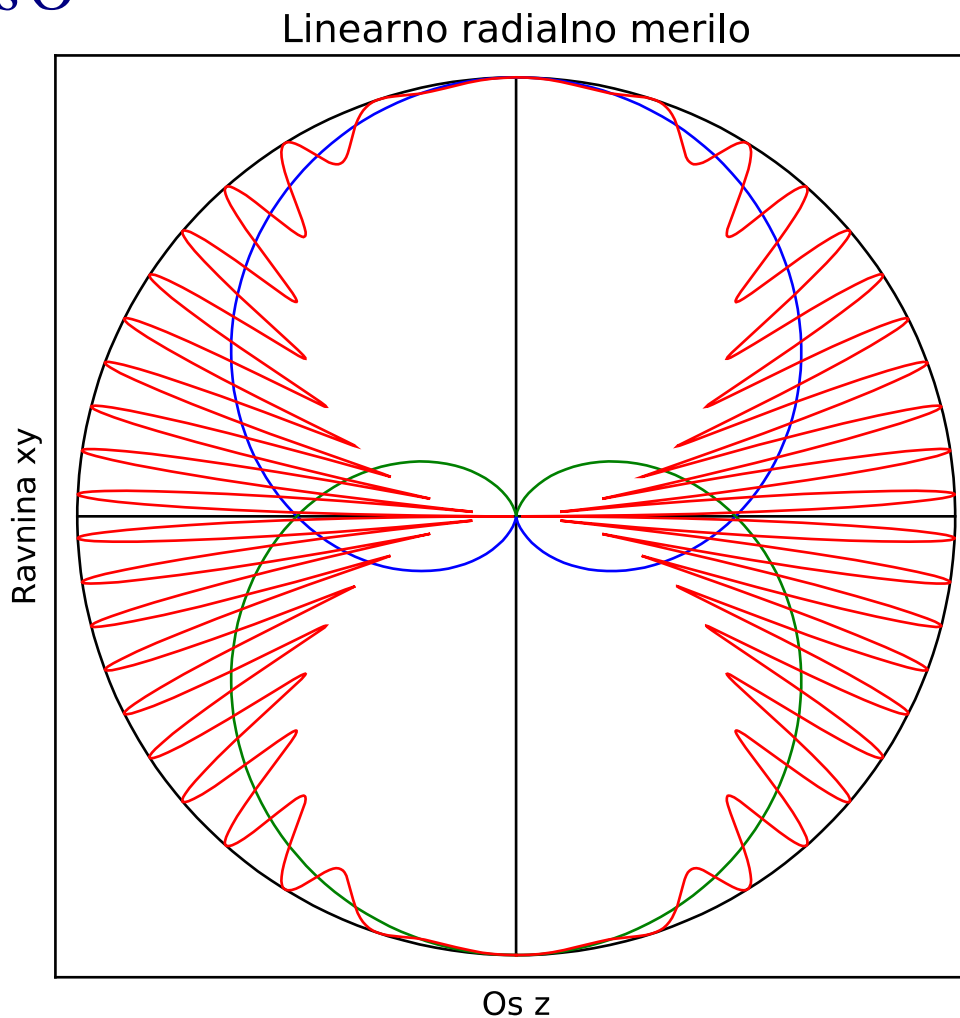
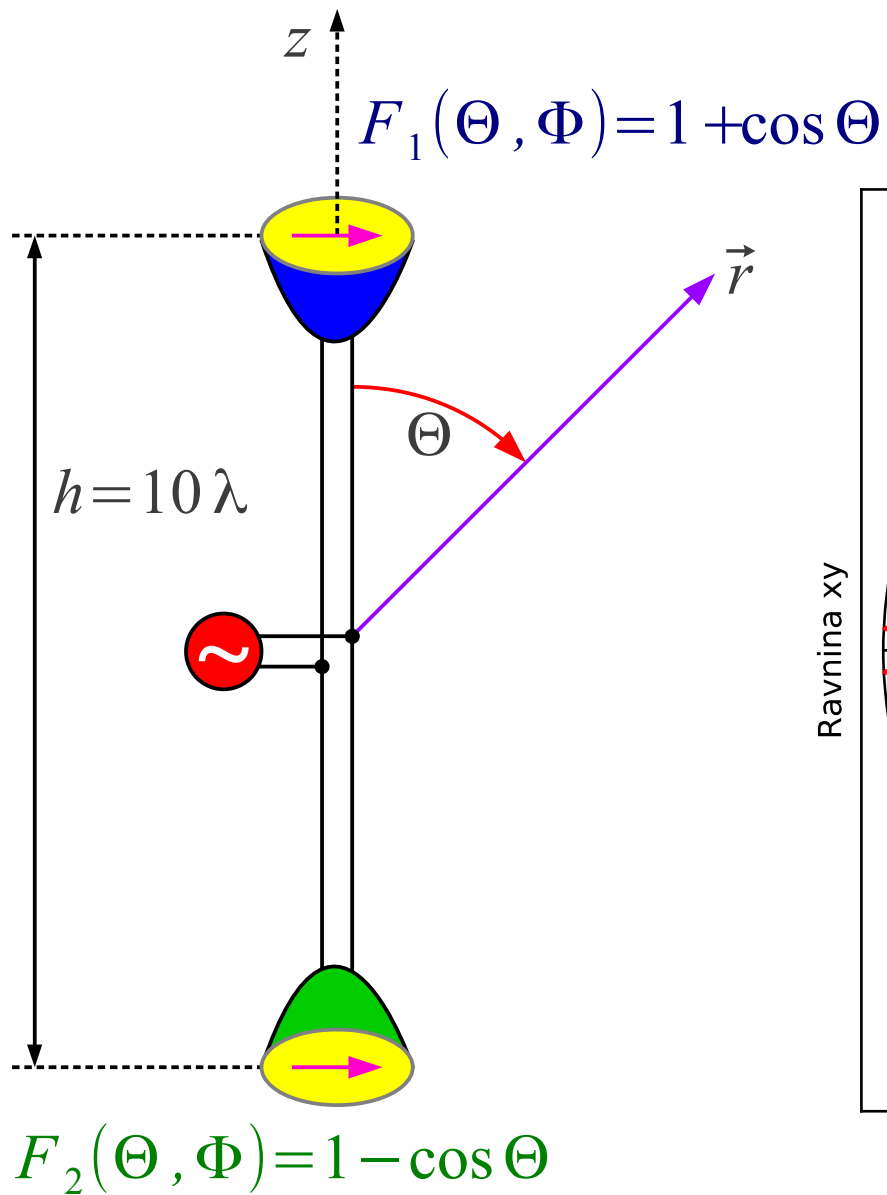
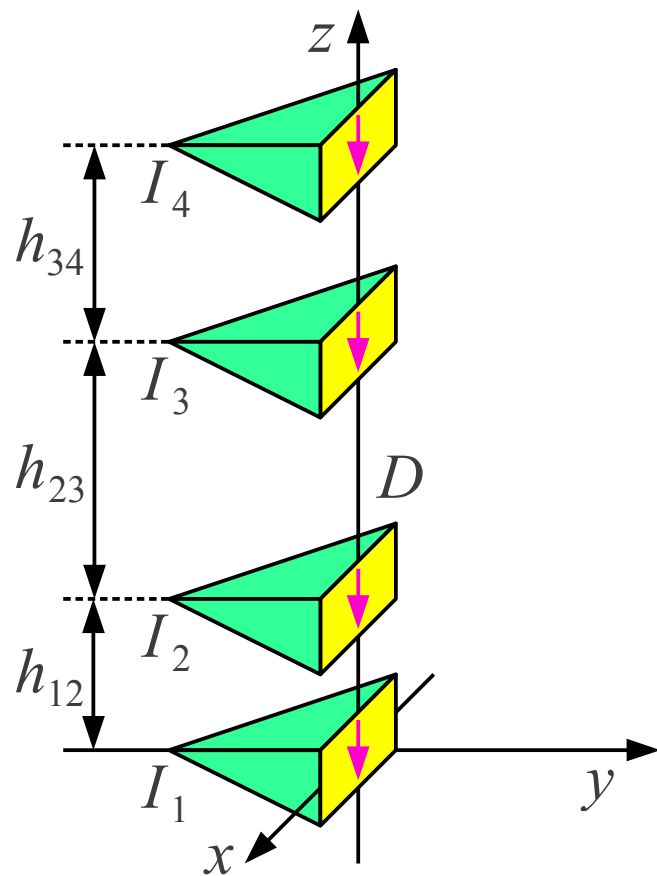


Skupine

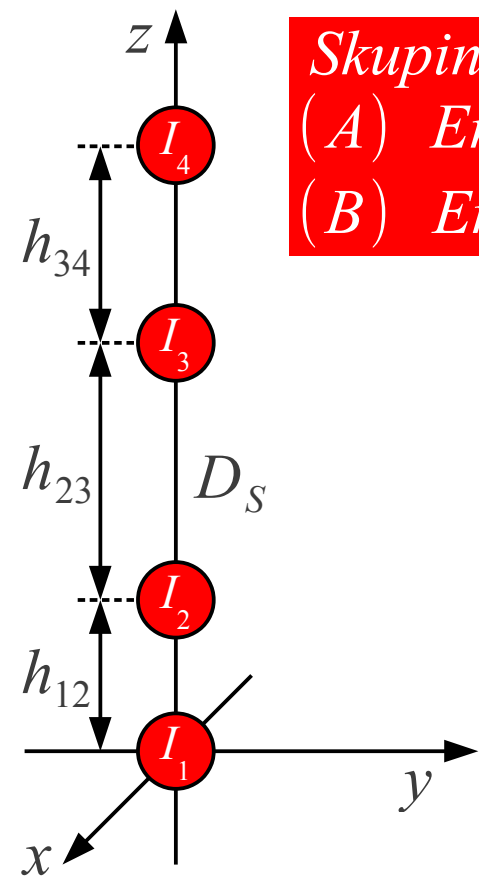


$$F(\Theta, \Phi) = F_1(\Theta, \Phi) e^{j \frac{kh}{2} \cos \Theta} - F_2(\Theta, \Phi) e^{-j \frac{kh}{2} \cos \Theta}$$

Nesmiselna skupina dveh anten

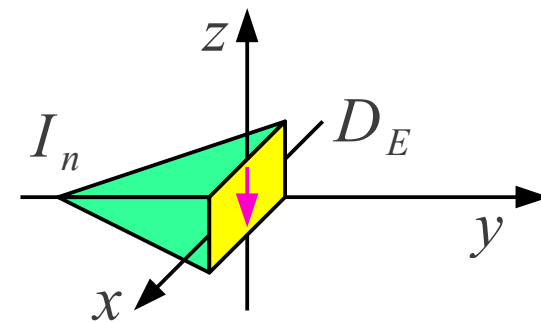


$F(\Theta, \Phi) \equiv$ smerni diagram skupine anten



$F_S(\Theta, \Phi) \equiv$ smerni diagram skupine neusmerjenih virov

Skupina neusmerjenih virov
 (A) Enaka razporeditev h_{mn}
 (B) Enako napajanje I_n



$F_E(\Theta, \Phi) \equiv$ smerni diagram elementa

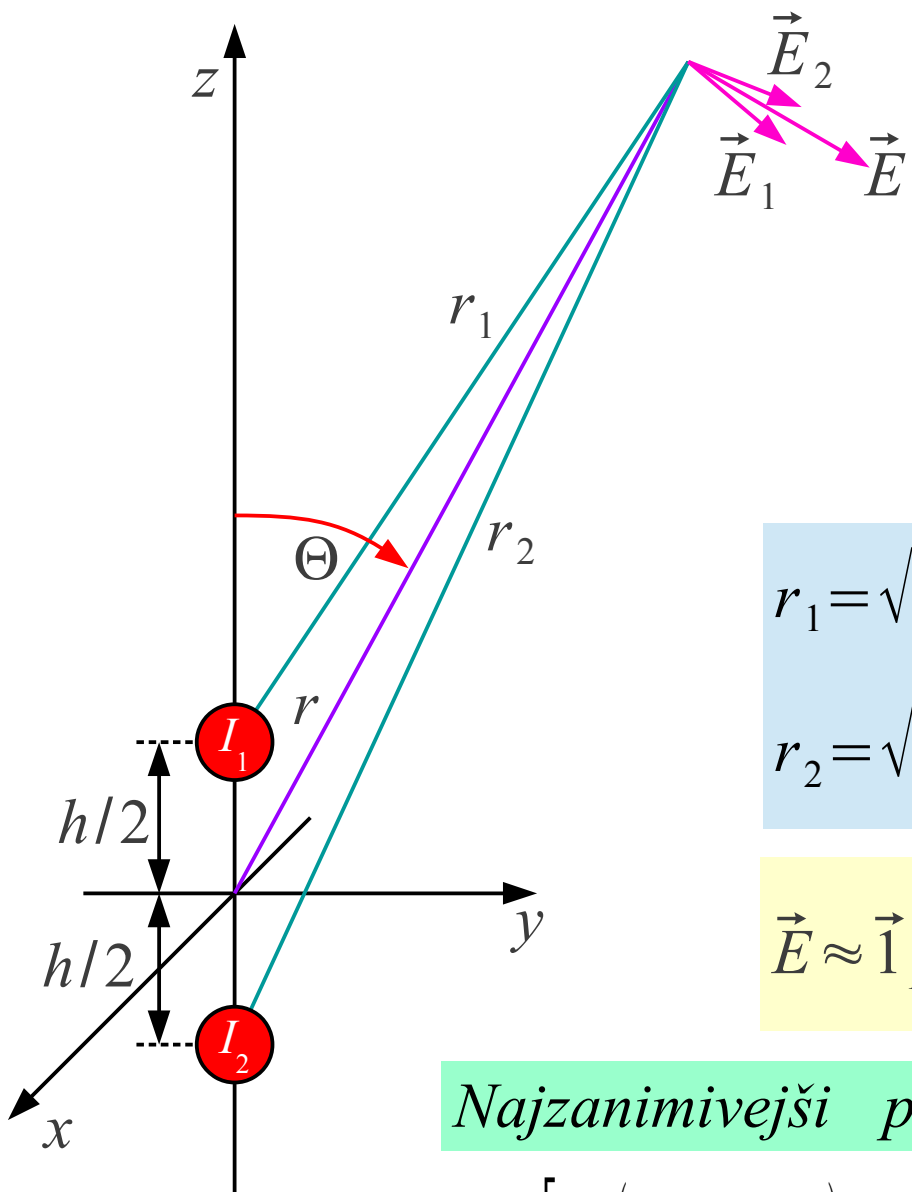
- (1) Skupina samih enakih anten
- (2) Vse antene enako orientirane
- (3) Vse antene enako polarizirane

$$F(\Theta, \Phi) = F_S(\Theta, \Phi) \cdot F_E(\Theta, \Phi)$$

Pravilo o množenju smernih diagramov

$$D \neq D_S \cdot D_E$$

Običajno $D_E, D_S < D < D_S \cdot D_E$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{1}_{E_1} \propto I_1 \frac{e^{-jkr_1}}{r_1} + \vec{1}_{E_2} \propto I_2 \frac{e^{-jkr_2}}{r_2}$$

