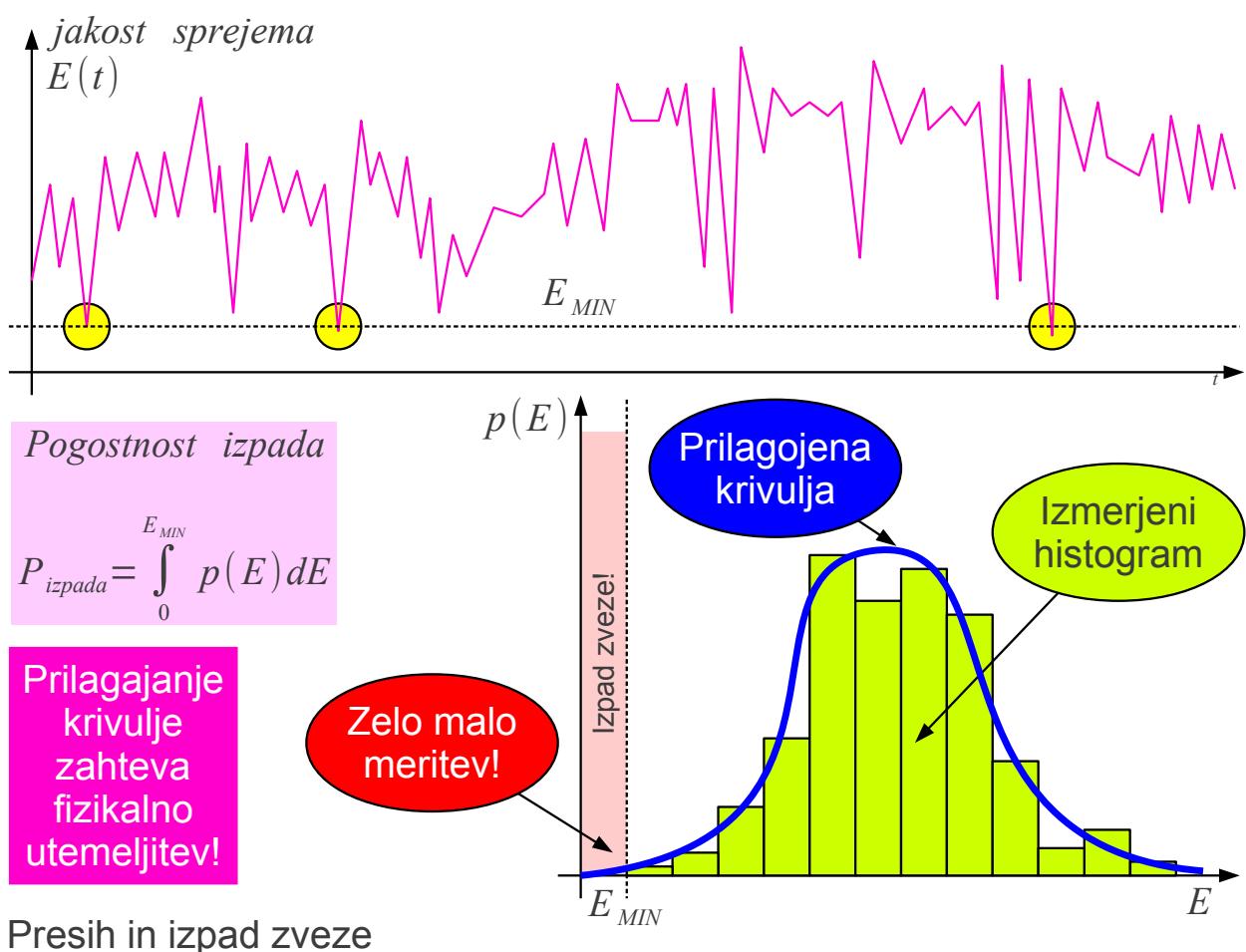
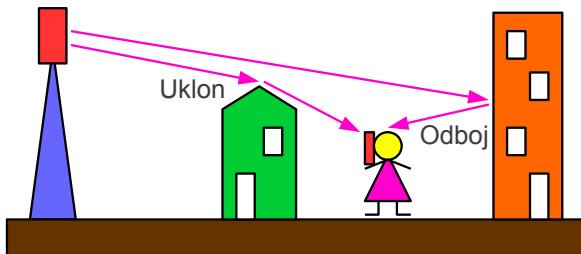


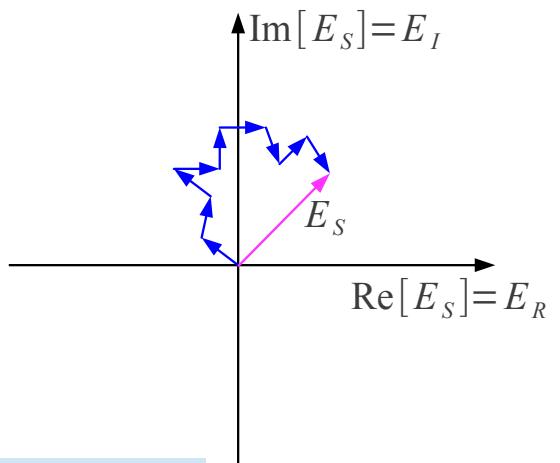
## 17. Večpotje v radijski zvezi

Večina nalog iz anten in razširjanje valov zahteva obravnavo v treh dimenzijah prostora. Tako skalarne kot tudi vektorske veličine so funkcije časa in vseh treh dimenzij prostora. Ozkopasovne signale  $B \ll f$  radija največkrat smemo v izračunih ponazoriti s harmonskim signalom ene same krožne frekvence  $\omega = 2\pi f$ , kar poenostavi časovne odvode v  $\partial/\partial t = j\omega$ .





Večpotje brez vidljivosti: Rayleigh ( $\langle E^2 \rangle$ )  
vsota mnogo naključnih malih kazalcev



Gaussova porazdelitev komponent

$$p(E_R) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_R^2}{2\sigma^2}}$$

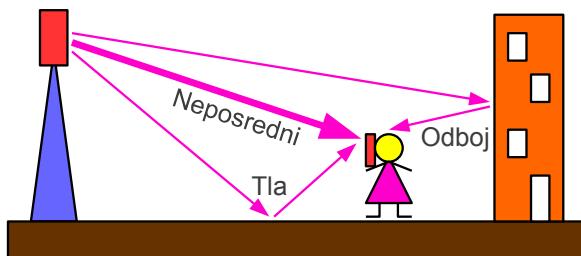
$$p(E_I) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_I^2}{2\sigma^2}}$$

$$E_S = E_R + jE_I = E e^{j\phi} \quad |E_S| = \sqrt{E_R^2 + E_I^2} \quad dE_R dE_I = E dE d\phi$$

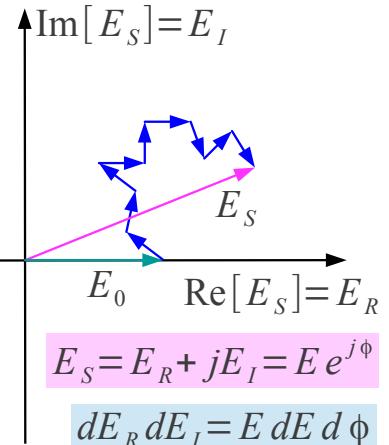
$$p(E_R, E_I) = p(E_R) p(E_I) = \frac{1}{\sigma^2 2\pi} e^{-\frac{E_R^2 + E_I^2}{2\sigma^2}} = p(E) p(\phi) \quad \langle E^2 \rangle = 2\sigma^2$$

$$p(E) = \int_0^{2\pi} p(E_R, E_I) E d\phi = \int_0^{2\pi} \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2}{2\sigma^2}} E d\phi = \frac{E}{\sigma^2} e^{-\frac{E^2}{2\sigma^2}} = \frac{2E}{\langle E^2 \rangle} e^{-\frac{E^2}{\langle E^2 \rangle}}$$

Rayleighjeva porazdelitev



Večpotje z neposrednim žarkom: Rice ( $E_0, \sigma$ )  
en velik in mnogo naključnih malih kazalcev



Gaussova  
porazdelitev  
komponent

$$p(E_R) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(E_R - E_0)^2}{2\sigma^2}}$$

