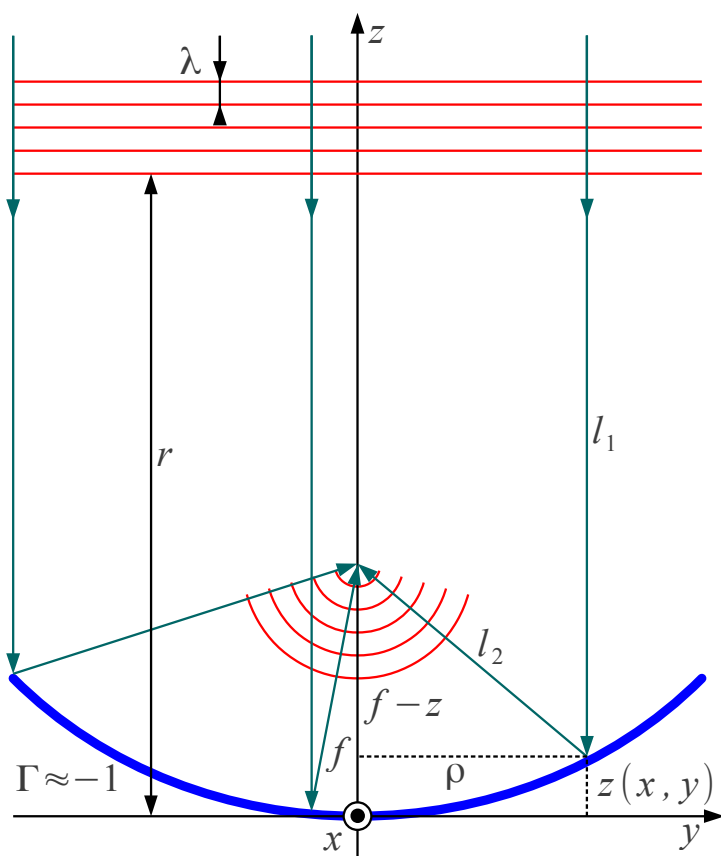# 10. Zbiralna zrcala

Omejevanje kvadratne napake faze lahko zahteva nepraktično dolge valovodne lijake. Kvadratno napako faze popravi zbiralna leča oziroma zbiralno zrcalo, ki pretvori krogelne valovne fronte v ravne valovne fronte.

Čeprav so osnove delovanja enake, se praktične izvedbe leč za radijske valove v marsičem razlikujejo od leč za vidno svetlobo. Bistvena razlika je v velikosti leče v primerjavi z valovno dolžino. Leče za vidno svetlobo so običajno dosti večje $d \gg \lambda$ od valovne dolžine. Izmere leč za radijske valove so pogosto primerljive $d \approx \lambda$ z valovno dolžino.

$$l_1 + l_2 = konst. = r + f$$

$$l_1 = r - z$$

$$l_2 = \sqrt{(f-z)^2 + \rho^2}$$

$$f \equiv goriščna \quad razdalja$$

$$r - z + \sqrt{(f-z)^2 + \rho^2} = r + f$$

$$\sqrt{(f-z)^2 + \rho^2} = f + z$$
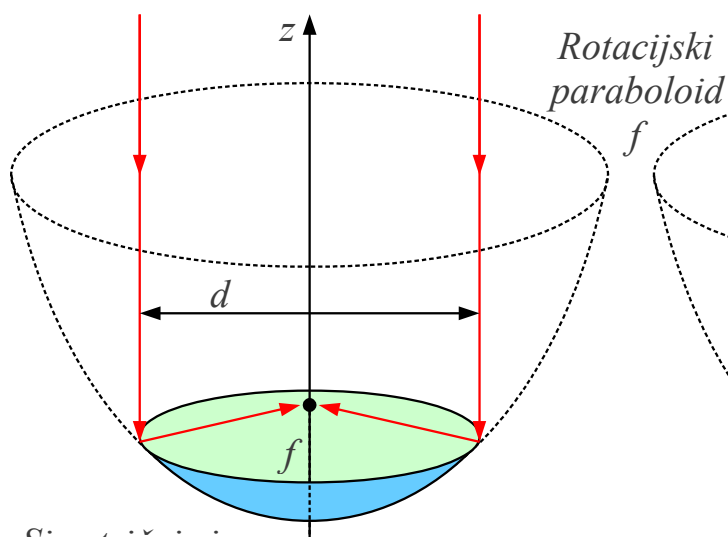
$$(f-z)^2 + \rho^2 = (f+z)^2$$