Fraunhofer $r > \frac{2h^2}{\lambda}$

$$\vec{1}_{E_1} \approx \vec{1}_{E_2} \approx \vec{1}_E \quad \frac{1}{r_1} \approx \frac{1}{r_2} \approx \frac{1}{r}$$

$$r_1 = \sqrt{r^2 + (h/2)^2 - rh \cos \Theta} \approx r - \frac{h}{2} \cos \Theta$$

$$r_2 = \sqrt{r^2 + (h/2)^2 + rh \cos \Theta} \approx r + \frac{h}{2} \cos \Theta$$

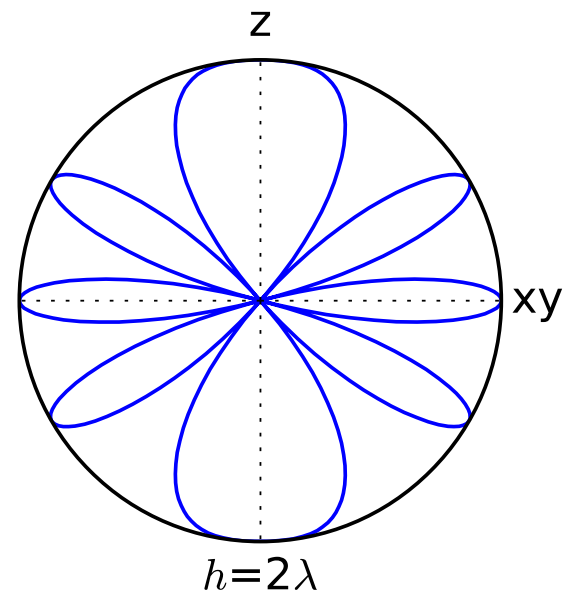
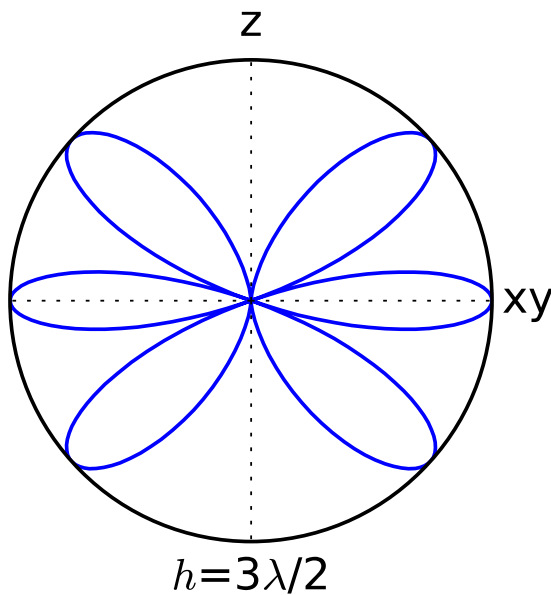
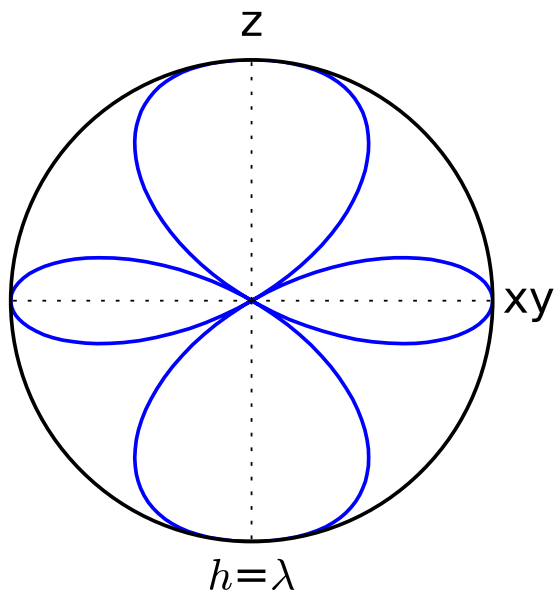
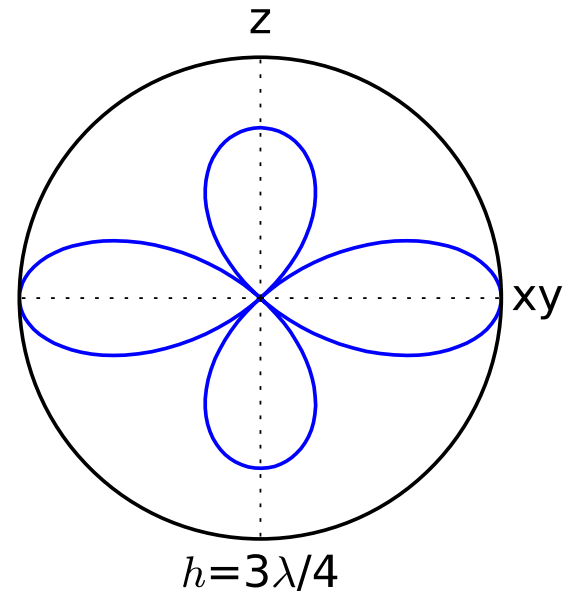
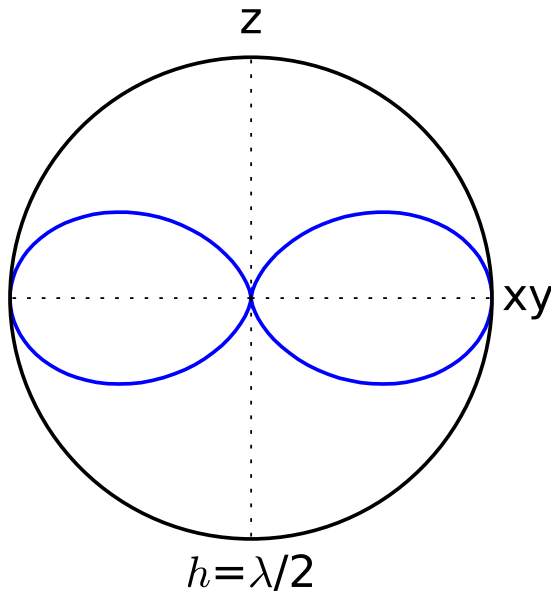
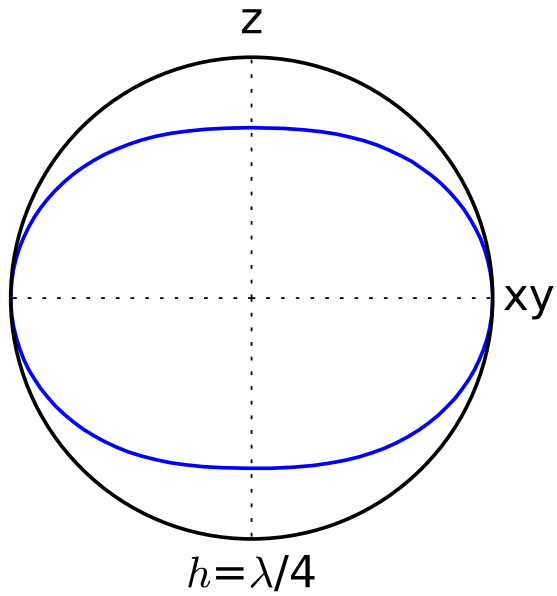
$$\vec{E} \approx \vec{1}_E \propto \frac{e^{-jkr}}{r} \left[I_1 e^{j\frac{kh}{2} \cos \Theta} + I_2 e^{-j\frac{kh}{2} \cos \Theta} \right]$$

Najzanimivejši primer $|I_1| = |I_2| \rightarrow I_1 = I e^{j\phi/2} \quad I_2 = I e^{-j\phi/2}$

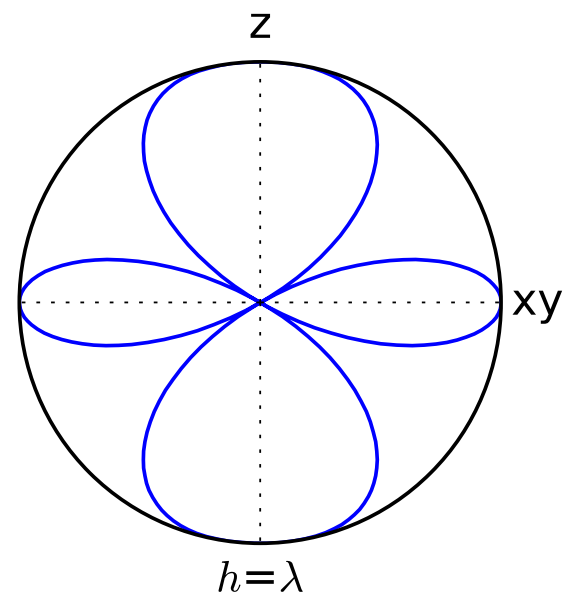
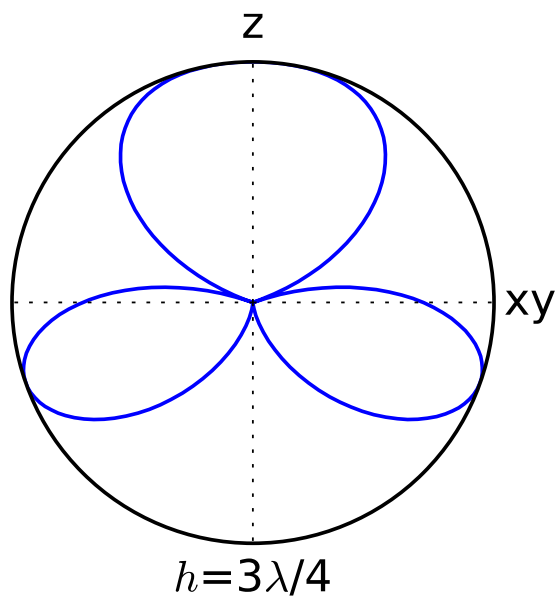
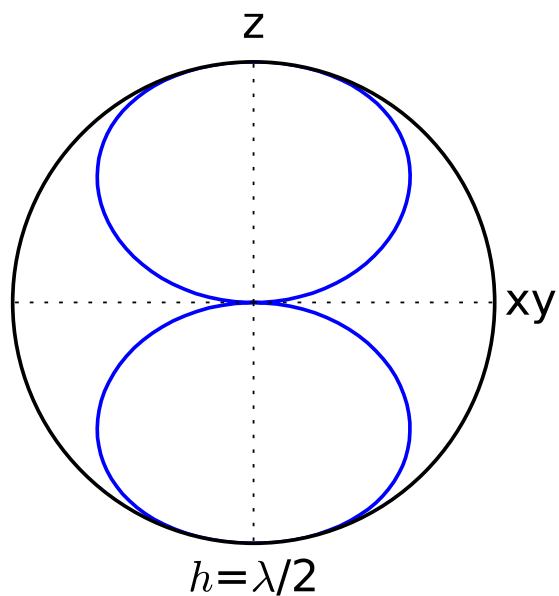
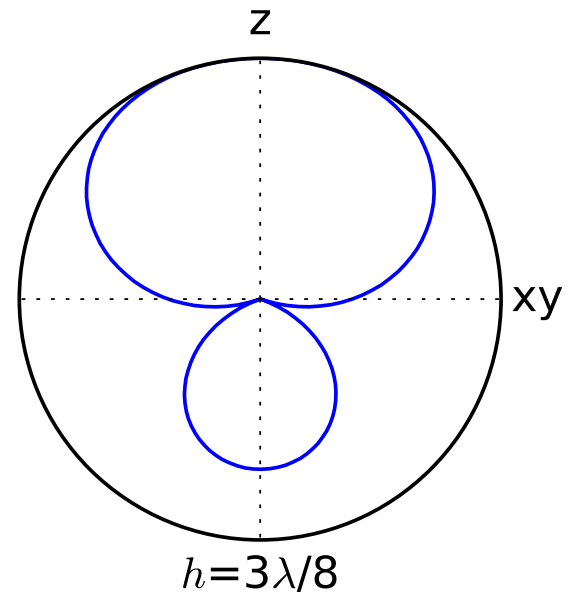
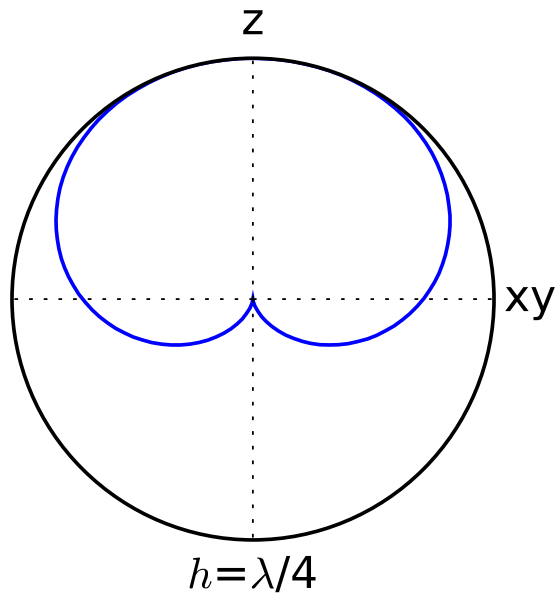
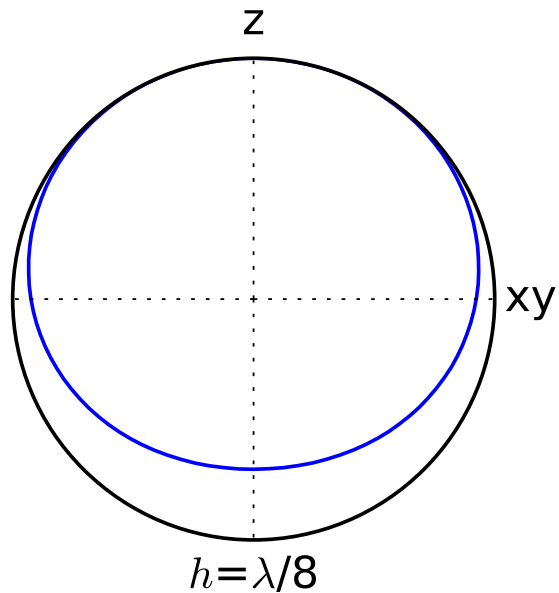
$$\vec{E} \approx \vec{1}_E \propto I \frac{e^{-jkr}}{r} \left[e^{j\left(\frac{\phi}{2} + \frac{kh}{2} \cos \Theta\right)} + e^{-j\left(\frac{\phi}{2} + \frac{kh}{2} \cos \Theta\right)} \right] = \vec{1}_E \propto I \frac{e^{-jkr}}{r} 2 \cos \left(\frac{\phi}{2} + \frac{kh}{2} \cos \Theta \right)$$

