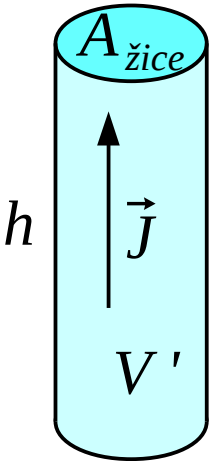


Žica



$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{V'} \vec{J}(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV'$$

$$\vec{J}(\vec{r}') = \vec{1}_z \frac{I}{A_{zice}}$$

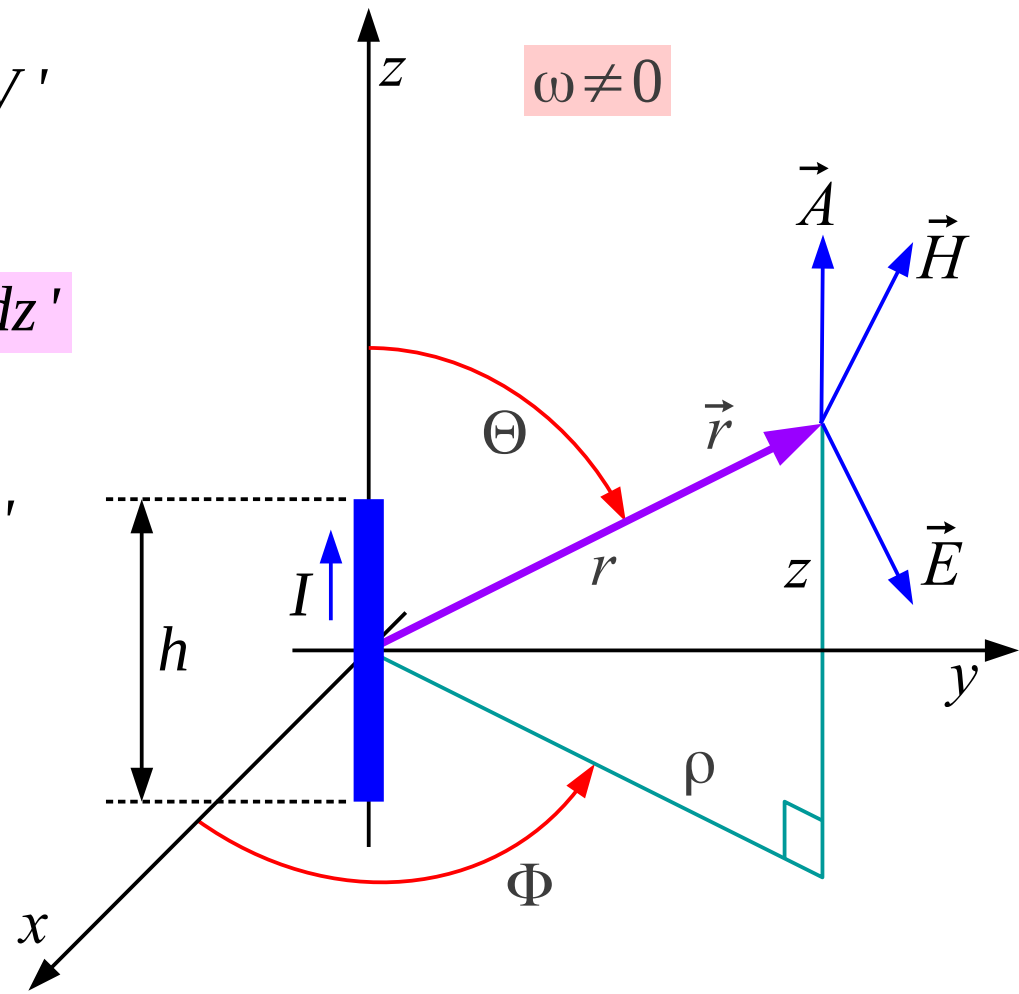
$$dV' = A_{zice} dz'$$

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{-h/2}^{h/2} \vec{1}_z I \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dz'$$

Poenostavitve:

(1)  $h \ll r \rightarrow \frac{1}{|\vec{r}-\vec{r}'|} \approx \frac{1}{r}$

(2)  $h \ll \lambda = \frac{2\pi}{k} \rightarrow e^{-jk|\vec{r}-\vec{r}'|} \approx e^{-jkr}$



$$\vec{A}(\vec{r}) = \vec{1}_z \frac{\mu I h e^{-jkr}}{4\pi r} = (\vec{1}_r \cos \Theta - \vec{1}_\Theta \sin \Theta) \frac{\mu I h e^{-jkr}}{4\pi r}$$

$$\vec{1}_z = \vec{1}_r \cos \Theta - \vec{1}_\Theta \sin \Theta$$

$$\vec{H}(\vec{r}) = \frac{1}{\mu} \text{rot } \vec{A}(\vec{r}) = \vec{1}_\Phi \frac{I h}{4\pi} e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin \Theta$$

Sevanje

Biot-Savart

Tokovni element

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\epsilon} \text{rot } \vec{H} = \frac{Ih}{4\pi j\omega\epsilon} e^{-jkr} \left[ \vec{1}_r \left( \frac{jk}{r^2} + \frac{1}{r^3} \right) 2\cos\Theta + \vec{1}_\Theta \left( -\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3} \right) \sin\Theta \right]$$

Zveznost  
toka/elektrine  
 $I = j\omega Q$

$$\vec{E}(\vec{r}) = \frac{Qh}{4\pi\epsilon} e^{-jkr} \left[ \vec{1}_r \left( \frac{jk}{r^2} + \frac{1}{r^3} \right) 2\cos\Theta + \vec{1}_\Theta \left( -\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3} \right) \sin\Theta \right]$$

Sevanje

Točkasti statični električni dipol

$$\frac{1}{\omega\epsilon} = \frac{1}{\omega\sqrt{\mu}\epsilon} \sqrt{\frac{\mu}{\epsilon}} = \frac{Z}{k}$$

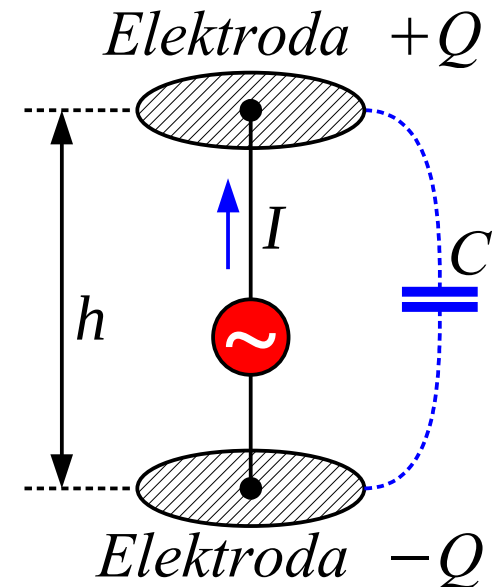
$$\vec{S}(\vec{r}) = \frac{1}{2} \vec{E}(\vec{r}) \times \vec{H}(\vec{r})^* = \frac{|I|^2 h^2 Z}{32\pi^2 k} \left[ \vec{1}_r \left( \frac{k^3}{r^2} - \frac{j}{r^5} \right) \sin^2\Theta + \vec{1}_\Theta \left( \frac{jk^2}{r^3} + \frac{j}{r^5} \right) 2\cos\Theta \sin\Theta \right]$$

Sevanje

$$P = \oint_{r \rightarrow \infty} \vec{S}(\vec{r}) \cdot \vec{1}_r r^2 \sin\Theta d\Theta d\Phi = \frac{|I|^2 h^2 Z k^2}{12\pi}$$

$$R_s = \frac{2P}{|I|^2} = \frac{Zk^2 h^2}{6\pi} = \frac{2\pi Z}{3} \left( \frac{h}{\lambda} \right)^2$$

$$h \ll \lambda \Rightarrow R_s \ll \frac{1}{\omega C}$$



Dinamični električni dipol

# Tokovna zanka

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{V'} \vec{J}(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV'$$

$\omega \neq 0$

$$|\vec{r}-\vec{r}'| = \sqrt{(r \sin \Theta \cos \Phi - a \cos \Phi')^2 + (r \sin \Theta \sin \Phi - a \sin \Phi')^2 + (r \cos \Theta)^2}$$

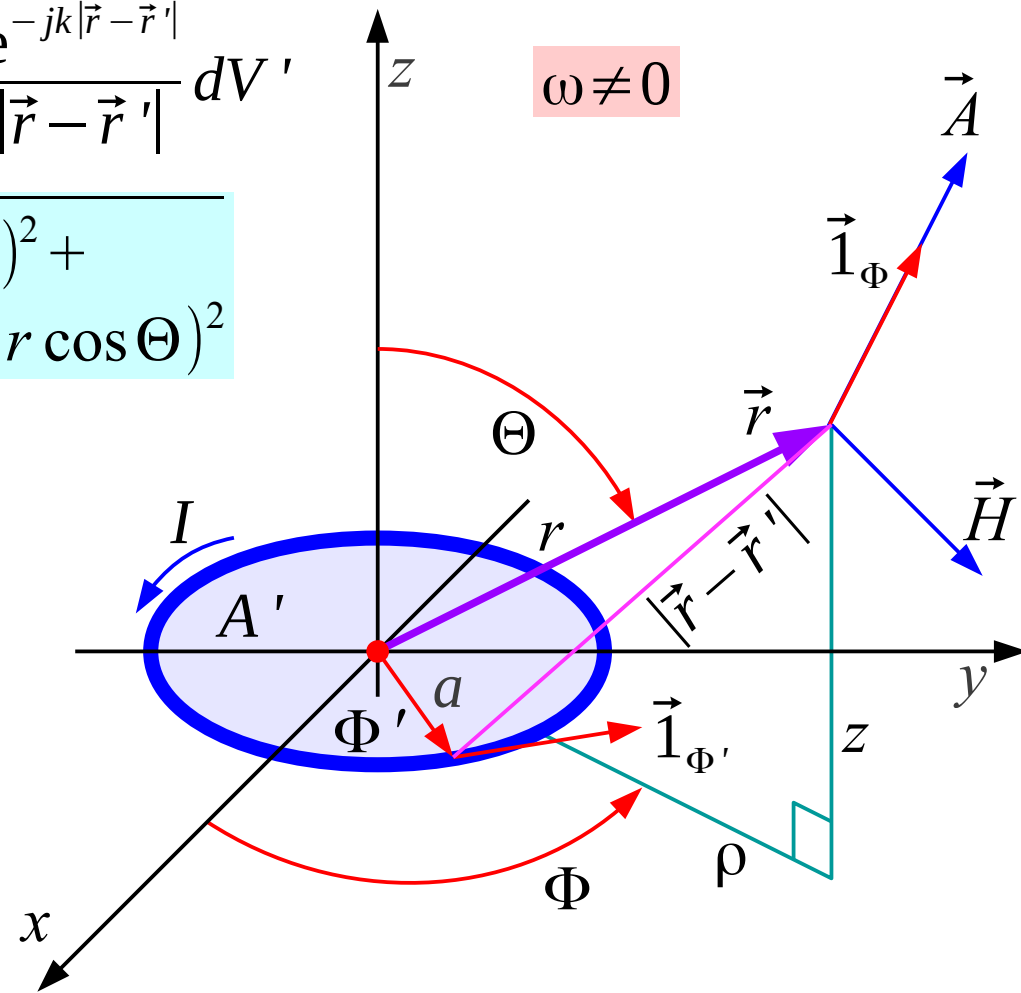
$$\vec{J}(\vec{r}') = \vec{1}_{\Phi'} \frac{I}{A_{\text{žice}}}$$

$$dV' = A_{\text{žice}} a d\Phi'$$

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_0^{2\pi} \vec{1}_{\Phi'} I \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} a d\Phi'$$

$$\vec{1}_{\Phi'} = -\vec{1}_x \sin \Phi' + \vec{1}_y \cos \Phi'$$

$$-\vec{1}_x \sin \Phi + \vec{1}_y \cos \Phi = \vec{1}_{\Phi}$$



Poenostavitve:

$$(1) \quad a \ll r \rightarrow \frac{1}{|\vec{r}-\vec{r}'|} \approx \frac{1}{r} \left[ 1 + \frac{a}{r} \sin \Theta \cos(\Phi - \Phi') \right]$$

$$(2) \quad a \ll \lambda \rightarrow e^{-jk|\vec{r}-\vec{r}'|} \approx e^{-jkr} \left[ 1 + jka \sin \Theta \cos(\Phi - \Phi') \right]$$

Površina zanke

$$A' = \pi a^2$$

$$\vec{A}(\vec{r}) = \vec{1}_{\Phi} \frac{\mu}{4\pi} I (\pi a^2) e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin \Theta = \vec{1}_{\Phi} \frac{\mu}{4\pi} I A' e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin \Theta$$

$$\vec{H}(\vec{r}) = \frac{1}{\mu} \text{rot } \vec{A}(\vec{r}) = \frac{IA'}{4\pi} e^{-jkr} \left[ \vec{1}_r \left( \frac{jk}{r^2} + \frac{1}{r^3} \right) 2\cos\Theta + \vec{1}_\Theta \left( -\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3} \right) \sin\Theta \right]$$