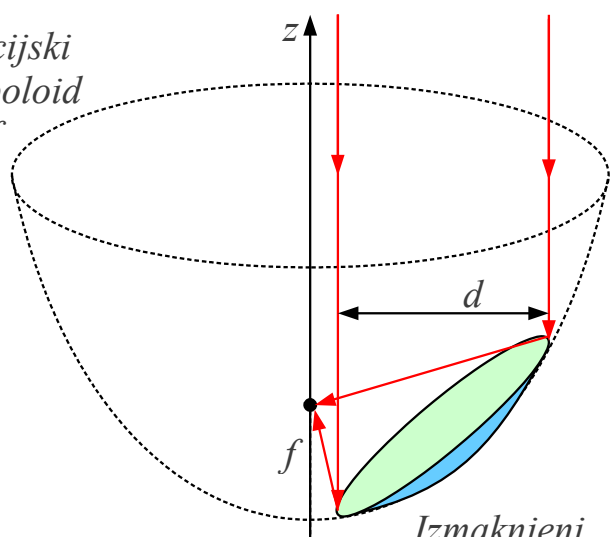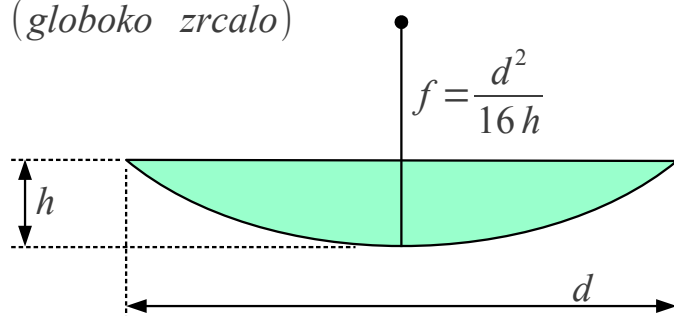$$f^2 - 2fz + z^2 + \rho^2 = f^2 + 2fz + z^2$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\rho^2 = 4fz = x^2 + y^2$$

$$Rotacijski \quad paraboloid:$$

$$z(x,y) = \frac{x^2 + y^2}{4f}$$

Zbiralno zrcalo

Rotacijski paraboloid
f

d

f

Simetrični izrez
$f/d \approx 0.3 \ldots 0.4$
$(globoko \;\; zrcalo)$

Izmaknjeni
$(offset)\; izrez$
$f/d \approx 0.6 \ldots 0.7$
$(plitvo \;\; zrcalo)$

d

f

$$f = \frac{d^2}{16\,h}$$

h

d

Fotoaparat
$f/d \approx 1.8 \ldots 22$

Izrezi zrcala

Rotacijsko − simetrično globoko zrcalo $f/d \approx 0.3 \ldots 0.4$

Neenakomerna osvetlitev

$d \geq 5\lambda$

Žarilec $F(\Theta, \Phi)$ $D \approx 8\text{dBi}$

FS

Senca žarilca

Senca kabla ? Senca nosilca ?