$$F(\Theta, \Phi) = \cos \left(\frac{\phi}{2} + \frac{kh}{2} \cos \Theta \right)$$

Dva neusmerjena (izotropna) vira



Smerni diagrami bočnih skupin $\phi = 0$



Smerni diagrami osnih skupin

$$\phi = -kh$$

$$D = \frac{4\pi |F(\Theta_{MAX}, \Phi_{MAX})|^2}{\oint_{4\pi} |F(\Theta, \Phi)|^2 d\Omega}$$

$$F(\Theta, \Phi) = \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \cos \Theta\right)$$

$$\oint_{4\pi} |F(\Theta, \Phi)|^2 d\Omega = \int_0^\pi \int_0^{2\pi} |F(\Theta, \Phi)|^2 \sin \Theta d\Theta d\Phi =$$

$$= \int_0^\pi \int_0^{2\pi} \left| \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \cos \Theta\right) \right|^2 \sin \Theta d\Theta d\Phi = 2\pi \int_0^\pi \left| \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \cos \Theta\right) \right|^2 \sin \Theta d\Theta =$$

$$= 2\pi \int_{-1}^1 \left[\cos\left(\frac{\Phi}{2} + \frac{khu}{2}\right) \right]^2 du = \pi \int_{-1}^1 [1 + \cos(\Phi + khu)] du =$$

$$= \pi \left[2 + \frac{\sin(\Phi + kh) - \sin(\Phi - kh)}{kh} \right] = 2\pi \left[1 + \frac{\sin(kh)}{kh} \cos \Phi \right]$$

$$D = \frac{2 |F(\Theta_{MAX}, \Phi_{MAX})|^2}{1 + \frac{\sin(kh)}{kh} \cos \Phi}$$

$$F(\Theta_{MAX} = \pi/2, \Phi_{MAX}) = 1$$

Bočna skupina $\rightarrow \Phi = 0$

$$D = \frac{2}{1 + \frac{\sin(kh)}{kh}}$$

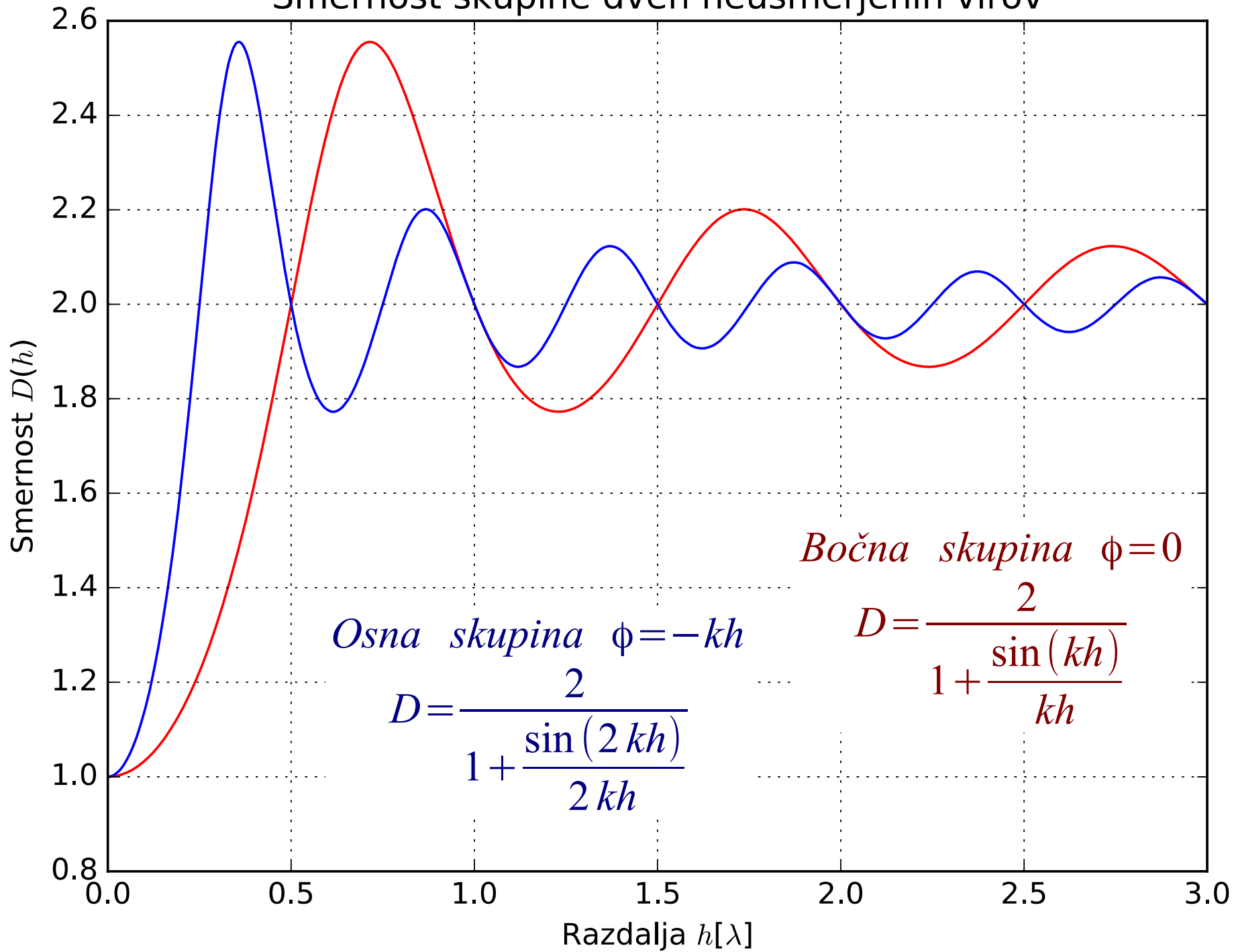
$$F(\Theta_{MAX} = 0, \Phi_{MAX}) = 1$$

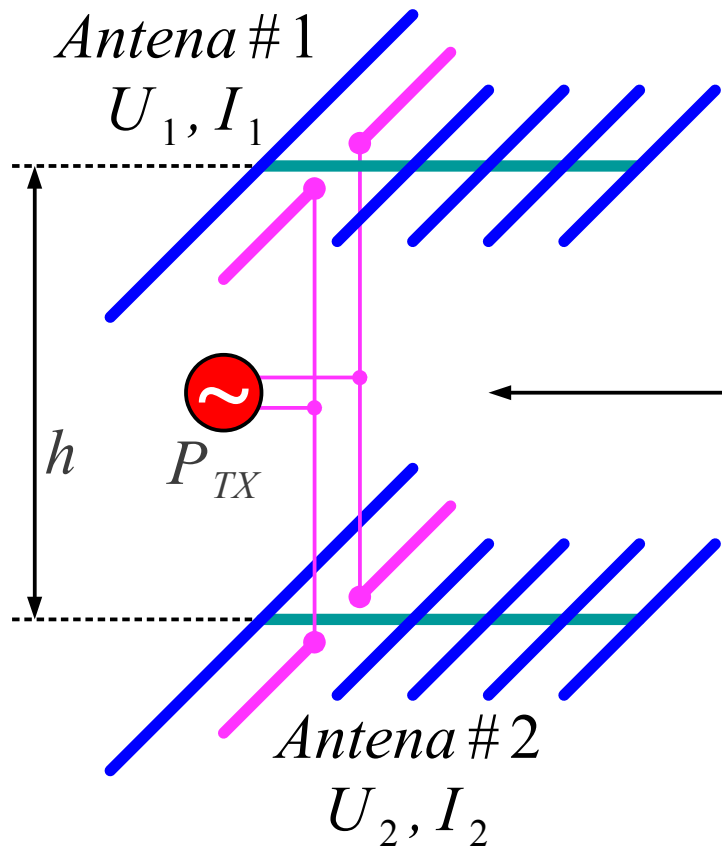
Osna skupina $\rightarrow \Phi = -kh$

$$D = \frac{2}{1 + \frac{\sin(2kh)}{2kh}}$$

Smernost dveh virov

Smernost skupine dveh neusmerjenih virov





Oddaja z anteno#1 $I_2=0$

$$P_{TXI} = \frac{1}{2} \operatorname{Re}[U_1 I_1^*] = \frac{1}{2} \operatorname{Re}[Z_{11}] |I_1|^2 \rightarrow P_{RXI}$$