Sevanje

Točkasti statični magnetni dipol

$$\rho(\vec{r}') = 0 \rightarrow \text{grad } V(\vec{r}) = 0 \rightarrow \vec{E}(\vec{r}) = -j\omega \vec{A}(\vec{r})$$

$$\omega\mu = \omega \sqrt{\mu\epsilon} \sqrt{\frac{\mu}{\epsilon}} = kZ$$

$$\vec{E}(\vec{r}) = -\vec{1}_\Phi \frac{j\omega\mu IA'}{4\pi} e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin\Theta = \vec{1}_\Phi \frac{ZIA'}{4\pi} e^{-jkr} \left( \frac{k^2}{r} - \frac{jk}{r^2} \right) \sin\Theta$$

$$\vec{S}(\vec{r}) = \frac{|I|^2 (A')^2 Z}{32\pi} \left[ \vec{1}_r \left( \frac{k^4}{r^2} + \frac{jk}{r^5} \right) \sin^2\Theta - \vec{1}_\Theta \left( \frac{jk^3}{r^3} + \frac{jk}{r^5} \right) 2\cos\Theta \sin\Theta \right]$$

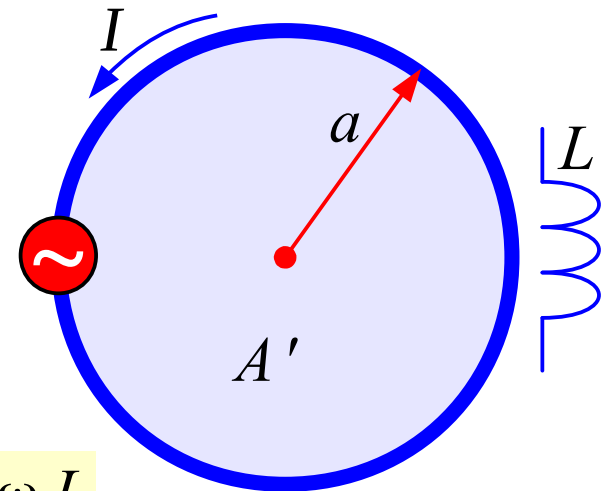
Sevanje

$$P = \oint_{r \rightarrow \infty} \vec{S}(\vec{r}) \cdot \vec{1}_r r^2 \sin\Theta d\Theta d\Phi = \frac{|I|^2 (A')^2 Z k^4}{12\pi}$$

$$R_s = \frac{2P}{|I|^2} = \frac{Zk^4 (A')^2}{6\pi} = \frac{8\pi^3 Z}{3} \left( \frac{A'}{\lambda^2} \right)^2$$

Dinamični magnetni dipol

$$\sqrt{A'} \ll \lambda \rightarrow R_s \ll \omega L$$



$$R_s = \frac{Z k^4 (N A')^2}{6 \pi} = \frac{8 \pi^3 Z}{3} \left( \frac{N A'}{\lambda^2} \right)^2$$

$f \approx 300 \text{ kHz}$

$A' \approx 1 \text{ m}^2$

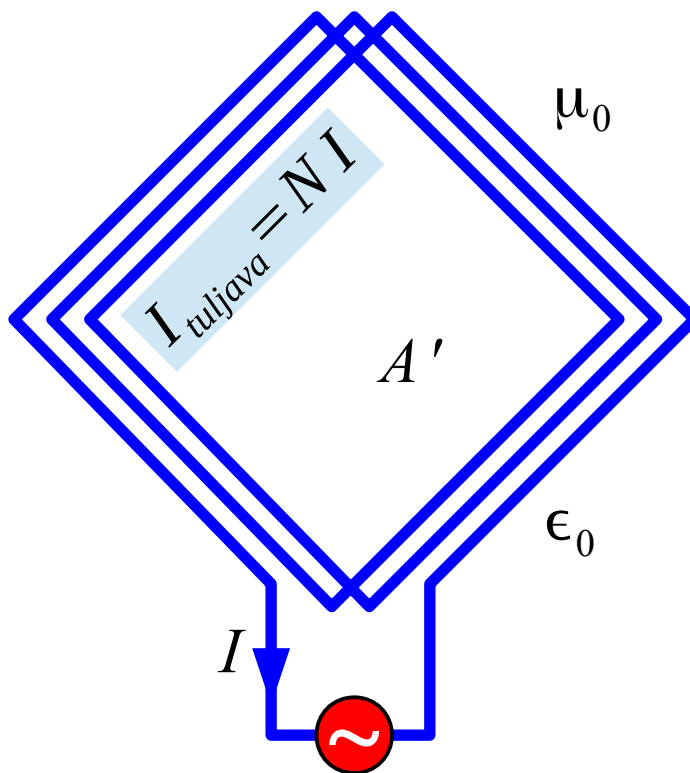
$N \approx 10$

Zrak

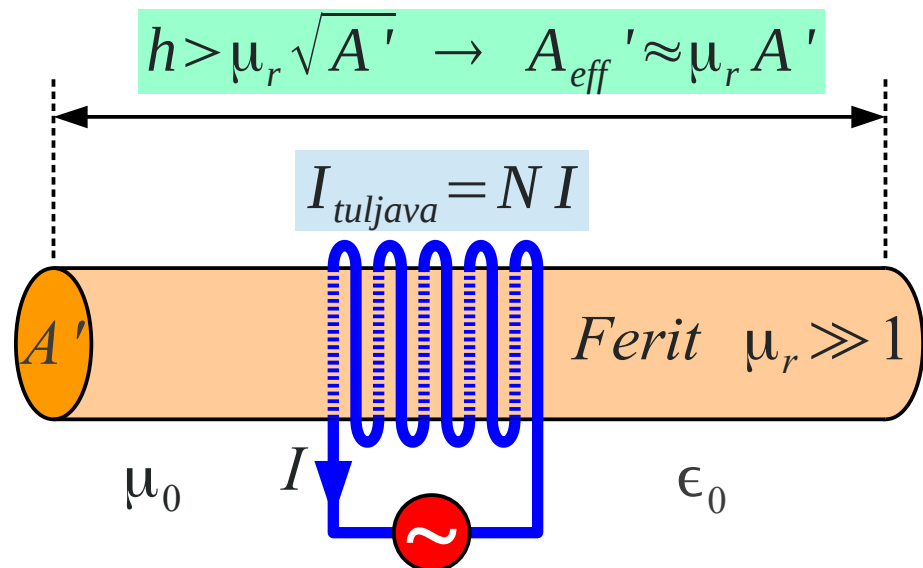
$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \Omega$$

$$\lambda = c_0 / f = 1 \text{ km}$$

$$R_s \approx 3.1 \mu \Omega$$



Okvirna antena ~1930



$$R_s = \frac{Z k^4 (\mu_r N A')^2}{6 \pi} = \frac{8 \pi^3 Z}{3} \left( \frac{\mu_r N A'}{\lambda^2} \right)^2$$

$f \approx 1 \text{ MHz}$

$A' \approx 1 \text{ cm}^2$

$h \approx 20 \text{ cm}$

$\mu_r \approx 100$

$N \approx 30$

Zrak

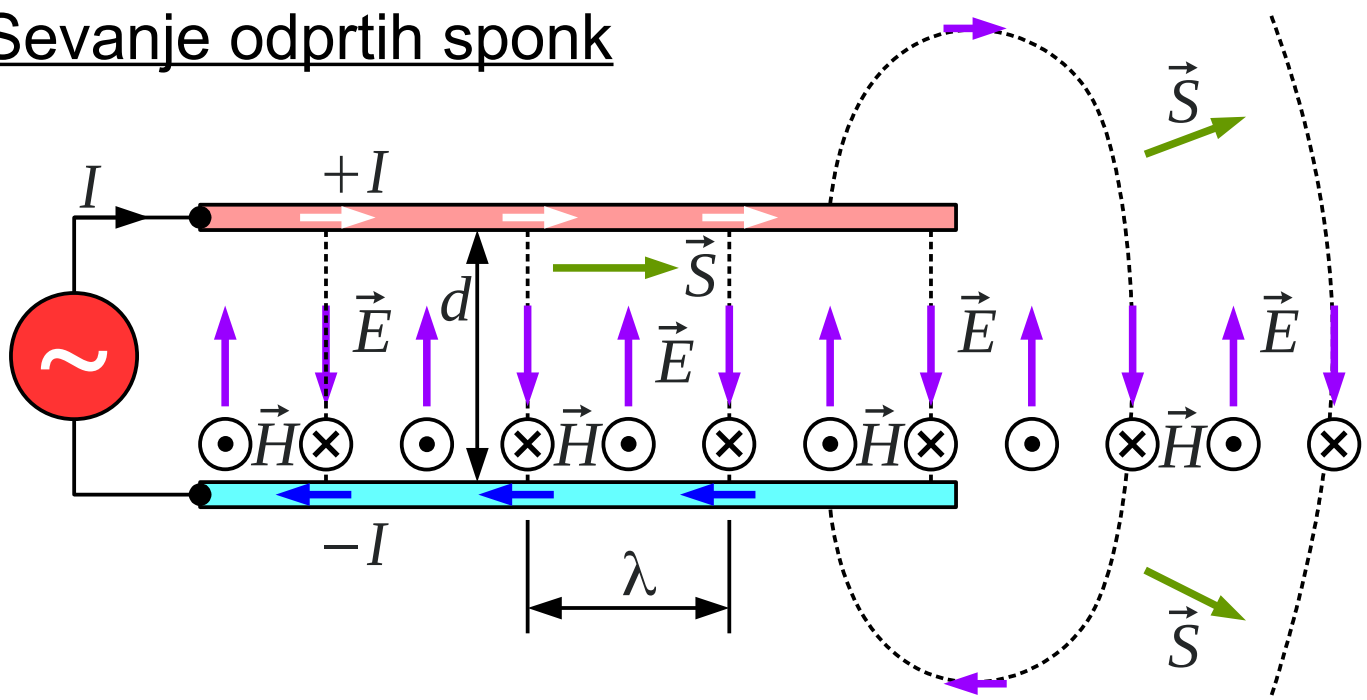
$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \Omega$$

$$\lambda = c_0 / f = 300 \text{ m}$$

$$R_s \approx 0.35 \mu \Omega$$

Feritna antena ~1970

# Sevanje odprtih sponk

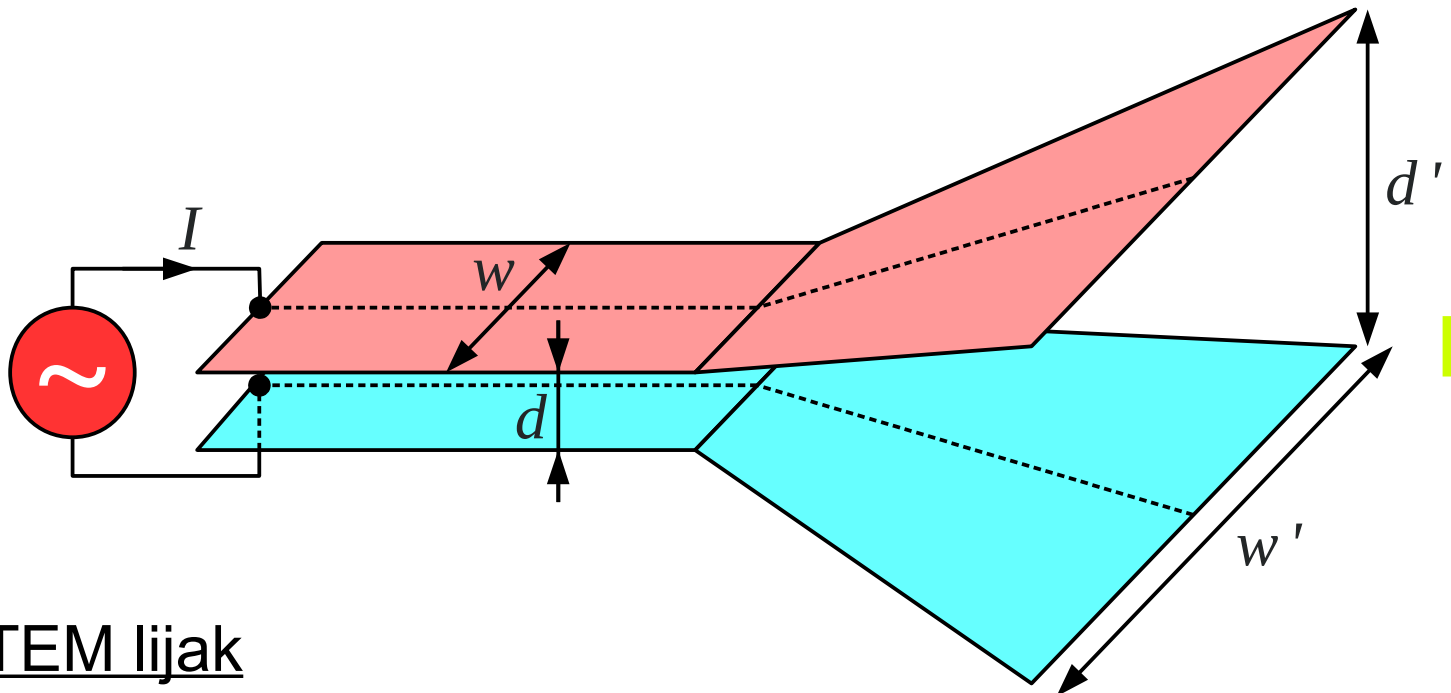


$$d \ll \lambda \rightarrow \Gamma \approx +1$$

$$d \approx \lambda/2 \rightarrow |\Gamma| \approx 0.3$$

$$d \gg \lambda \rightarrow \Gamma \approx 0$$

Trakasti dvovod  $Z_K \approx \frac{d}{w} \sqrt{\frac{\mu_0}{\epsilon_0}}$



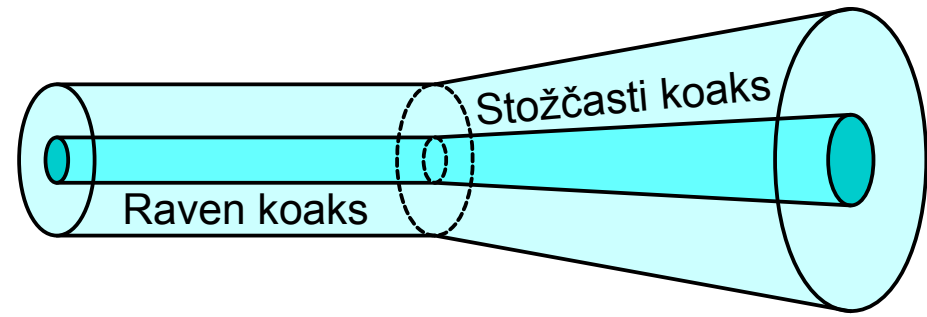
Sevanje

$$Z_K \approx \frac{d}{w} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{d'}{w'} \sqrt{\frac{\mu_0}{\epsilon_0}}$$

