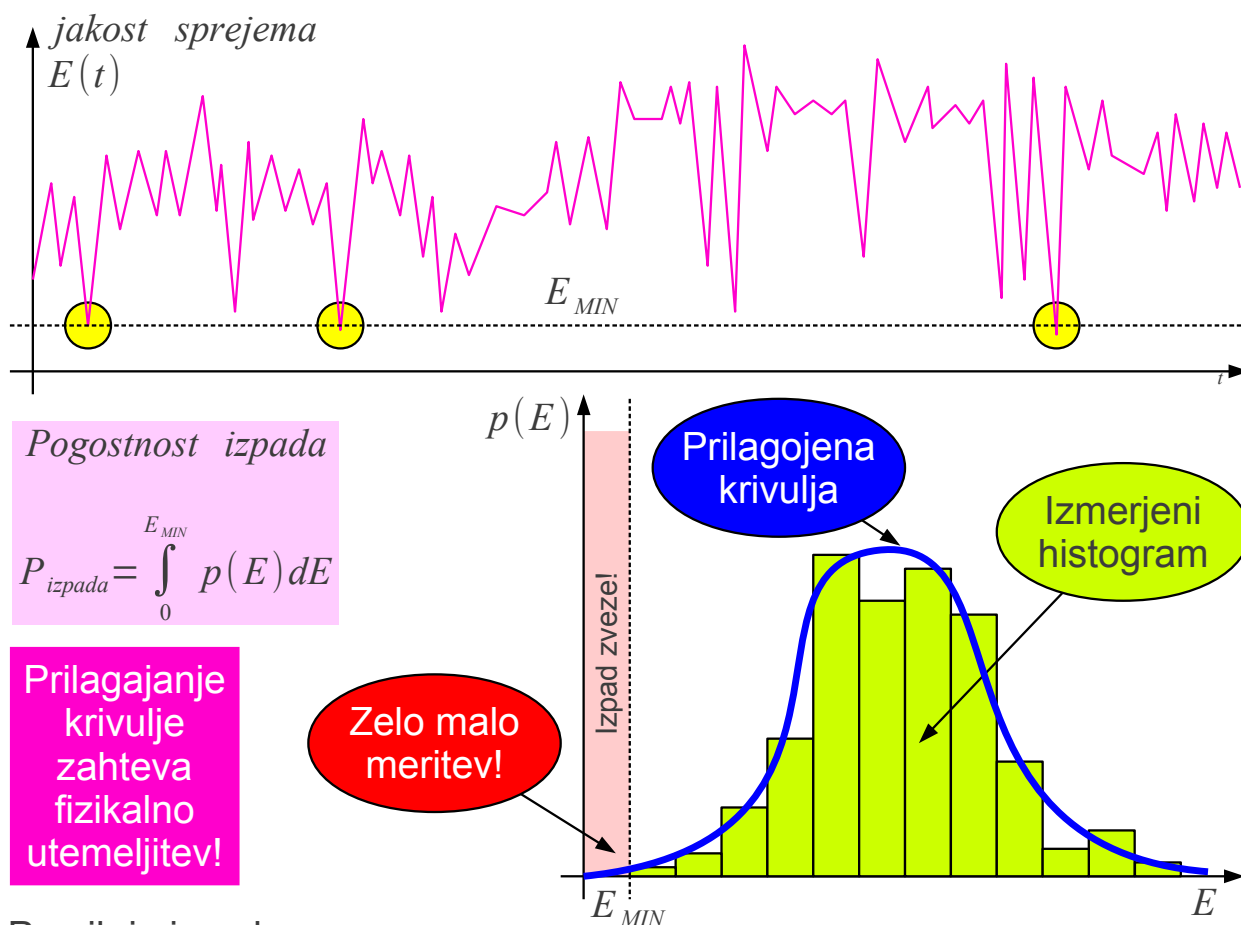
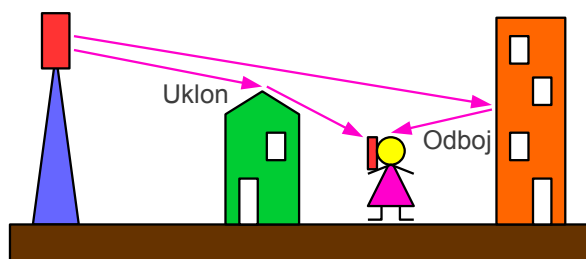


## 17. Večpotje v radijski zvezi

Večina nalog iz anten in razširjanje valov zahteva obravnavo v treh dimenzijah prostora. Tako skalarne kot tudi vektorske veličine so funkcije časa in vseh treh dimenzij prostora. Ozkopasovne signale  $B \ll f$  radia največkrat smemo v izračunih ponazoriti s harmonskim signalom ene same krožne frekvence  $\omega = 2\pi f$ , kar poenostavi časovne odvode v  $\partial/\partial t = j\omega$ .



Presih in izpad zveze



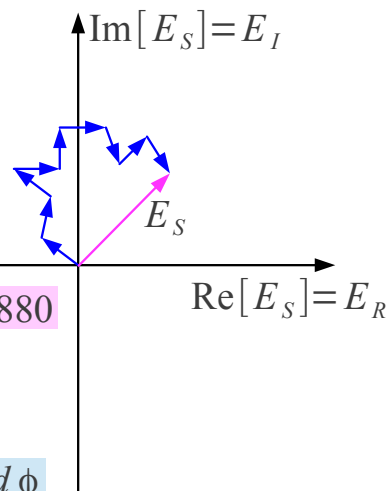
Večpotje brez vidljivosti: Rayleigh ( $\langle E^2 \rangle$ )  
vsota mnogo naključnih malih kazalcev

Gaussova porazdelitev komponent

$$p(E_R) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_R^2}{2\sigma^2}}$$

$$p(E_I) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_I^2}{2\sigma^2}}$$

Lord Rayleigh 1880



$$E_S = E_R + jE_I = E e^{j\phi}$$

$$E = |E_S|$$

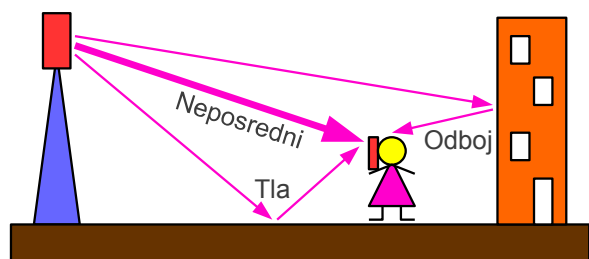
$$dE_R dE_I = E dE d\phi$$

$$p(E_R, E_I) = p(E_R) p(E_I) = \frac{1}{\sigma^2 2\pi} e^{-\frac{E_R^2 + E_I^2}{2\sigma^2}} = p(E) p(\phi)$$

$$\langle E^2 \rangle = 2\sigma^2$$

$$p(E) = \int_0^{2\pi} p(E_R, E_I) E d\phi = \int_0^{2\pi} \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2}{2\sigma^2}} E d\phi = \frac{E}{\sigma^2} e^{-\frac{E^2}{2\sigma^2}} = \frac{2E}{\langle E^2 \rangle} e^{-\frac{E^2}{\langle E^2 \rangle}}$$

Rayleighjeva porazdelitev



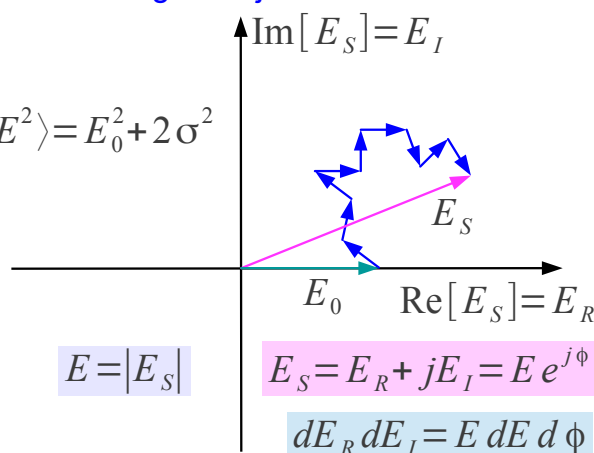
Večpotje z neposrednim žarkom: Rice ( $E_0, \sigma$ )  
en velik in mnogo naključnih malih kazalcev