Oddaja z dvema antenama $I_1=I_2$

$$P_{TX} = 2 \cdot \frac{1}{2} \operatorname{Re}[U_1 I_1^*] = \operatorname{Re}[Z_{11} + Z_{12}] |I_1|^2$$

$$\vec{E}_{RX} = 2 \vec{E}_{RXI} \rightarrow P_{RX} = 4 P_{RXI}$$

Medsebojni vpliv v skupini

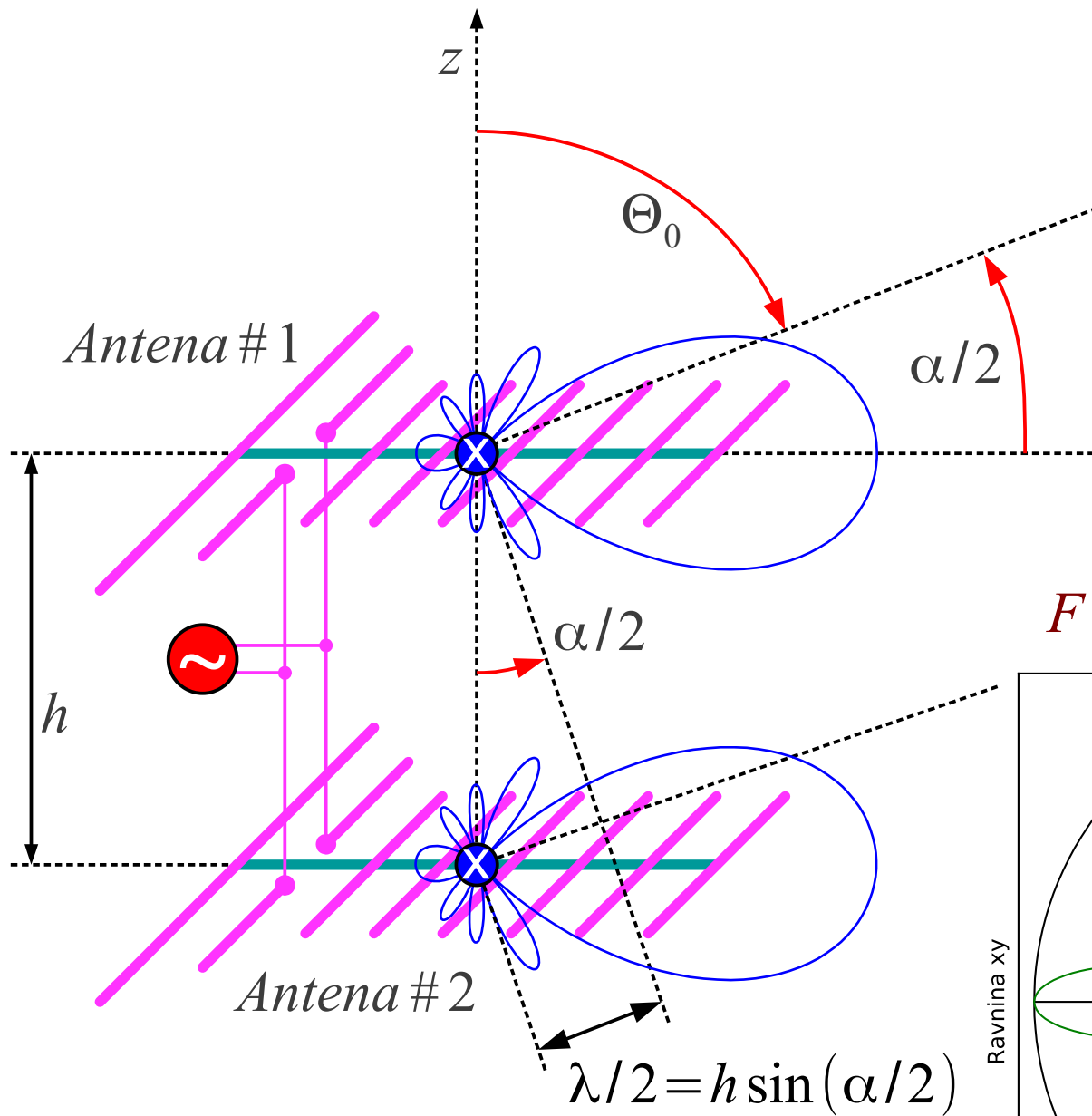
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Recipročnost $Z_{12}(h) = Z_{21}(h)$

Enaki anteni $Z_{11} = Z_{22}$

$$\Delta D = \frac{\left(\frac{P_{RX}}{P_{RXI}} \right)}{\left(\frac{P_{TX}}{P_{TXI}} \right)} = \frac{4}{\frac{\operatorname{Re}[Z_{11} + Z_{12}] |I_1|^2}{\frac{1}{2} \operatorname{Re}[Z_{11}] |I_1|^2}} = \frac{2 \operatorname{Re}[Z_{11}]}{\operatorname{Re}[Z_{11} + Z_{12}]}$$

Medsebojna impedanca v bočni skupini

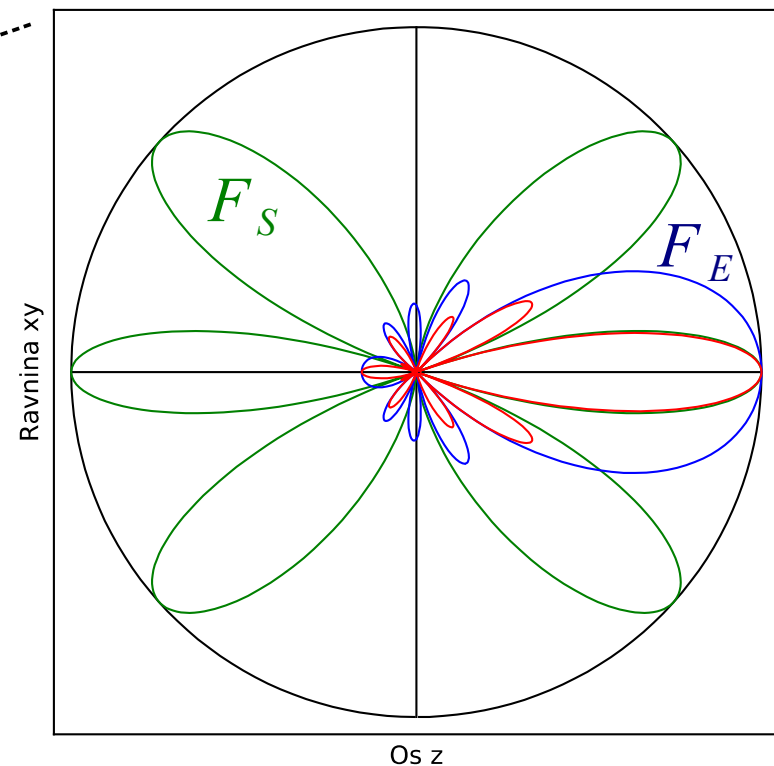


Ničla skupine

$$F_S = \cos\left(\frac{kh}{2} \cos \Theta_0\right) = 0$$

$$\frac{\alpha}{2} + \Theta_0 = \frac{\pi}{2}$$

$$F(\Theta, \Phi) = F_S(\Theta, \Phi) \cdot F_E(\Theta, \Phi)$$



Približno pravilo za bočno skupino

Osna skupina $|F(\Theta_{MAX}=0, \Phi_{MAX})| < 1$

$$F(\Theta, \Phi) = \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \cos \Theta\right)$$

$$D = \frac{2|F(\Theta_{MAX}, \Phi_{MAX})|^2}{1 + \frac{\sin(kh)}{kh} \cos \phi} = \frac{2\left|\cos\left(\frac{\phi}{2} + \frac{kh}{2}\right)\right|^2}{1 + \frac{\sin(kh)}{kh} \cos \phi} = \frac{1 + \cos(\phi + kh)}{1 + \frac{\sin(kh)}{kh} \cos \phi}$$

$$\frac{\partial D}{\partial \phi} = 0 = \frac{-\sin(\phi + kh) \left[1 + \frac{\sin(kh)}{kh} \cos \phi\right] - \left[1 + \cos(\phi + kh)\right] \left[-\frac{\sin(kh)}{kh} \sin \phi\right]}{\left[1 + \frac{\sin(kh)}{kh} \cos \phi\right]^2}$$

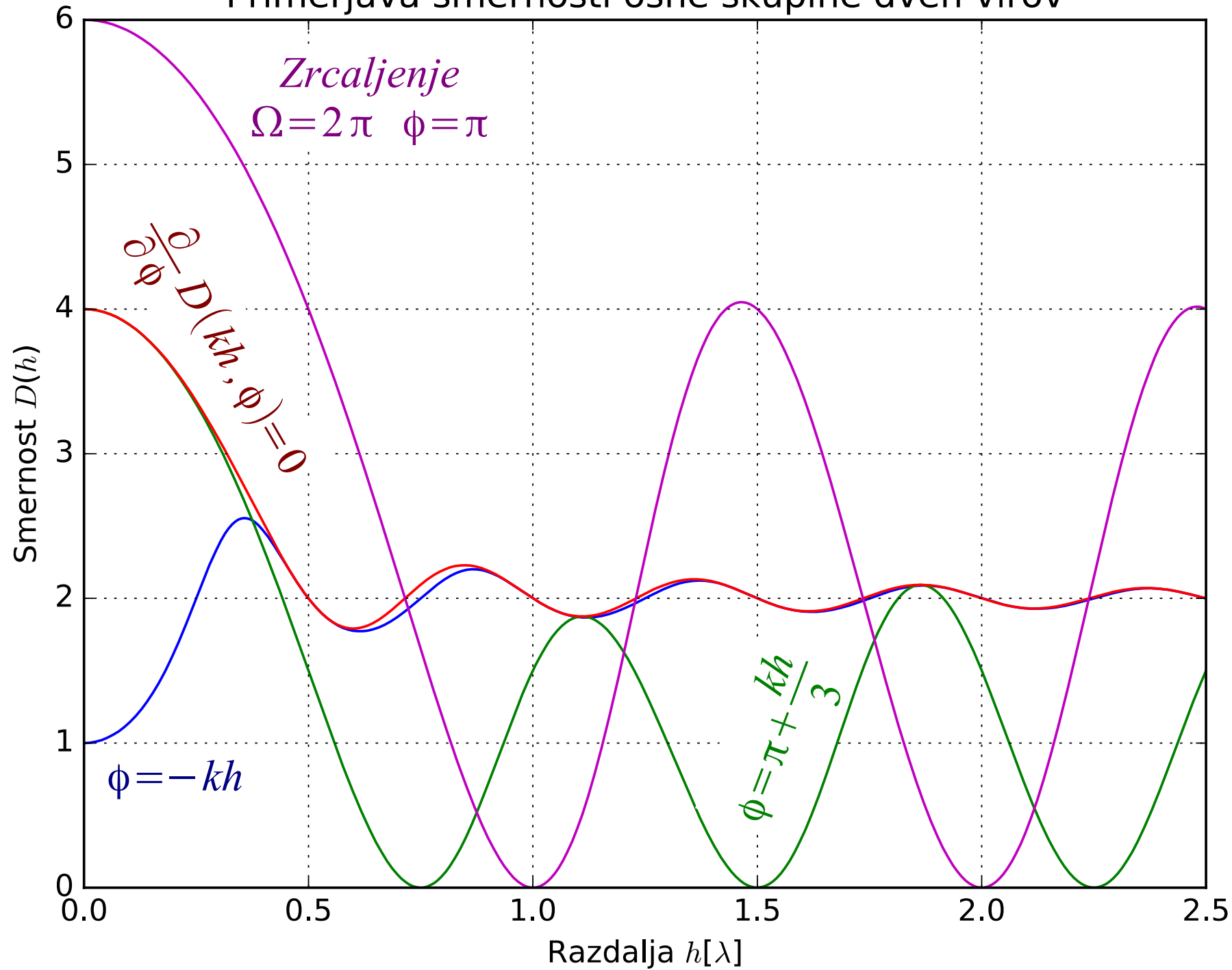
$$0 = \left[\frac{\sin^2(kh)}{(kh)^2} - 2\frac{\sin(kh)}{kh} \cos(kh) + 1\right] \sin^2 \phi -$$
$$- 2\frac{\sin^2(kh)}{kh} \left[\frac{\sin(kh)}{kh} - \cos(kh)\right] \sin \phi + \left[\frac{\sin^4(kh)}{(kh)^2} - \sin^2(kh)\right]$$

$u = \sin \phi \rightarrow \phi = \arcsin u$ ali $\phi = \pi - \arcsin u$

Največja smernost osne skupine

Približek $h < \frac{\lambda}{4} \rightarrow \phi \approx \pi + \frac{kh}{3}$

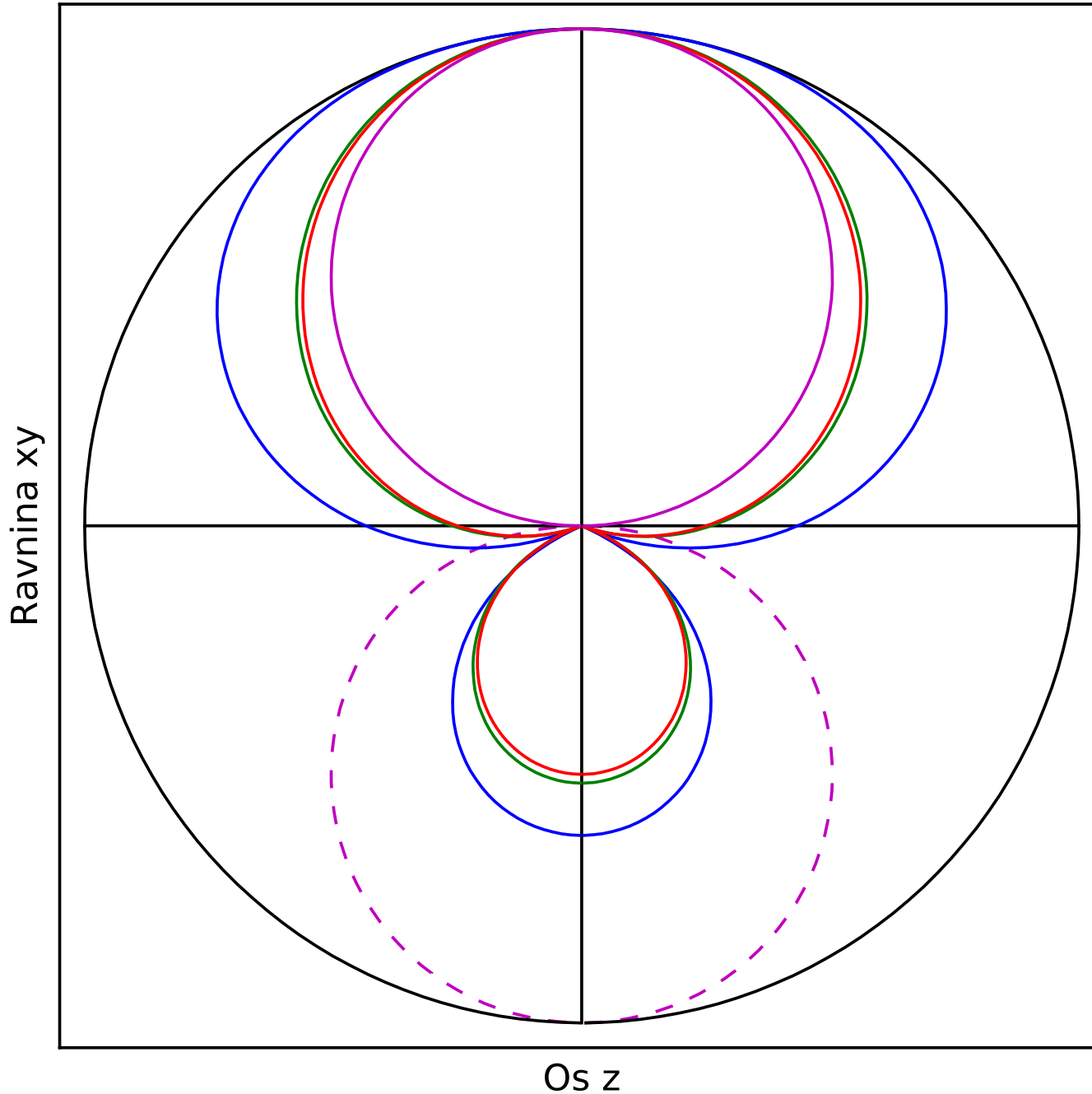
Primerjava smernosti osne skupine dveh virov