TEM lijak

# Stožčasti vod

$C [V] \equiv \text{konstanta}$



$$\vec{E}(\vec{r}) = \vec{1}_\Theta \frac{C}{r \sin \Theta} e^{\mp jkr}$$

$$\vec{H}(\vec{r}) = \frac{j}{\omega \mu} \text{rot } \vec{E}(\vec{r}) = \pm \vec{1}_\Phi \frac{C/Z_0}{r \sin \Theta} e^{\mp jkr}$$

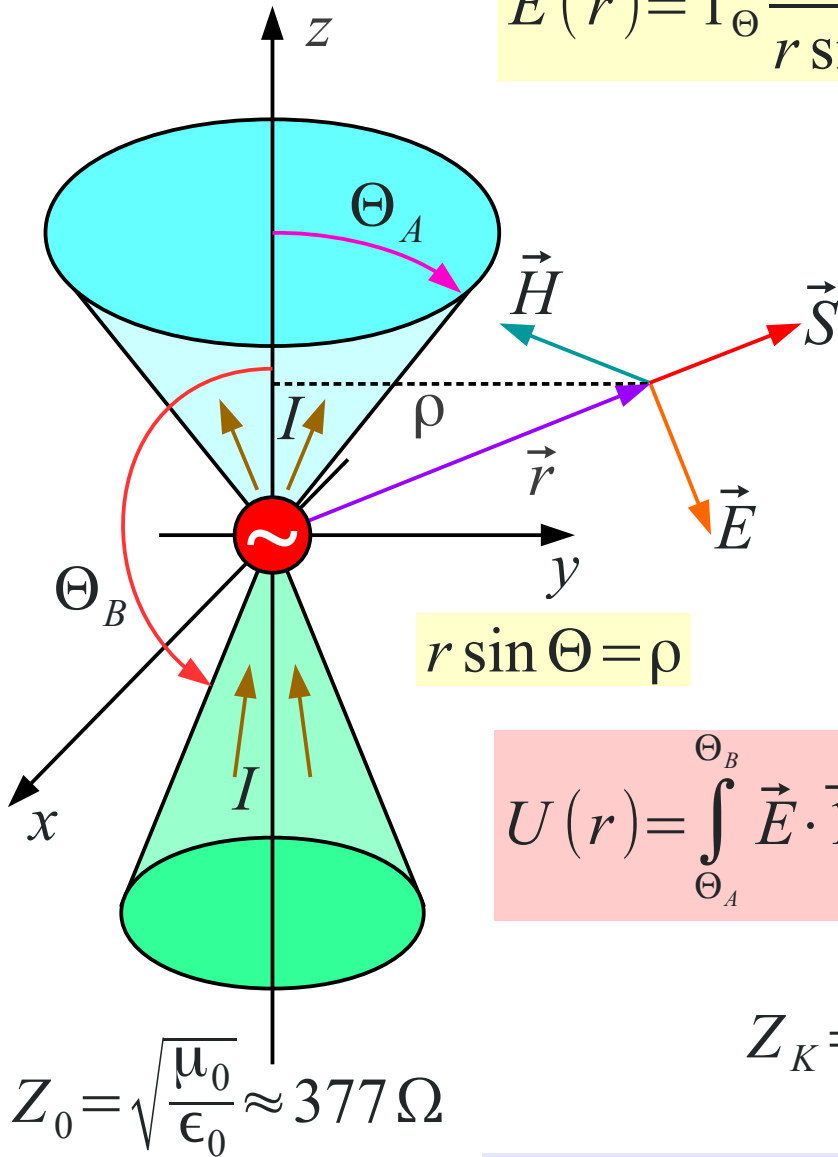
$$\vec{S}(\vec{r}) = \frac{1}{2} \vec{E} \times \vec{H}^* = \pm \vec{1}_r \frac{|C|^2}{2Z_0} \left( \frac{1}{r \sin \Theta} \right)^2$$

$$I(r) = \int_0^{2\pi} \vec{H} \cdot \vec{1}_\Phi r \sin \Theta d\Phi = \pm \frac{2\pi C}{Z_0} e^{\mp jkr}$$

$$U(r) = \int_{\Theta_A}^{\Theta_B} \vec{E} \cdot \vec{1}_\Theta r d\Theta = C e^{\mp jkr} \int_{\Theta_A}^{\Theta_B} \frac{d\Theta}{\sin \Theta} = C e^{\mp jkr} \ln \left( \frac{\text{tg}(\Theta_B/2)}{\text{tg}(\Theta_A/2)} \right)$$

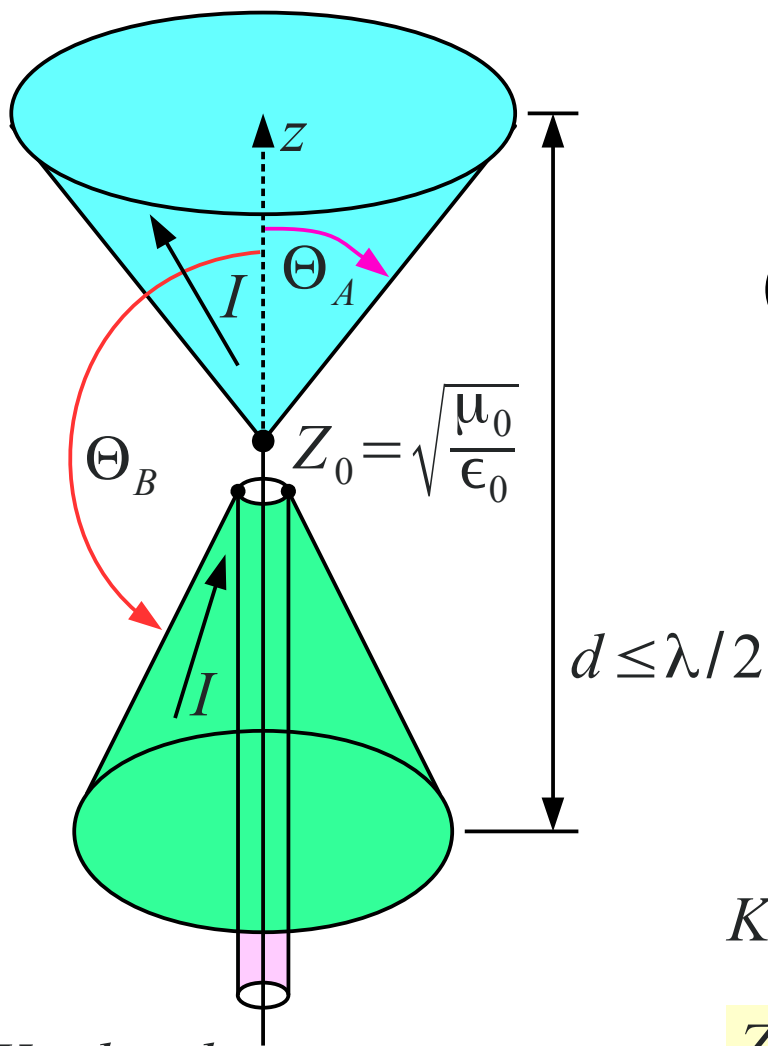
$$Z_K = \pm \frac{U}{I} = \frac{Z_0}{2\pi} \ln \left( \frac{\text{tg}(\Theta_B/2)}{\text{tg}(\Theta_A/2)} \right) \approx 60 \Omega \ln \left( \frac{\text{tg}(\Theta_B/2)}{\text{tg}(\Theta_A/2)} \right)$$

$$P = \int_{\Theta_A}^{\Theta_B} \int_0^{2\pi} \vec{S} \cdot \vec{1}_r r^2 \sin \Theta d\Theta d\Phi = \frac{U I^*}{2} = \pm \frac{\pi |C|^2}{Z_0} \ln \left( \frac{\text{tg}(\Theta_B/2)}{\text{tg}(\Theta_A/2)} \right)$$





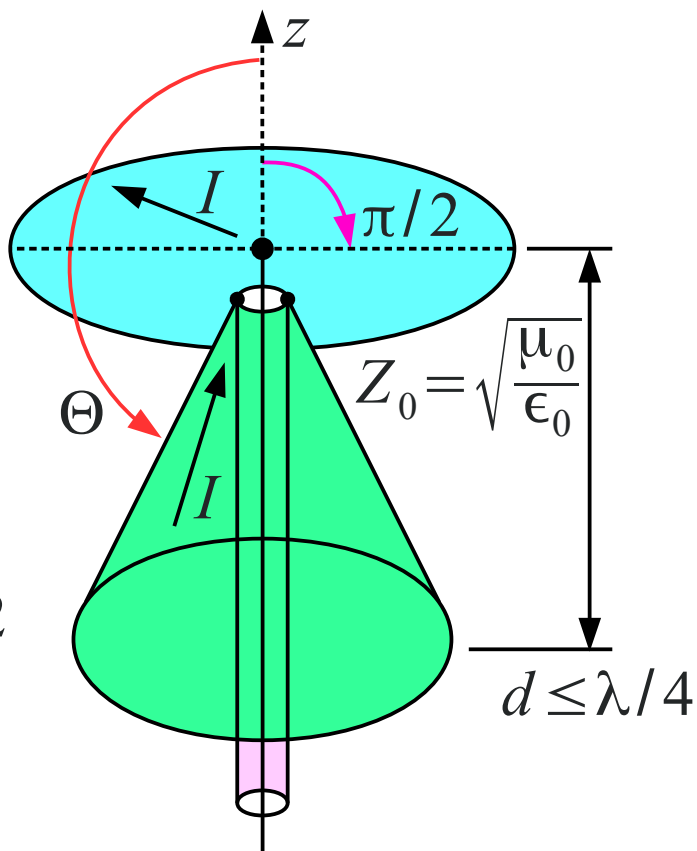
## Bikonična antena



*Koaksialno napajanje*

$$Z_K \approx 60 \Omega \ln \left( \frac{\operatorname{tg}(\Theta_B/2)}{\operatorname{tg}(\Theta_A/2)} \right)$$

