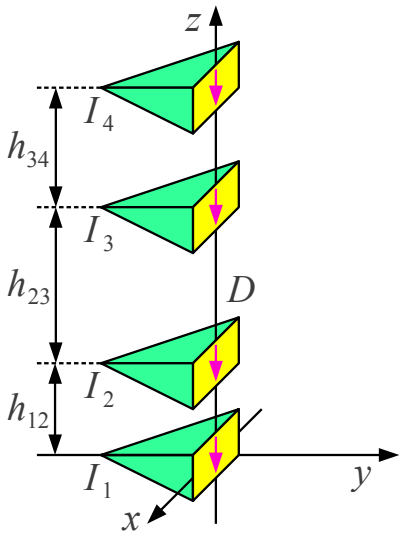
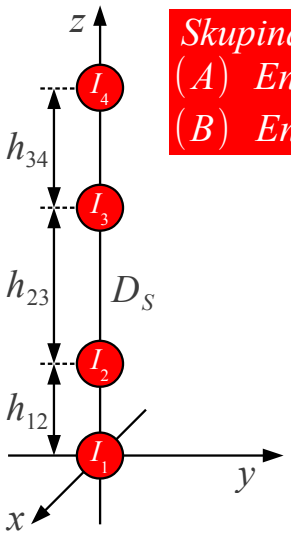
11. Skupine anten

Omejevanje kvadratne napake faze lahko zahteva nepraktično dolge valovodne lijake. Kvadratno napako faze popravi zbiralna leča oziroma zbiralno zrcalo, ki pretvori krogelne valovne fronte v ravne valovne fronte.

Čeprav so osnove delovanja enake, se praktične izvedbe leč za radijske valove v marsičem razlikujejo od leč za vidno svetlobo. Bistvena razlika je v velikosti leče v primerjavi z valovno dolžino. Leče za vidno svetlobo so običajno dosti večje $d \gg \lambda$ od valovne dolžine. Izmere leč za radijske valove so pogosto primerljive $d \approx \lambda$ z valovno dolžino.

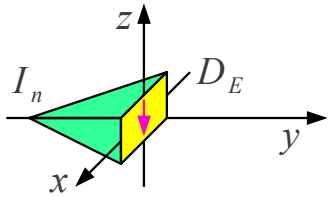


$F(\Theta,\Phi) \equiv$ *smerni diagram skupine anten*



$F_S(\Theta,\Phi) \equiv$ *smerni diagram skupine neusmerjenih virov*

Skupina neusmerjenih virov
(A) Enaka razporeditev h_{mn}
(B) Enako napajanje I_n



$F_E(\Theta,\Phi) \equiv$ *smerni diagram elementa*

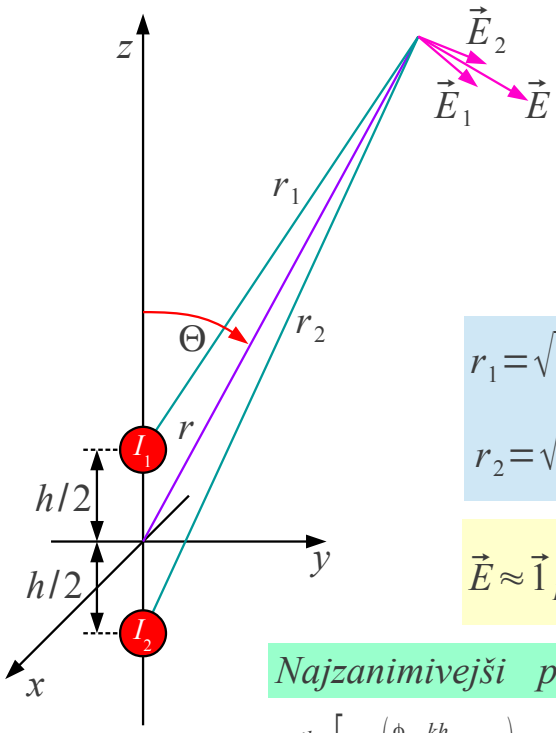
(1) Skupina samih enakih anten
(2) Vse antene enako orientirane
(3) Vse antene enako polarizirane

$$F(\Theta,\Phi) = F_S(\Theta,\Phi) \cdot F_E(\Theta,\Phi)$$

Pravilo o množenju smernih diagramov

$$D \neq D_S \cdot D_E$$

Običajno $D_E, D_S < D < D_S \cdot D_E$



$$\vec{E}=\vec{E}_1+\vec{E}_2=\vec{1}_{E_1}\alpha I_1\frac{e^{-jkr_1}}{r_1}+\vec{1}_{E_2}\alpha I_2\frac{e^{-jkr_2}}{r_2}$$

$$Fraunhofer \quad r>\frac{2h^2}{\lambda}$$

$$\vec{1}_{E_1}\approx\vec{1}_{E_2}\approx\vec{1}_E \qquad \frac{1}{r_1}\approx\frac{1}{r_2}\approx\frac{1}{r}$$

$$r_1=\sqrt{r^2+(h/2)^2-rh\cos\Theta}\approx r-\frac{h}{2}\cos\Theta$$

$$r_2=\sqrt{r^2+(h/2)^2+rh\cos\Theta}\approx r+\frac{h}{2}\cos\Theta$$

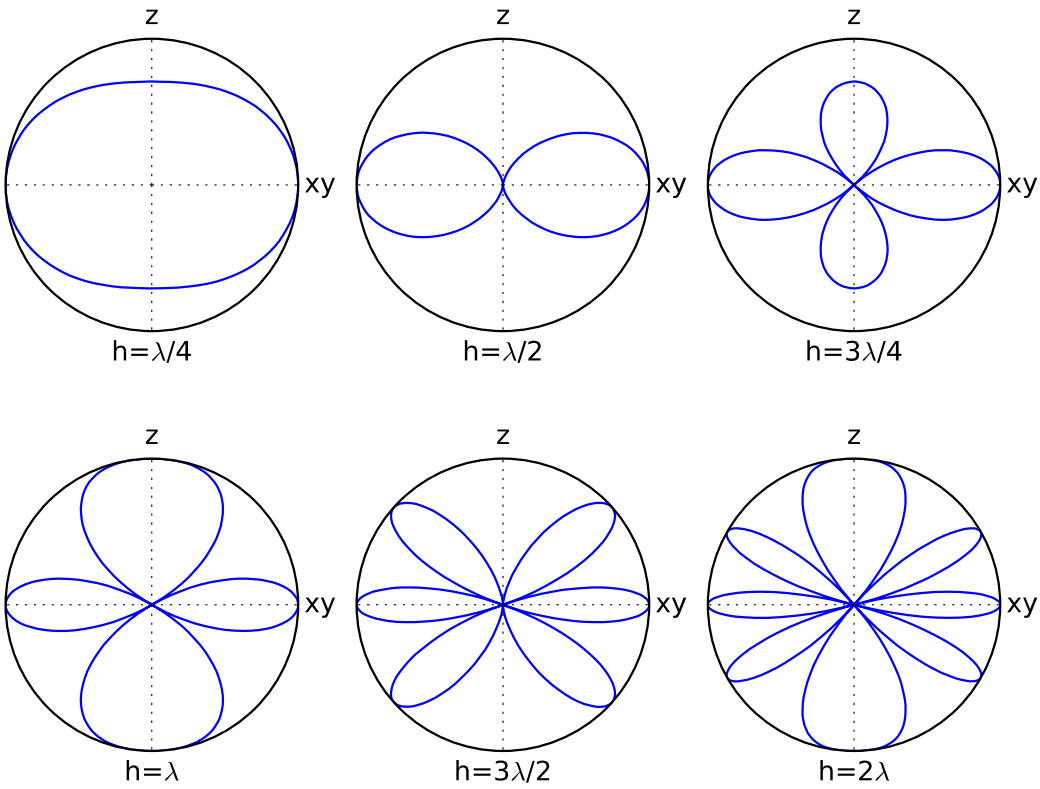
$$\vec{E}\approx\vec{1}_E\alpha\frac{e^{-jkr}}{r}\left[I_1e^{j\frac{kh}{2}\cos\Theta}+I_2e^{-j\frac{kh}{2}\cos\Theta}\right]$$

$$Najzanimivejši \quad primer \quad |I_1|=|I_2| \;\rightarrow\; I_1=I\,e^{j\phi/2} \qquad I_2=I\,e^{-j\phi/2}$$