Gaussova porazdelitev komponent

$$p(E_R) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(E_R - E_0)^2}{2\sigma^2}}$$

$$p(E_I) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{E_I^2}{2\sigma^2}}$$

$$\langle E^2 \rangle = E_0^2 + 2\sigma^2$$



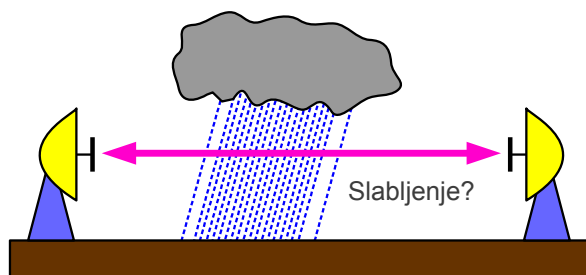
$$p(E_R, E_I) = \frac{1}{\sigma^2 2\pi} e^{-\frac{(E_R - E_0)^2 + E_I^2}{2\sigma^2}} = \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} e^{\left(\frac{E_R E_0}{\sigma^2}\right)} = \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} e^{\left(\frac{E_0 E}{\sigma^2}\right) \cos \phi}$$

$$p(E) = \int_0^{2\pi} p(E_R, E_I) E d\phi = \frac{E}{\sigma^2} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} I_0\left(\frac{E_0 E}{\sigma^2}\right)$$

Stephen O. Rice 1948

Riceova porazdelitev

$$\int_0^{2\pi} e^{\left(\frac{E_0 E}{\sigma^2}\right) \cos \phi} d\phi = 2\pi I_0\left(\frac{E_0 E}{\sigma^2}\right)$$



Vremenski pojavi: log-normalna ( $\langle E_{dB} \rangle, \sigma_{dB}$ )  
 produkt mnogo naključnih prispevkov  
 brez interference večpotja

$$p(E_{dB}) = \frac{1}{\sigma_{dB} \sqrt{2\pi}} e^{-\frac{(E_{dB} - \langle E_{dB} \rangle)^2}{2\sigma_{dB}^2}}$$

Fizikalno utemeljeno?

Francis Galton ~ 1880

$$E_{dB} = 20 \log_{10} \left( \frac{|E_S|}{E_{REF}} \right) = 20 \log_{10} \left( \frac{E}{E_{REF}} \right) = \frac{20}{\ln 10} \ln \left( \frac{E}{E_{REF}} \right)$$

$$\sigma_{dB} = \sqrt{\langle (E_{dB} - \langle E_{dB} \rangle)^2 \rangle}$$

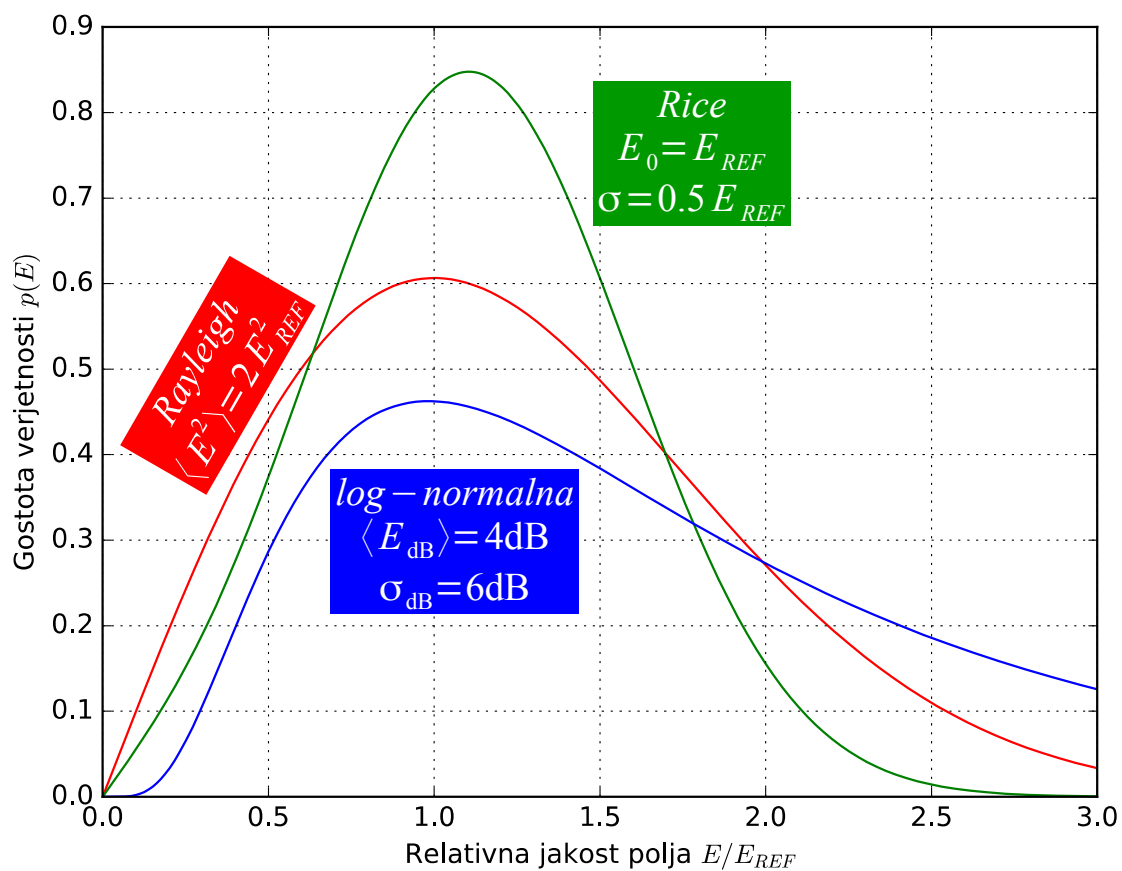
$$p(E_{dB}) dE_{dB} = p(E) dE$$

$$\frac{dE_{dB}}{dE} = \frac{20}{\ln 10} \left( \frac{E_{REF}}{E} \right) \frac{1}{E_{REF}} = \frac{20}{E \ln 10}$$

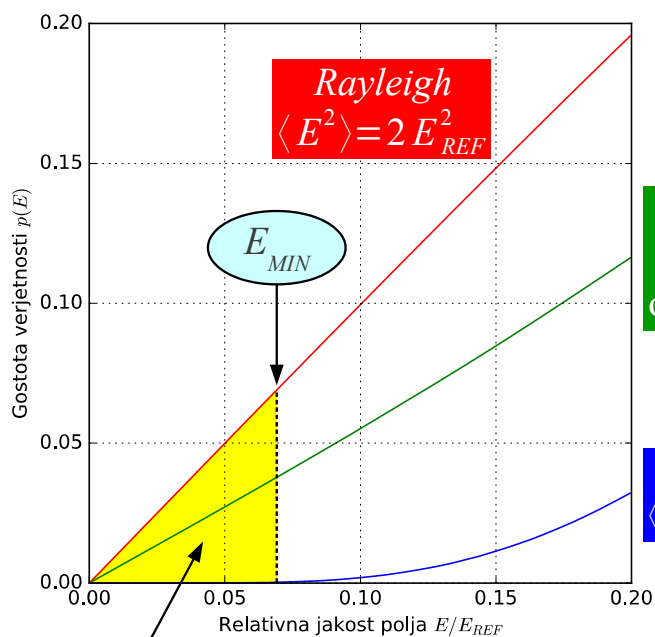
$$p(E) = p(E_{dB}) \frac{dE_{dB}}{dE} = p(E_{dB}) \frac{20}{E \ln 10} = \frac{20}{E (\ln 10) \sigma_{dB} \sqrt{2\pi}} e^{-\frac{\left[ 20 \log_{10} \left( \frac{E}{E_{REF}} \right) - \langle E_{dB} \rangle \right]^2}{2\sigma_{dB}^2}}$$

Log-normalna porazdelitev

# Rayleighjeva, Riceova in log-normalna porazdelitev



Rayleighjeva, Riceova in log-normalna porazdelitev



*Rayleigh*  
 $\langle E^2 \rangle = 2 E_{REF}^2$

*Rice*  
 $E_0 = E_{REF}$   
 $\sigma = 0.5 E_{REF}$

*log-normalna*  
 $\langle E_{dB} \rangle = 4\text{dB}$   $\sigma_{dB} = 6\text{dB}$

$P_{izpada}$

$$P = \alpha E^2 \rightarrow dP = \alpha 2 E dE$$

$$P_{izpada} = \int_0^{P_{MIN}} \frac{1}{\langle P \rangle} e^{-\frac{P}{\langle P \rangle}} dP = 1 - e^{-\frac{P_{MIN}}{\langle P \rangle}}$$