## Discone antena



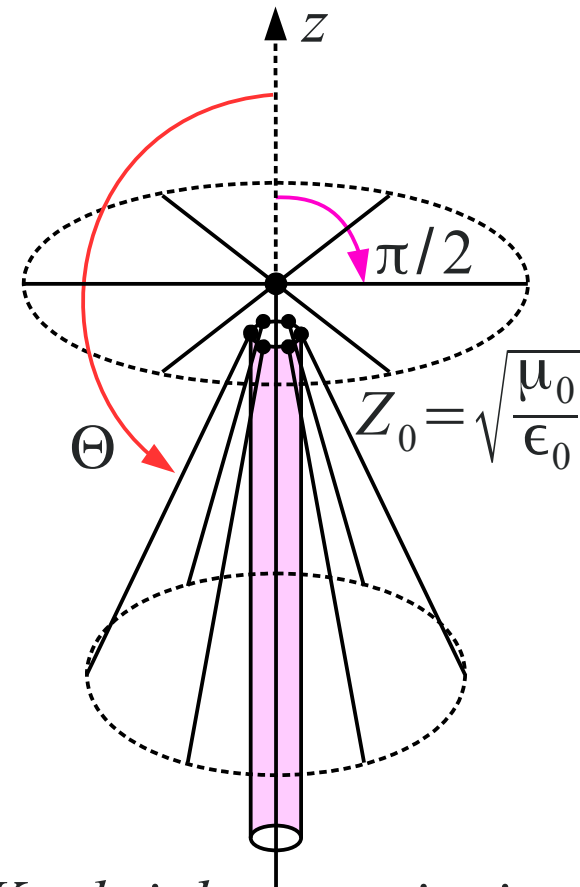
*Koaksialno napajanje*

$$Z_K \approx 60 \Omega \ln(\operatorname{tg}(\Theta/2))$$

$$\Theta \approx 2 \operatorname{arctg} \left( e^{Z/60\Omega} \right)$$

$$Z_K = 50 \Omega \rightarrow \Theta \approx 133^\circ$$

## Discone iz palčk



*Koaksialno napajanje*

$$\lambda/2\pi = a$$

$$w \ll a = \lambda/2\pi$$

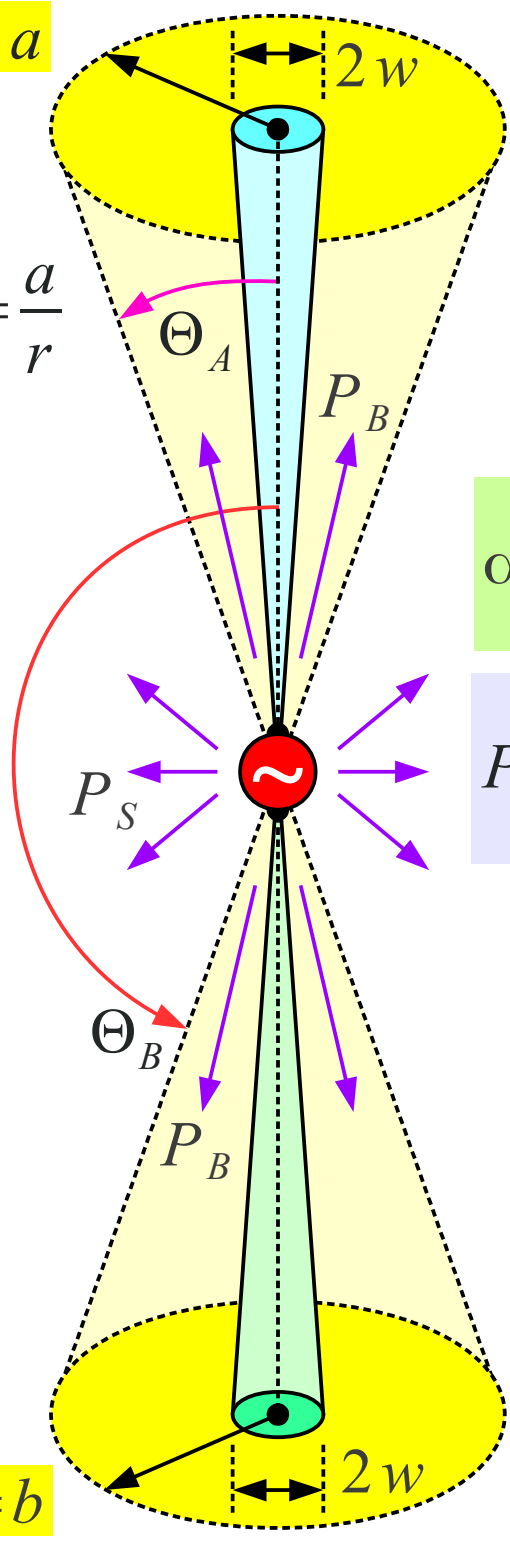
$$\sin \Theta_w = \frac{w}{r}$$

Točna rešitev  $\vec{E}(\vec{r}) = \vec{1}_\Theta \frac{C}{r \sin \Theta} e^{\mp jkr}$

$$\sin \Theta_A = \frac{a}{r}$$

$$\alpha = \pm \frac{\pi |C|^2}{Z_0}$$

Zgled  $w = 0.001 \lambda$   $r = \lambda/2$



$$P_S = \alpha \ln \left( \frac{\text{tg}(\Theta_B/2)}{\text{tg}(\Theta_A/2)} \right) = -2\alpha \ln \text{tg}(\Theta_A/2) \approx 3.623 \alpha$$

$$\sin \Theta_B = \frac{b}{r}$$

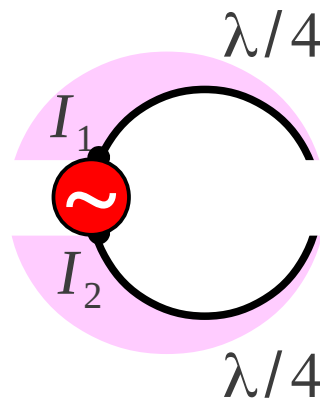
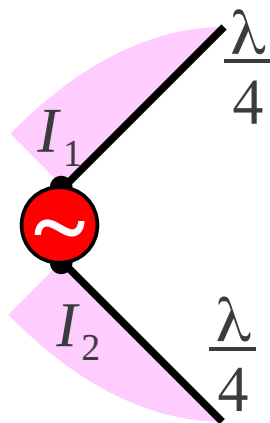
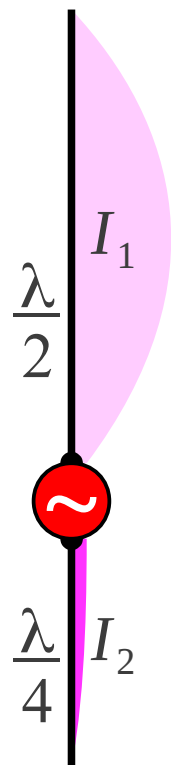
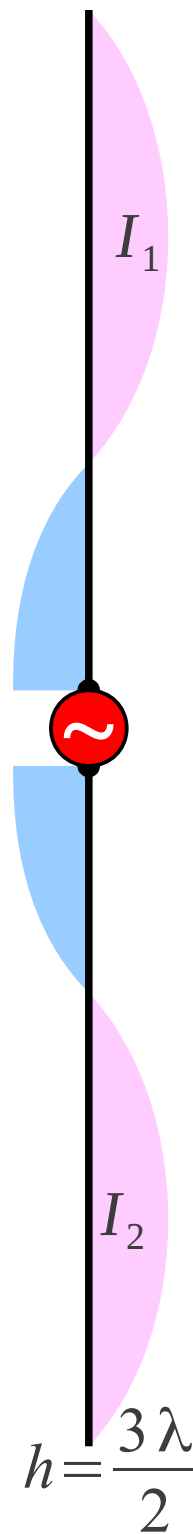
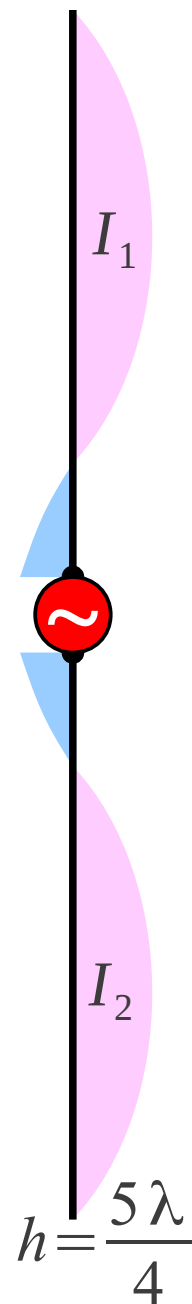
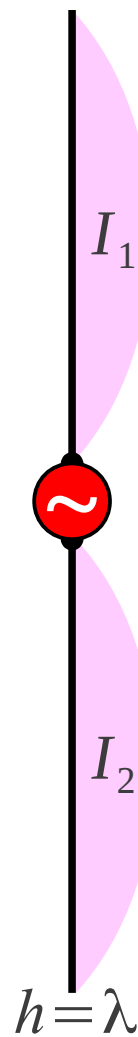
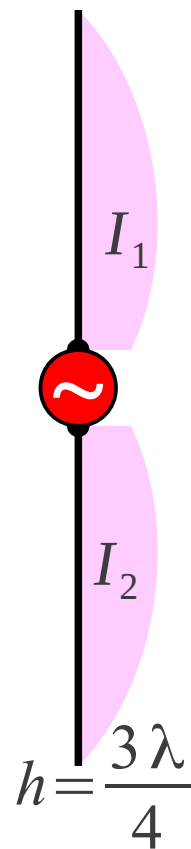
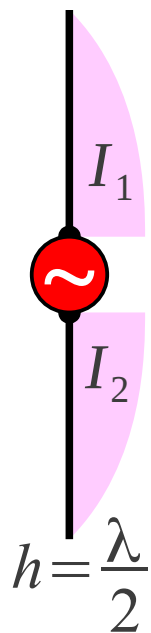
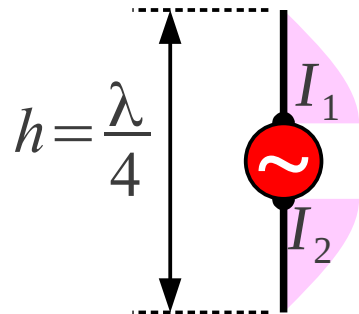
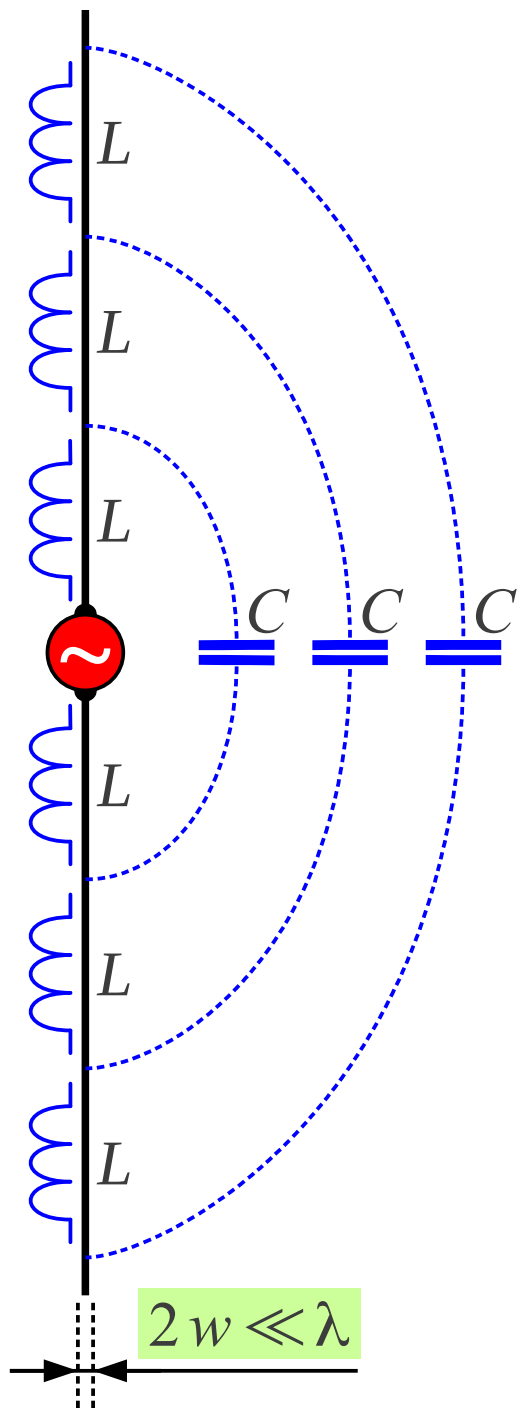
$$P_B = \alpha \left[ \ln \left( \frac{\text{tg}(\Theta_A/2)}{\text{tg}(\Theta_w/2)} \right) + \ln \left( \frac{\text{tg}((\pi - \Theta_w)/2)}{\text{tg}(\Theta_B/2)} \right) \right] = 2\alpha \ln \left( \frac{\text{tg}(\Theta_A/2)}{\text{tg}(\Theta_w/2)} \right) \approx 10.19 \alpha$$