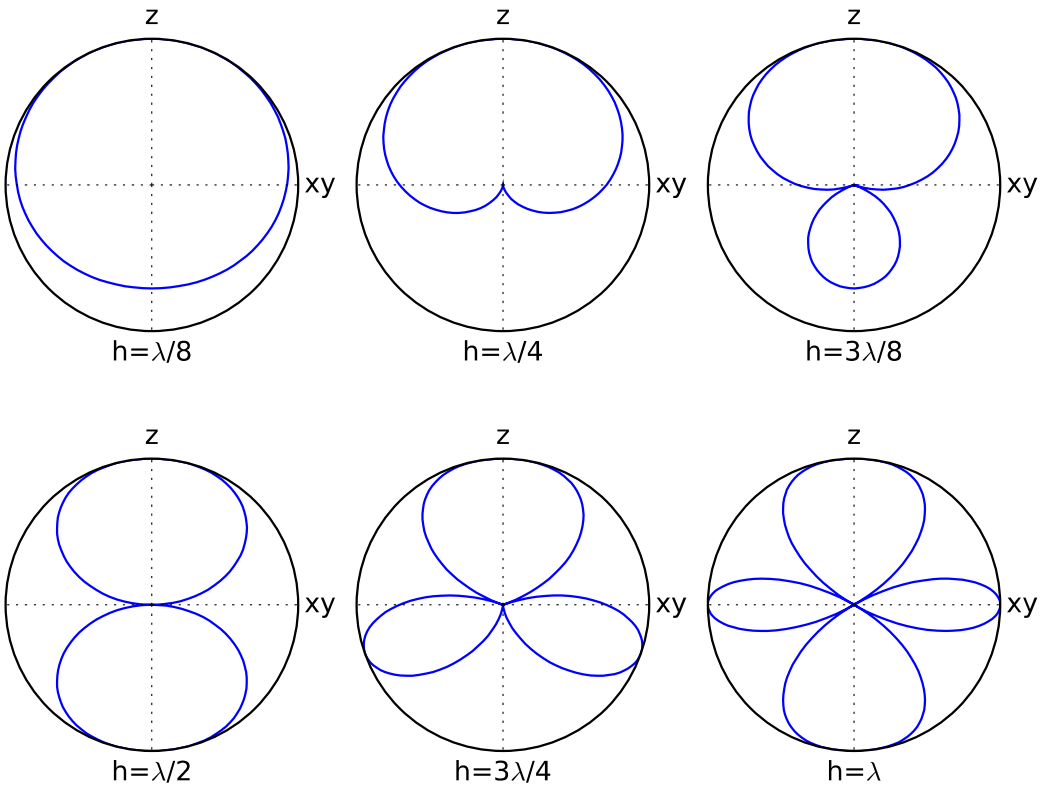
$$\vec{E}\approx\vec{1}_E\alpha I\frac{e^{-jkr}}{r}\left[e^{j\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)}+e^{-j\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)}\right]=\vec{1}_E\alpha I\frac{e^{-jkr}}{r}2\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)$$

$$F(\Theta,\Phi)=\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)$$

Dva neusmerjena (izotropna) vira



Smerni diagrami bočnih skupin $\phi = 0$



Smerni diagrami osnih skupin $\phi = -kh$

$$D=\frac{4\pi\left|F\left(\Theta_{MAX},\Phi_{MAX}\right)\right|^2}{\iint\limits_{4\pi}\left|F\left(\Theta,\Phi\right)\right|^2 d\,\Omega}$$

$$F\left(\Theta,\Phi\right)=\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)$$

$$\iint\limits_{4\pi}\left|F\left(\Theta,\Phi\right)\right|^2 d\,\Omega=\int_0^{\pi}\int_0^{2\pi}\left|F\left(\Theta,\Phi\right)\right|^2\sin\Theta\,d\,\Theta\,d\,\Phi=$$

$$=\int_0^{\pi}\int_0^{2\pi}\left|\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)\right|^2\sin\Theta\,d\,\Theta\,d\,\Phi=2\pi\int_0^{\pi}\left|\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)\right|^2\sin\Theta\,d\,\Theta=$$

$$=2\pi\int_{-1}^{1}\left[\cos\left(\frac{\phi}{2}+\frac{khu}{2}\right)\right]^2 du=\pi\int_{-1}^{1}\left[1+\cos\left(\phi+khu\right)\right]du=$$

$$=\pi\left[2+\frac{\sin\left(\phi+kh\right)-\sin\left(\phi-kh\right)}{kh}\right]=2\pi\left[1+\frac{\sin\left(kh\right)}{kh}\cos\phi\right]$$

$$D=\frac{2\left|F\left(\Theta_{MAX},\Phi_{MAX}\right)\right|^2}{1+\frac{\sin\left(kh\right)}{kh}\cos\phi}$$