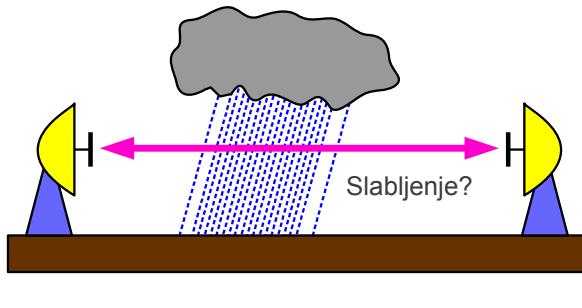
$$p(E_I) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_I^2}{2\sigma^2}}$$

$$p(E_R, E_I) = \frac{1}{\sigma^2 2\pi} e^{-\frac{(E_R - E_0)^2 + E_I^2}{2\sigma^2}} = \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} e^{\left(\frac{E_R E_0}{\sigma^2}\right)} = \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} e^{\left(\frac{E_0 E}{\sigma^2}\right) \cos \phi}$$

$$p(E) = \int_0^{2\pi} p(E_R, E_I) E d\phi = \frac{E}{\sigma^2} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} I_0\left(\frac{E_0 E}{\sigma^2}\right)$$

$$\int_0^{2\pi} e^{\left(\frac{E_0 E}{\sigma^2}\right) \cos \phi} d\phi = 2\pi I_0\left(\frac{E_0 E}{\sigma^2}\right)$$

Riceova porazdelitev



Neznane razmere: log-normalna ( $\langle E_{\text{dB}} \rangle, \sigma_{\text{dB}}$ )

Fizikalno neutemeljeno!

$$p(E_{\text{dB}}) = \frac{1}{\sigma_{\text{dB}} \sqrt{2\pi}} e^{-\frac{(E_{\text{dB}} - \langle E_{\text{dB}} \rangle)^2}{2\sigma_{\text{dB}}^2}}$$

$$E_{\text{dB}} = 20 \log_{10} \left( \frac{|E_S|}{E_{\text{REF}}} \right) = 20 \log_{10} \left( \frac{E}{E_{\text{REF}}} \right) = \frac{20}{\ln 10} \ln \left( \frac{E}{E_{\text{REF}}} \right)$$

$$\sigma_{\text{dB}} = \sqrt{\langle (E_{\text{dB}} - \langle E_{\text{dB}} \rangle)^2 \rangle}$$

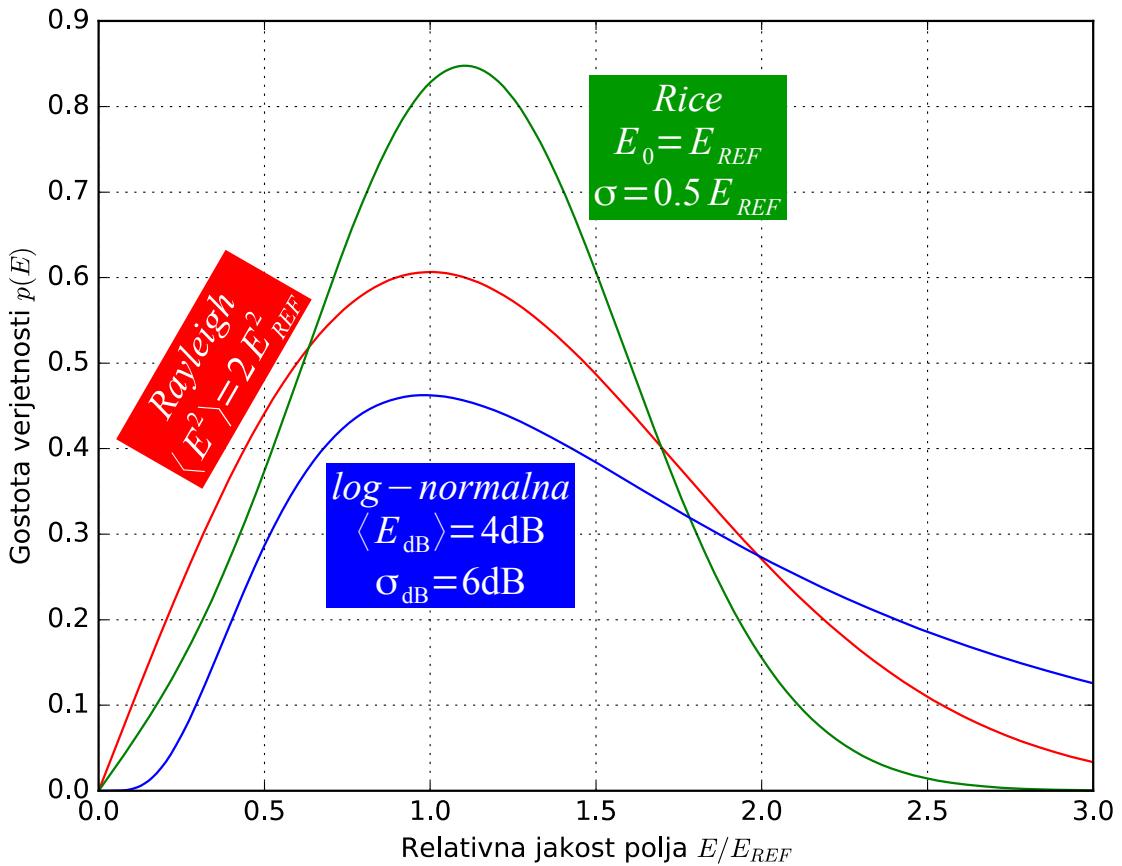
$$p(E_{\text{dB}}) dE_{\text{dB}} = p(E) dE$$

$$\frac{dE_{\text{dB}}}{dE} = \frac{20}{\ln 10} \left( \frac{E_{\text{REF}}}{E} \right) \frac{1}{E_{\text{REF}}} = \frac{20}{E \ln 10}$$

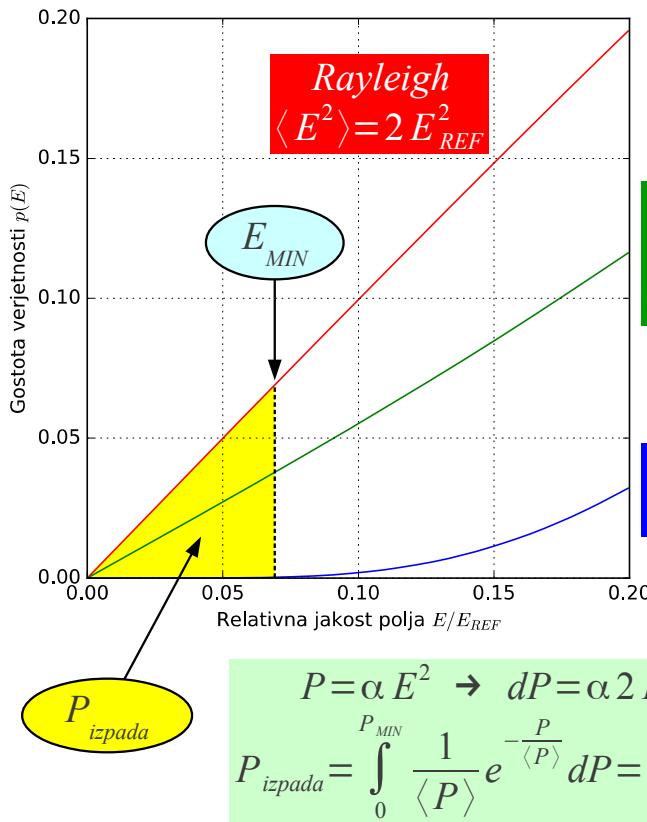
$$p(E) = p(E_{\text{dB}}) \frac{dE_{\text{dB}}}{dE} = p(E_{\text{dB}}) \frac{20}{E \ln 10} = \frac{20}{E (\ln 10) \sigma_{\text{dB}} \sqrt{2\pi}} e^{-\frac{\left[ 20 \log_{10} \left( \frac{E}{E_{\text{REF}}} \right) - \langle E_{\text{dB}} \rangle \right]^2}{2\sigma_{\text{dB}}^2}}$$