Primerjava smernih diagramov osnih skupin

$h = 0.357\lambda$
 $\phi = -kh$
 $D = 2.56$

$h = 0.001\lambda$
 $\phi = \pi + \frac{kh}{3}$
 $D = 4.00$

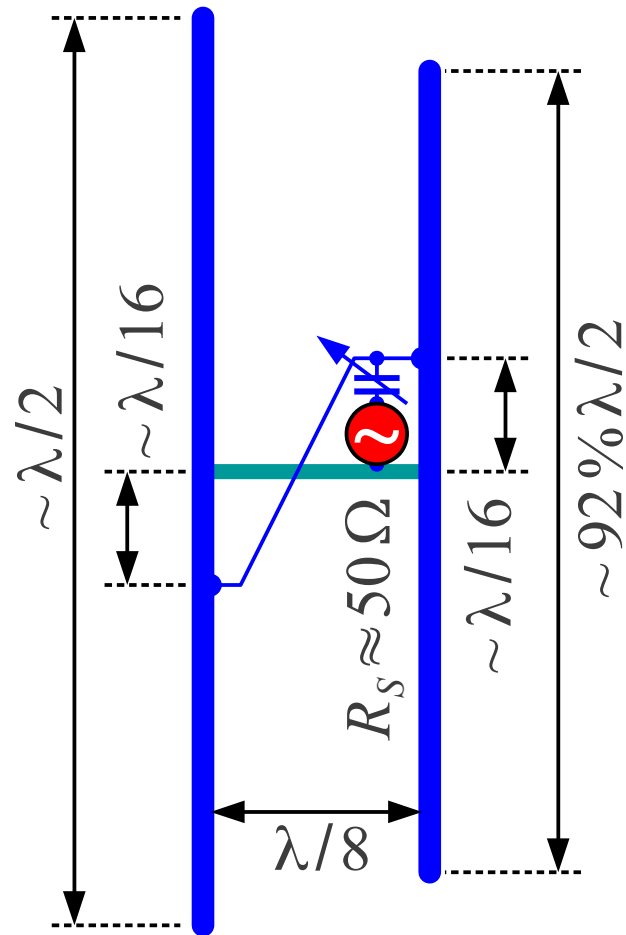


$h = \lambda/8$
 $\phi = \pi + \frac{kh}{3}$
 $D = 3.84$

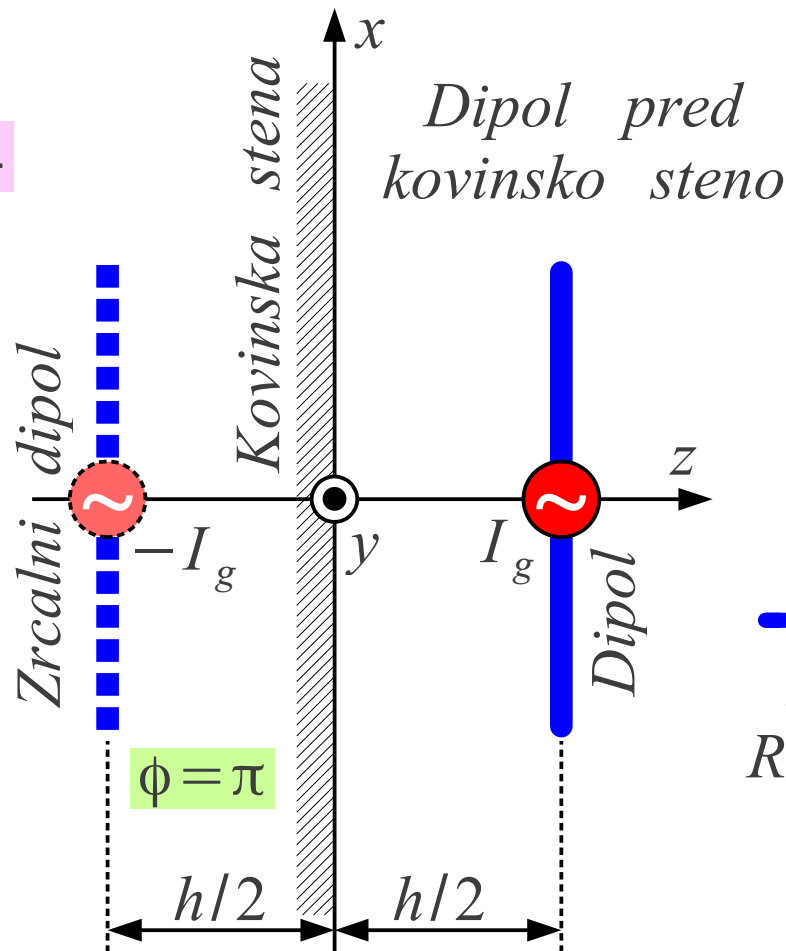
Zrcaljenje
 $h = 0.1\lambda$
 $\phi = \pi$
 $D = 5.92$

Rudolf Baumgartner ~ 1954

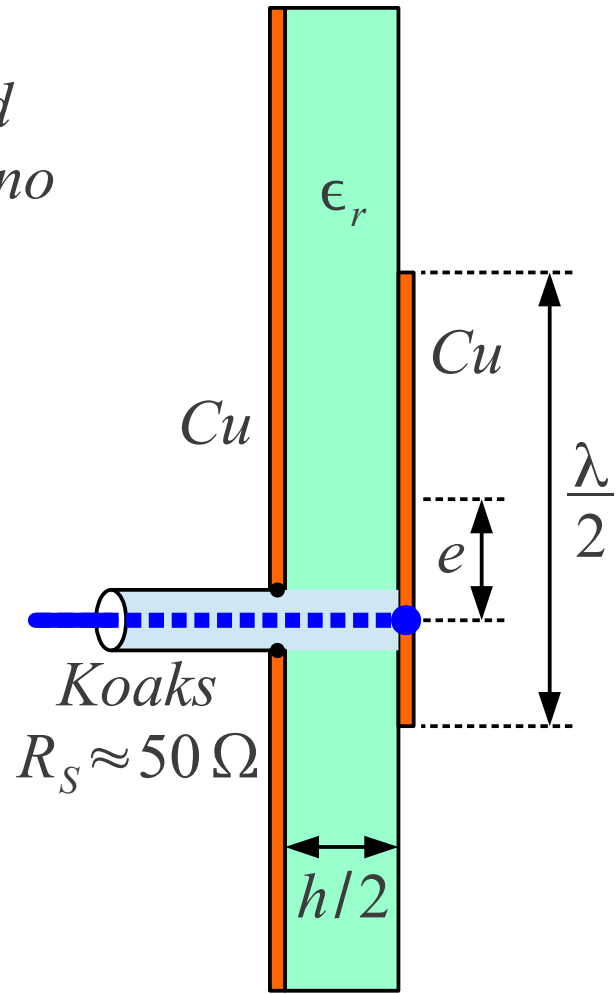
Antena HB9CV



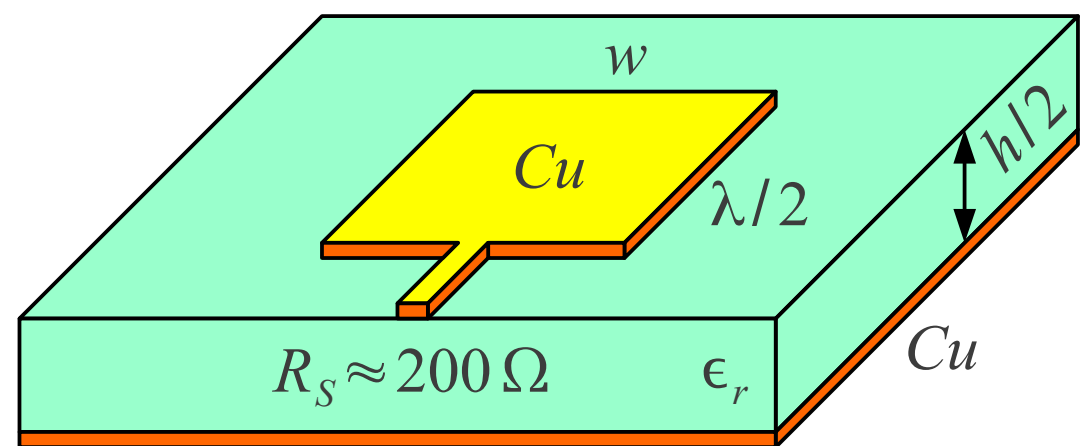
Izvedbe osnih skupin

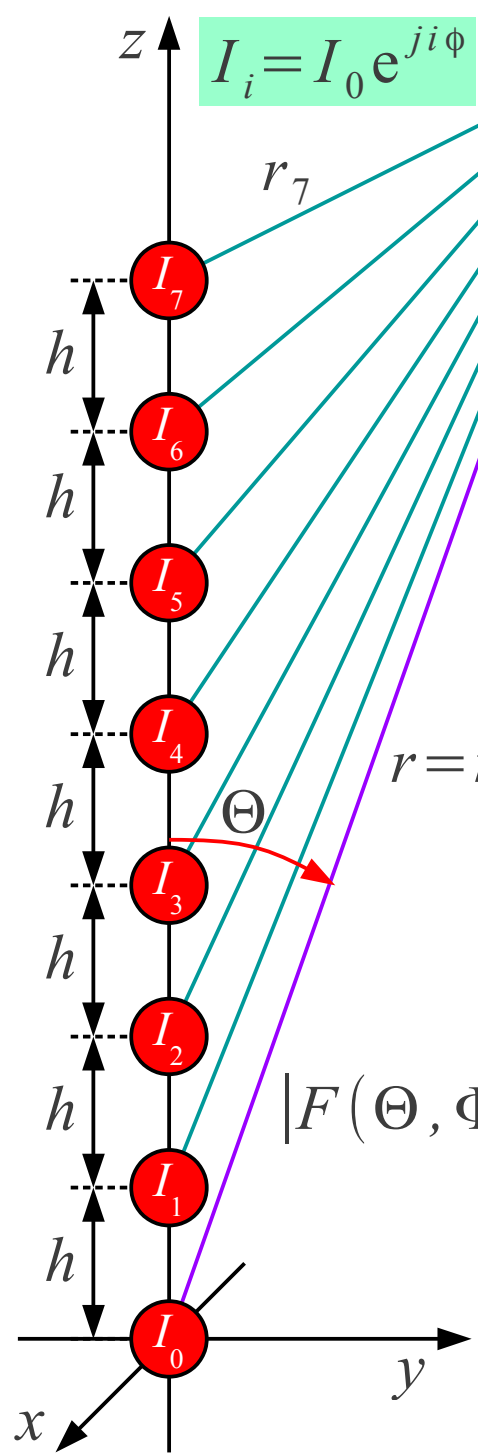


Dipol pred kovinsko steno



Mikrotrakasta krpica





$$I_i = I_0 e^{ji\phi}$$

$$\vec{E} = \sum_{i=0}^{m-1} \vec{E}_i$$

$$\vec{E}_i = \vec{1}_{E_i} \alpha I_i \frac{e^{-jkr_i}}{r_i} \approx \vec{1}_E \frac{\alpha I_0}{r} e^{ji\phi} e^{-jkr_i}$$

$$r_i = \sqrt{r^2 + (ih)^2 - 2rih \cos \Theta} \approx r - ih \cos \Theta$$

$$\vec{E} \approx \vec{1}_E \frac{\alpha I_0}{r} \sum_{i=0}^{m-1} e^{-j(kr_i - i\phi)} \approx \vec{1}_E \alpha I_0 \frac{e^{-jkr}}{r} \sum_{i=0}^{m-1} e^{ji(\phi + kh \cos \Theta)}$$