Smernost dveh virov

$F\left(\Theta_{MAX}=\pi/2,\Phi_{MAX}\right)=1$

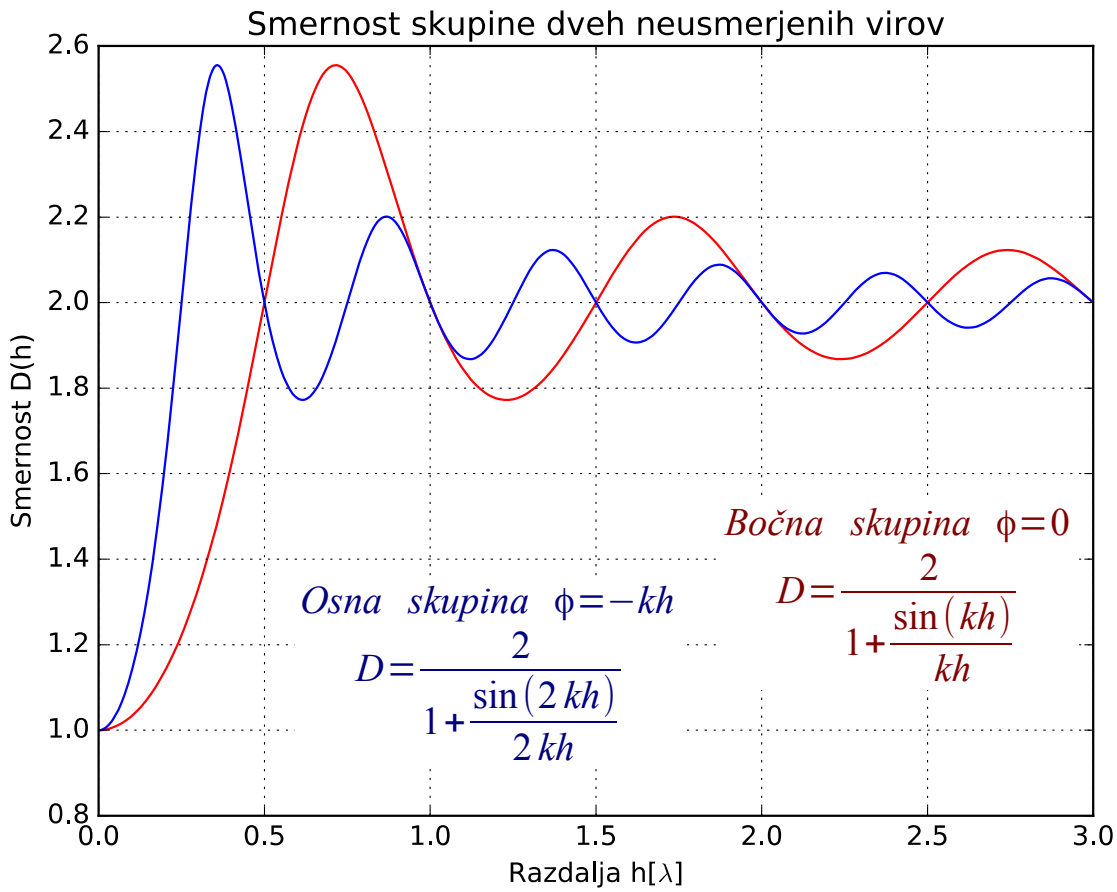
Bočna skupina → $\phi=0$

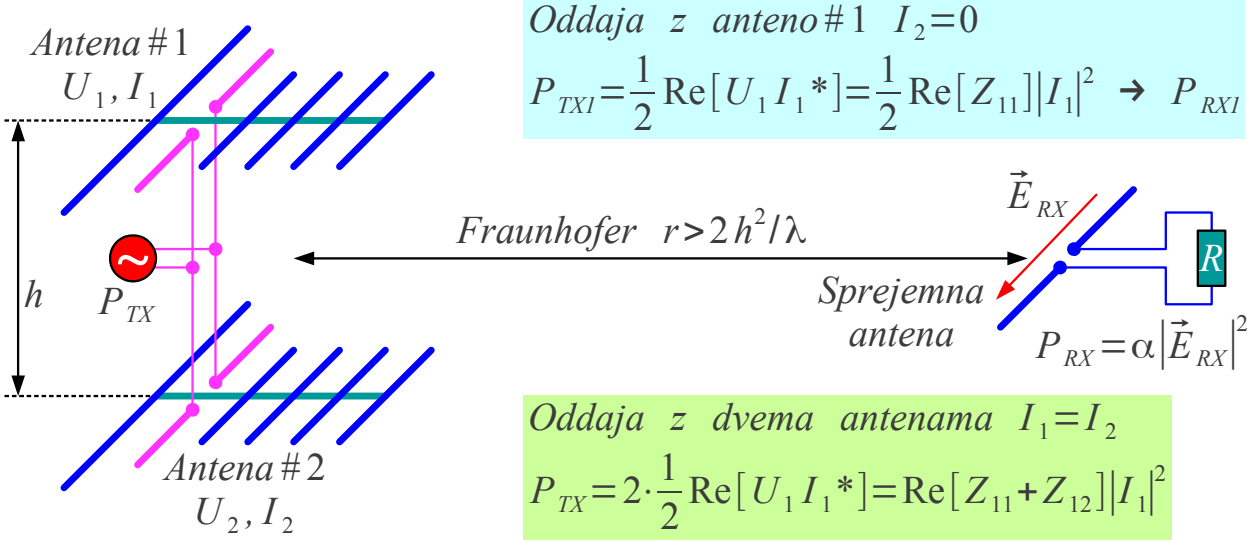
$$D=\frac{2}{1+\frac{\sin\left(kh\right)}{kh}}$$

$F\left(\Theta_{MAX}=0,\Phi_{MAX}\right)=1$

Osna skupina → $\phi=-kh$

$$D=\frac{2}{1+\frac{\sin\left(2\,kh\right)}{2\,kh}}$$


Smernost skupine dveh neusmerjenih virov
Smernost D(h)
Razdalja h[λ]
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Osna skupina ϕ=−kh
D=2/(1+sin(2kh)/(2kh))
Bočna skupina ϕ=0
D=2/(1+sin(kh)/(kh))



Oddaja z anteno #1 $I_2 = 0$

$$P_{TX1} = \frac{1}{2}\mathrm{Re}[U_1 I_1^*] = \frac{1}{2}\mathrm{Re}[Z_{11}]|I_1|^2 \rightarrow P_{RX1}$$

Oddaja z dvema antenama $I_1 = I_2$

$$P_{TX} = 2 \cdot \frac{1}{2}\mathrm{Re}[U_1 I_1^*] = \mathrm{Re}[Z_{11} + Z_{12}]|I_1|^2$$

$$\vec{E}_{RX} = 2\vec{E}_{RX1} \rightarrow P_{RX} = 4 P_{RX1}$$

Medsebojni vpliv v skupini

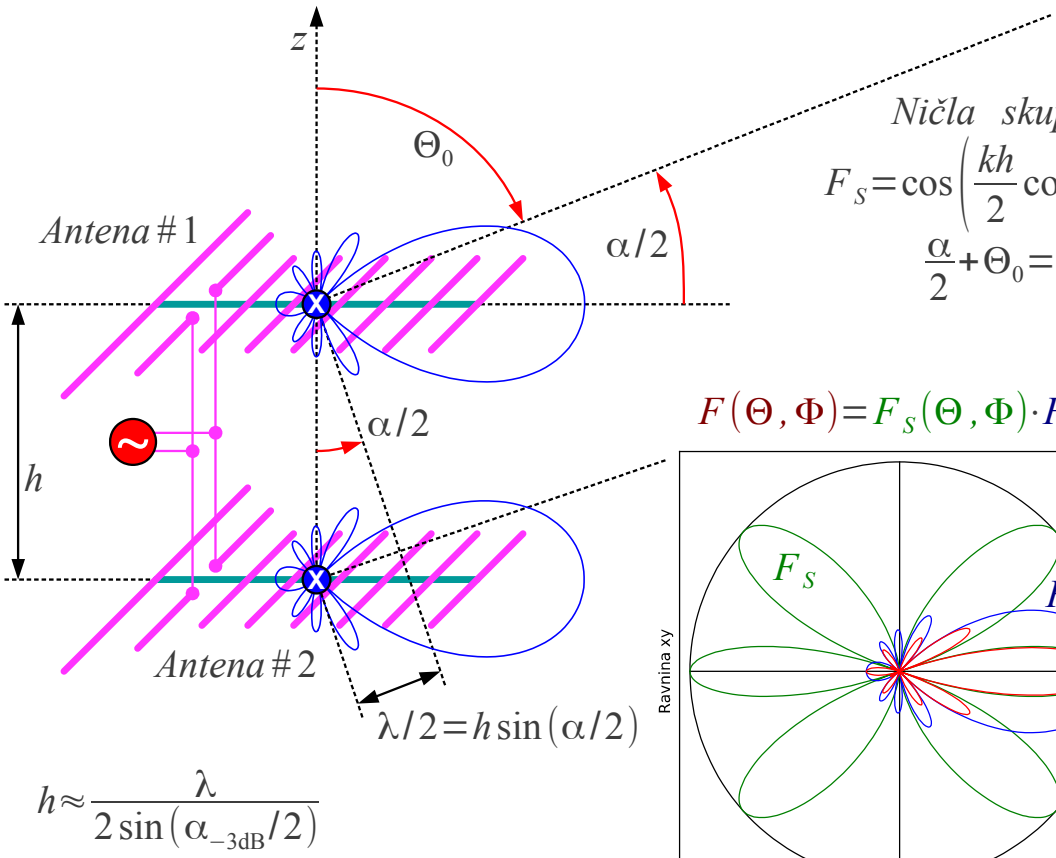
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} . \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Recipročnost $Z_{12}(h) = Z_{21}(h)$