$$\Theta_B = \pi - \Theta_A$$

$$\lambda/2\pi = b$$

$$\sin(\pi - \Theta_w) = \frac{w}{r}$$

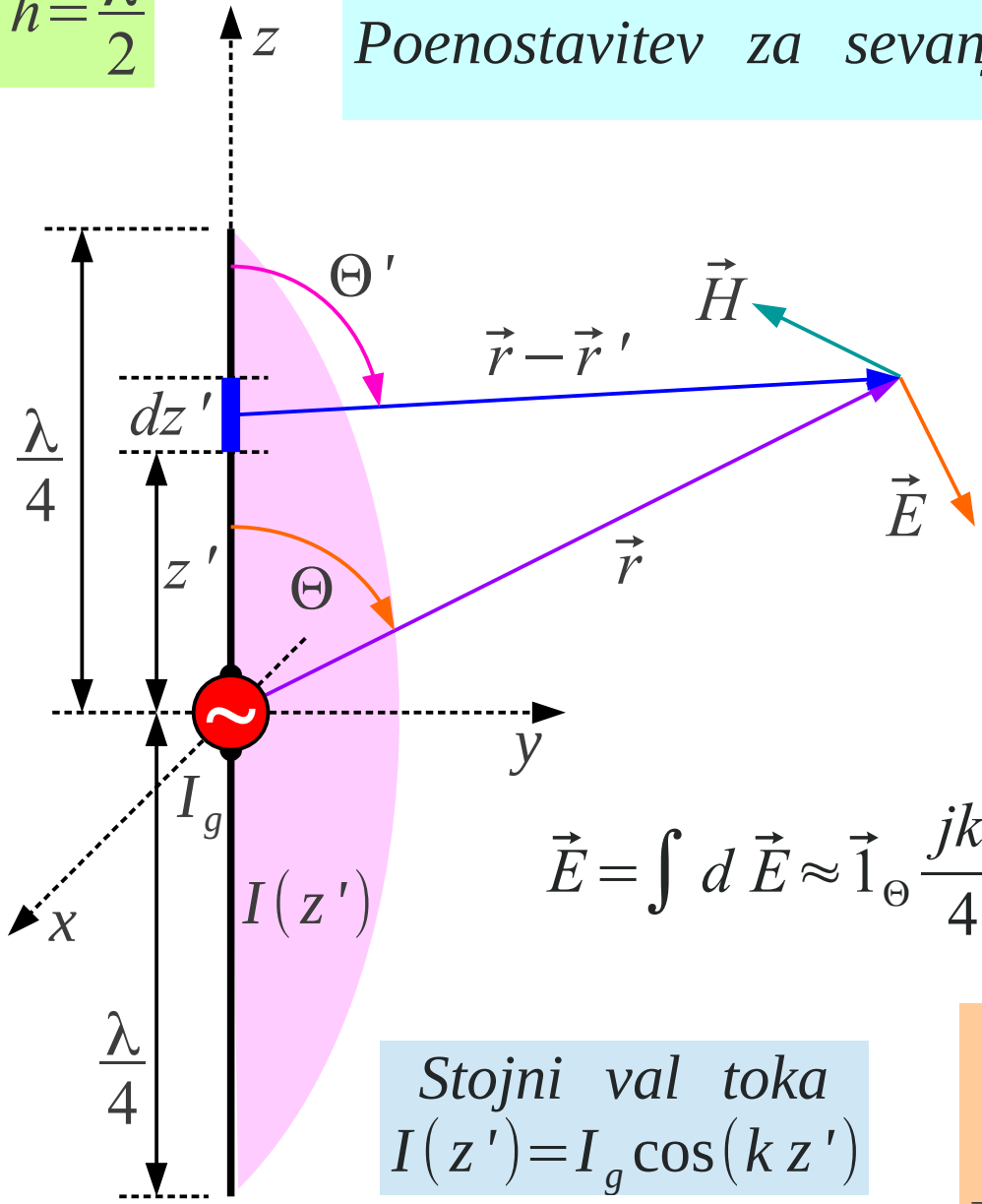
Vitka stožca



Stojni val na tankih žičnih dipolih

$$h = \frac{\lambda}{2}$$

Poenostavitev za sevanje  $d\vec{E} \approx \vec{1}_{\Theta} \frac{jkZ_0}{4\pi} I(z') dz' \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \sin \Theta'$



$$|\vec{r} - \vec{r}'| = \sqrt{r^2 + z'^2 - 2rz' \cos \Theta}$$

$$r \gg h \rightarrow |\vec{r} - \vec{r}'| \approx r - z' \cos \Theta$$

Daljnje polje  $r \gg \frac{2h^2}{\lambda}$   $\vec{1}_{\Theta'} \approx \vec{1}_{\Theta}$   
 $\frac{1}{|\vec{r} - \vec{r}'|} \approx \frac{1}{r}$   $\sin \Theta' \approx \sin \Theta$   
 $e^{-jk|\vec{r} - \vec{r}'|} \approx e^{-jkr} e^{jkz' \cos \Theta}$

$$\vec{E} = \int d\vec{E} \approx \vec{1}_{\Theta} \frac{jkZ_0}{4\pi} I_g \frac{e^{-jkr}}{r} \sin \Theta \int_{-\lambda/4}^{\lambda/4} \cos(kz') e^{jkz' \cos \Theta} dz'$$

Stojni val toka  
 $I(z') = I_g \cos(kz')$

$$\int_{-\lambda/4}^{\lambda/4} \cos(kz') e^{jkz' \cos \Theta} dz' = \frac{2 \cos\left(\frac{\pi}{2} \cos \Theta\right)}{k \sin^2 \Theta}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \Omega$$

Polvalovni dipol

$$\vec{E} \approx \vec{1}_{\Theta} \frac{jZ_0}{2\pi} I_g \frac{e^{-jkr}}{r} \frac{\cos\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta}$$

$$\vec{S} = \vec{1}_r \frac{|\vec{E}|^2}{2Z_0} = \vec{1}_r \frac{Z_0}{8\pi^2} |I_g|^2 \frac{1}{r^2} \left[ \frac{\cos\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta} \right]^2$$

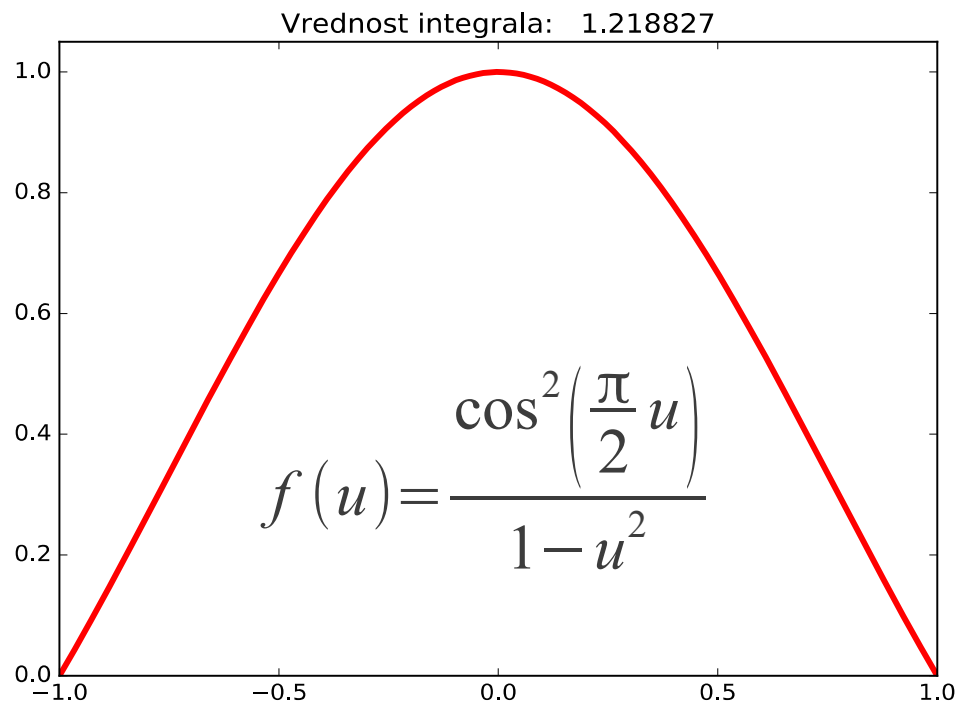
$$\int_0^{2\pi} d\Phi = 2\pi$$

$$P = \int_0^\pi \int_0^{2\pi} \vec{S} \cdot \vec{1}_r r^2 \sin \Theta d\Theta d\Phi = \frac{Z_0}{4\pi} |I_g|^2 \int_0^\pi \left[ \frac{\cos\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta} \right]^2 \sin \Theta d\Theta$$

$$I_{\lambda/2} = \int_0^\pi \left[ \frac{\cos\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta} \right]^2 \sin \Theta d\Theta = \int_{-1}^1 \frac{\cos^2\left(\frac{\pi}{2} u\right)}{1-u^2} du \approx 1.218827$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \Omega \approx 120 \pi \Omega$$

$$R_S = \frac{2P}{|I_g|^2} = \frac{Z_0}{2\pi} I_{\lambda/2} \approx 60 \Omega I_{\lambda/2} \approx 73.1 \Omega$$



Upornost polvalovnega dipola

$$h = \frac{\lambda}{2} \rightarrow F(\Theta, \Phi) = \frac{\cos\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta}$$

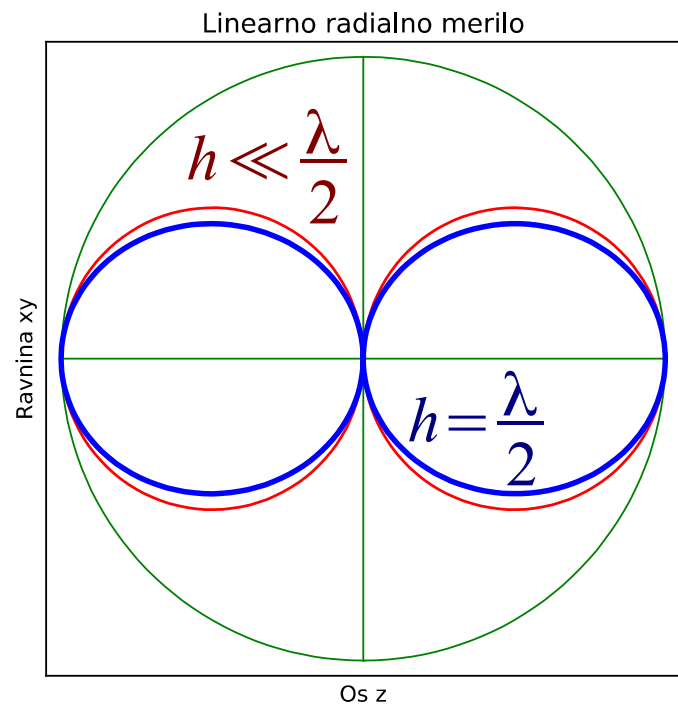
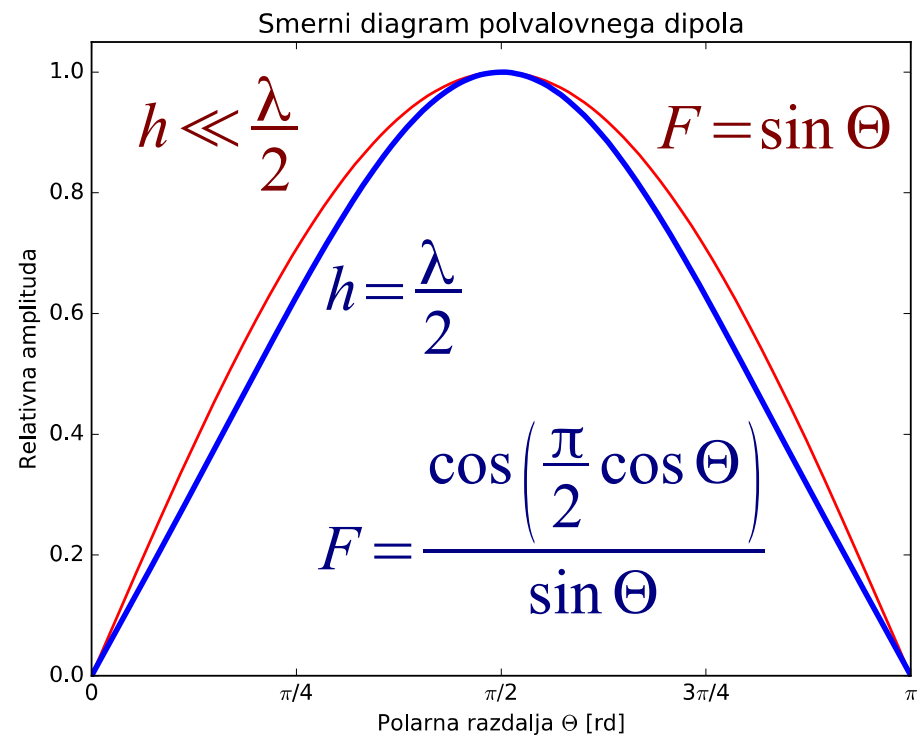
$$D = \frac{4\pi |F(\Theta_{MAX}, \Phi_{MAX})|^2}{\iint_{4\pi} |F(\Theta, \Phi)|^2 d\Omega}$$

$$F(\Theta_{MAX} = \pi/2) = 1$$

$$D = \frac{4\pi}{\int_0^\pi \int_0^{2\pi} \left[ \frac{\cos\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta} \right]^2 \sin \Theta d\Theta d\Phi}$$