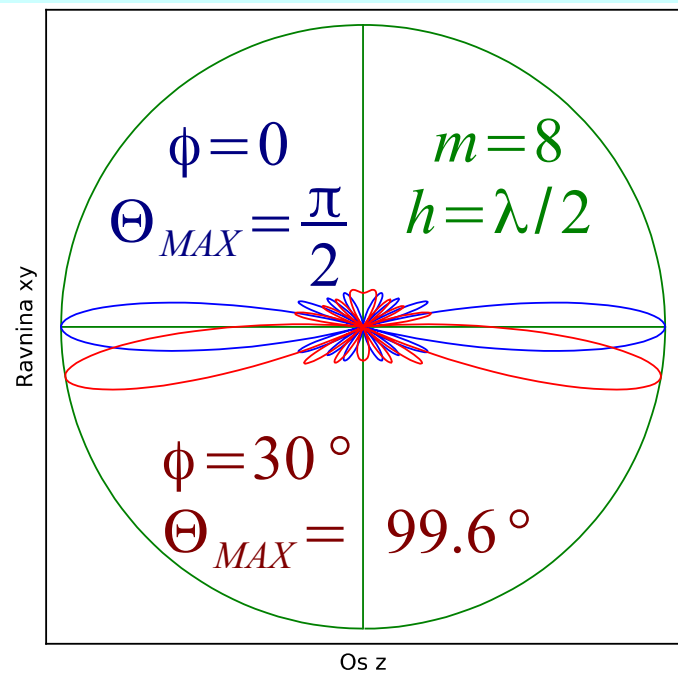
$$b = e^{j(\phi + kh \cos \Theta)}$$

$$|b| = 1$$

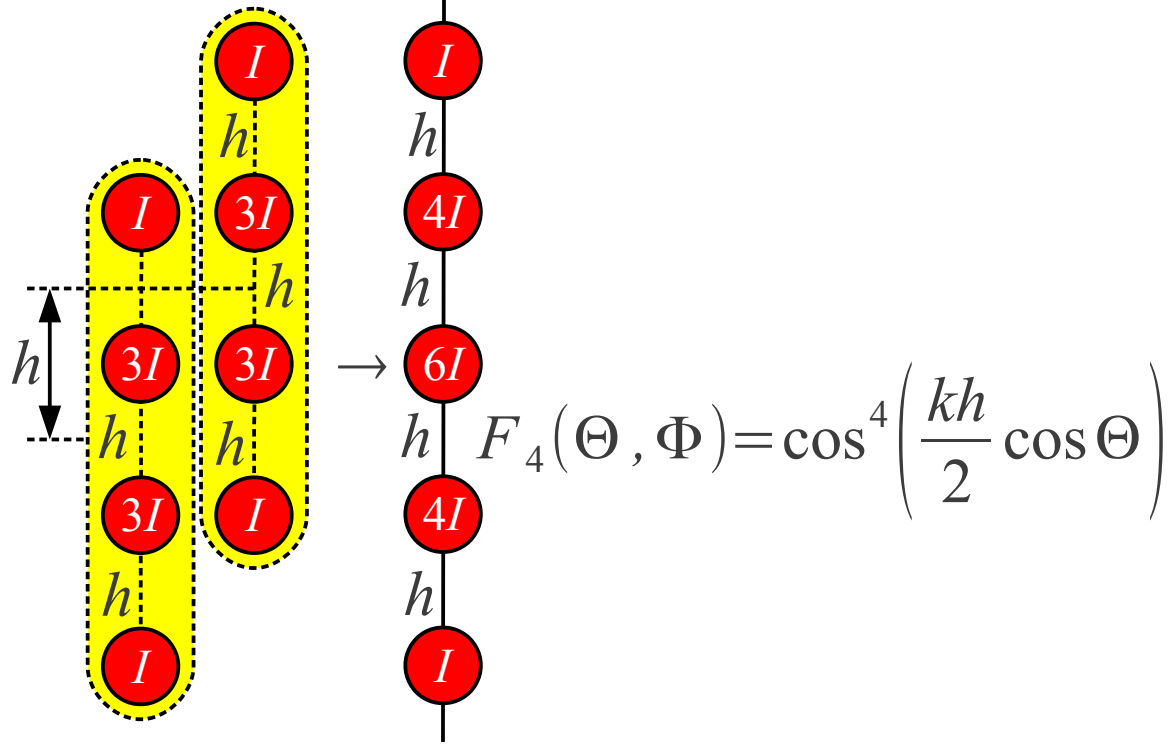
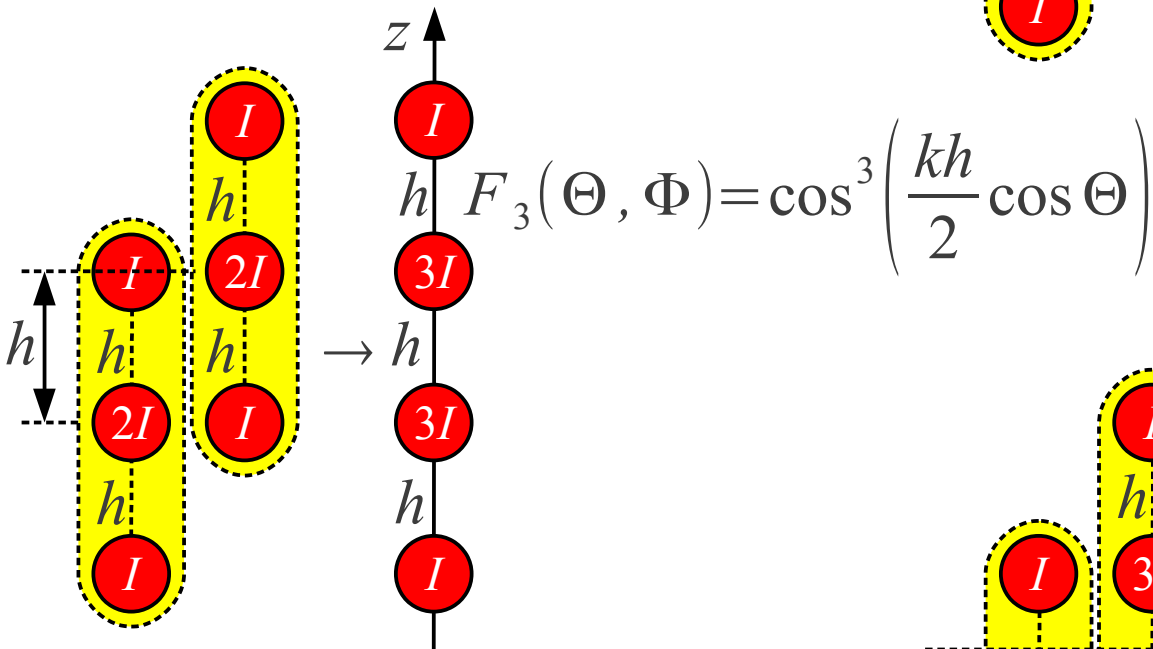
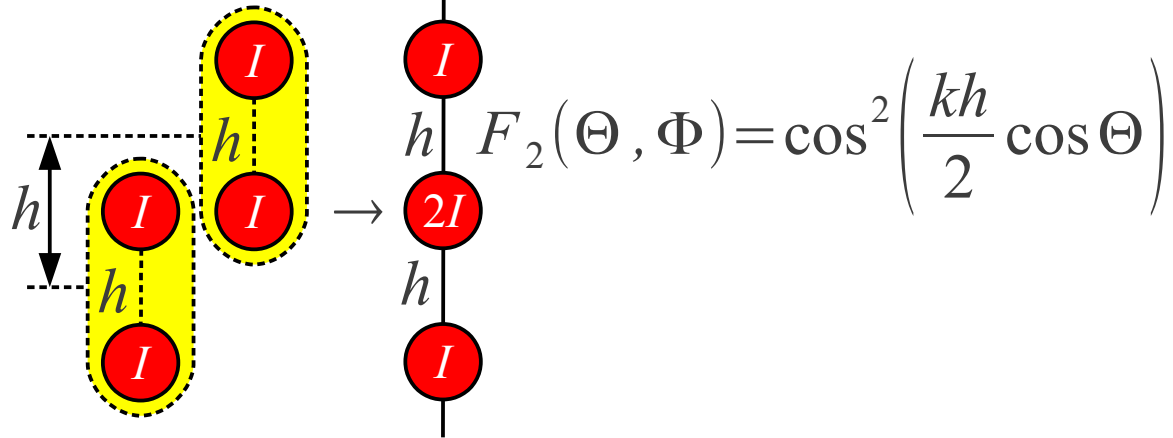
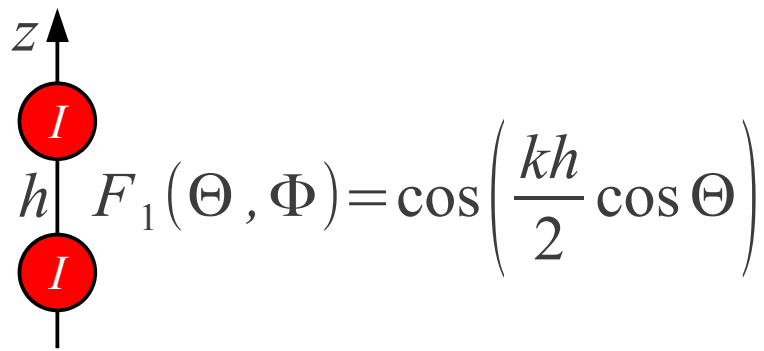
$$\sum_{i=0}^{m-1} b^i = \frac{b^m - 1}{b - 1} = \left(b^{\frac{m-1}{2}} \right) \frac{\sin \left(m \frac{\phi + kh \cos \Theta}{2} \right)}{\sin \left(\frac{\phi + kh \cos \Theta}{2} \right)}$$

$$|F(\Theta, \Phi)| = \frac{1}{m} \left| \frac{\sin \left(m \frac{\phi + kh \cos \Theta}{2} \right)}{\sin \left(\frac{\phi + kh \cos \Theta}{2} \right)} \right|$$

$$\Theta_{MAX} = \arccos \left(\frac{-\phi}{kh} \right)$$



Enakomerna skupina izvorov

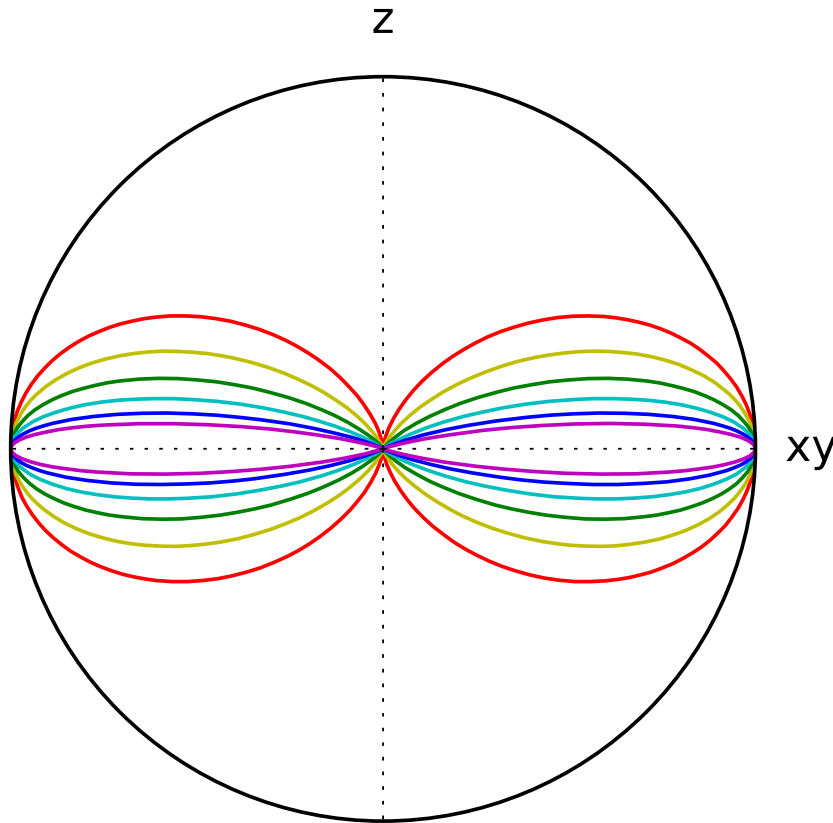


$$F_N(\Theta, \Phi) = \cos^N\left(\frac{kh}{2} \cos \Theta\right)$$

Binomska bočna skupina

Bočna binomska skupina

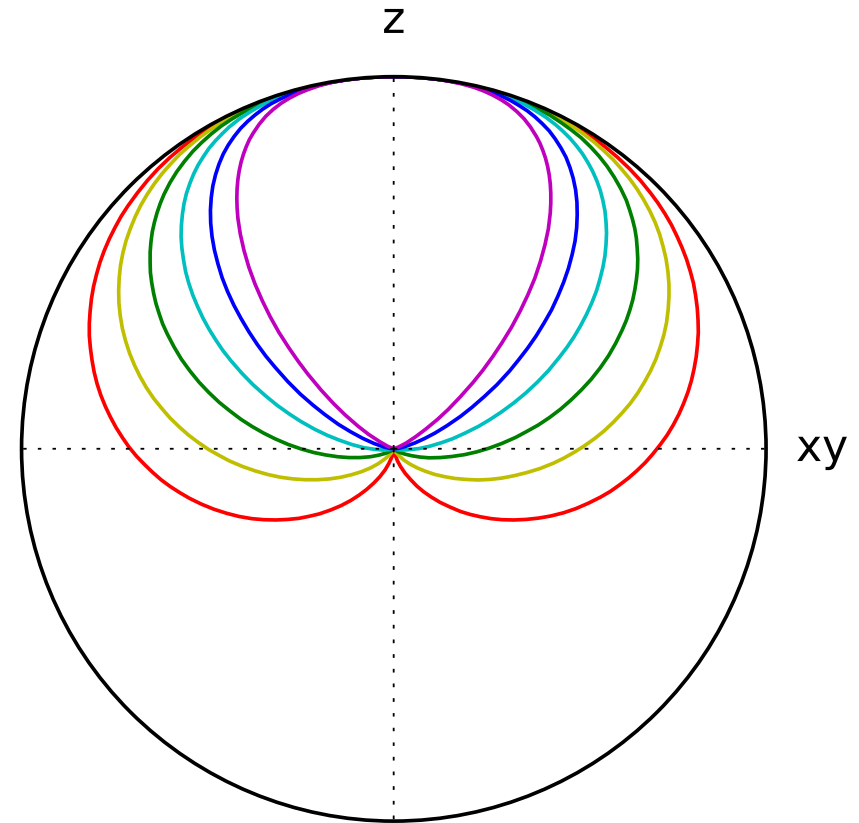
$$F_N(\Theta, \Phi) = \cos^N \left(\frac{kh}{2} \cos \Theta \right)$$