*Pošten račun: Rayleigh*

$$P_{izpada} = \int_0^{E_{MIN}} p(E) dE$$

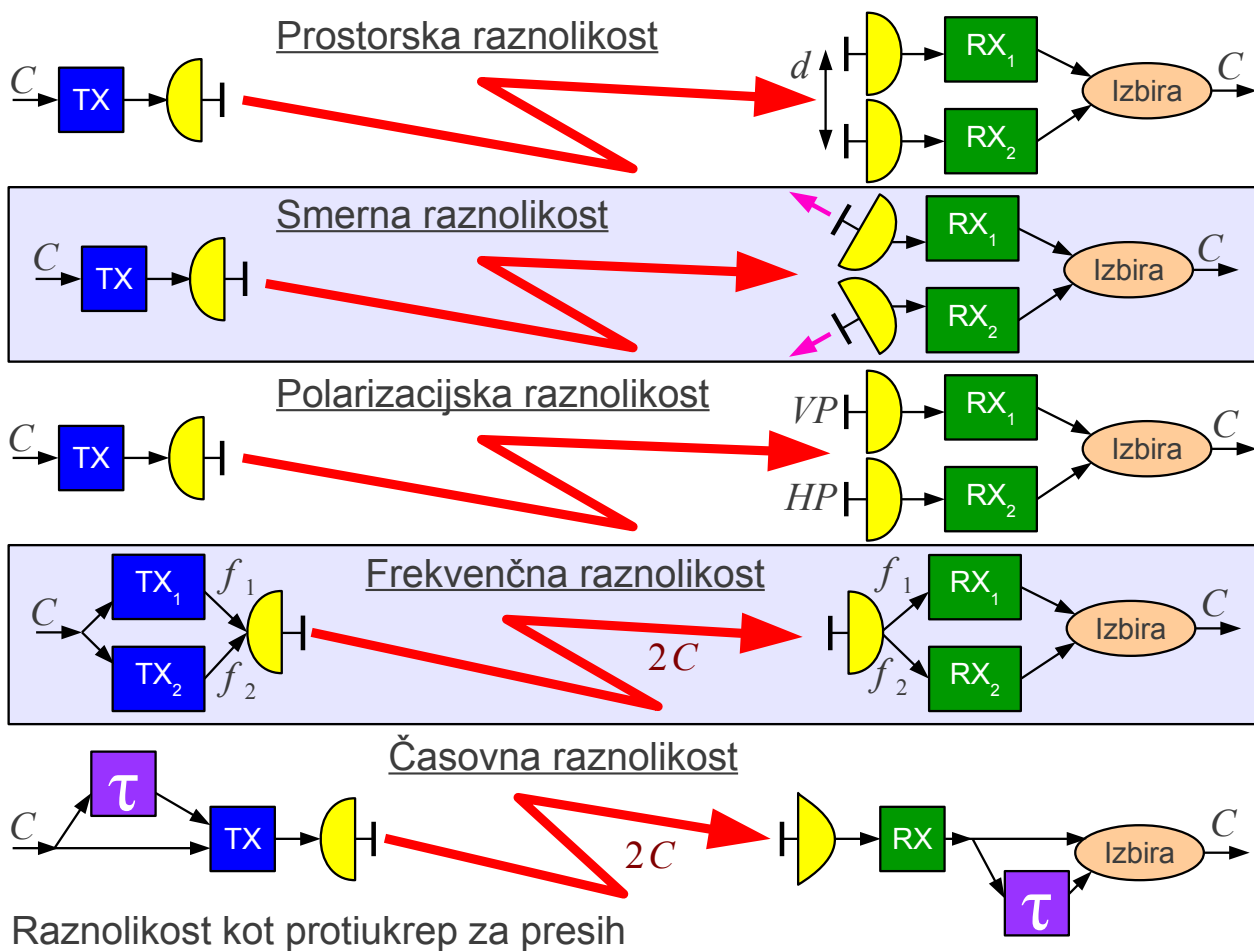
$$P_{izpada} = \int_0^{E_{MIN}} \frac{2 E}{\langle E^2 \rangle} e^{-\frac{E^2}{\langle E^2 \rangle}} dE$$

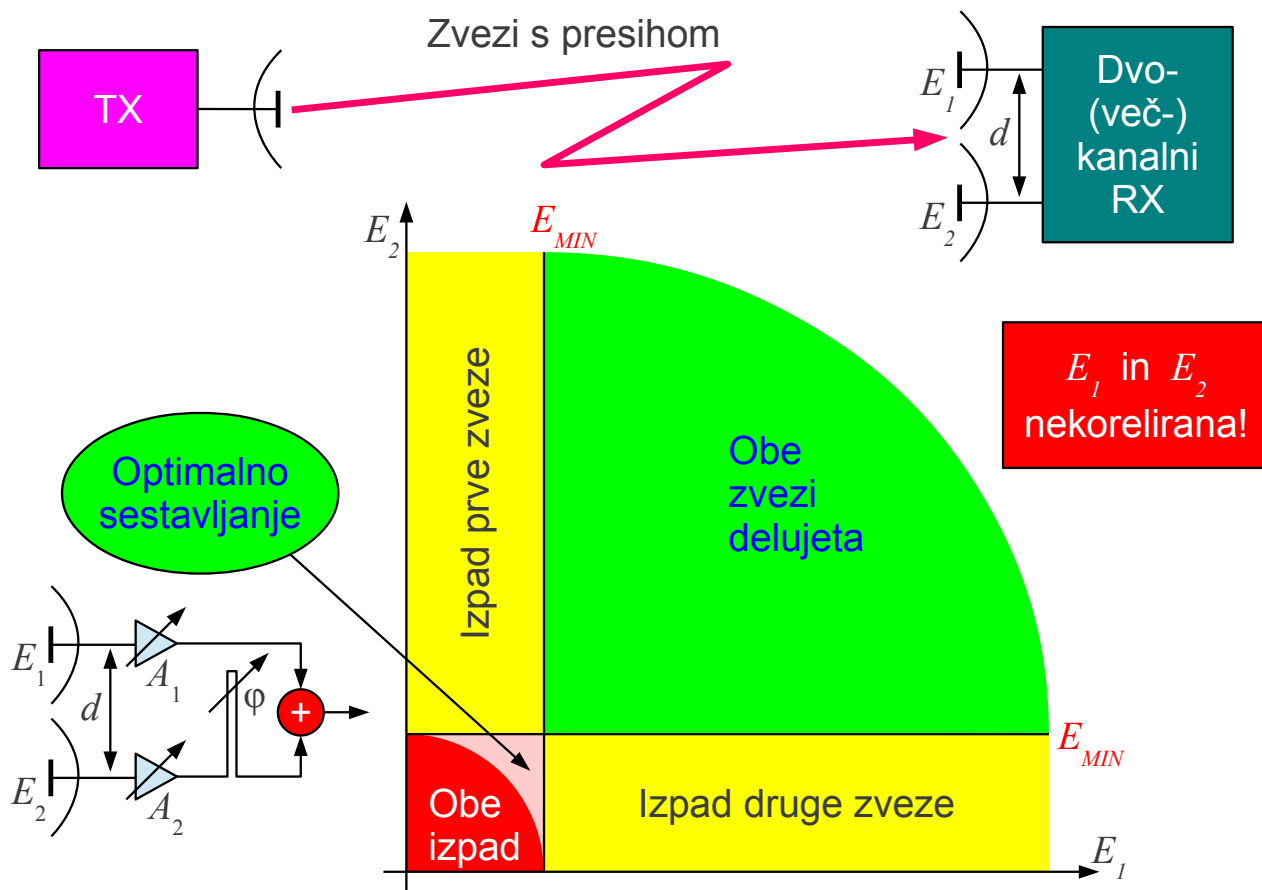
$$P_{izpada} = 1 - e^{-\frac{E_{MIN}^2}{\langle E^2 \rangle}}$$

$$P_{MIN} \ll \langle P \rangle \rightarrow P_{izpada} \approx \frac{P_{MIN}}{\langle P \rangle}$$

*Zgled: mobilni telefon*  
 $\langle P \rangle = -90\text{dBm} = 1\text{pW}$   
 $P_{MIN} = -105\text{dBm} = 0.032\text{pW}$   
 $P_{izpada} \approx 0.032 \approx 3\%$