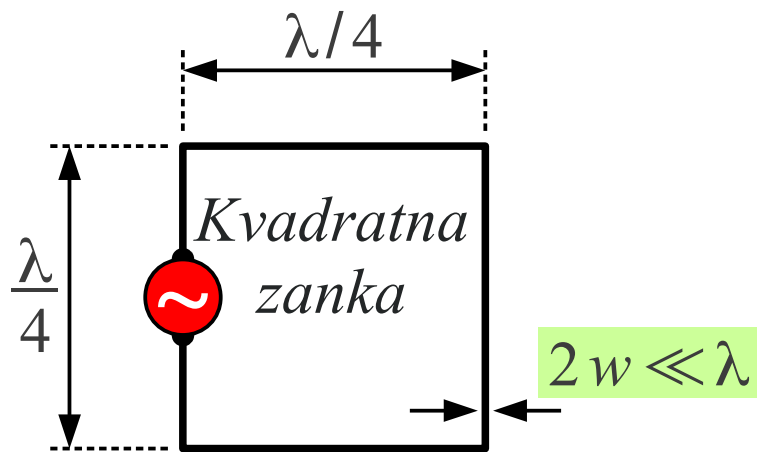
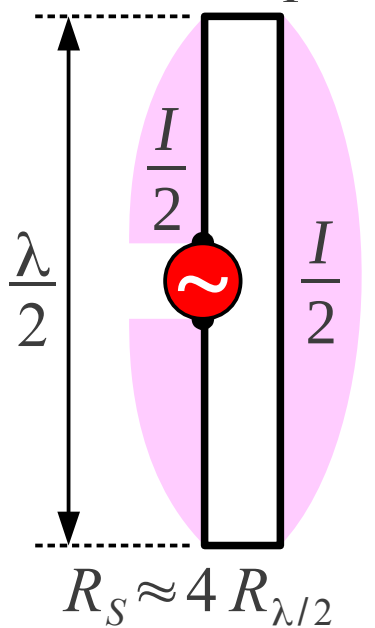
$$D = \frac{4\pi}{2\pi I_{\lambda/2}} = \frac{2}{1.218827} = 1.640922$$

$$D_{dB_i} = 10 \log_{10} 1.640922 = 2.150879 \text{ dB}_i$$

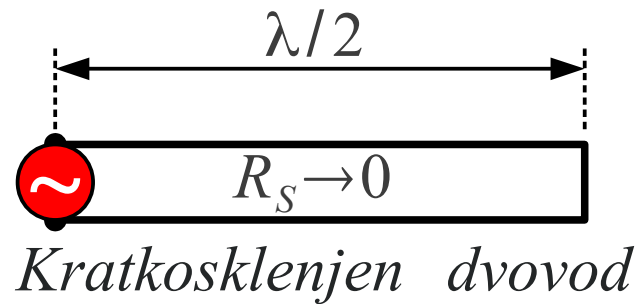


Smernost polvalovnega dipola

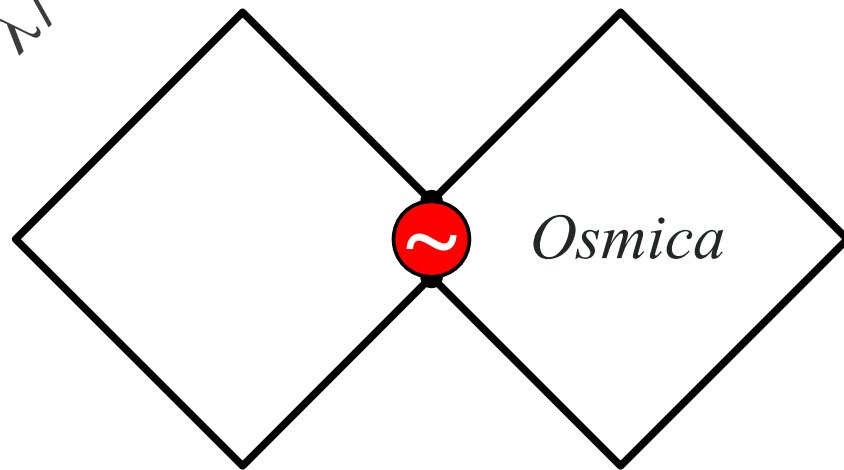
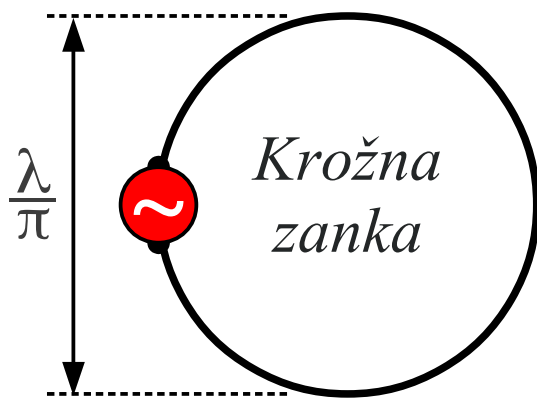
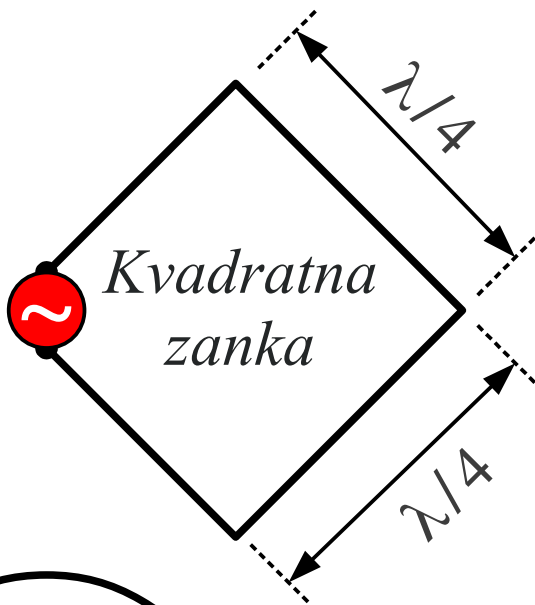
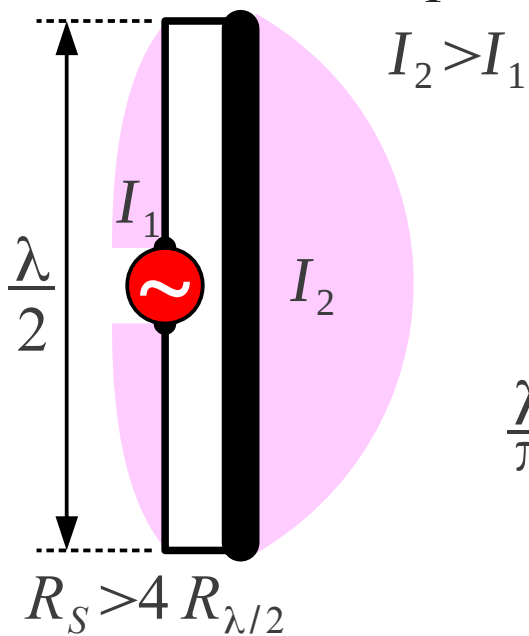
*Zaviti dipol*



Enovalovne zanke



*Nesimetrični dipol*

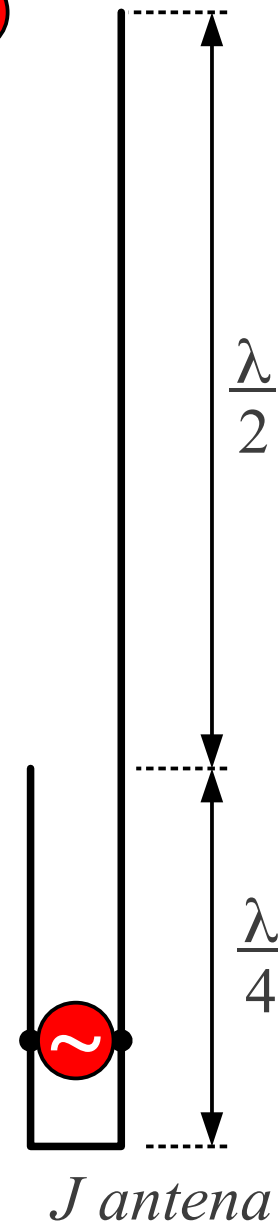
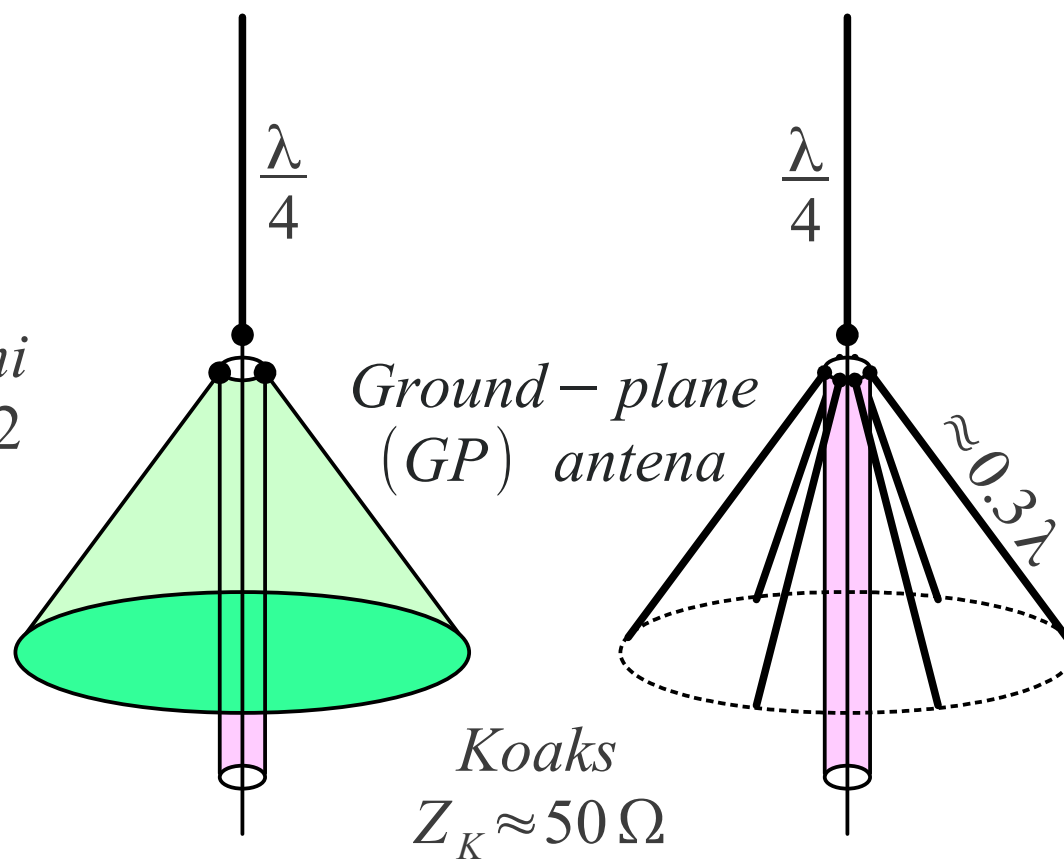
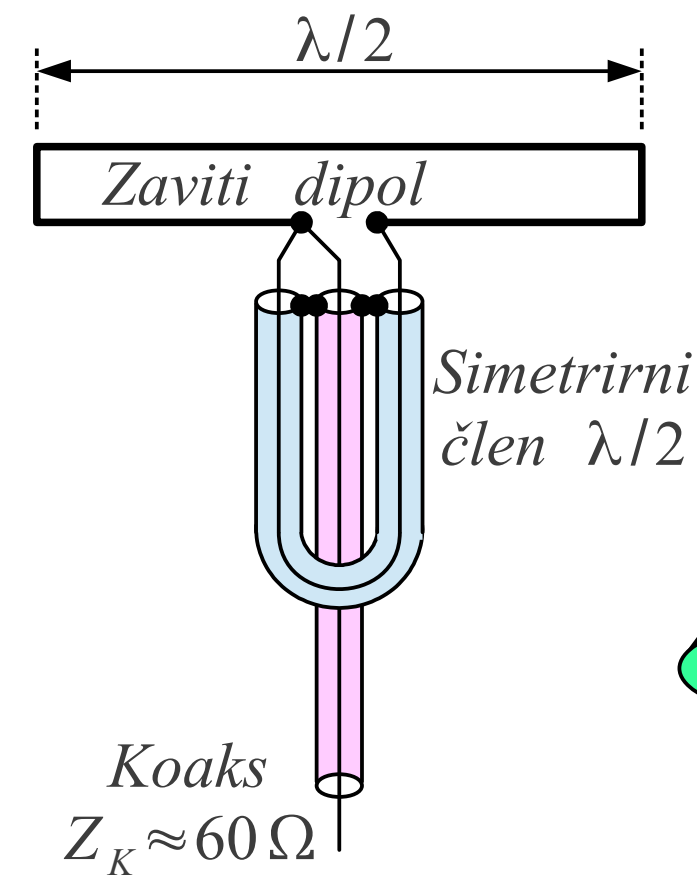
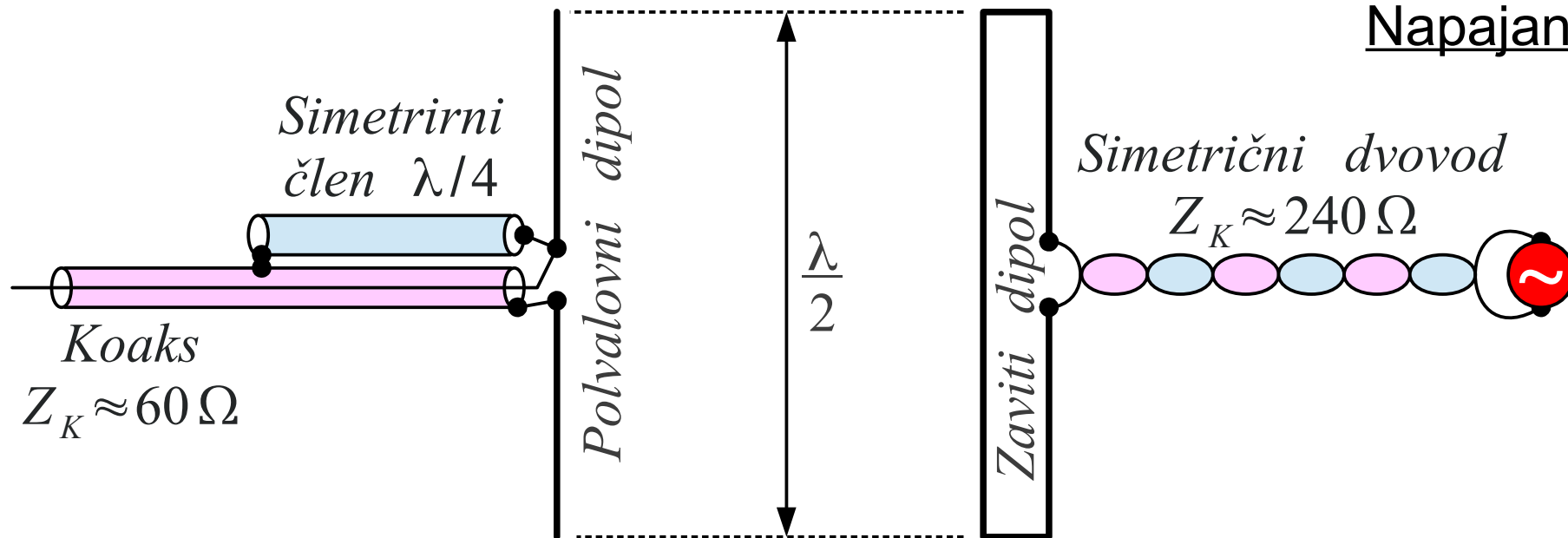