Log-normalna porazdelitev

### Rayleighjeva, Riceova in log-normalna porazdelitev



Rayleighjeva, Riceova in log-normalna porazdelitev



Pošten račun: Rayleigh

$$P_{izpada} = \int_0^{E_{MIN}} p(E) dE$$

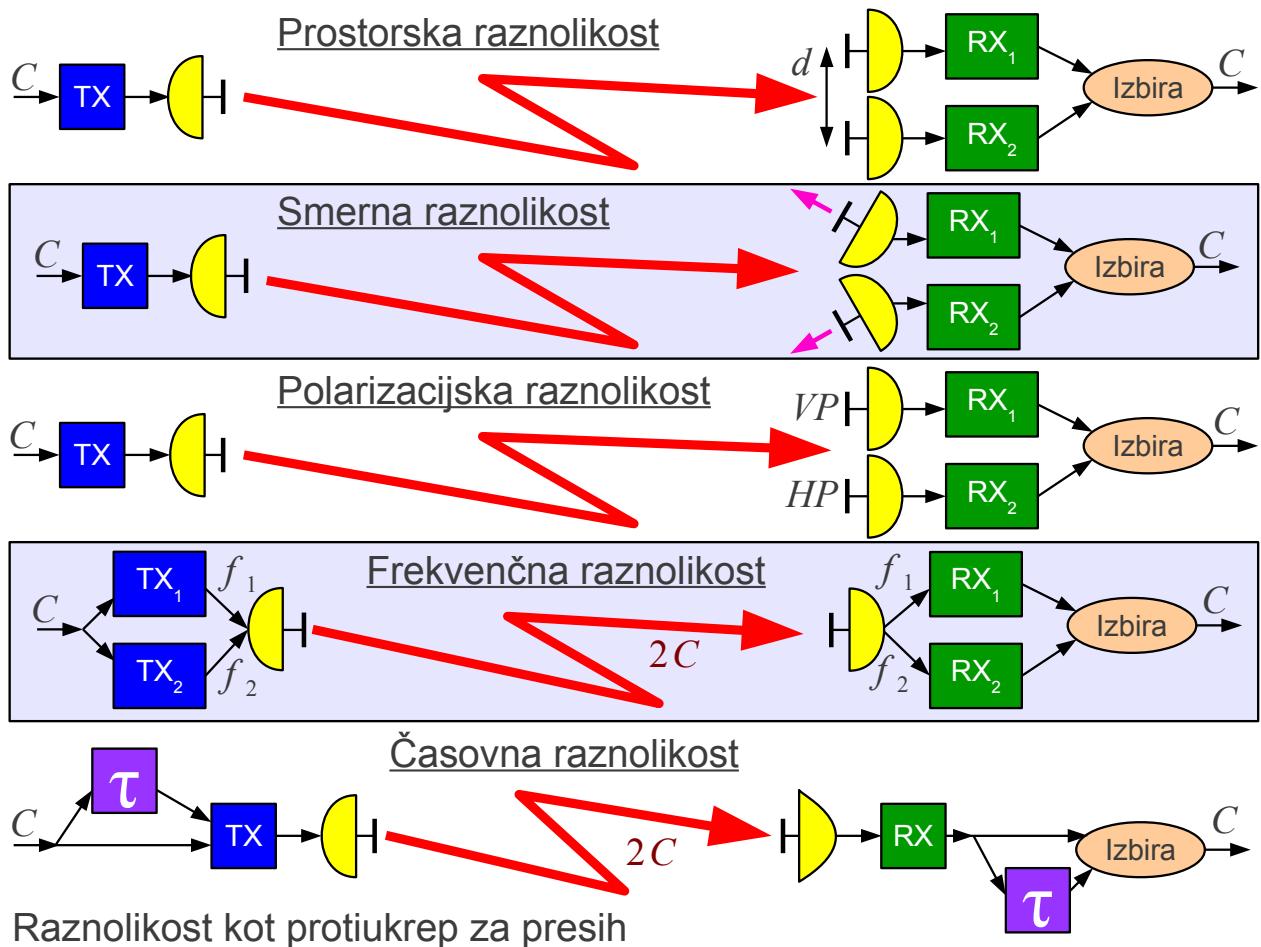
$$P_{izpada} = \int_0^{E_{MIN}} \frac{2E}{\langle E^2 \rangle} e^{-\frac{E^2}{\langle E^2 \rangle}} dE$$

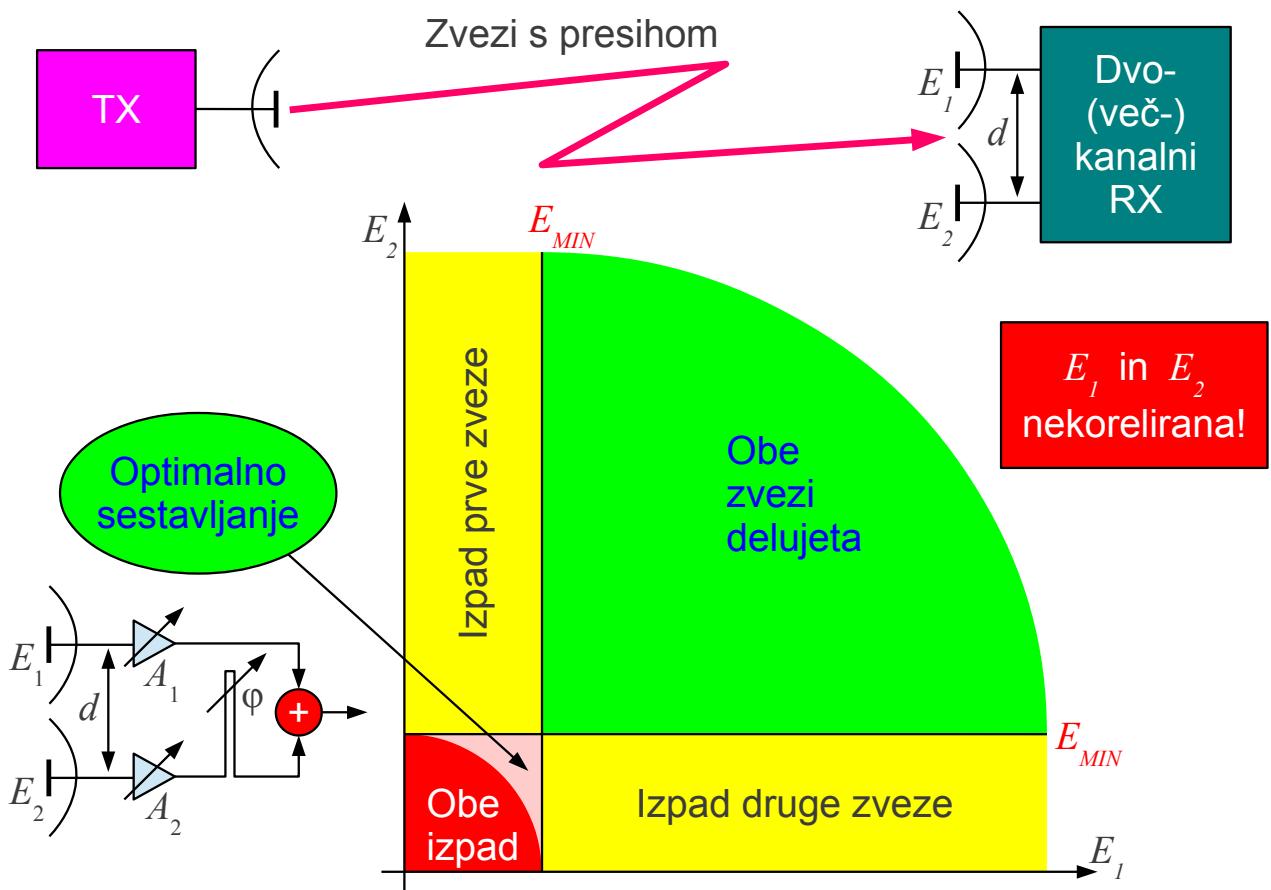
$$P_{izpada} = 1 - e^{-\frac{E_{MIN}^2}{\langle E^2 \rangle}}$$

Izračun verjetnosti izpada zvez

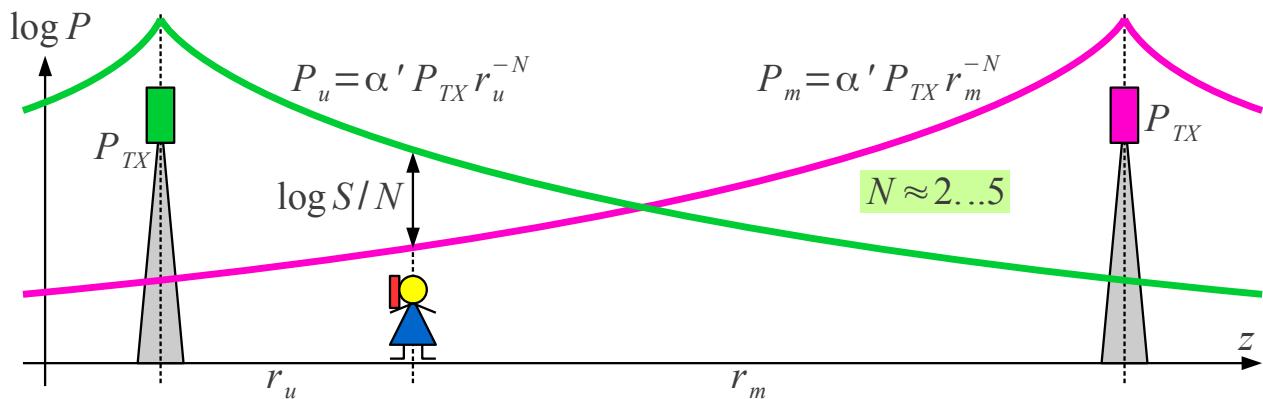
$$P_{MIN} \ll \langle P \rangle \rightarrow P_{izpada} \approx \frac{P_{MIN}}{\langle P \rangle}$$