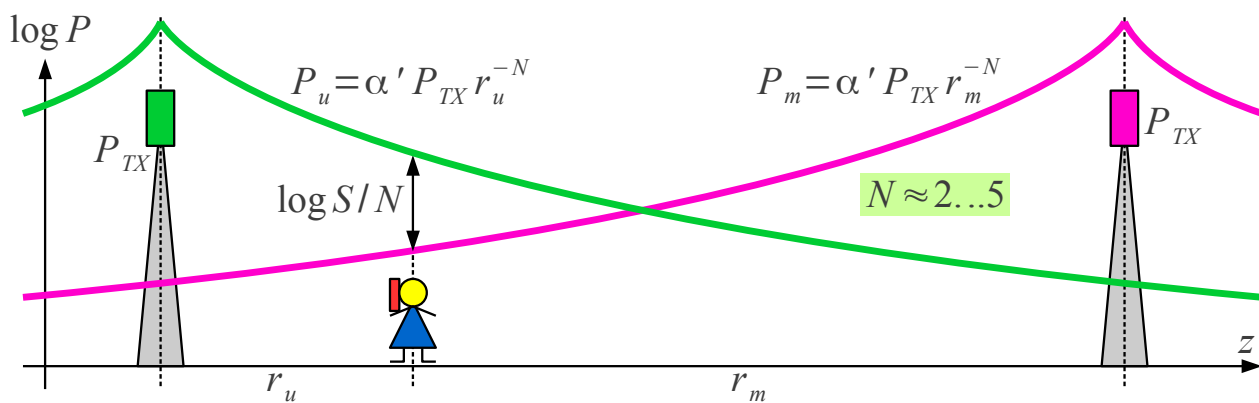
Izračun verjetnosti izpada zveze





Pogostnost izpada pri nekoreliranem sprejemu





Mestno okolje brez vidljivosti

$$3 \leq N \leq 5$$

$$P_{RX} = P_{TX} G_{TX} G_{RX} \alpha(\lambda) h_{TX}^2 h_{RX}^2 d^{-N}$$

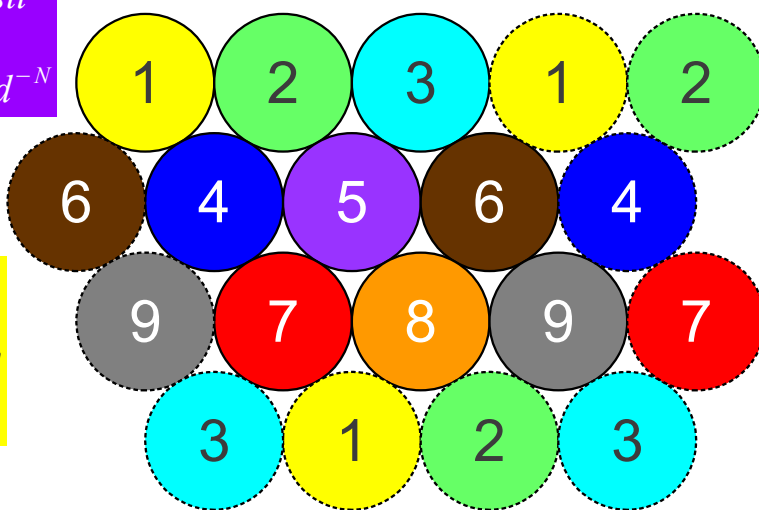
$$S/N = \frac{P_u}{P_m} = \left( \frac{r_m}{r_u} \right)^N$$

Primer:  $N=4$

$$S/N = 28\text{dB} = 625$$

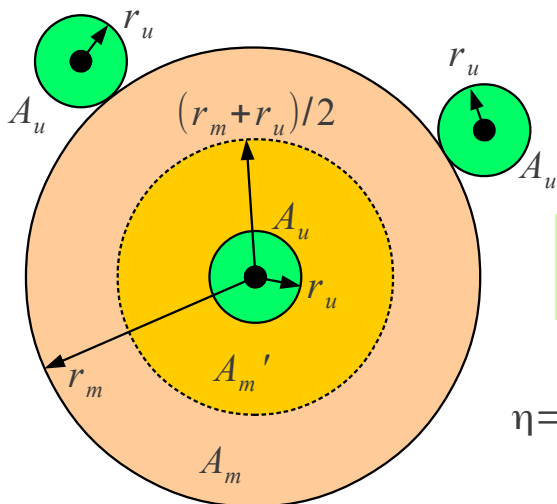
z rezervo presiha!

$$r_m = r_u \sqrt[N]{S/N} \approx 5 r_u$$



Ponovna uporaba spektra

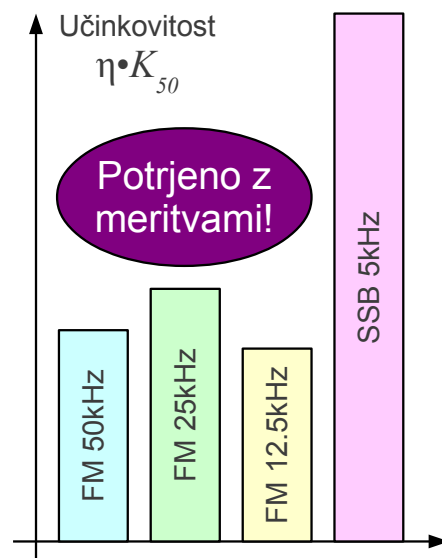
Modulacija	$K_{50}$	Koleb $\Delta f$	$m=\Delta f/B_u$	$3m^2$	$r_m/r_u$	$\eta$	$\eta \cdot K_{50}$	Ocena
FM 50kHz	1	$\pm 15\text{kHz}$	5	75	1.08	0.929	0.929	FM prag?
FM 25kHz	2	$\pm 6\text{kHz}$	2	12	1.70	0.549	1.098	Dober!
FM 12.5kHz	4	$\pm 1.5\text{kHz}$	0.5	0.75	3.40	0.207	0.827	Slab!
SSB 5kHz	10	$B_R=B_u=3\text{kHz}$		1	3.16	0.231	2.309	Presluh?



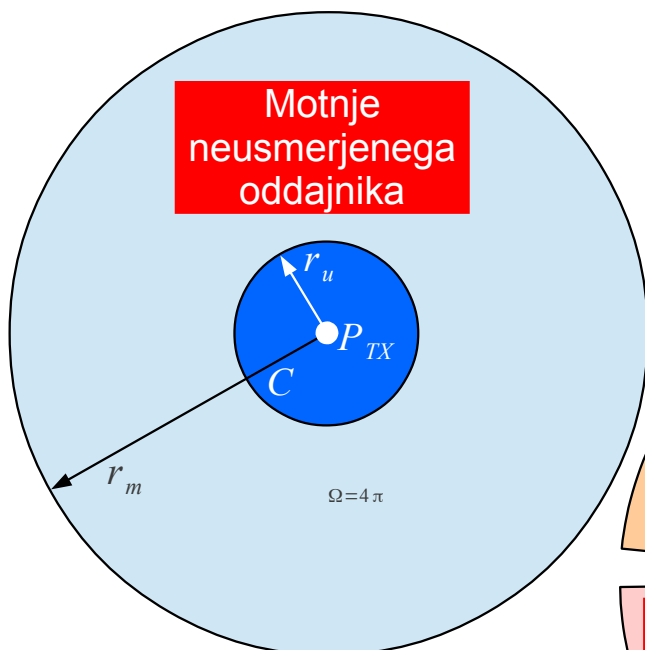
*Primer:*  
 $N=4$   
 $S/N=100$

$$\frac{r_m}{r_u} = \sqrt[4]{\frac{S/N}{3m^2}}$$

$$\eta = \frac{A_u}{A_m'} = \left( \frac{2}{1 + \frac{r_m}{r_u}} \right)^2$$



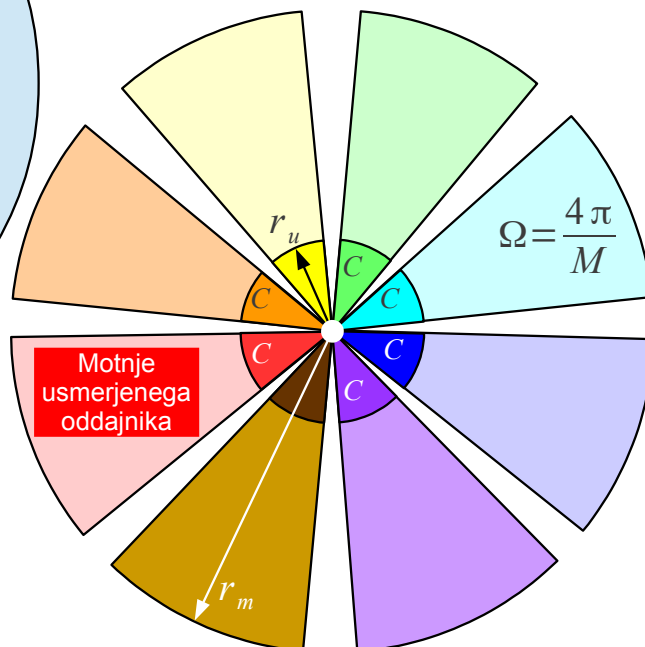
Izbira učinkovite modulacije



Uporaba  
usmerjenih  
anten:

$$D = \frac{4\pi}{\Omega} = M$$

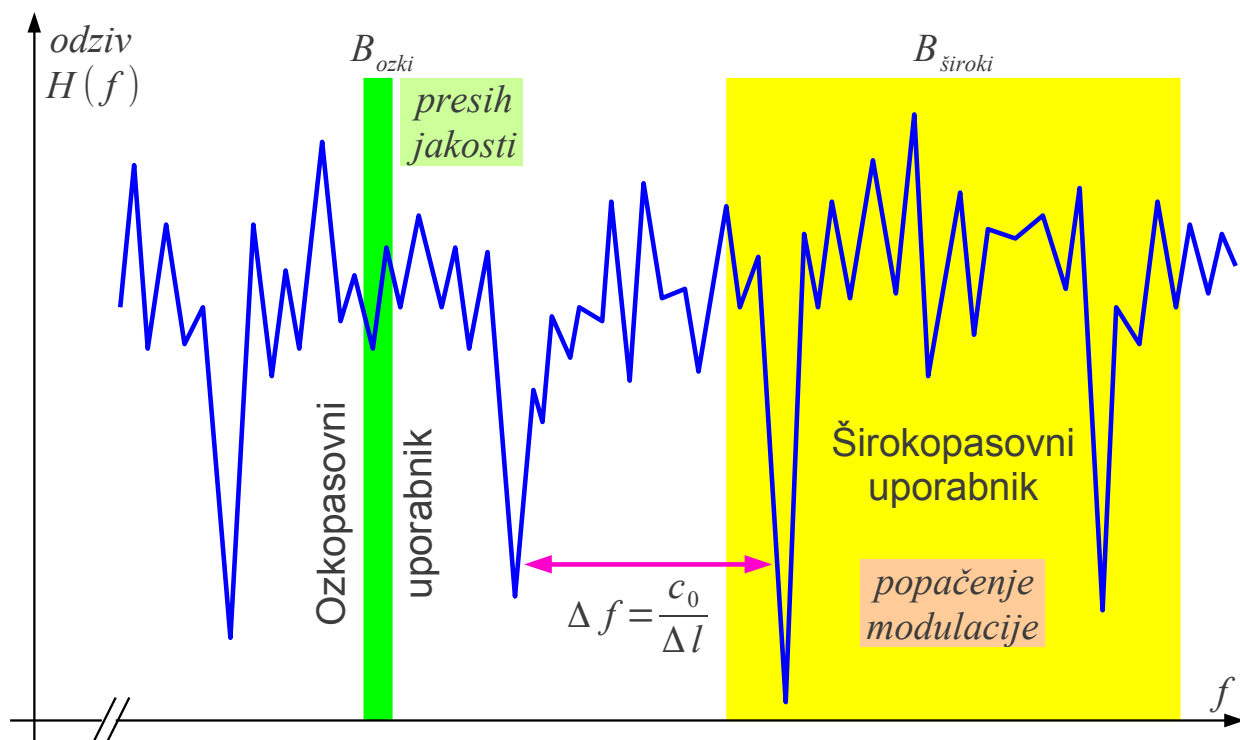
$$\sum C = MC$$



Omejitev **EIRP** je škodljiva!

Smiselna je omejitev  $\sum P_{TX}$

## Povečanje zmogljivosti z usmerjenimi antenami



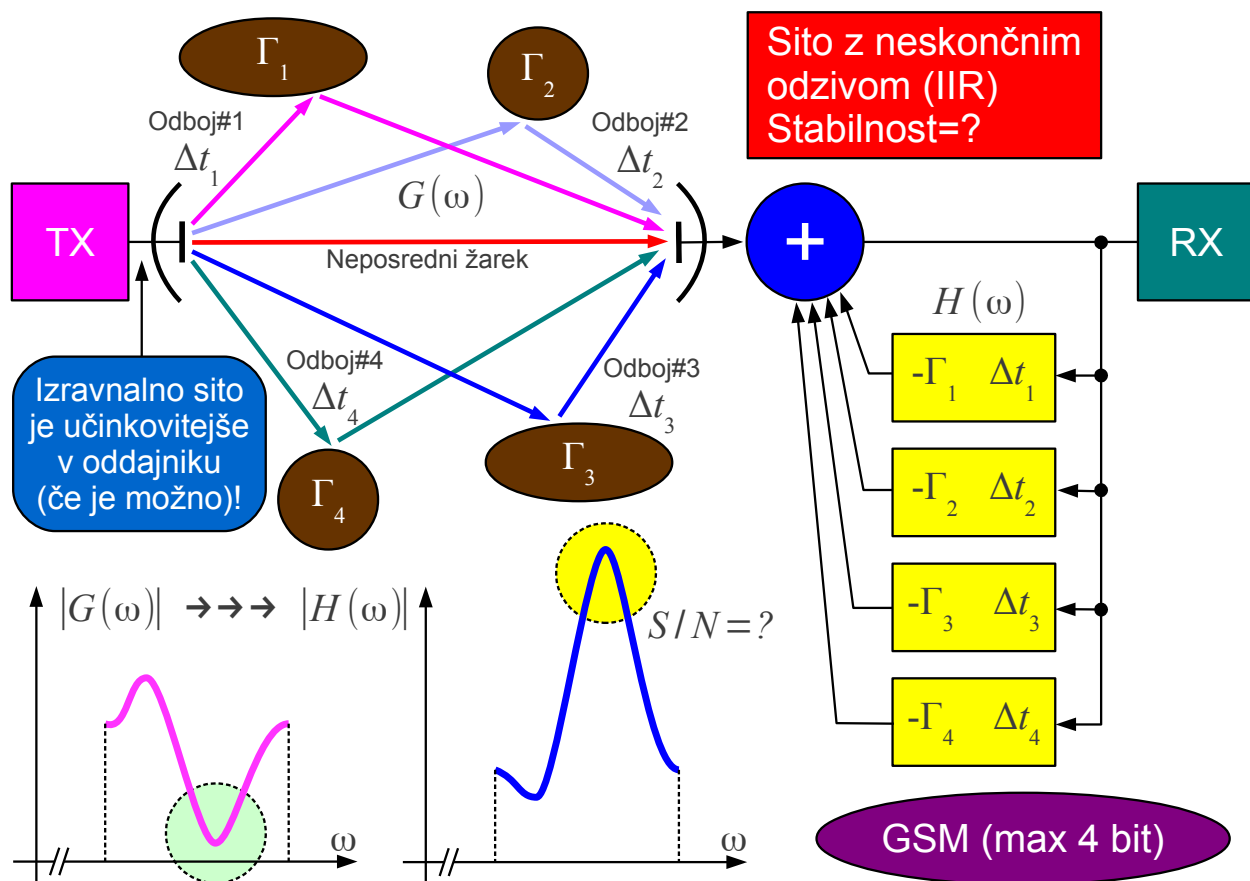
$$B_{ozki} \ll \Delta f \ll B_{široki}$$