*Vse zanke*

*polarizacija  $\vec{E}$*

$\vec{S}_{MAX}$

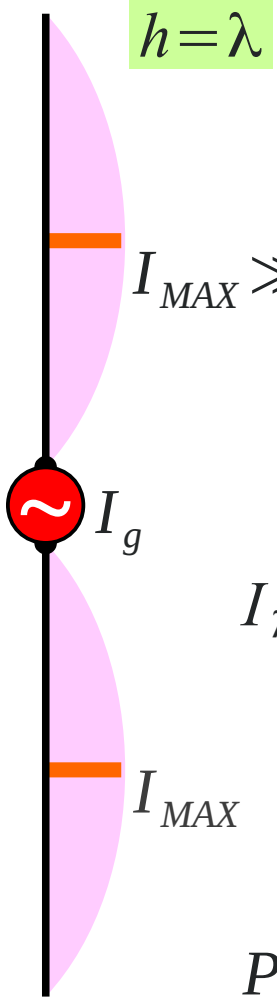
# Napajanje dipolov





*Stojni val toka*  
 $I(z') = I_{MAX} |\sin(kz')|$

$$\vec{E} = \vec{E}_{\lambda/2} 2 \cos\left(\frac{\pi}{2} \cos \Theta\right)$$



$$\vec{E} \approx \vec{1}_{\Theta} \frac{jZ_0}{\pi} I_{MAX} \frac{e^{-jkr}}{r} \frac{\cos^2\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta}$$

$$F(\Theta, \Phi) = \frac{\cos^2\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta}$$

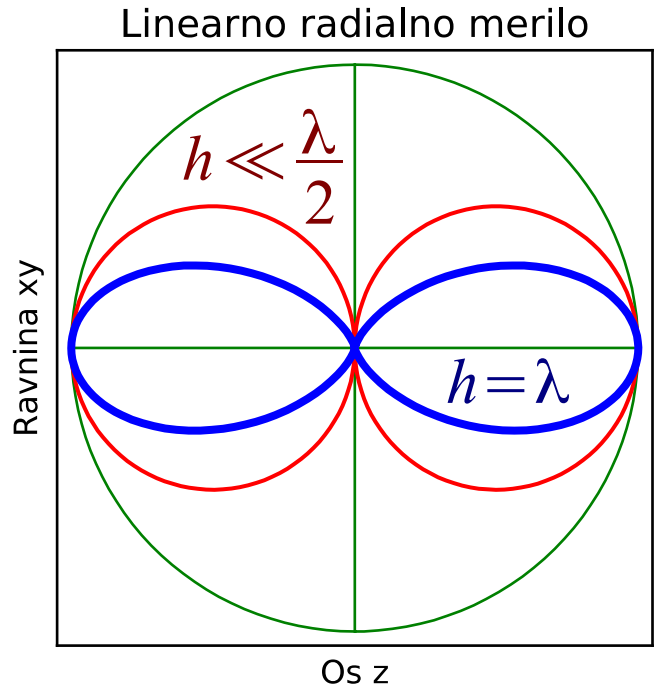
$$I_{\lambda} = \int_0^{\pi} \left[ \frac{\cos^2\left(\frac{\pi}{2} \cos \Theta\right)}{\sin \Theta} \right]^2 \sin \Theta d\Theta \approx 0.829532$$

$$D = \frac{2}{I_{\lambda}} \approx 2.41$$

$$D_{dBi} = 10 \log_{10} \frac{2}{I_{\lambda}} \approx 3.82 \text{ dBi}$$

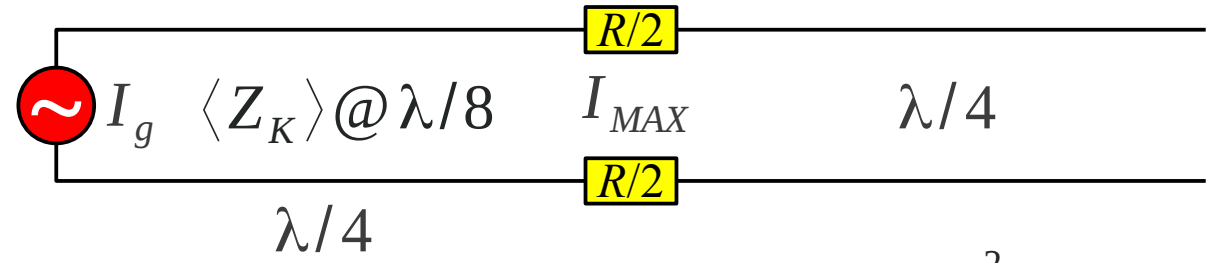
$$P = \frac{Z_0}{\pi} |I_{MAX}|^2 I_{\lambda}$$

$$R = \frac{2P}{|I_{MAX}|^2} = \frac{2Z_0}{\pi} I_{\lambda} \approx 199 \Omega$$



Tanka žica  $w = 0.001 \lambda$

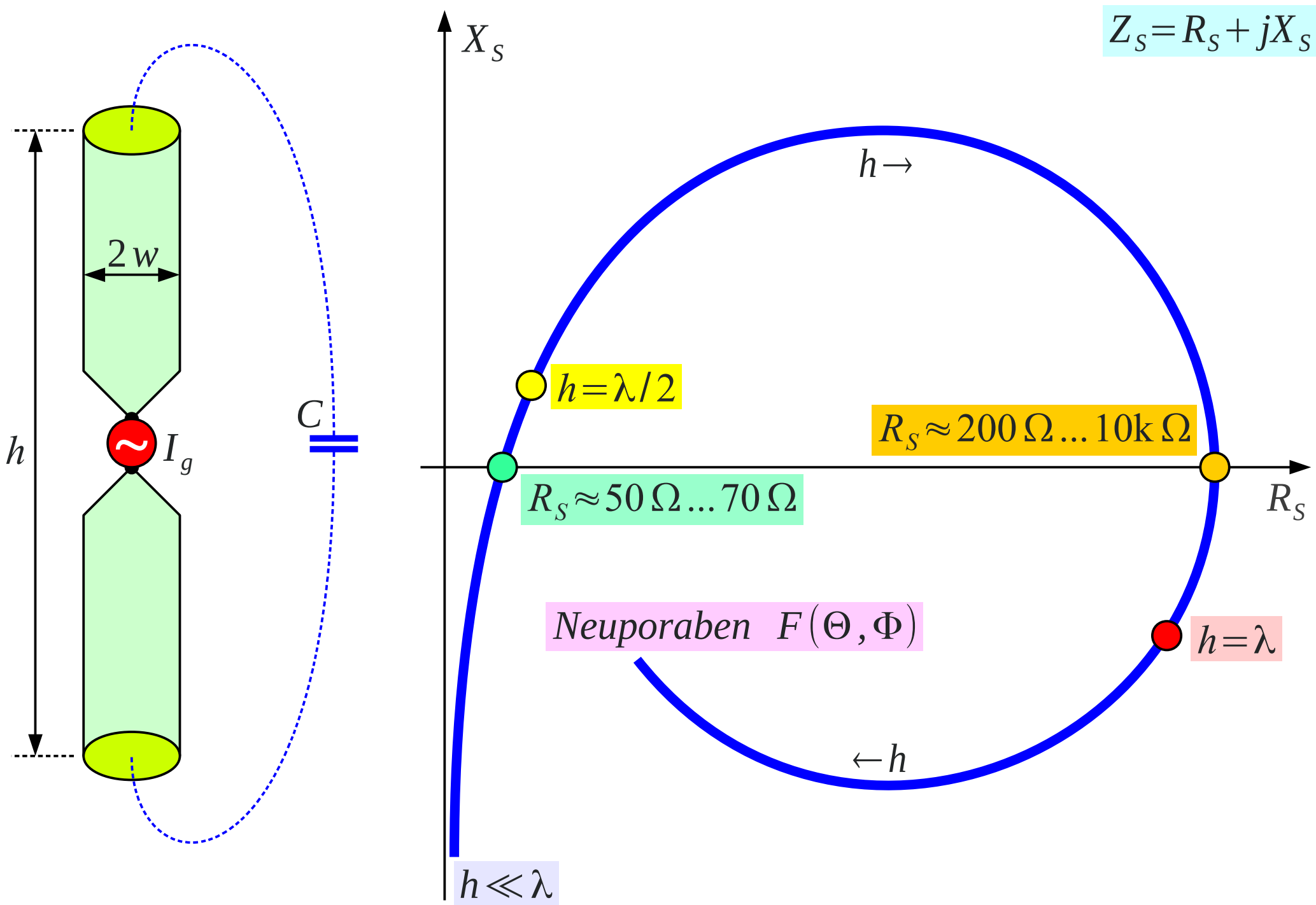
$$\Theta_A = \pi - \Theta_B \approx \frac{w}{\lambda/8}$$



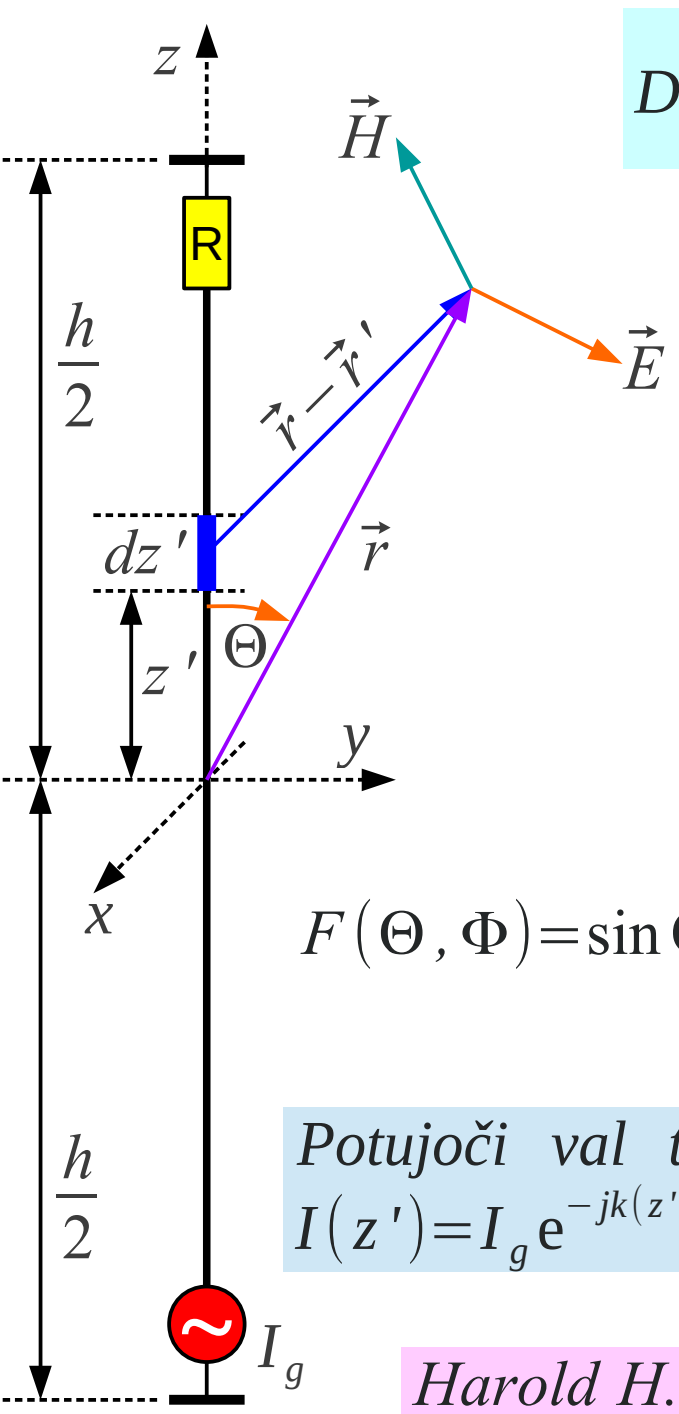
$$Z_K \approx 120 \Omega \ln \frac{\lambda}{4w} \approx 663 \Omega$$

$$R_S = \frac{Z_K^2}{R} \approx 2.2 \text{ k}\Omega$$

Enovalovni dipol



Impedanca debelega dipola



Daljnje polje  $d\vec{E} \approx \vec{1}_\Theta \frac{jkZ_0}{4\pi} I(z') dz' \frac{e^{-jkr}}{r} e^{jkz' \cos \Theta} \sin \Theta$