Enaki anteni $Z_{11} = Z_{22}$

$$\Delta D = \frac{\left(\frac{P_{RX}}{P_{RX1}}\right)}{\left(\frac{P_{TX}}{P_{TX1}}\right)} = \frac{4}{\frac{\mathrm{Re}[Z_{11} + Z_{12}]|I_1|^2}{\frac{1}{2}\mathrm{Re}[Z_{11}]|I_1|^2}} = \frac{2\,\mathrm{Re}[Z_{11}]}{\mathrm{Re}[Z_{11} + Z_{12}]}$$

Medsebojna impedanca v bočni skupini



Ničla skupine

$$F_S = \cos\left(\frac{kh}{2}\cos\Theta_0\right) = 0$$

$$\frac{\alpha}{2} + \Theta_0 = \frac{\pi}{2}$$

$$F(\Theta,\Phi) = F_S(\Theta,\Phi)\cdot F_E(\Theta,\Phi)$$

$$h \approx \frac{\lambda}{2\sin(\alpha_{-3\mathrm{dB}}/2)}$$

Približno pravilo za bočno skupino

Osna skupina $|F(\Theta_{MAX}=0,\Phi_{MAX})|<1$

$$F(\Theta,\Phi)=\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta\right)$$

$$D=\frac{2|F(\Theta_{MAX},\Phi_{MAX})|^2}{1+\frac{\sin(kh)}{kh}\cos\phi}=\frac{2\left|\cos\left(\frac{\phi}{2}+\frac{kh}{2}\right)\right|^2}{1+\frac{\sin(kh)}{kh}\cos\phi}=\frac{1+\cos(\phi+kh)}{1+\frac{\sin(kh)}{kh}\cos\phi}$$

$$\frac{\partial D}{\partial\phi}=0=\frac{-\sin(\phi+kh)\left[1+\frac{\sin(kh)}{kh}\cos\phi\right]-\left[1+\cos(\phi+kh)\right]\left[-\frac{\sin(kh)}{kh}\sin\phi\right]}{\left[1+\frac{\sin(kh)}{kh}\cos\phi\right]^2}$$

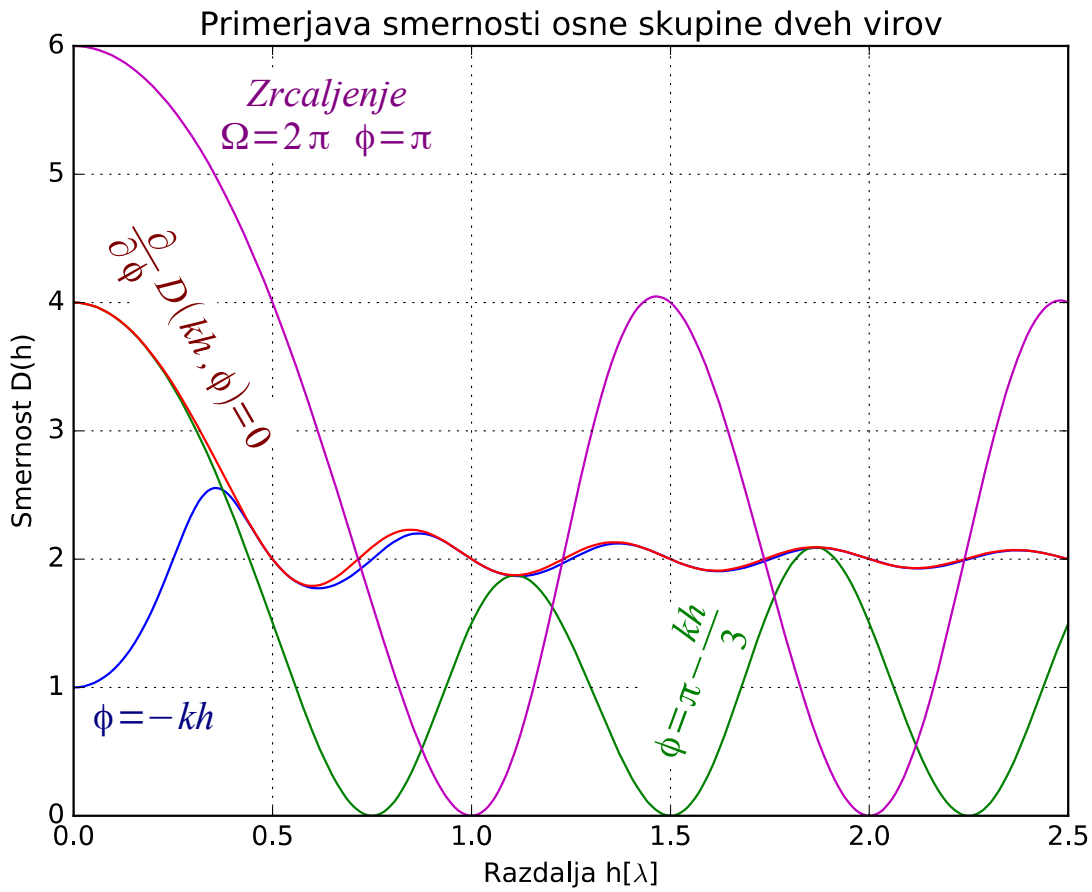
$$0=\left[\frac{\sin^2(kh)}{(kh)^2}-2\frac{\sin(kh)}{kh}\cos(kh)+1\right]\sin^2\phi-$$