Mestno okolje  $f_0 \approx 450\text{MHz}$

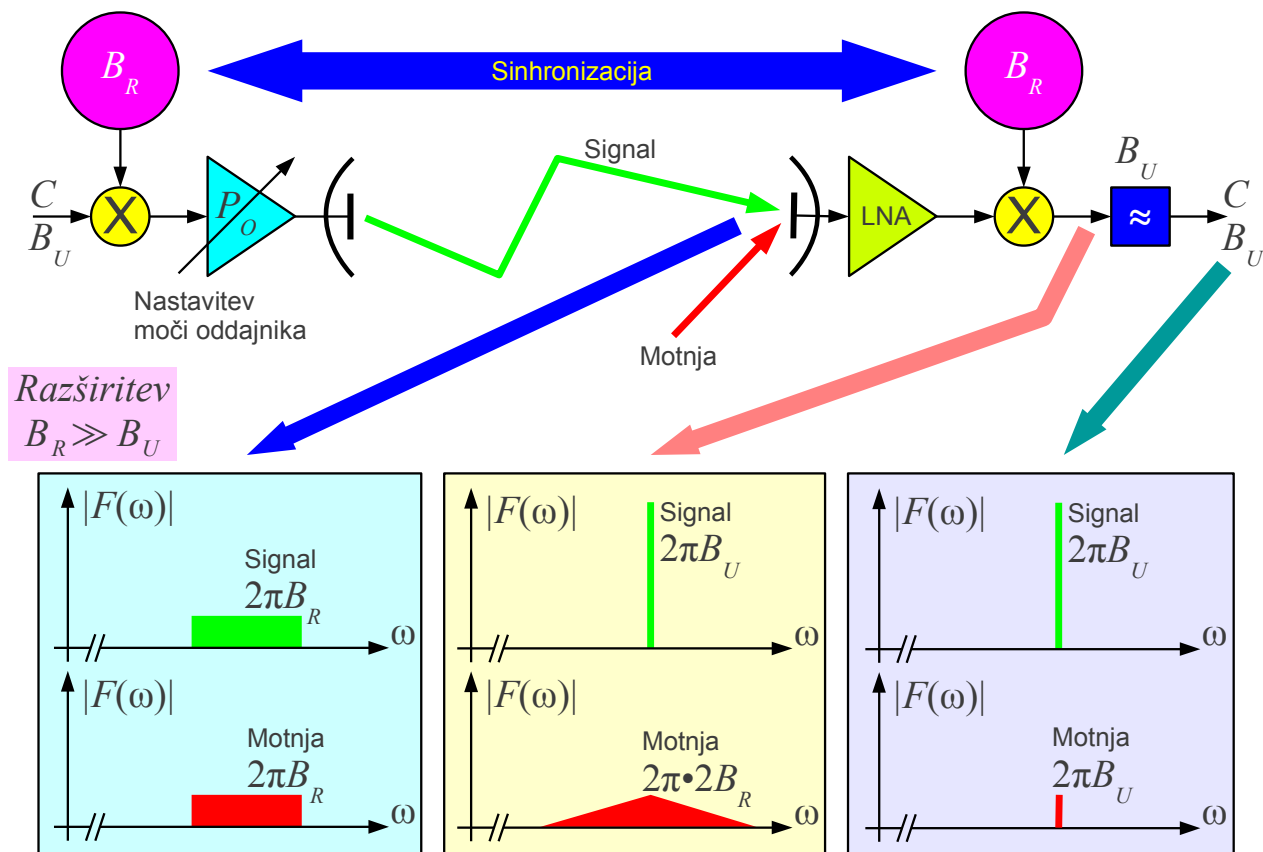
$\Delta l = 200\text{m} \dots 1.5\text{km}$

$\Delta f = 1.5\text{MHz} \dots 200\text{kHz}$

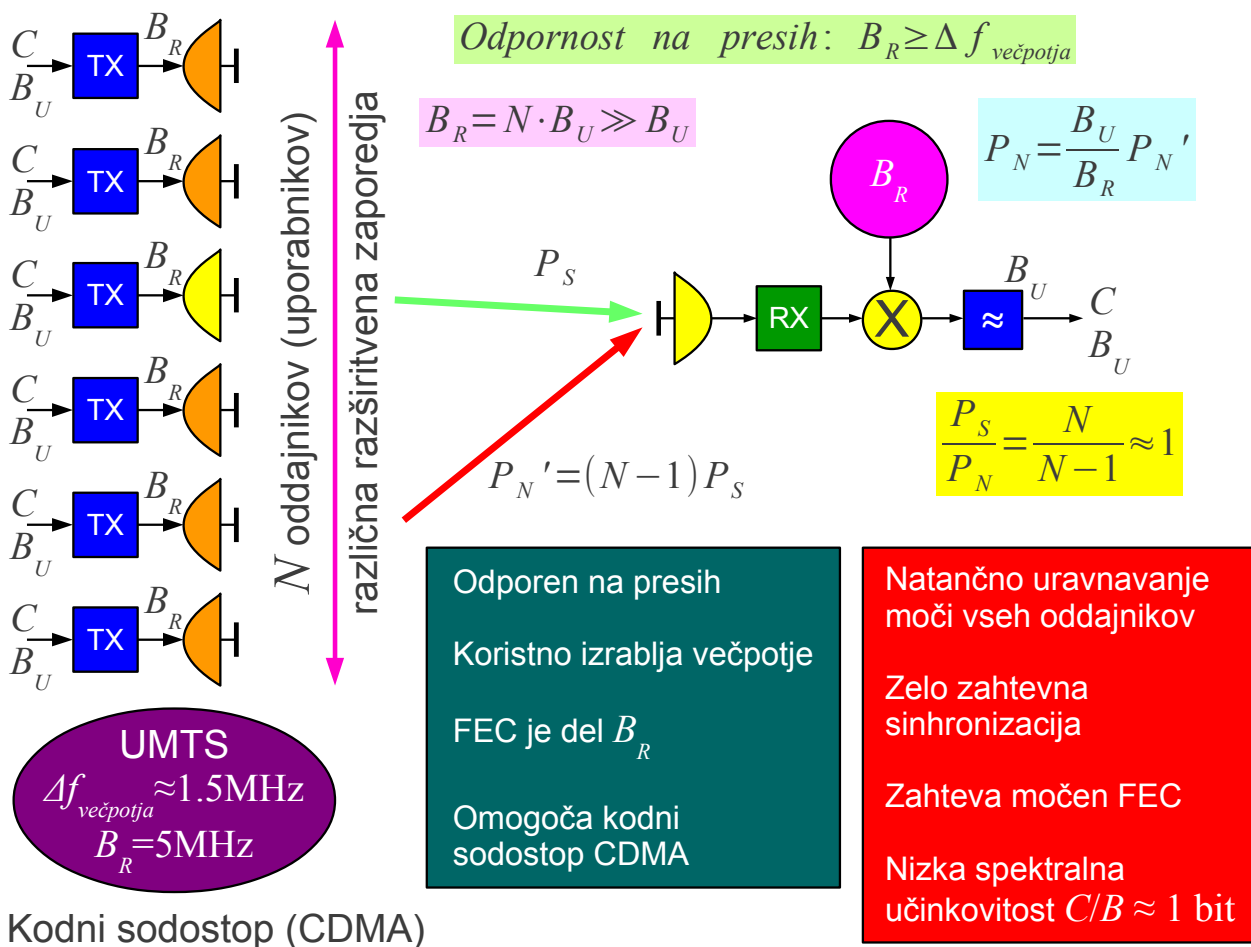
Večpotje v frekvenčnem prostoru

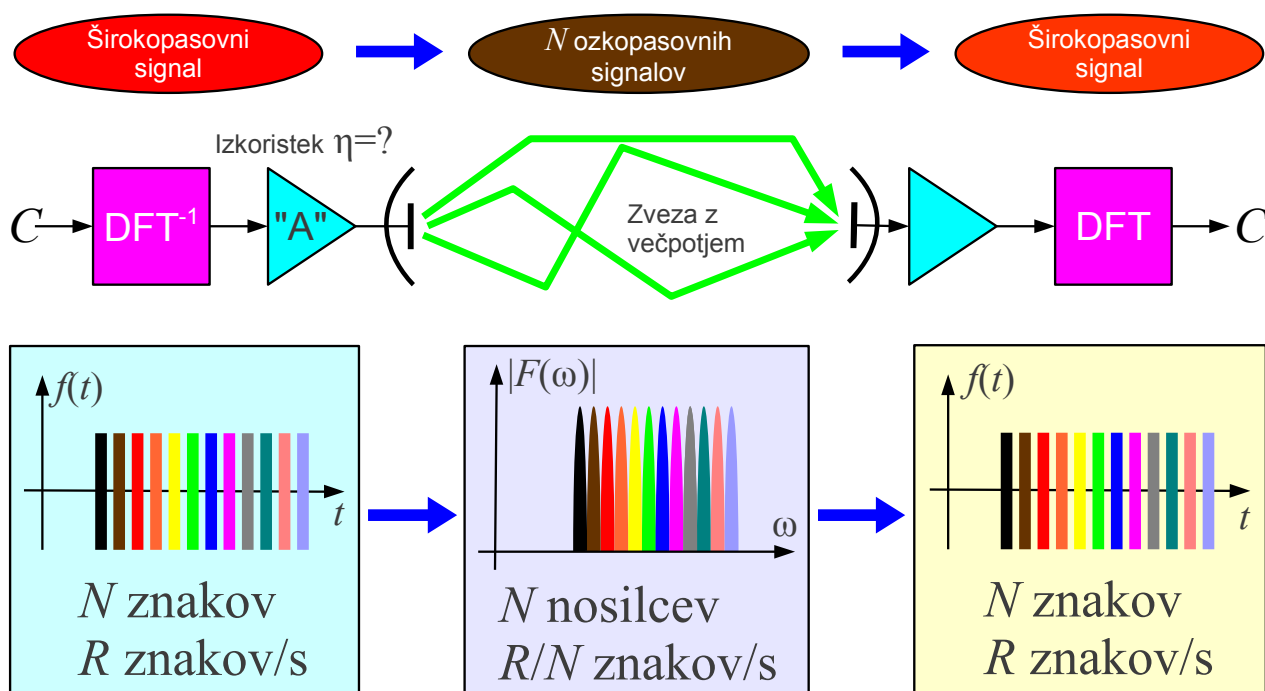


Odpravljanje popačenja večpotja z izravnalnim sitom



Razširjeni spekter (Spread spectrum)