$h = \lambda/2 \quad \phi = 0 \quad N = 1, 2, 4, 8, 16, 32$

Osna binomska skupina

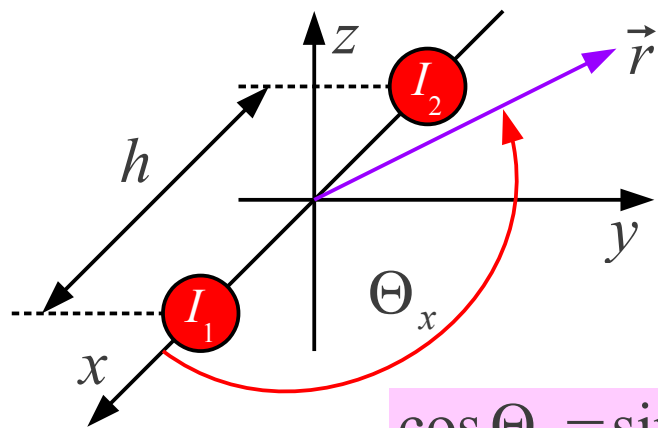
$$F_N(\Theta, \Phi) = \cos^N \left(\frac{kh}{2} (\cos \Theta - 1) \right)$$



$h = \lambda/4 \quad \phi = -kh \quad N = 1, 2, 4, 8, 16, 32$

Smerni diagrami binomskih skupin

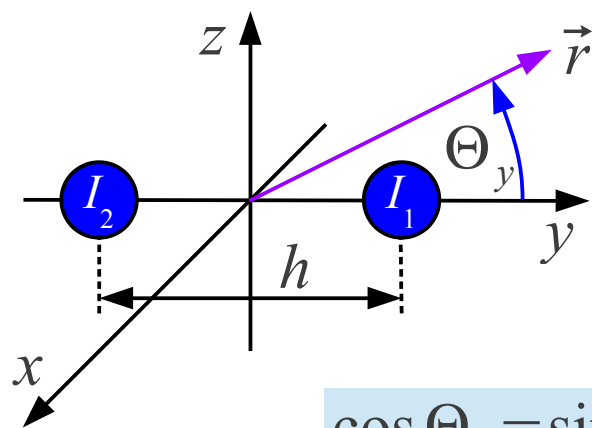
$$F_S(\Theta, \Phi) = \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \cos \Theta_x\right)$$



$$\cos \Theta_x = \sin \Theta \cos \Phi$$

$$F_S(\Theta, \Phi) = \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \sin \Theta \cos \Phi\right)$$

$$F_S(\Theta, \Phi) = \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \cos \Theta_y\right)$$



$$\cos \Theta_y = \sin \Theta \sin \Phi$$

$$F_S(\Theta, \Phi) = \cos\left(\frac{\Phi}{2} + \frac{kh}{2} \sin \Theta \sin \Phi\right)$$

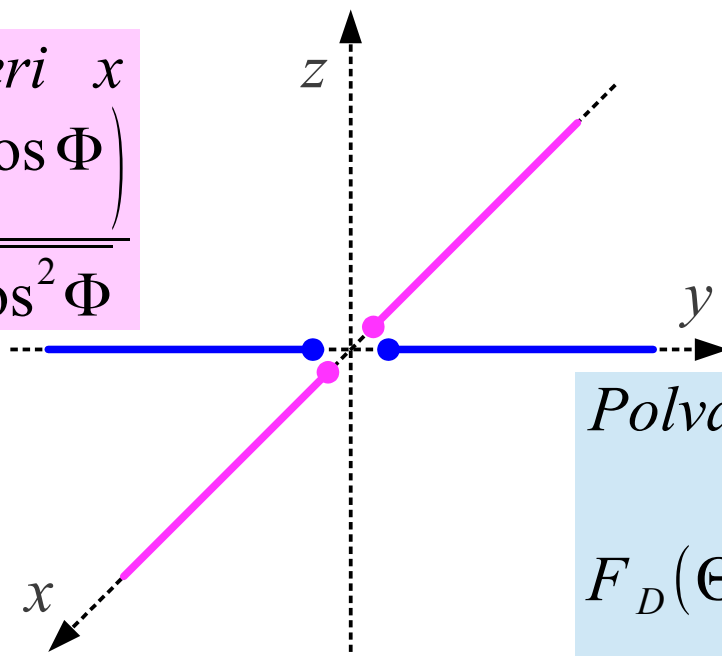
Polvalovni dipol v smeri x

$$\cos\left(\frac{\pi}{2} \sin \Theta \cos \Phi\right)$$

$$F_D(\Theta, \Phi) = \frac{\cos\left(\frac{\pi}{2} \sin \Theta \cos \Phi\right)}{\sqrt{1 - \sin^2 \Theta \cos^2 \Phi}}$$

$$\sin \Theta_x = \sqrt{1 - \sin^2 \Theta \cos^2 \Phi}$$

Obračanje skupin

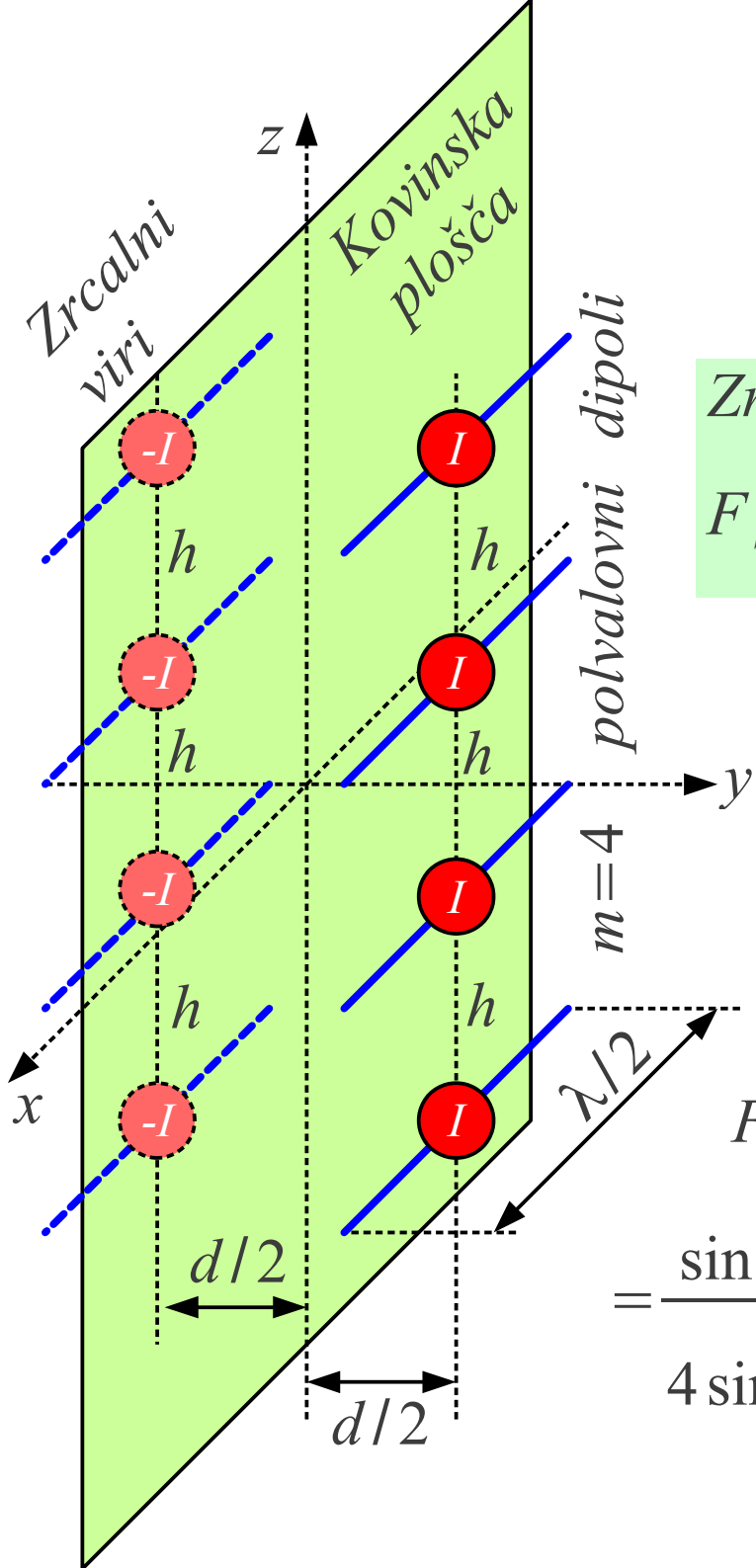


$$\sin \Theta_y = \sqrt{1 - \sin^2 \Theta \sin^2 \Phi}$$

Polvalovni dipol v smeri y

$$\cos\left(\frac{\pi}{2} \sin \Theta \sin \Phi\right)$$

$$F_D(\Theta, \Phi) = \frac{\cos\left(\frac{\pi}{2} \sin \Theta \sin \Phi\right)}{\sqrt{1 - \sin^2 \Theta \sin^2 \Phi}}$$



Polvalovni dipol v smeri osi x

$$F_E(\Theta, \Phi) = \frac{\cos\left(\frac{\pi}{2} \cos \Theta_x\right)}{\sin \Theta_x} = \frac{\cos\left(\frac{\pi}{2} \sin \Theta \cos \Phi\right)}{\sqrt{1 - \sin^2 \Theta \cos^2 \Phi}}$$

Zrcaljenje v smeri osi y → φ = -π

$$F_{SI}(\Theta, \Phi) = \cos\left(\frac{\phi}{2} + \frac{kd}{2} \cos \Theta_y\right) = \sin\left(\frac{kd}{2} \sin \Theta \sin \phi\right)$$

Enakomerna bočna skupina v smeri osi z

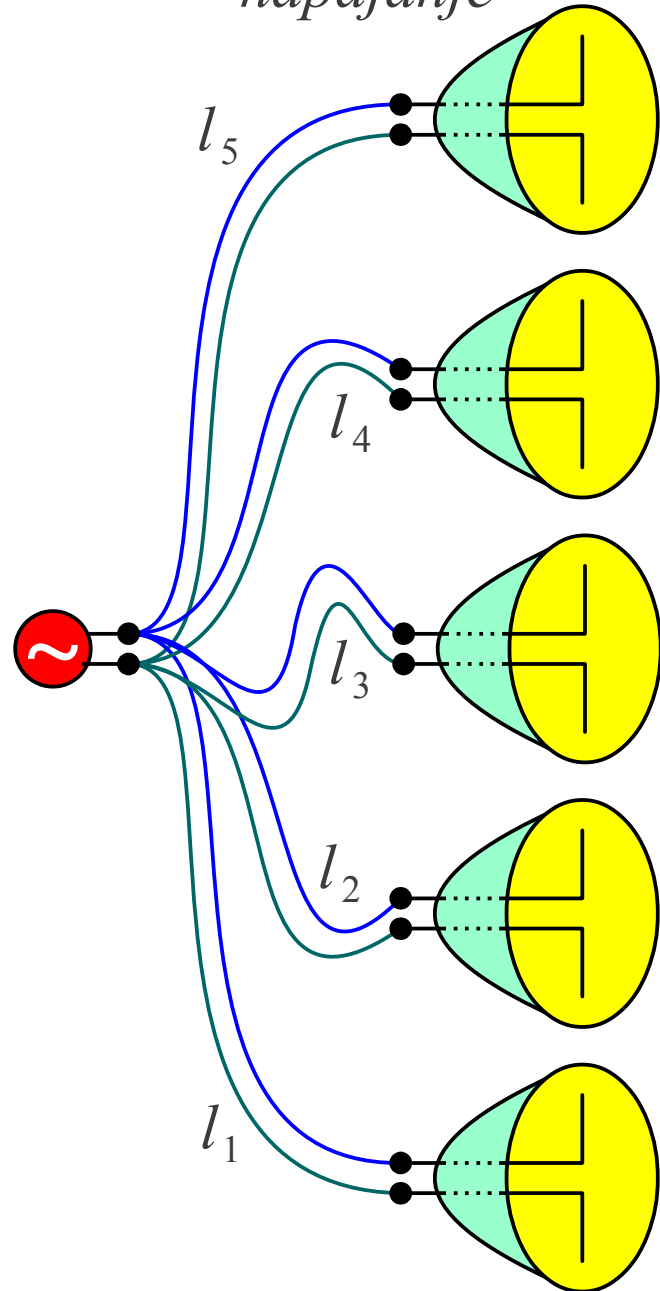
$$F_{S2}(\Theta, \Phi) = \frac{\sin\left(m \frac{kh}{2} \cos \Theta\right)}{m \sin\left(\frac{kh}{2} \cos \Theta\right)} = \frac{\sin(2kh \cos \Theta)}{4 \sin\left(\frac{kh}{2} \cos \Theta\right)}$$

$$F(\Theta, \Phi) = F_{S2}(\Theta, \Phi) \cdot F_{SI}(\Theta, \Phi) \cdot F_E(\Theta, \Phi) =$$