Zgled: mobilni telefon  
 $\langle P \rangle = -90\text{dBm} = 1\text{pW}$   
 $P_{MIN} = -105\text{dBm} = 0.032\text{pW}$   
 $P_{izpada} \approx 0.032 \approx 3\%$





Pogostnost izpada pri nekoreliranem sprejemu

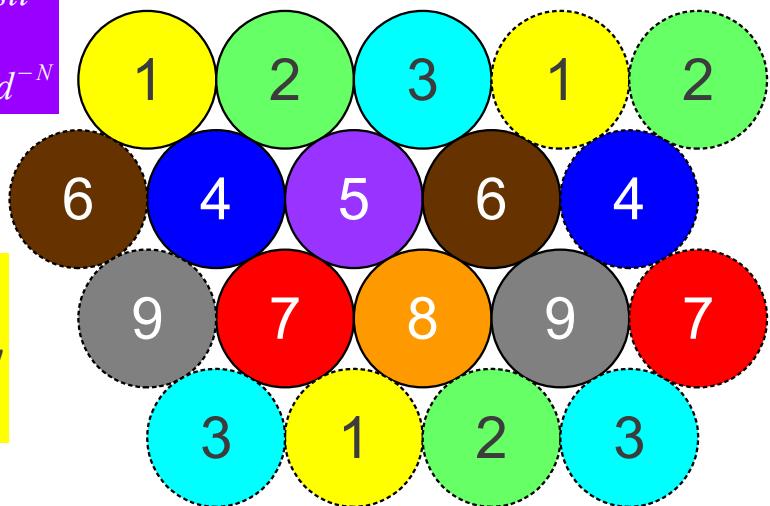


Mestno okolje brez vidljivosti  
 $3 \leq N \leq 5$

$$P_{RX} = P_{TX} G_{TX} G_{RX} \alpha(\lambda) h_{TX}^2 h_{RX}^2 d^{-N}$$

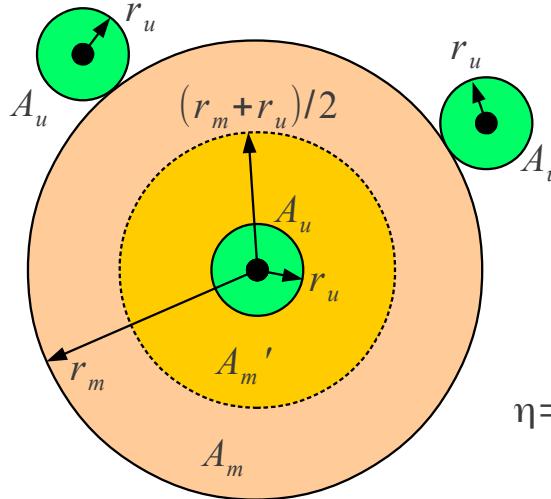
$$S/N = \frac{P_u}{P_m} = \left( \frac{r_m}{r_u} \right)^N$$

Primer:  $N=4$   
 $S/N = 28 \text{ dB} = 625$   
 $z$  rezervo presiha!  
 $r_m = r_u \sqrt[N]{S/N} \approx 5 r_u$



Ponovna uporaba spektra

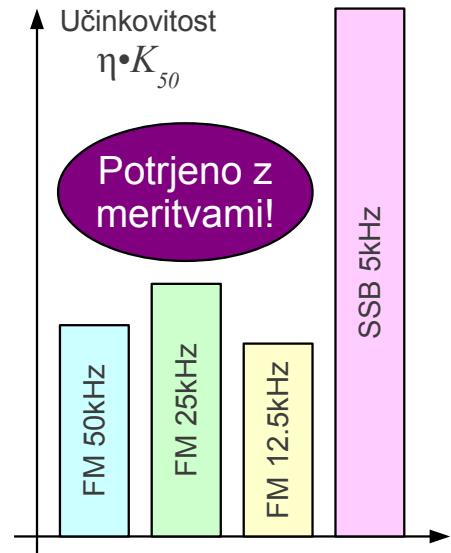
Modulacija	$K_{50}$	Koleb $\Delta f$	$m = \Delta f / B_u$	$3m^2$	$r_m / r_u$	$\eta$	$\eta \cdot K_{50}$	Ocena
FM 50kHz	1	$\pm 15\text{kHz}$	5	75	1.08	0.929	0.929	FM prag?
FM 25kHz	2	$\pm 6\text{kHz}$	2	12	1.70	0.549	1.098	Dober!
FM 12.5kHz	4	$\pm 1.5\text{kHz}$	0.5	0.75	3.40	0.207	0.827	Slab!
SSB 5kHz	10	$B_R = B_u = 3\text{kHz}$	1	3.16	0.231	2.309		Presluh?



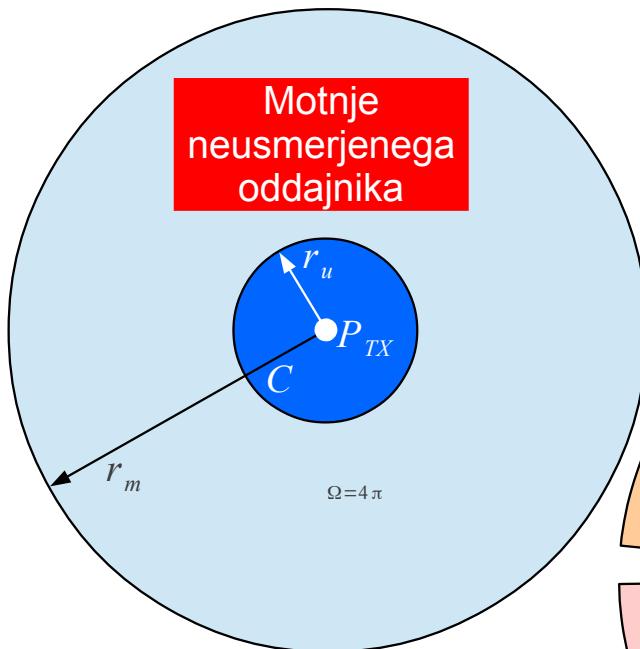
Primer:  
 $N=4$   
 $S/N=100$

$$\frac{r_m}{r_u} = \sqrt[N]{\frac{S/N}{3m^2}}$$

$$\eta = \frac{A_u}{A_m'} = \left( \frac{2}{1 + \frac{r_m}{r_u}} \right)^2$$



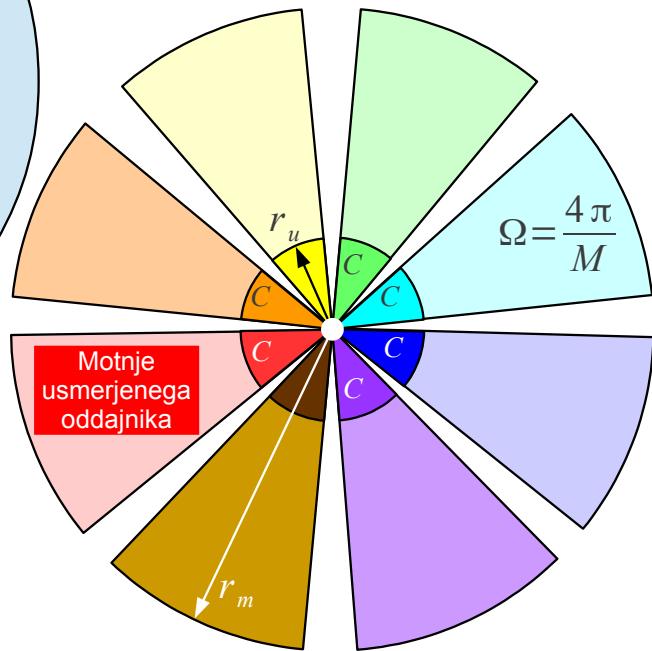
Izbira učinkovite modulacije



Uporaba usmerjenih anten:

$$D = \frac{4\pi}{\Omega} = M$$

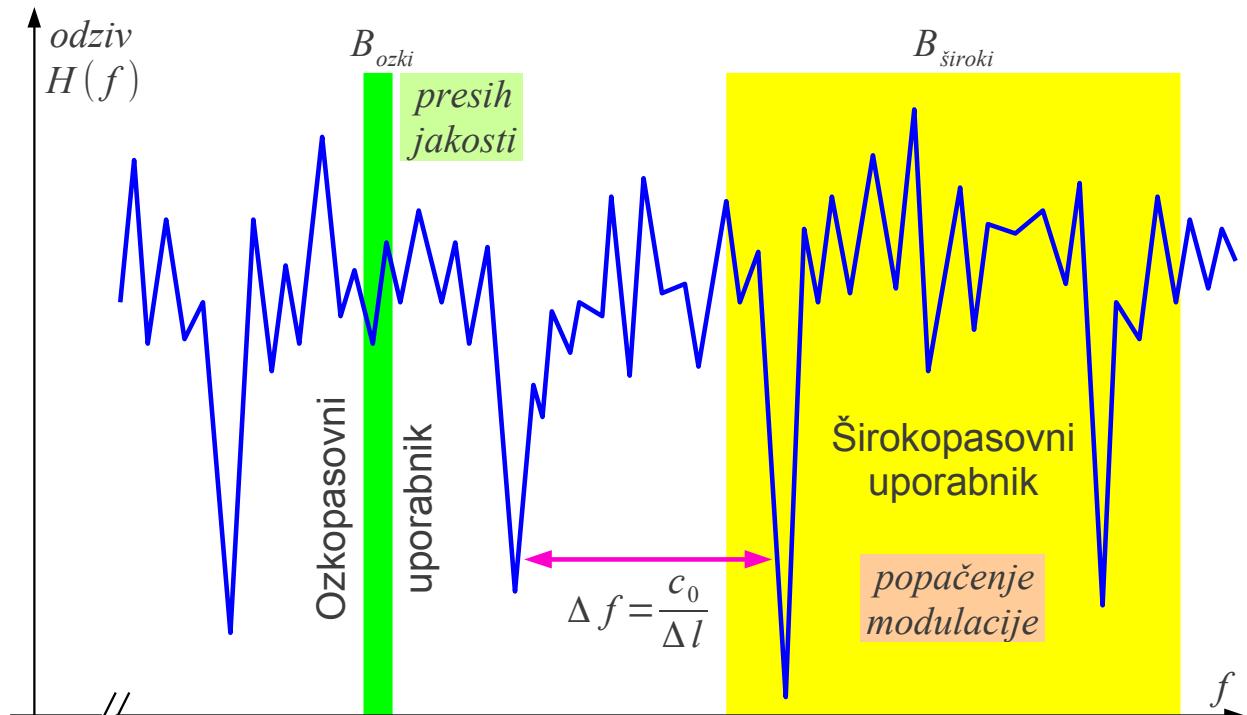
$$\sum C = M C$$



Omejitev EIRP je škodljiva!

Smiselna je omejitev  $\sum P_{TX}$

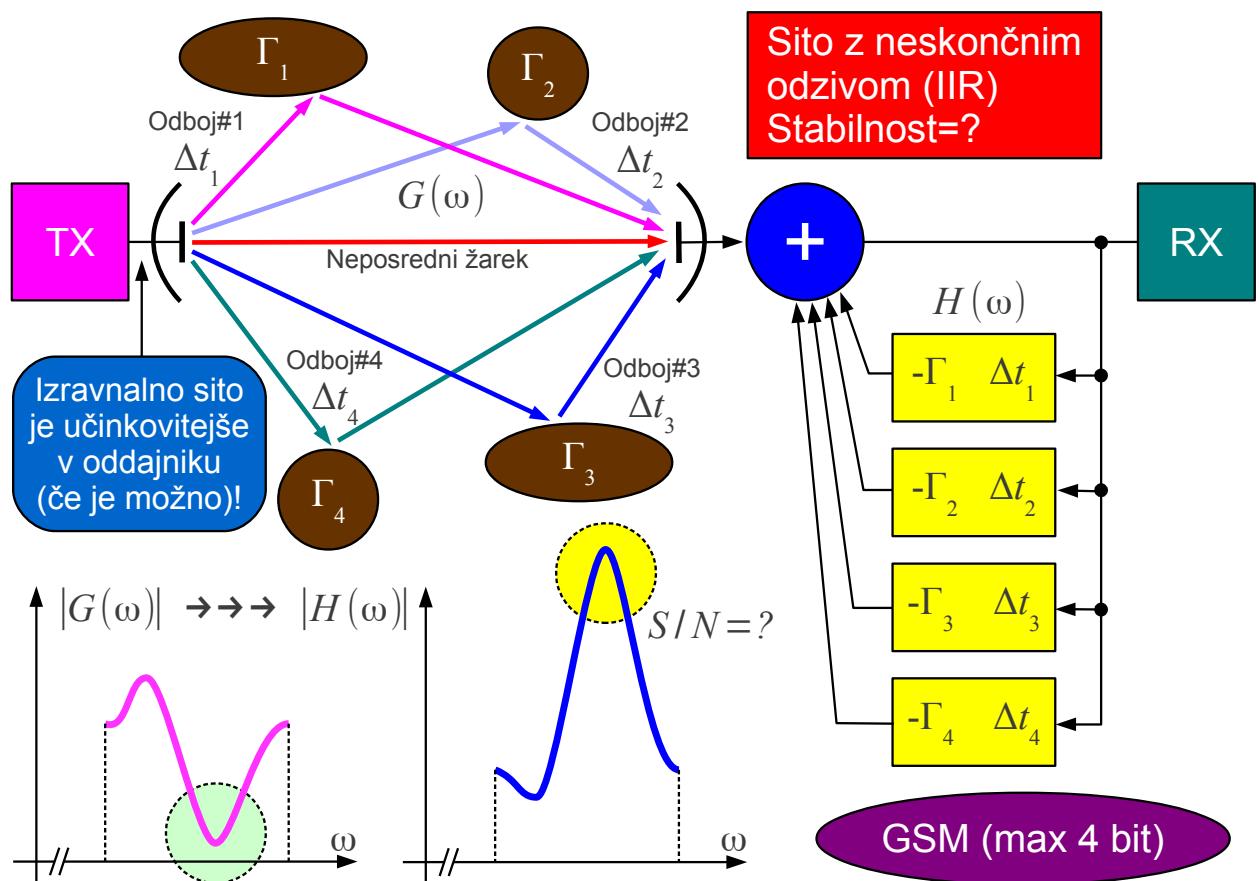
Povečanje zmogljivosti z usmerjenimi antenami



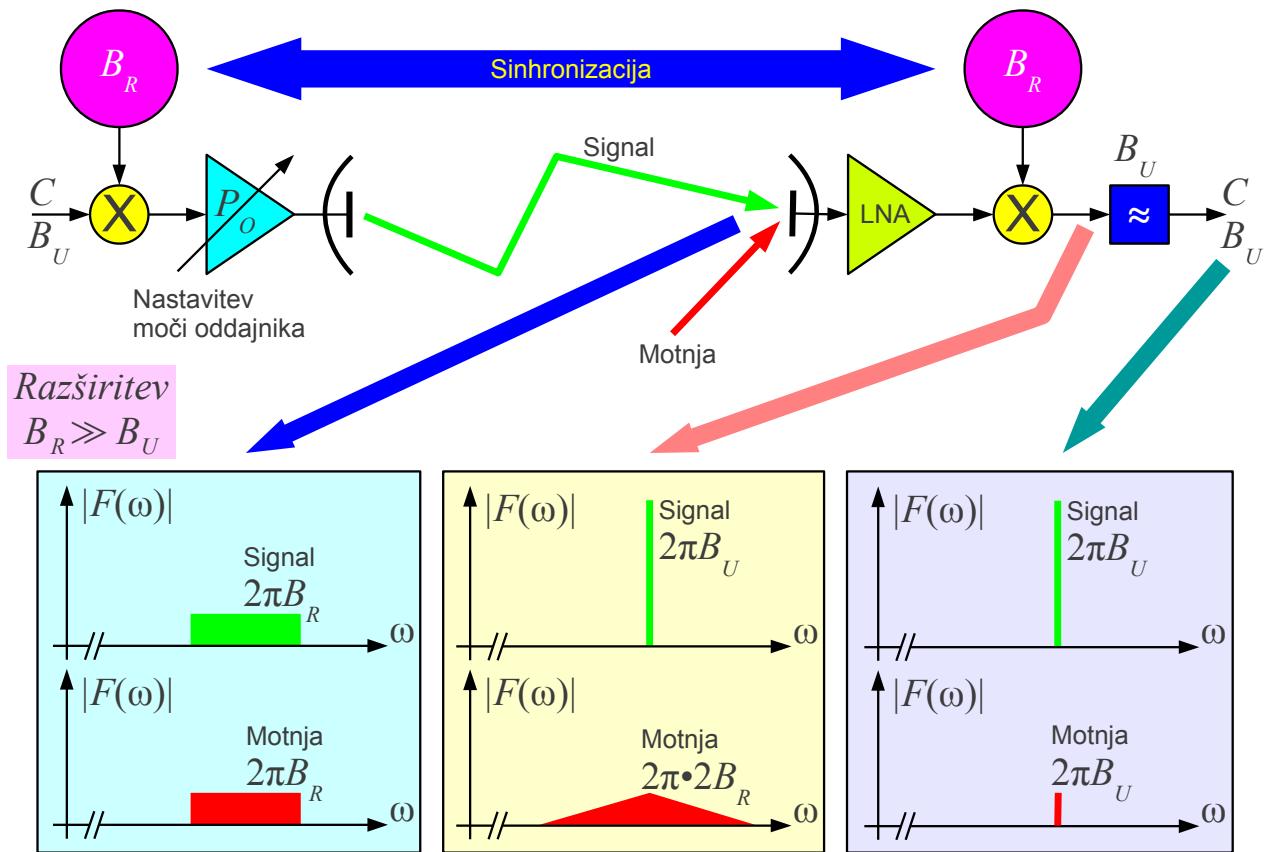
$$B_{ozki} \ll \Delta f \ll B_{široki}$$