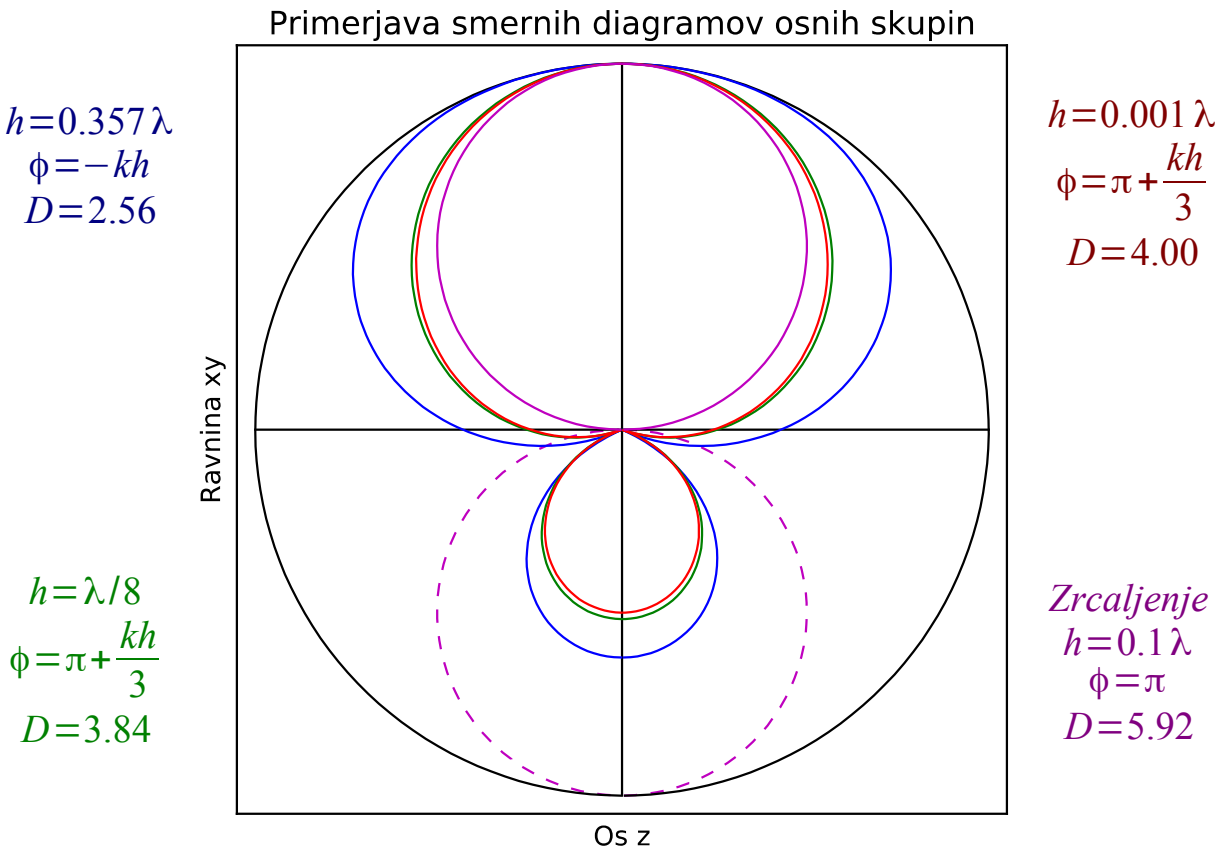
$$-2\frac{\sin^2(kh)}{kh}\left[\frac{\sin(kh)}{kh}-\cos(kh)\right]\sin\phi+\left[\frac{\sin^4(kh)}{(kh)^2}-\sin^2(kh)\right]$$

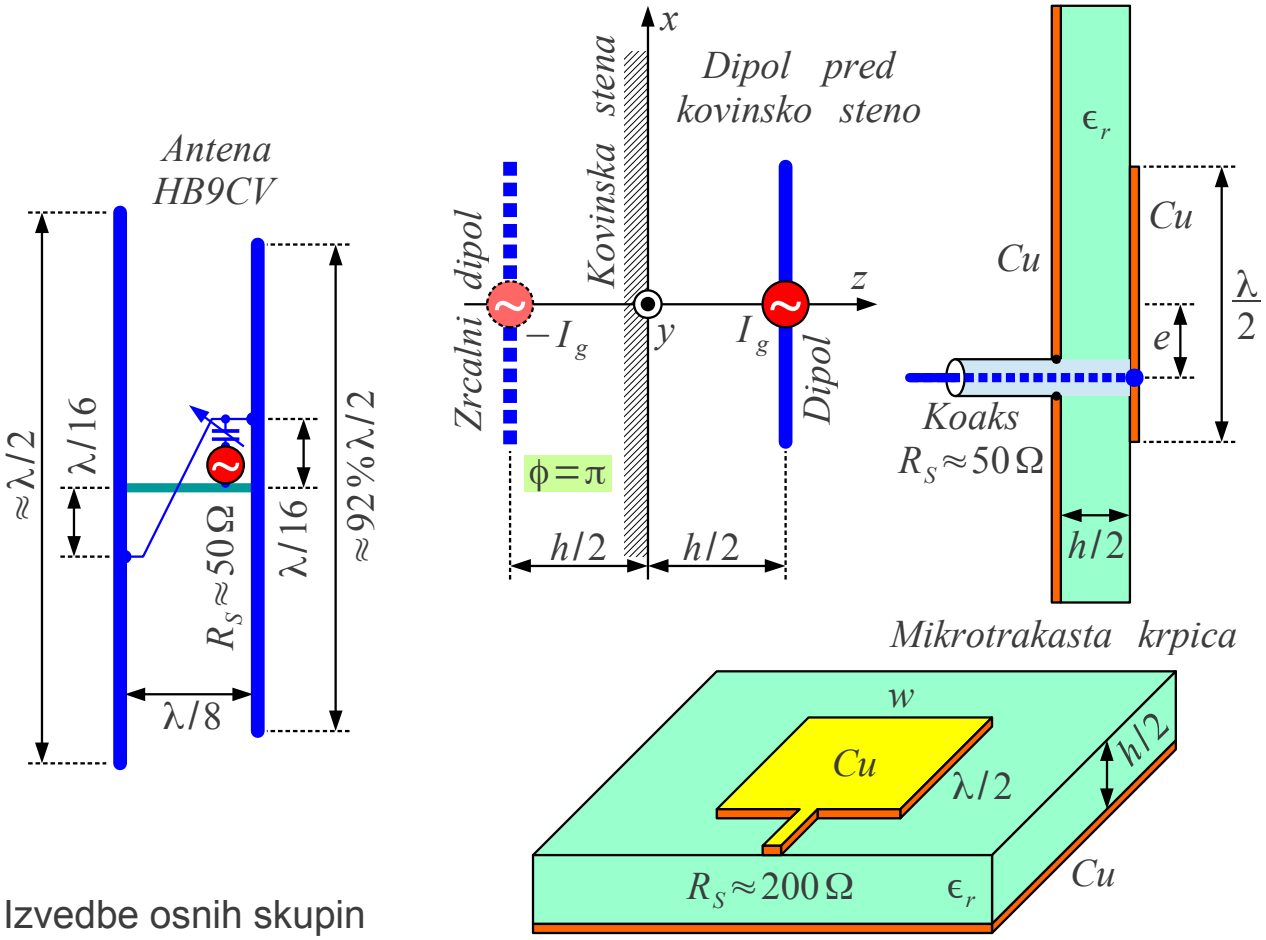
$u=\sin\phi \rightarrow \phi=\arcsin u$ *ali* $\phi=\pi-\arcsin u$

Približek $h<\frac{\lambda}{4} \rightarrow \phi\approx\pi+\frac{kh}{3}$

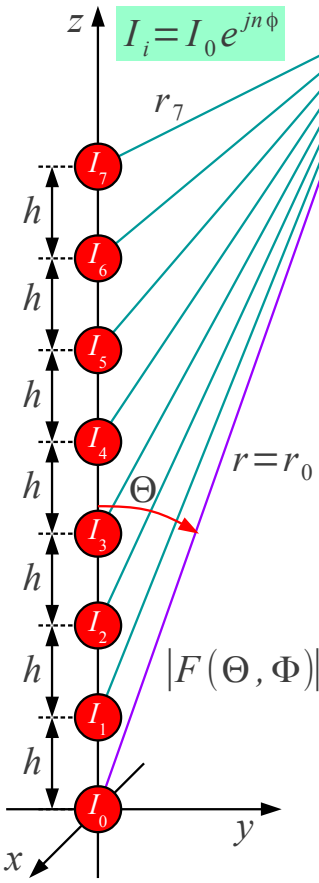
Največja smernost osne skupine


Primerjava smernosti osne skupine dveh virov
Zrcaljenje
Ω=2π ϕ=π
∂/∂ϕ D(kh,ϕ)=0
ϕ=−kh
ϕ=π−kh/3
Smernost D(h)
Razdalja h[λ]
0
1
2
3
4
5
6
0.0
0.5
1.0
1.5
2.0
2.5