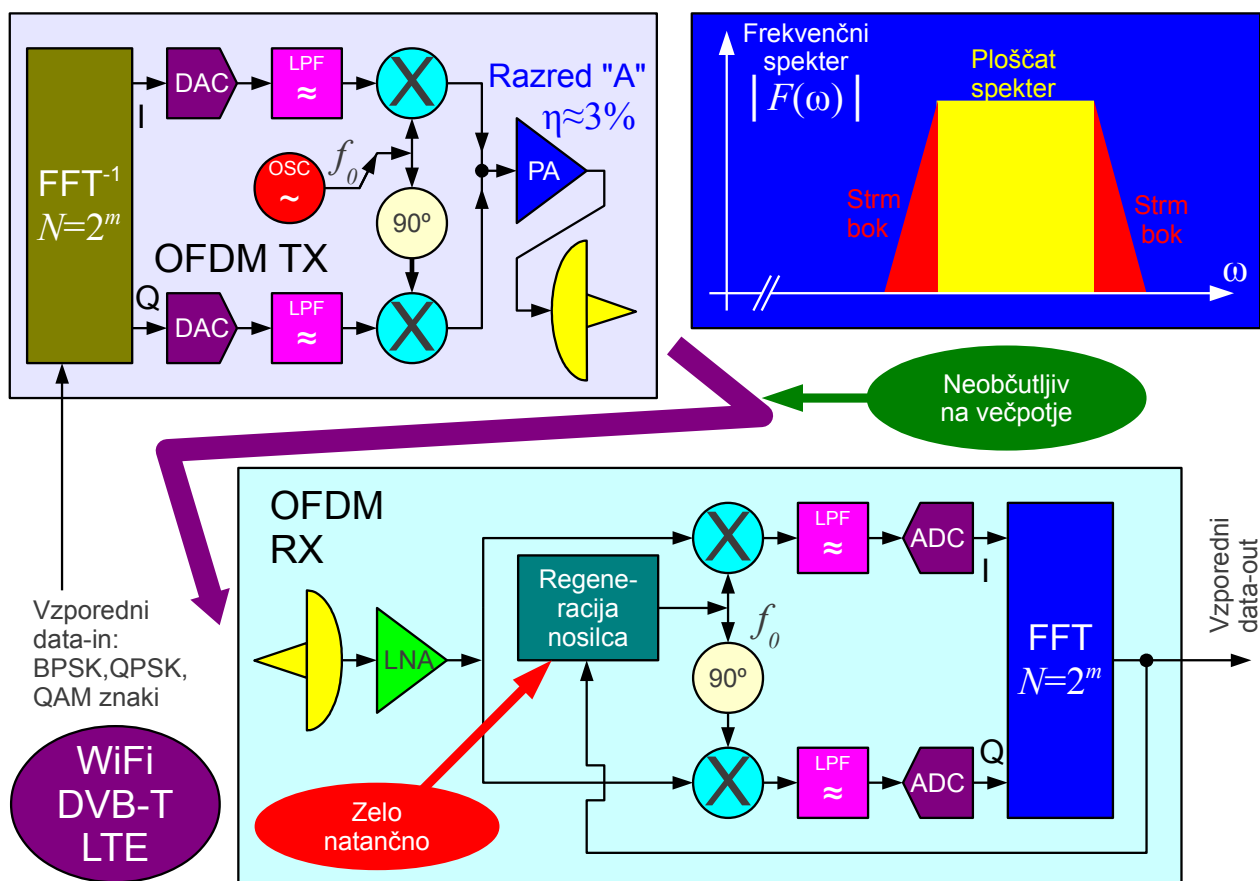


~1950 analogni večtonski modem za ionosferske zveze

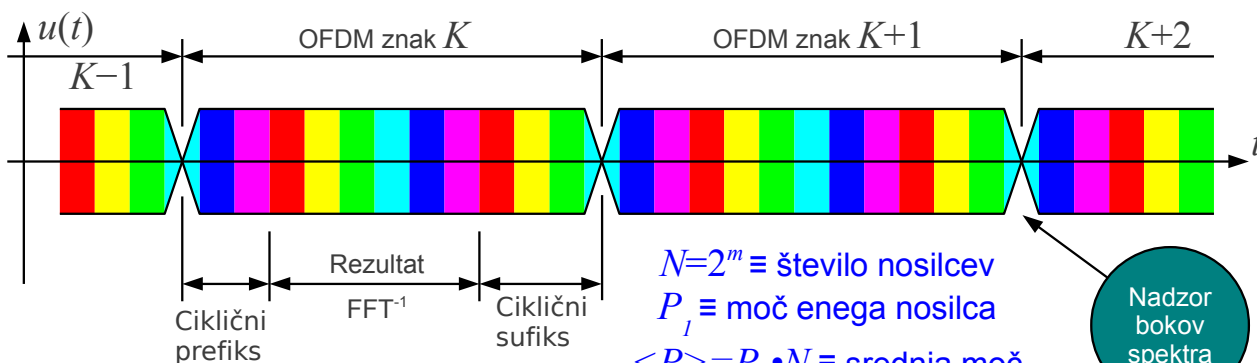
~2000 številski DFT  $\rightarrow$  OFDM WLAN (WiFi) 802.11a (FFT)

Večtonski modem kot protiukrep za popačenje večpotja





Orthogonal Frequency-Division Multiplex (OFDM)



$$t_{\text{prefiks}} + t_{\text{sufiks}} \geq \Delta t_{\text{večpotja}}$$

$$N = 2^m \equiv \text{število nosilcev}$$

$$P_I \equiv \text{moč enega nosilca}$$

$$\langle P \rangle = P_I \cdot N \equiv \text{srednja moč}$$

$$P_{MAX} = P_I \cdot N^2 \equiv \text{vršna moč}$$

Nastavljiva odpornost na  $\Delta t_{\text{večpotja}}$   
 Skoraj pravokoten frekvenčni spekter  
 Zadošča šibek FEC  
 Spektralni izkoristek  $C/B$  dosega teoretske vrednosti BPSK, QPSK, QAM  
 Omogoča enofrekvenčna omrežja SFN (Single-Frequency Network)

Visoko razmerje  $P_{MAX}/\langle P \rangle = N$  pogojuje slab izkoristek oddajnika  $\eta \approx 3\%$

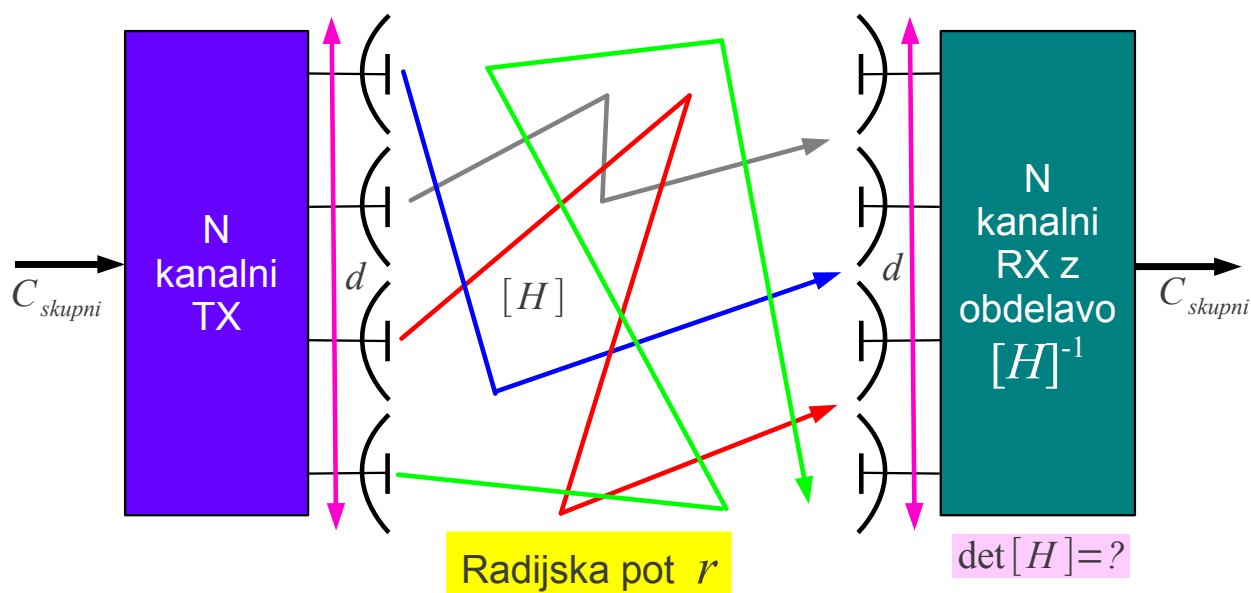
FFT zahteva  $N \cdot \log_2 N$  računskih operacij