Večpotje v frekvenčnem prostoru

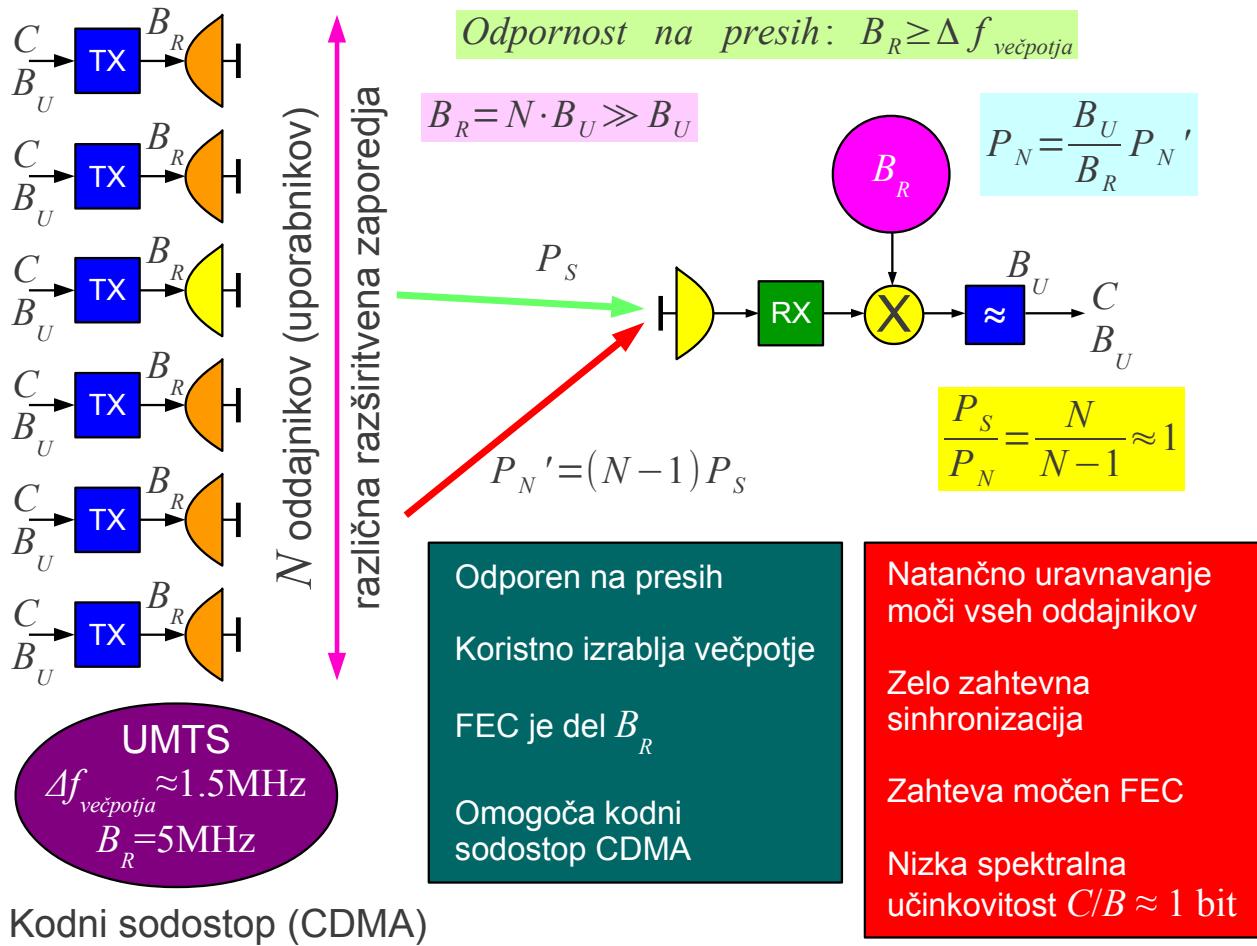
Mestno okolje  $f_0 \approx 450\text{MHz}$   
 $\Delta l = 200\text{m} \dots 1.5\text{km}$   
 $\Delta f = 1.5\text{MHz} \dots 200\text{kHz}$