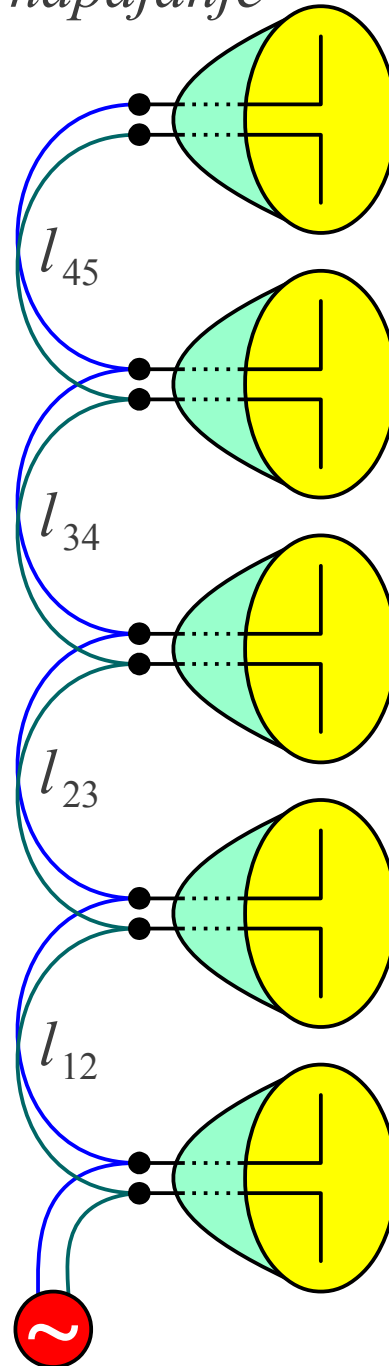
$$= \frac{\sin(2kh \cos \Theta)}{4 \sin\left(\frac{kh}{2} \cos \Theta\right)} \sin\left(\frac{kd}{2} \sin \Theta \sin \phi\right) \frac{\cos\left(\frac{\pi}{2} \sin \Theta \cos \Phi\right)}{\sqrt{1 - \sin^2 \Theta \cos^2 \Phi}}$$

Sestavljanje skupin

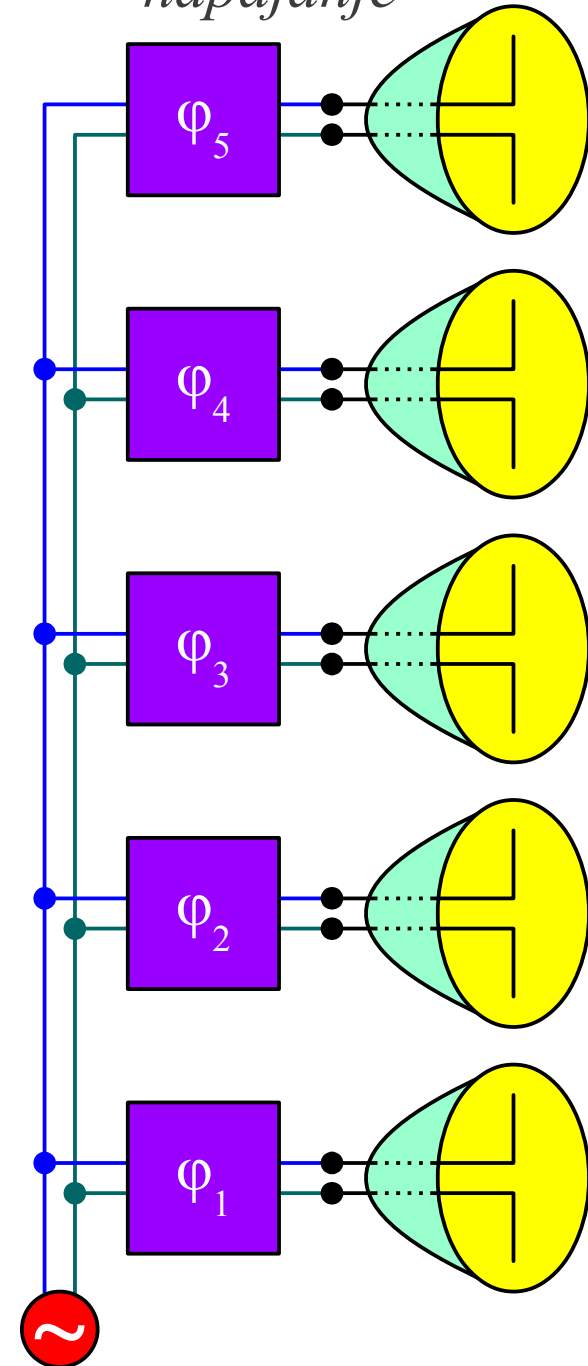
Vzporedno napajanje



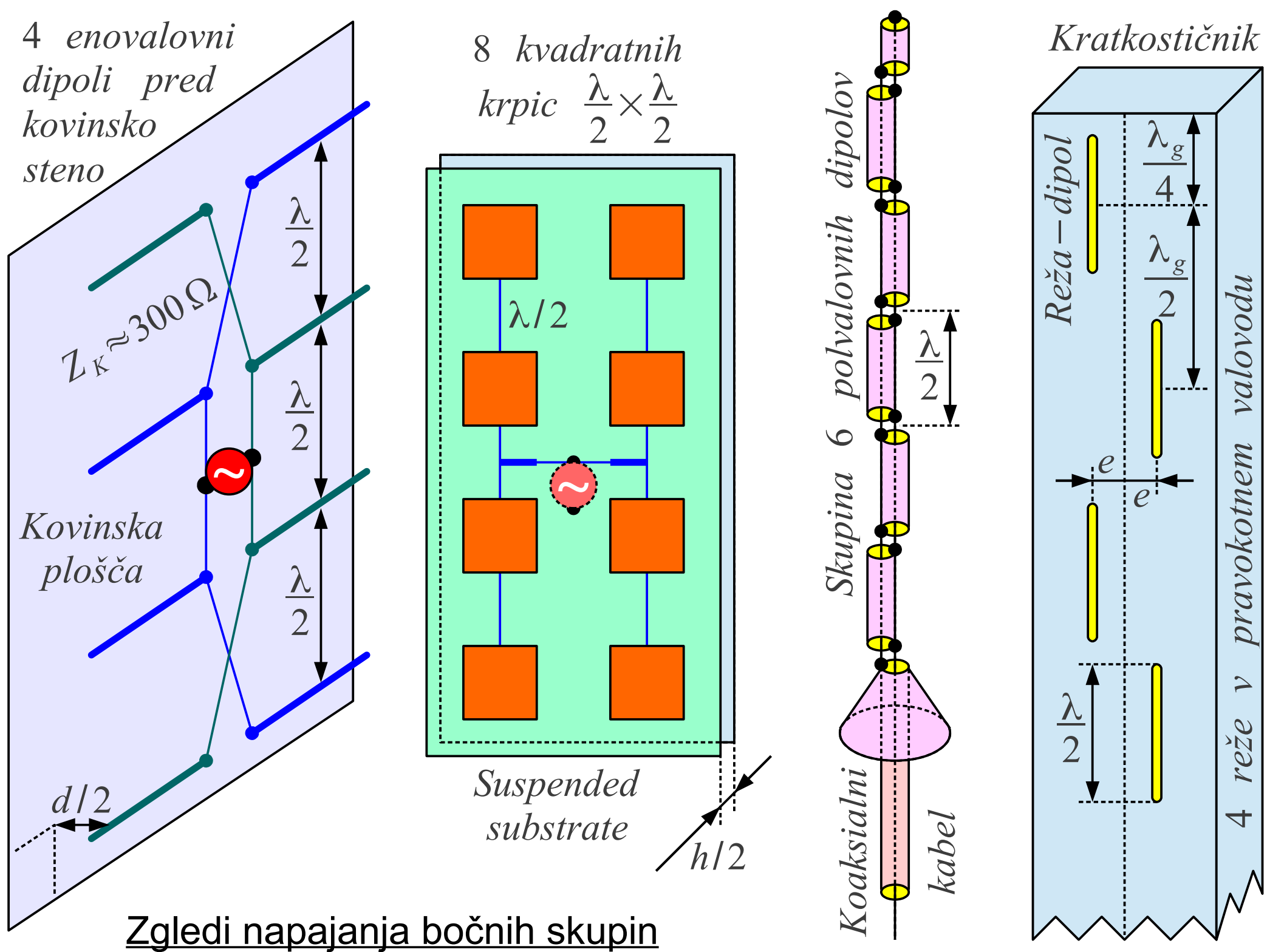
Zaporedno napajanje

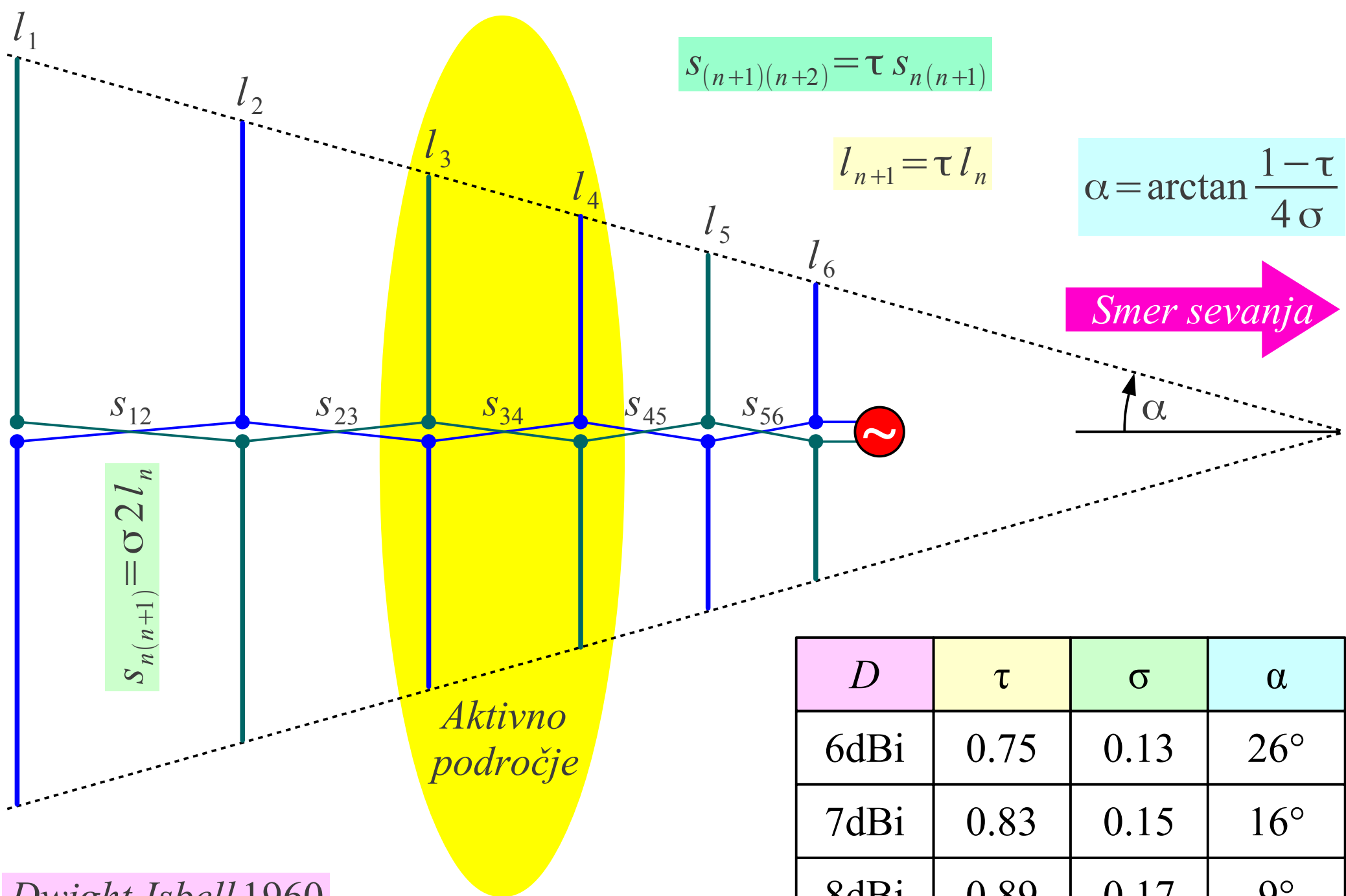


Nastavljivo napajanje



Napajanje skupine anten





Dwight Isbell 1960

Logaritmično-periodična skupina dipolov

D	τ	σ	α
6dBi	0.75	0.13	26°
7dBi	0.83	0.15	16°
8dBi	0.89	0.17	9°
9dBi	0.94	0.19	5°