Sevanje preko roba

Sevanje preko roba

$d \geq 10\lambda$

Neenakomerna osvetlitev

Žarilec $F(\Theta, \Phi)$ $D \approx 12\text{dBi}$

FS

Izmaknjeno (*offset*) *plitvo zrcalo* $f/d \approx 0.6 \ldots 0.7$
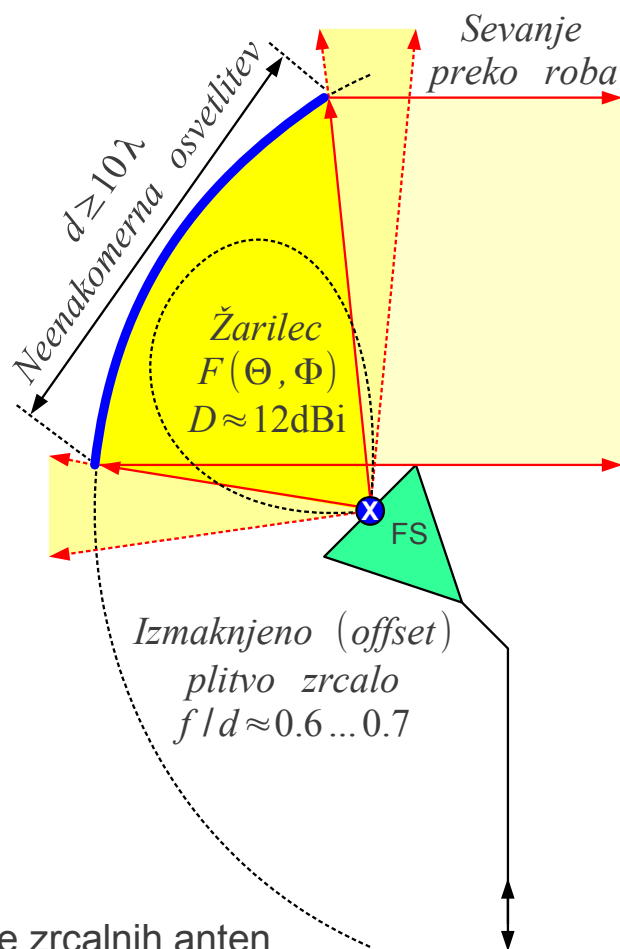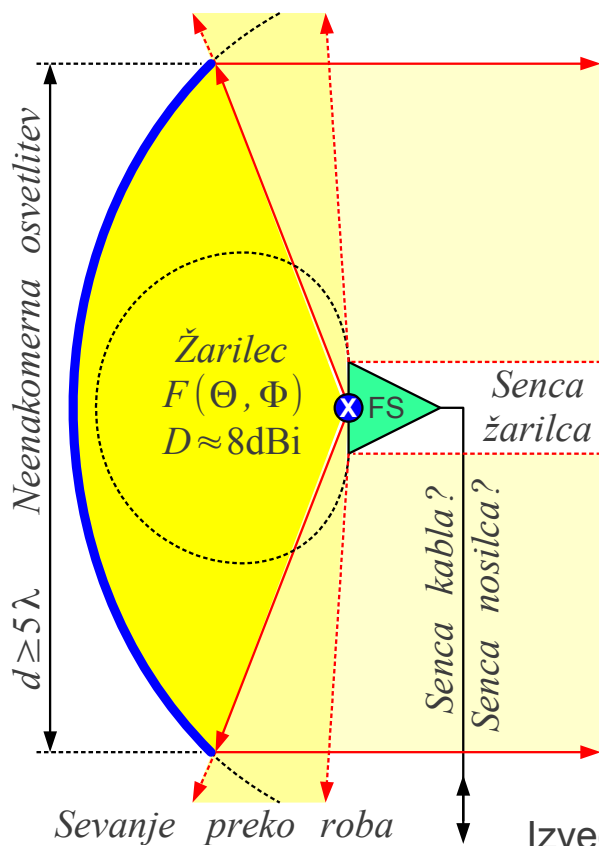
Izvedbe zrcalnih anten
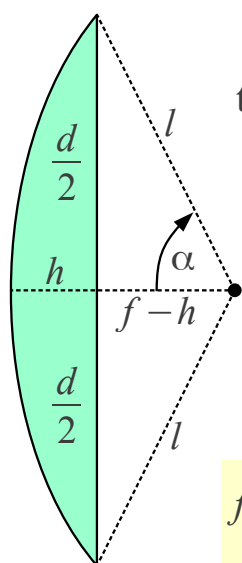
$$\tan\alpha = \frac{\dfrac{d}{2}}{f-h} = \frac{\dfrac{d}{2}}{f-\dfrac{d^2}{16f}} = \frac{2\cdot\dfrac{d}{4f}}{1-\left(\dfrac{d}{4f}\right)^2}$$

$$h = \frac{d^2}{16f}$$

$$\tan\alpha = \frac{2\tan\dfrac{\alpha}{2}}{1-\left(\tan\dfrac{\alpha}{2}\right)^2}$$

$$\tan\frac{\alpha}{2} = \frac{d}{4f}$$
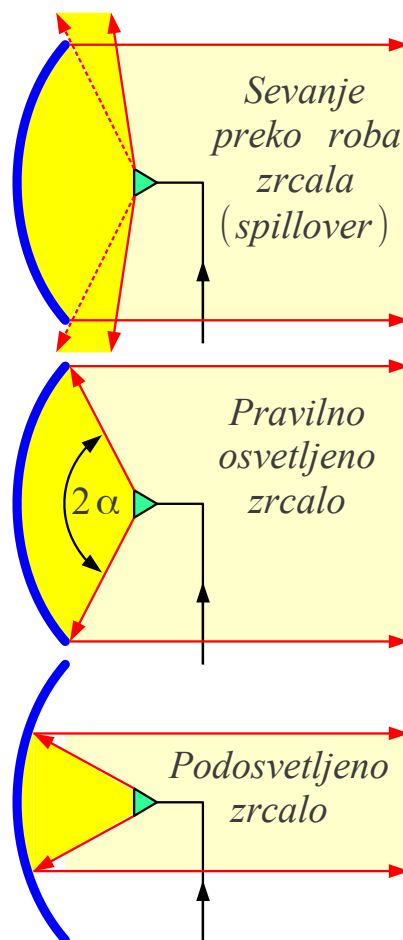
$$f/d = \frac{1}{4\tan\dfrac{\alpha}{2}}$$