Primerjava smernih diagramov osnih skupin
$h=0.357\lambda$
$\phi=-kh$
$D=2.56$
$h=0.001\lambda$
$\phi=\pi+\frac{kh}{3}$
$D=4.00$
$h=\lambda/8$
$\phi=\pi+\frac{kh}{3}$
$D=3.84$
Zrcaljenje
$h=0.1\lambda$
$\phi=\pi$
$D=5.92$
Ravnina xy
Os z



Izvedbe osnih skupin



$$\vec{E} = \sum_{n=0}^{m-1} \vec{E}_i$$

$$\vec{E}_i = \vec{1}_{E_i}\, \alpha\, I_i \frac{e^{-jkr_i}}{r_i} \approx \vec{1}_E \frac{\alpha\, I_0}{r} e^{jn\phi} e^{-jkr_n}$$

$$r_i = \sqrt{r^2 + (ih)^2 - 2\, rih \cos\Theta} \approx r - ih\cos\Theta$$

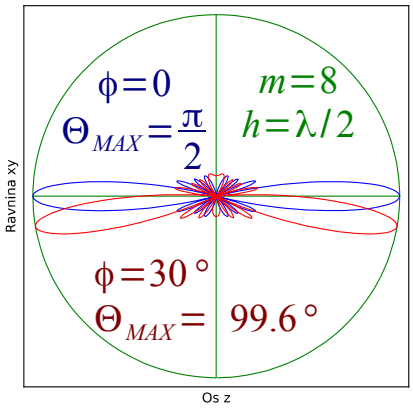
$$\vec{E} \approx \vec{1}_E \frac{\alpha\, I_0}{r} \sum_{n=0}^{m-1} e^{-j(kr_n - n\phi)} \approx \vec{1}_E\, \alpha\, I_0 \frac{e^{-jkr}}{r} \sum_{n=0}^{m-1} e^{jn(\phi + kh\cos\Theta)}$$

$$b = e^{j(\phi + kh\cos\Theta)}$$
$$|b| = 1$$

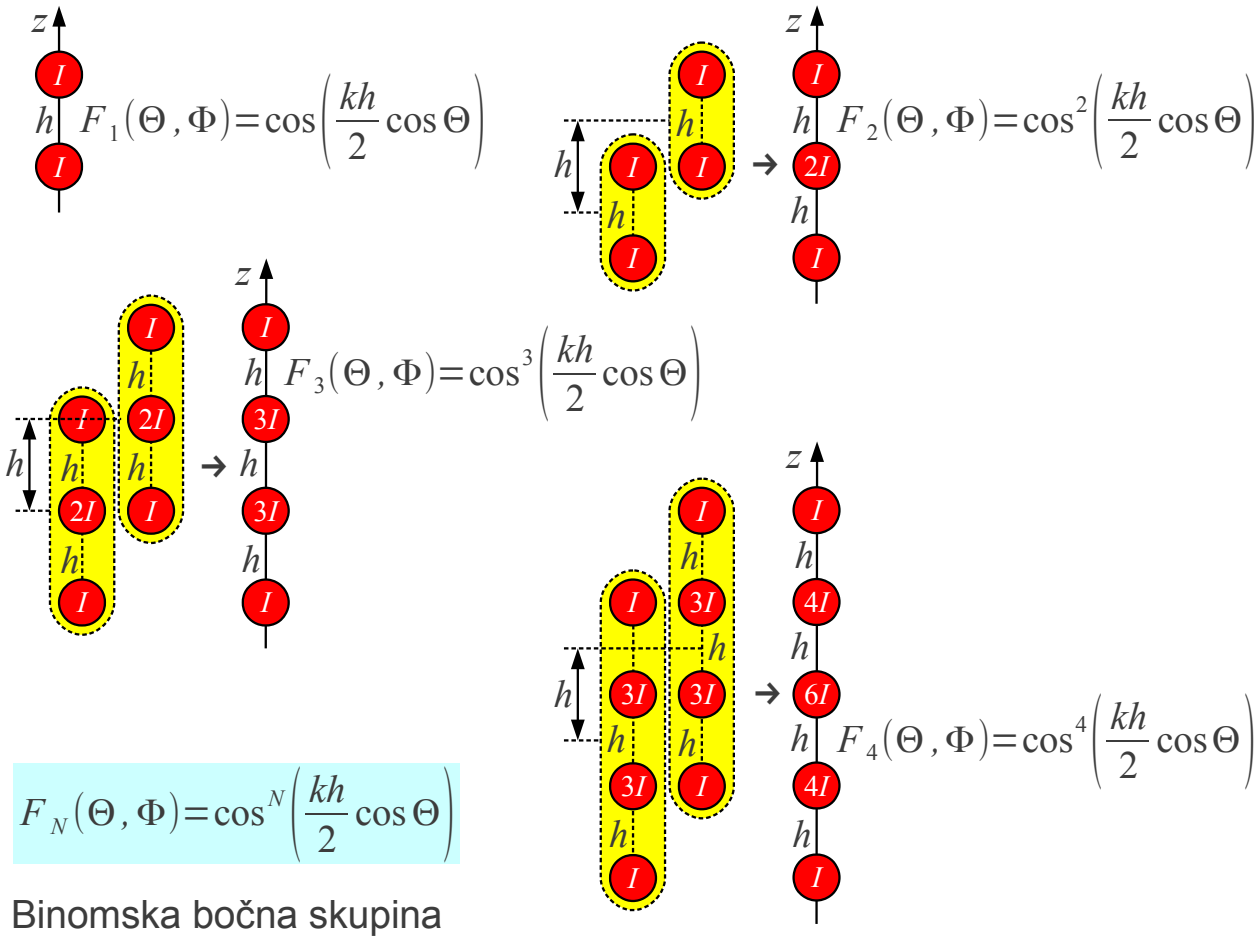
$$\sum_{n=0}^{m-1} b^n = \frac{b^m - 1}{b - 1} = \left(b^{\frac{m-1}{2}}\right) \frac{\sin\left(m\frac{\phi + kh\cos\Theta}{2}\right)}{\sin\left(\frac{\phi + kh\cos\Theta}{2}\right)}$$

$$|F(\Theta, \Phi)| = \frac{1}{m} \left| \frac{\sin\left(m\frac{\phi + kh\cos\Theta}{2}\right)}{\sin\left(\frac{\phi + kh\cos\Theta}{2}\right)} \right|$$