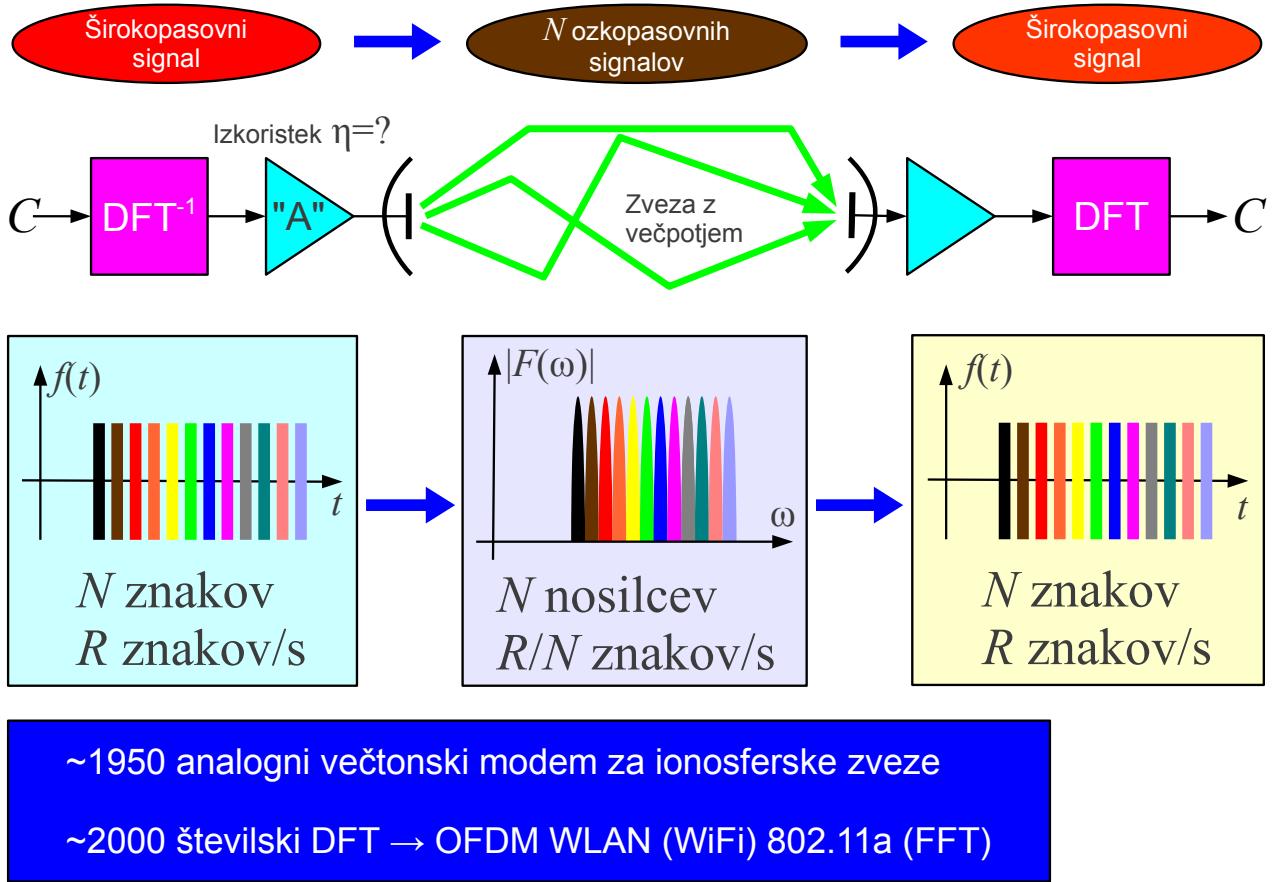


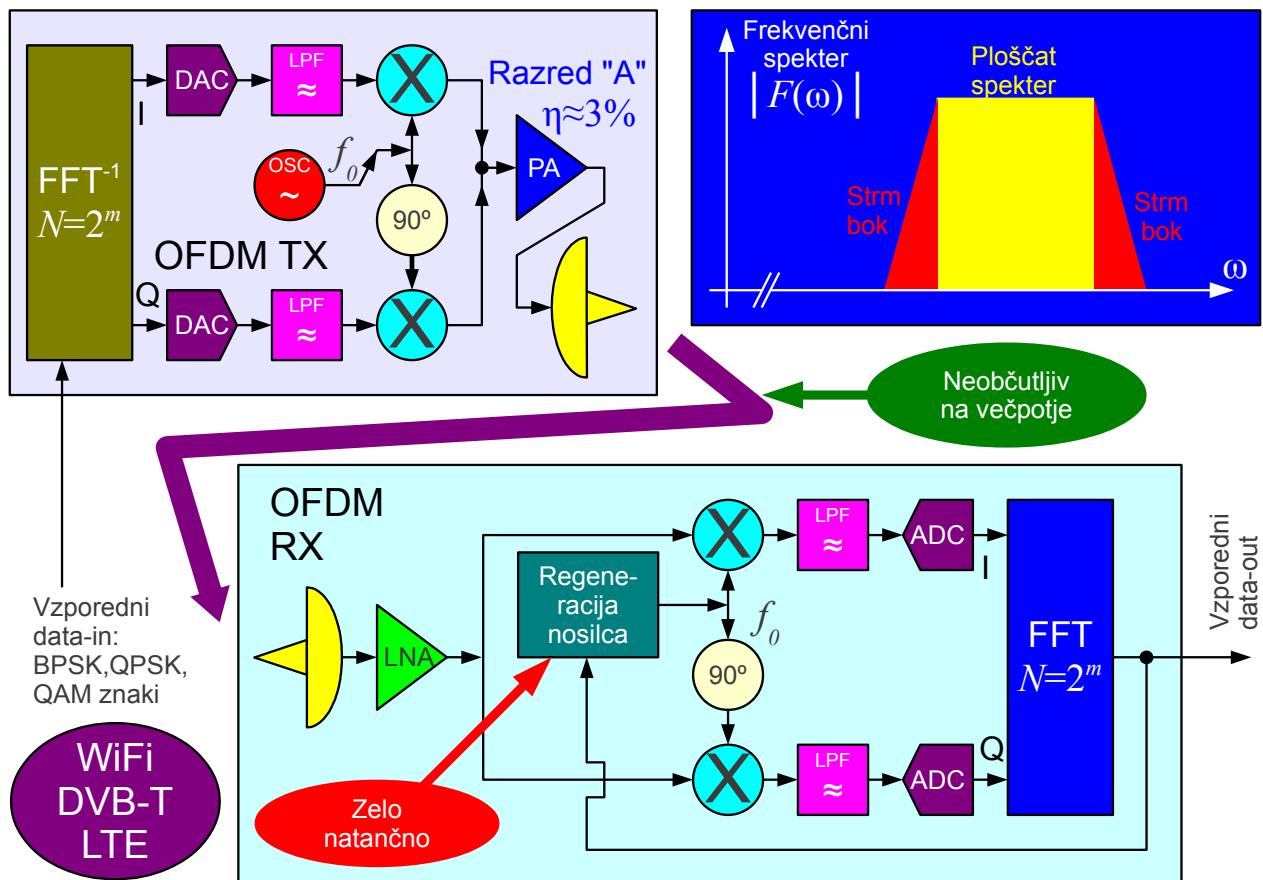
Odpavljanje popačenja večpotja z izravnalnim sitom



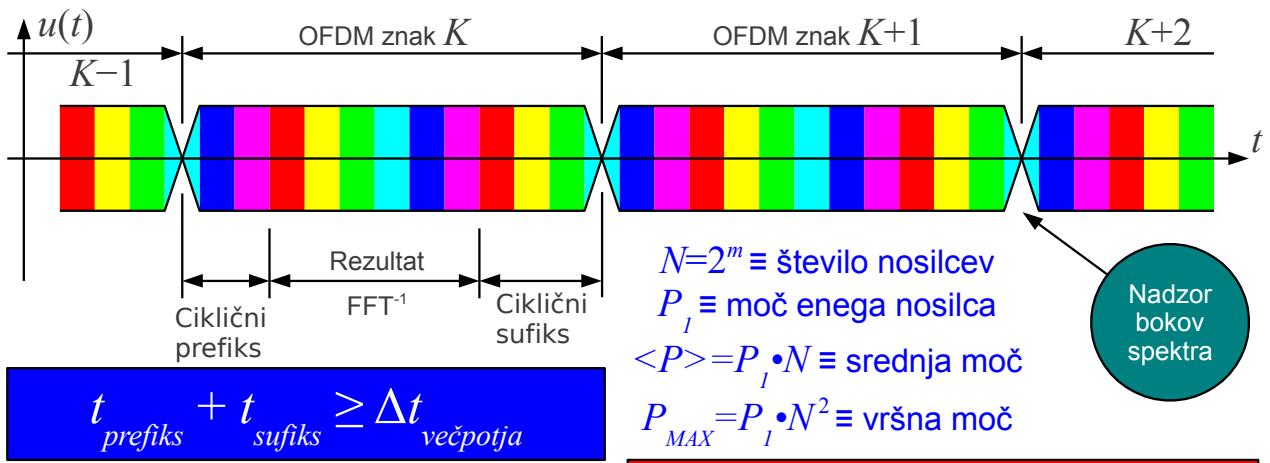
Razširjeni spekter (Spread spectrum)







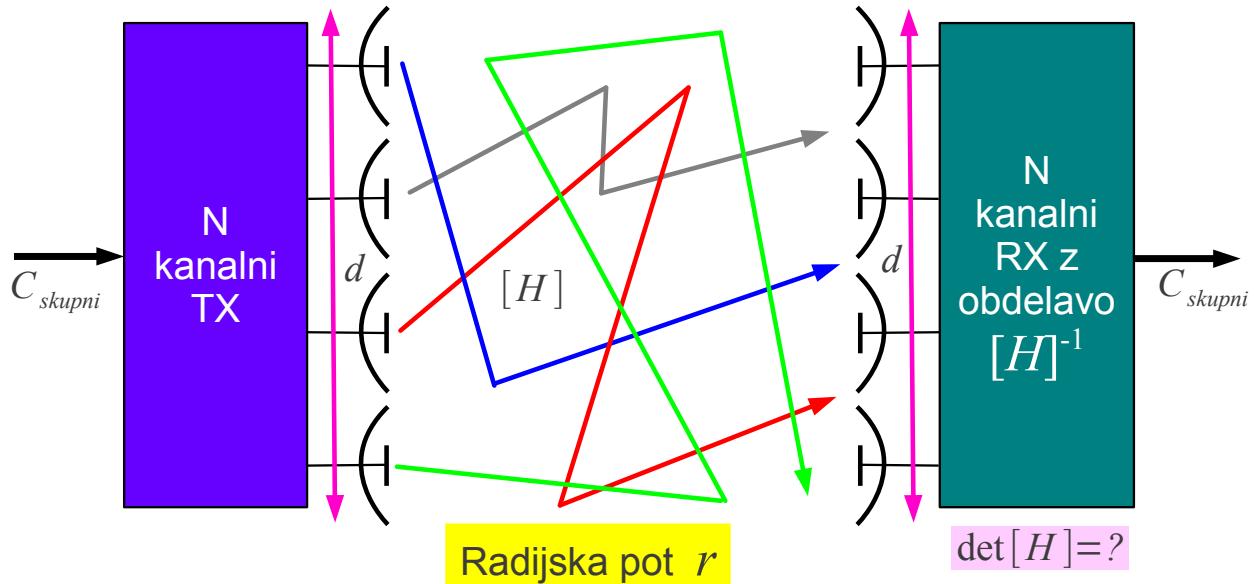
Orthogonal Frequency-Division Multiplex (OFDM)