Ozkopasovni nosilci zahtevajo visoko frekvenčno stabilnost  $\Delta f \leq 10\% R/N$

Preveliki znaki  $\sim 12000$  bit ( $N \approx 2000$ ,  $C/B \approx 6$  bit) za nekatere protokole

Ozkopasovne motnje rušijo sinhronizacijo

Lastnosti OFDM



Koristna uporaba večpotja!

$$C_{skupni} = N \cdot C_{kanala} = N \cdot B \cdot \log_2 \left( 1 + \frac{P_s}{P_n} \right)$$

(+) visoka spektralna učinkovitost:  $C/B \approx 10$  bit

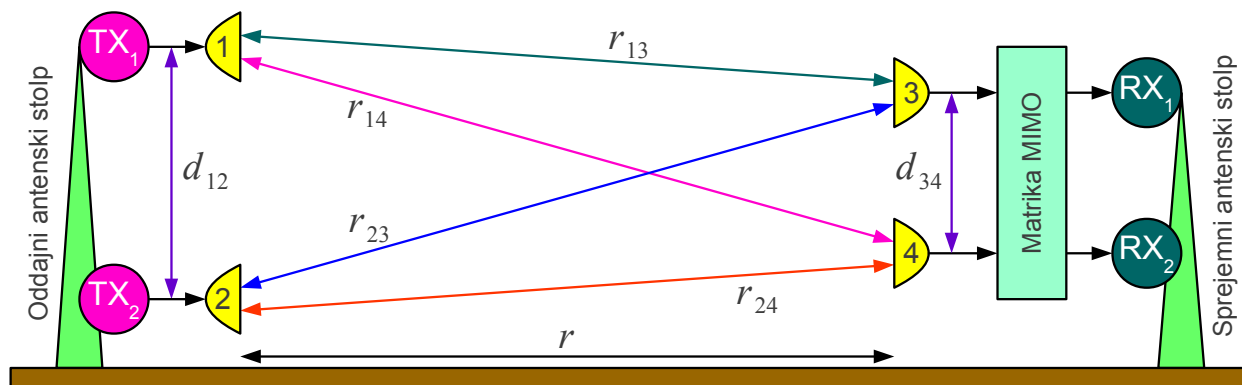
(-) zahteva  $N$  oddajnih anten in  $N$  sprejemnih anten

(+) preprosta rešitev MIMO 2x2: uporaba obeh polarizacij

(-) več kot dve polarizaciji  $\det[H] \neq 0$  le na kratkih poteh  $r \approx 2d^2/\lambda$

MIMO (Multiple-In Multiple-Out)

Podvojevanje C/B mikrovalovne zveze  $\equiv$  Line-Of-Sight MIMO



Pogoj za  $\max \det[MIMO]$ :  $r_{14} - r_{13} - r_{24} + r_{23} = \lambda/2$

$C/B > 40\text{bit}$

$$r_{13} = r_{24} = \sqrt{r^2 + ((d_{12} - d_{34})/2)^2} \approx r + \frac{d_{12}^2 - 2d_{12}d_{34} + d_{34}^2}{8r}$$

$$r_{14} = r_{23} = \sqrt{r^2 + ((d_{12} + d_{34})/2)^2} \approx r + \frac{d_{12}^2 + 2d_{12}d_{34} + d_{34}^2}{8r}$$

$$r_{14} - r_{13} - r_{24} + r_{23} \approx \frac{d_{12}d_{34}}{r} \rightarrow d_{12}d_{34} = r \cdot \lambda/2$$

MIMO brez večpotja

Zgled:

$$r = 10\text{km} \quad f = 15\text{GHz}$$

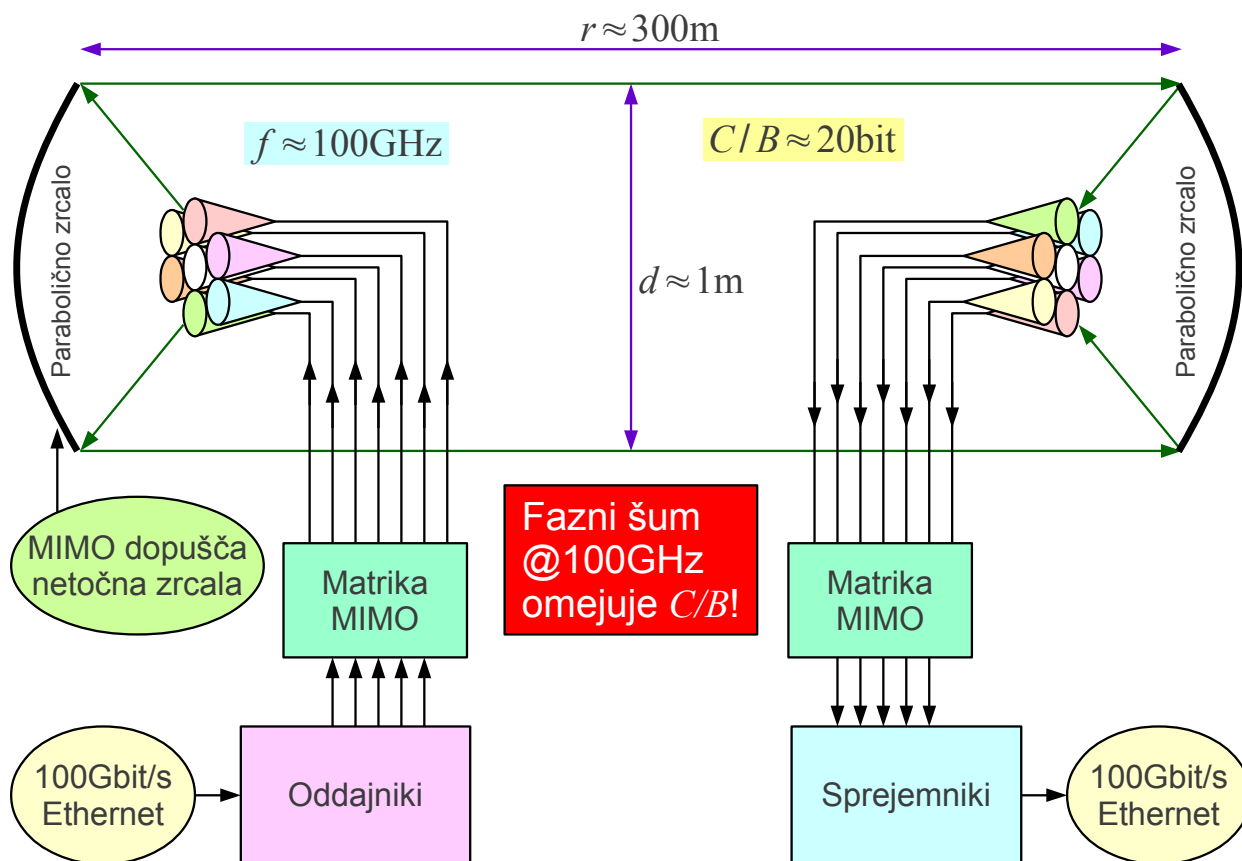
$$\lambda = c_0/f = 2\text{cm}$$

$$\langle d \rangle = \sqrt{d_{12}d_{34}}$$

$$\langle d \rangle = \sqrt{r \cdot \lambda/2} = 10\text{m}$$

Preizkus:

$$r = \frac{2\langle d \rangle^2}{\lambda} = 10\text{km}$$



Visokozmogljiva zveza na kratko razdaljo

\* \* \* \* \*