$$\Theta_{MAX} = \arccos\left(\frac{-\phi}{kh}\right)$$



Enakomerna skupina izvorov



$$F_N(\Theta,\Phi)=\cos^N\left(\frac{kh}{2}\cos\Theta\right)$$

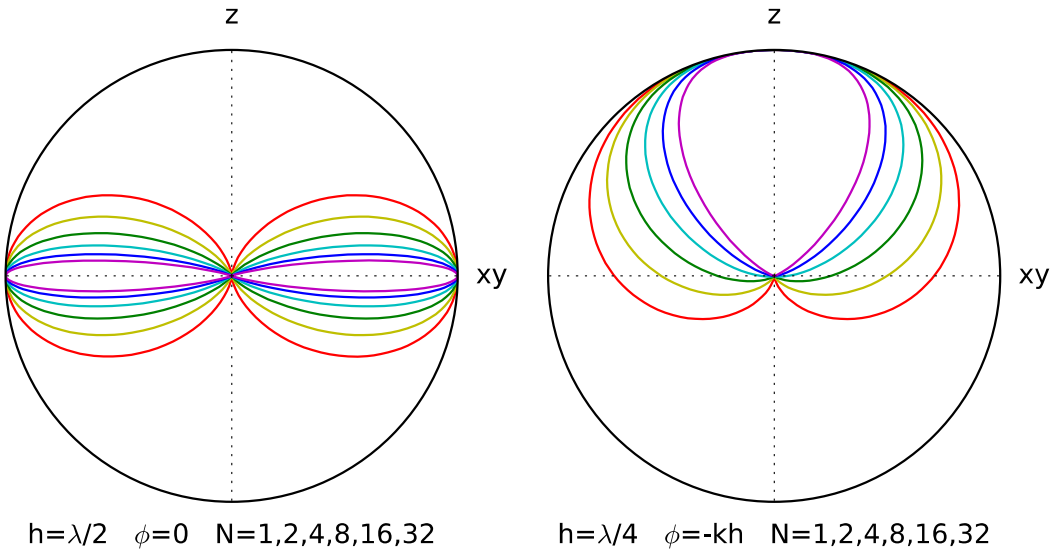
Binomska bočna skupina

Bočna binomska skupina

$$F_N(\Theta,\Phi)=\cos^N\left(\frac{kh}{2}\cos\Theta\right)$$

Osna binomska skupina

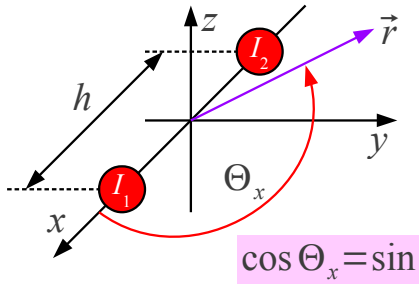
$$F_N(\Theta,\Phi)=\cos^N\left(\frac{kh}{2}(\cos\Theta-1)\right)$$



h=λ/2 ϕ=0 N=1,2,4,8,16,32 h=λ/4 ϕ=-kh N=1,2,4,8,16,32

Smerni diagrami binomskih skupin

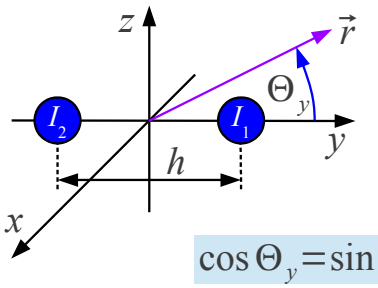
$$F_S(\Theta,\Phi)=\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta_x\right)$$



$$\cos\Theta_x=\sin\Theta\cos\Phi$$

$$F_S(\Theta,\Phi)=\cos\left(\frac{\phi}{2}+\frac{kh}{2}\sin\Theta\cos\Phi\right)$$

$$F_S(\Theta,\Phi)=\cos\left(\frac{\phi}{2}+\frac{kh}{2}\cos\Theta_y\right)$$

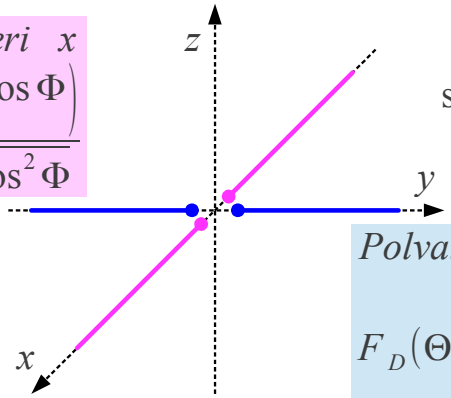


$$\cos\Theta_y=\sin\Theta\sin\Phi$$

$$F_S(\Theta,\Phi)=\cos\left(\frac{\phi}{2}+\frac{kh}{2}\sin\Theta\sin\Phi\right)$$

Polvalovni dipol v smeri x

$$F_D(\Theta,\Phi)=\frac{\cos\left(\frac{\pi}{2}\sin\Theta\cos\Phi\right)}{\sqrt{1-\sin^2\Theta\cos^2\Phi}}$$



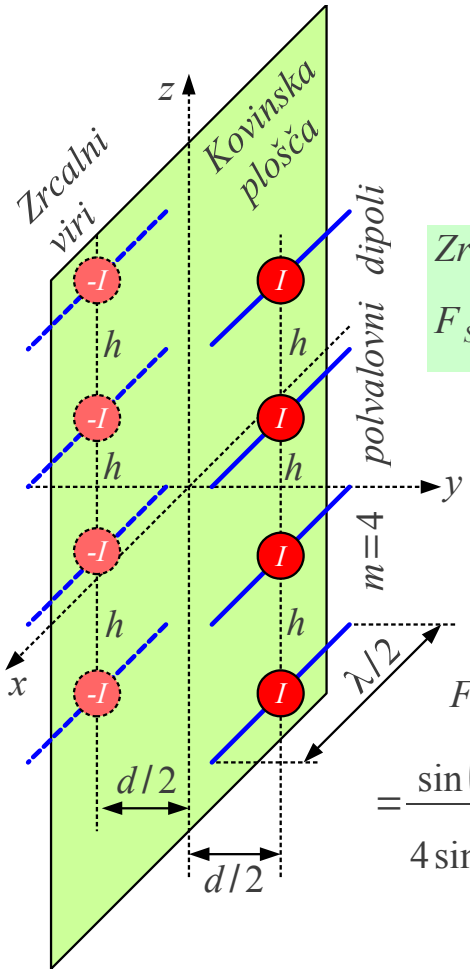
$$\sin\Theta_y=\sqrt{1-\sin^2\Theta\sin^2\Phi}$$

$$\sin\Theta_x=\sqrt{1-\sin^2\Theta\cos^2\Phi}$$

Polvalovni dipol v smeri y

$$F_D(\Theta,\Phi)=\frac{\cos\left(\frac{\pi}{2}\sin\Theta\sin\Phi\right)}{\sqrt{1-\sin^2\Theta\sin^2\Phi}}$$

Obračanje anten



Polvalovni dipol v smeri osi x

$$F_E(\Theta,\Phi)=\frac{\cos\left(\frac{\pi}{2}\cos\Theta_x\right)}{\sin\Theta_x}=\frac{\cos\left(\frac{\pi}{2}\sin\Theta\cos\Phi\right)}{\sqrt{1-\sin^2\Theta\cos^2\Phi}}$$

Zrcaljenje v smeri osi y → $\phi=-\pi$

$$F_{S1}(\Theta,\Phi)=\cos\left(\frac{\phi}{2}+\frac{kd}{2}\cos\Theta_y\right)=\sin\left(\frac{kd}{2}\sin\Theta\sin\phi\right)$$