Nastavljiva odpornost na  $\Delta t_{\text{večpotja}}$   
 Skoraj pravokoten frekvenčni spekter  
 Zadošča šibek FEC  
 Spektralni izkoristek  $C/B$  dosega teoretske vrednosti BPSK, QPSK, QAM  
 Omogoča enofrekvenčna omrežja SFN (Single-Frequency Network)

Visoko razmerje  $P_{\text{MAX}}/\langle P \rangle = N$  pogojuje slab izkoristek oddajnika  $\eta \approx 3\%$   
 FFT zahteva  $N \cdot \log_2 N$  računskih operacij  
 Ozkopasovni nosilci zahtevajo visoko frekvenčno stabilnost  $\Delta f \leq 10\% R/N$   
 Preveliki znaki  $\sim 12000$  bit ( $N \approx 2000$ ,  $C/B \approx 6$  bit) za nekatere protokole  
 Ozkopasovne motnje rušijo sinhronizacijo

## Lastnosti OFDM



Koristna uporaba večpotja!

$$C_{skupni} = N \cdot C_{kanala} = N \cdot B \cdot \log_2 \left( 1 + \frac{P_s}{P_n} \right)$$

(+) visoka spektralna učinkovitost:  $C/B \approx 10$  bit

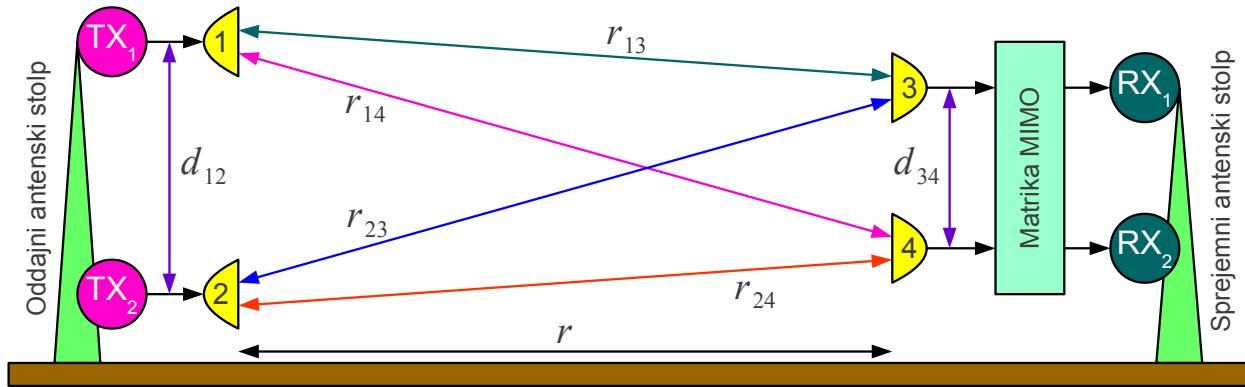
(-) zahteva N oddajnih anten in N sprejemnih anten

(+) preprosta rešitev MIMO 2x2: uporaba obeh polarizacij

(-) več kot dve polarizaciji  $\det[H] \neq 0$  le na kratkih poteh  $r \approx 2d^2/\lambda$

MIMO (Multiple-In Multiple-Out)

Podvojevanje C/B mikrovalovne zveze  $\equiv$  Line-Of-Sight MIMO



$$\text{Pogoj za max } \det[\text{MIMO}]: \quad r_{14} - r_{13} - r_{24} + r_{23} = \lambda/2$$

$C/B > 40\text{bit}$

$$r_{13} = r_{24} = \sqrt{r^2 + ((d_{12} - d_{34})/2)^2} \approx r + \frac{d_{12}^2 - 2d_{12}d_{34} + d_{34}^2}{8r}$$

$$r_{14} = r_{23} = \sqrt{r^2 + ((d_{12} + d_{34})/2)^2} \approx r + \frac{d_{12}^2 + 2d_{12}d_{34} + d_{34}^2}{8r}$$

$$r_{14} - r_{13} - r_{24} + r_{23} \approx \frac{d_{12}d_{34}}{r} \rightarrow d_{12}d_{34} = r \cdot \lambda/2$$

MIMO brez večpotja

Zgled:

$$r = 10\text{km} \quad f = 15\text{GHz}$$

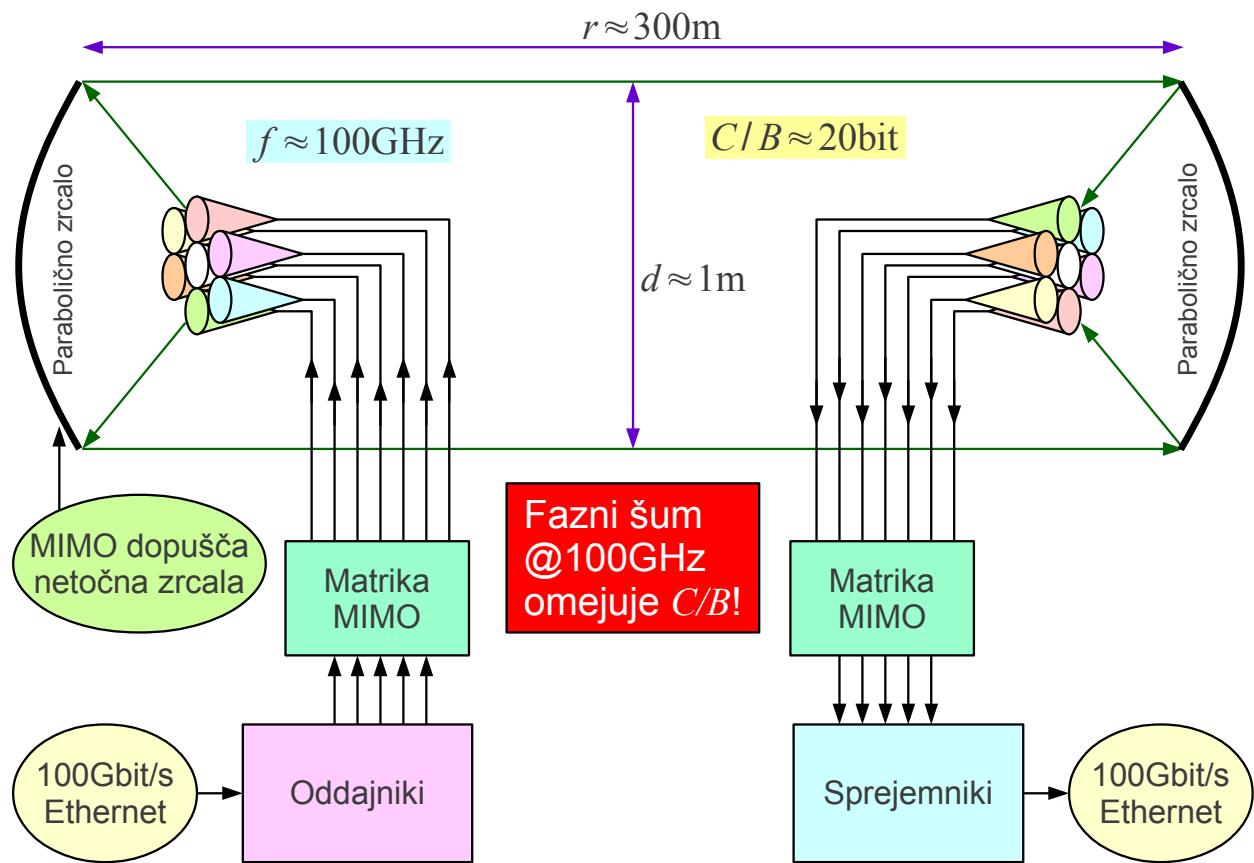
$$\lambda = c_0/f = 2\text{cm}$$

$$\langle d \rangle = \sqrt{d_{12}d_{34}}$$

$$\langle d \rangle = \sqrt{r \cdot \lambda/2} = 10\text{m}$$

Preizkus:

$$r = \frac{2\langle d \rangle^2}{\lambda} = 10\text{km}$$



Visokozmogljiva zveza na kratko razdaljo

\* \* \* \* \*