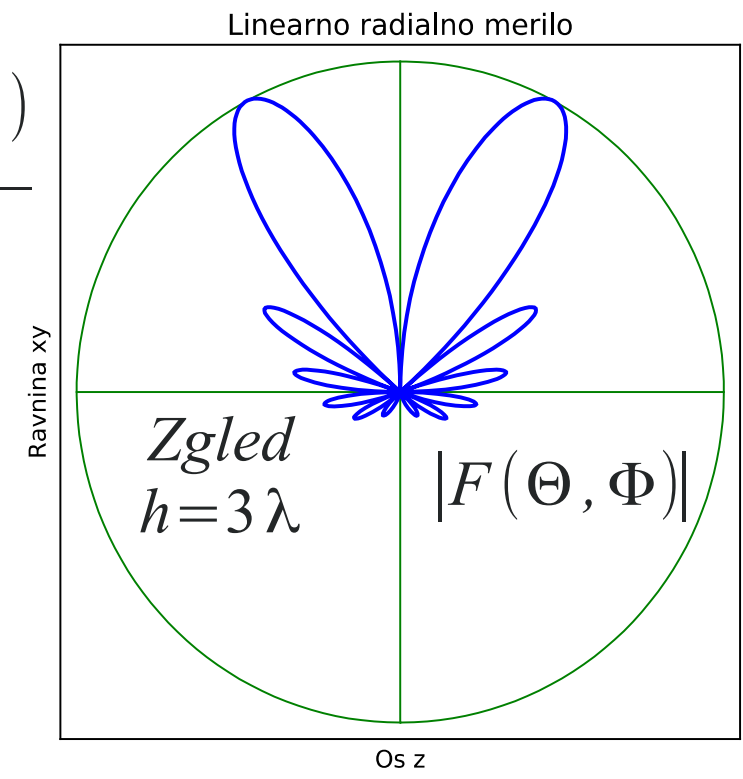
$$\vec{E} \approx \vec{1}_\Theta \frac{jZ_0}{2\lambda} I_g e^{-j\frac{kh}{2}} \frac{e^{-jkr}}{r} \sin \Theta \int_{-h/2}^{h/2} e^{jkz'(\cos \Theta - 1)} dz'$$

$$\vec{E} = \vec{1}_\Theta \frac{jZ_0}{2\lambda} I_g h e^{-j\frac{kh}{2}} \frac{e^{-jkr}}{r} \sin \Theta \frac{\sin \frac{kh}{2} (\cos \Theta - 1)}{\frac{kh}{2} (\cos \Theta - 1)}$$

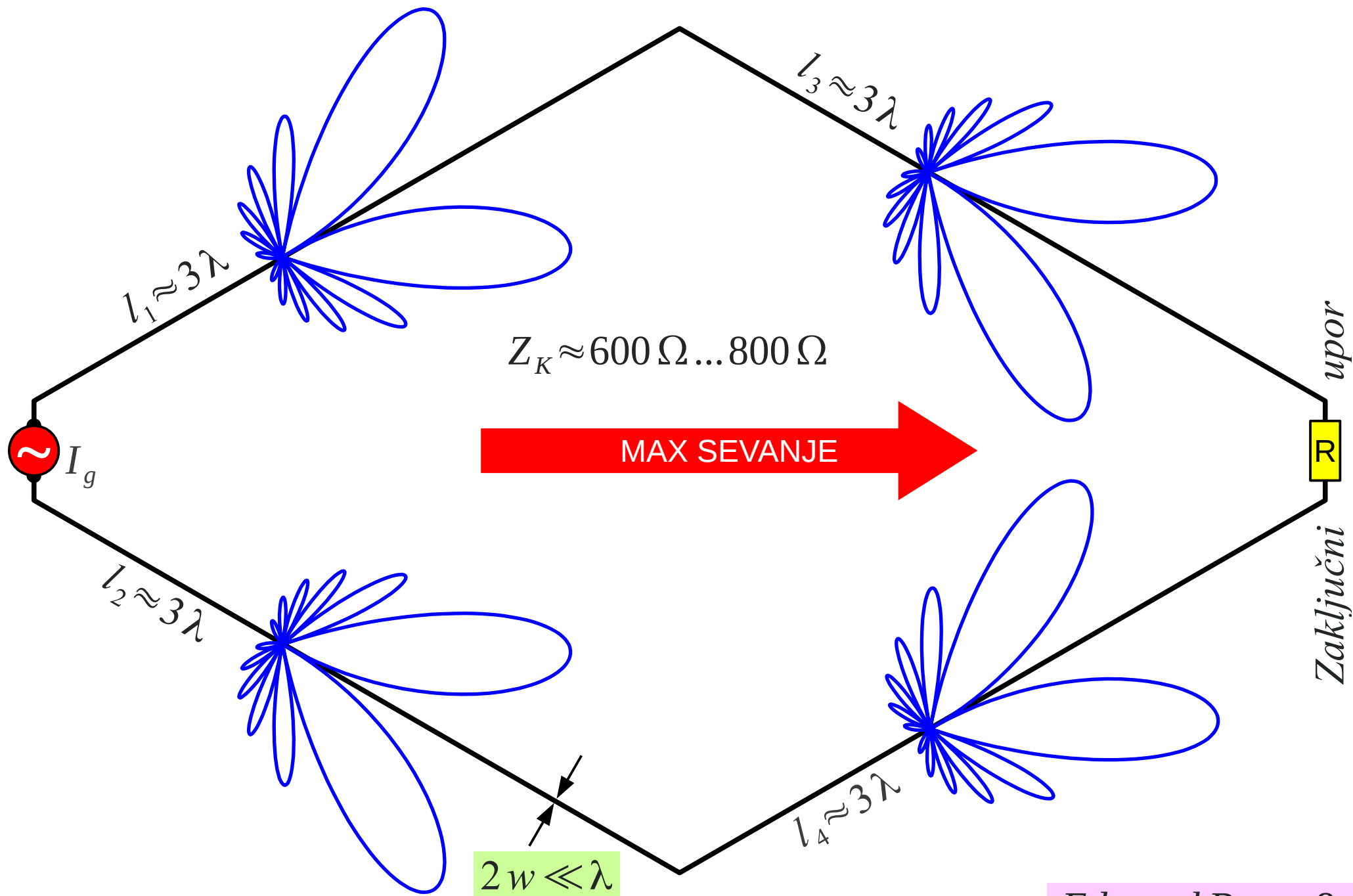
$$F(\Theta, \Phi) = \sin \Theta \frac{\sin \frac{kh}{2} (\cos \Theta - 1)}{\frac{kh}{2} (\cos \Theta - 1)}$$

Potujoči val toka  
 $I(z') = I_g e^{-jk(z'+h/2)}$

Harold H. Beverage 1921



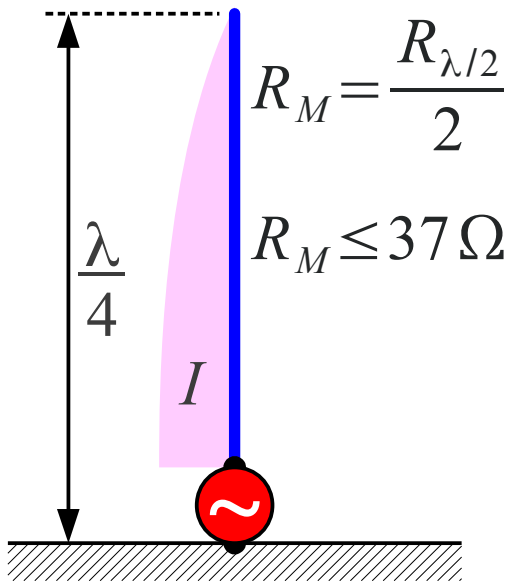
Sevanje potujočega vala na žici



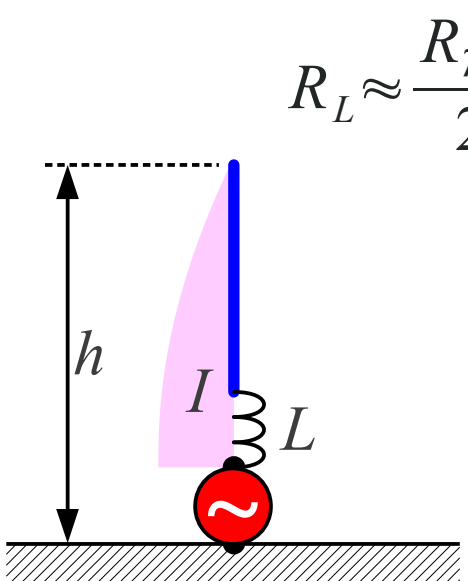
Romb antenna

*Edmond Bruce & Harald Friis 1931*

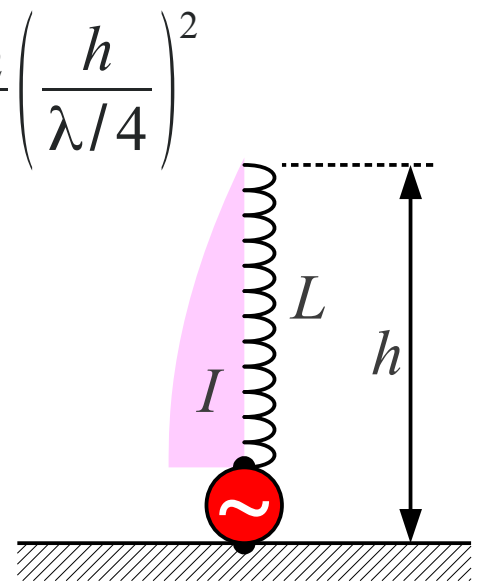
$\lambda/4$  monopol nad prevodno ravnino



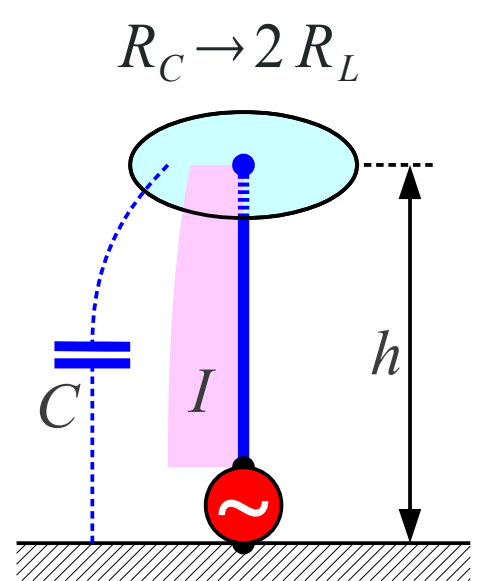
Koncentrirana tuljava



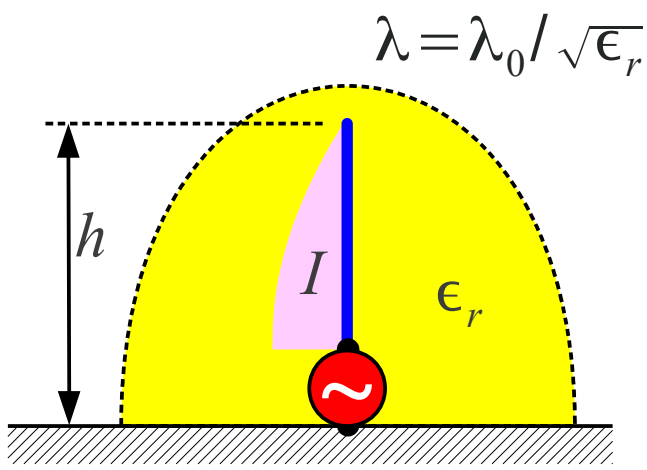
Porazdeljena tuljava



Kapacitivni klobuk



Monopol v dielektriku



Skrajšane antene

Kapacitivni klobuk s transformacijo impedance