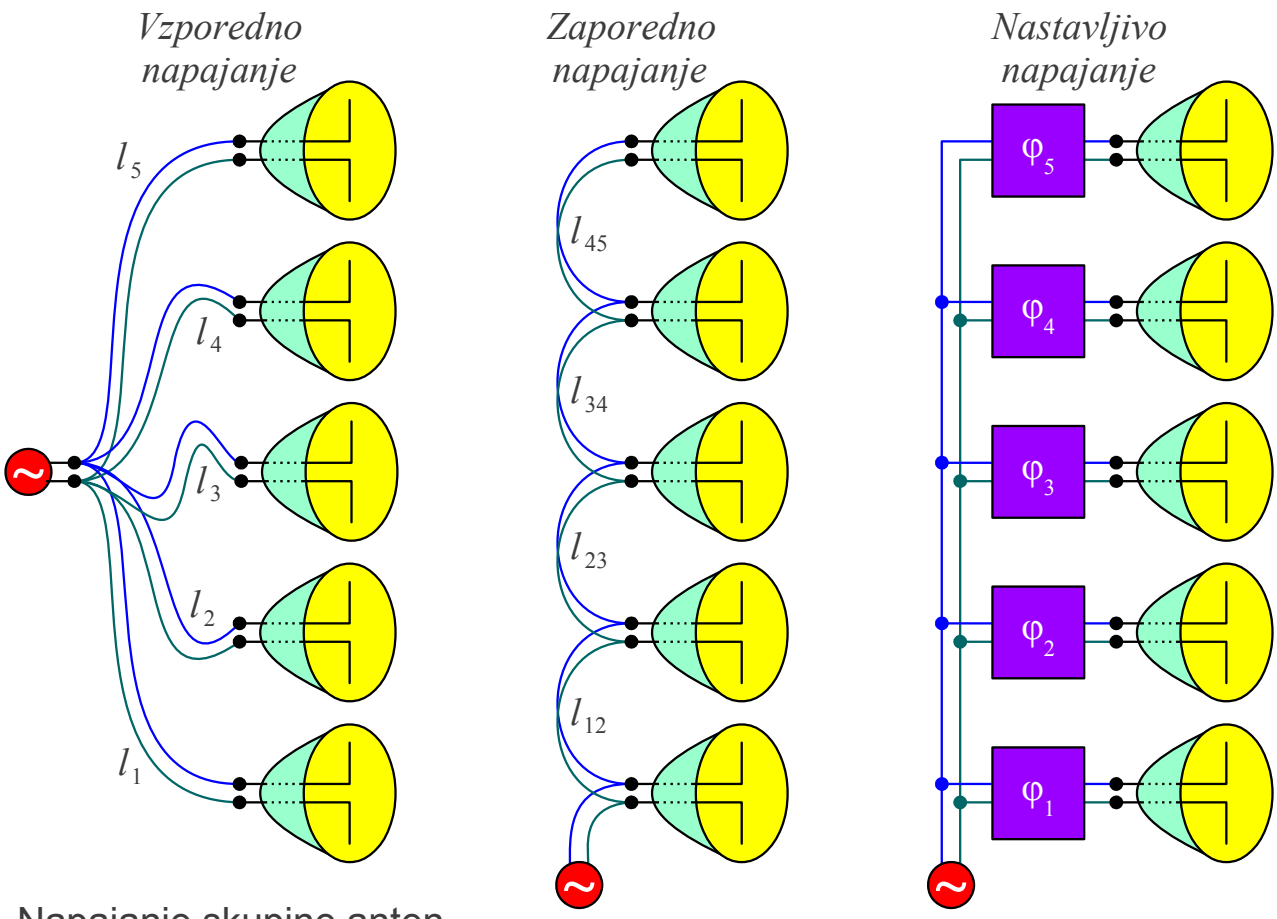
Enakomerna bočna skupina v smeri osi z

$$F_{S2}(\Theta,\Phi)=\frac{\sin\left(m\frac{kh}{2}\cos\Theta\right)}{m\sin\left(\frac{kh}{2}\cos\Theta\right)}=\frac{\sin\left(2\,kh\cos\Theta\right)}{4\sin\left(\frac{kh}{2}\cos\Theta\right)}$$

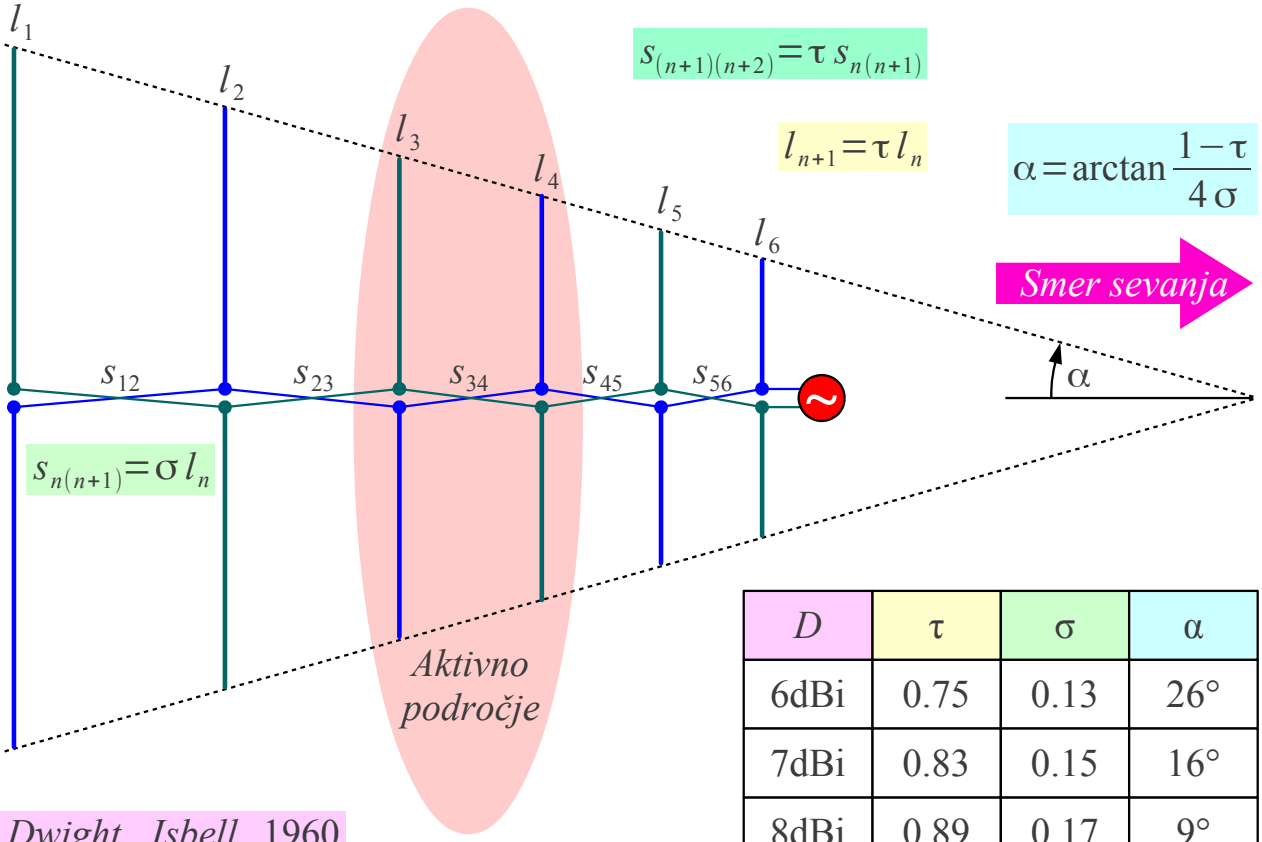
$$F(\Theta,\Phi)=F_{S2}(\Theta,\Phi)\cdot F_{S1}(\Theta,\Phi)\cdot F_E(\Theta,\Phi)=$$

$$=\frac{\sin\left(2\,kh\cos\Theta\right)}{4\sin\left(\frac{kh}{2}\cos\Theta\right)}\sin\left(\frac{kd}{2}\sin\Theta\sin\phi\right)\frac{\cos\left(\frac{\pi}{2}\sin\Theta\cos\Phi\right)}{\sqrt{1-\sin^2\Theta\cos^2\Phi}}$$

Sestavljanje skupin



Napajanje skupine anten



Dwight Isbell 1960

Logaritmično-periodična skupina dipolov

D	τ	σ	α
6dBi	0.75	0.13	26°
7dBi	0.83	0.15	16°
8dBi	0.89	0.17	9°
9dBi	0.94	0.19	5°

* * * * *