$$\alpha = 2\arctan\frac{1}{4(f/d)}$$

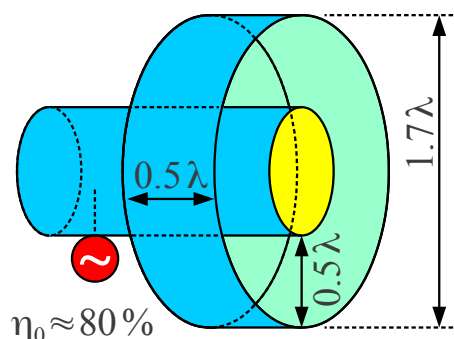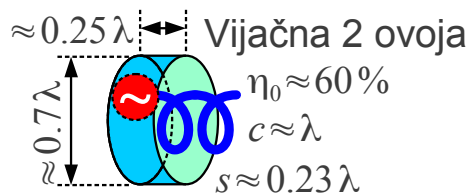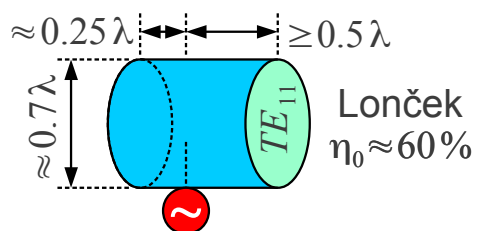$$l = \sqrt{(d/2)^2 + (f-h)^2} = f\left[1 + \left(\frac{1}{4(f/d)}\right)^2\right]$$

Zgled $f/d = 0.4$ ➔ $\alpha \approx 1.12\,\text{rd} \approx 64°$ $2\alpha \approx 128°$

$l \approx 1.39\,f$ ➔ $20\log_{10}(l/f) \approx 2.86\,\text{dB}$

Osvetlitev zrcala

Sevanje preko roba zrcala (*spillover*)

2α *Pravilno osvetljeno zrcalo*

Podosvetljeno zrcalo

$\approx 0.25\,\lambda$ $\geq 0.5\,\lambda$ $\approx 0.7\,\lambda$

Lonček
$\eta_0 \approx 60\%$

TE_{11}

$\approx 0.25\,\lambda$ Vijačna 2 ovoja
$\eta_0 \approx 60\%$
$c \approx \lambda$
$s \approx 0.23\,\lambda$
$\approx 0.7\,\lambda$

Žarilci za globoka zrcala

TE_{11}

$\approx 0.7\,\lambda$ $\approx 1.2\,\lambda$ $\approx 1.7\,\lambda$ $\approx 2.2\,\lambda$

$\approx 0.25\,\lambda$

$0.5\,\lambda$
$0.5\,\lambda$
$1.7\,\lambda$

$\eta_0 \approx 80\%$
Ovratnik Kumar/VE4MA
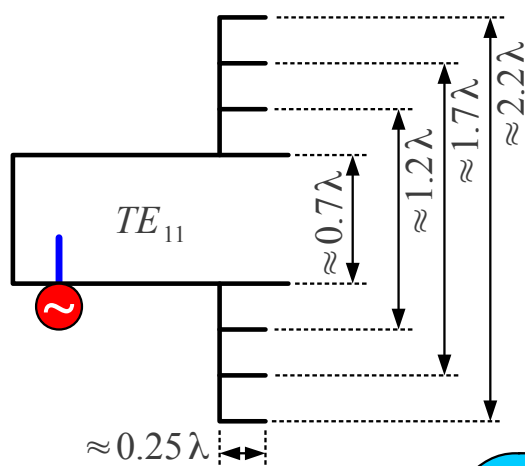
Korugirana
prirobnica
$\eta_0 \approx 80\%$

$$Daljnje \quad polje \quad r > \frac{2 d^2}{\lambda}$$

$\vec{E}(\vec{r})$

$\vec{r} - \vec{r}\,'$

$\vec{r}$

Θ

Odprtina

$E_0(\rho, \phi)$

$d\phi$

ϕ

$d\rho$

ρ

A

d

$$dE \approx \frac{j}{2\lambda} E_0(\rho, \phi) \rho \, d\rho \, d\phi \frac{e^{-jk|\vec{r} - \vec{r}\,'|}}{|\vec{r} - \vec{r}\,'|}(1 + \cos\Theta)$$

$$\vec{r}\,' = \vec{1}_x \rho \cos\phi + \vec{1}_y \rho \sin\phi$$

$$|\vec{r} - \vec{r}\,'| = \sqrt{(x - \rho\cos\phi)^2 + (y - \rho\sin\phi)^2 + z^2}$$

$$Faza \quad k|\vec{r} - \vec{r}\,'| \approx k(r - \rho\sin\Theta\cos(\Phi - \phi))$$
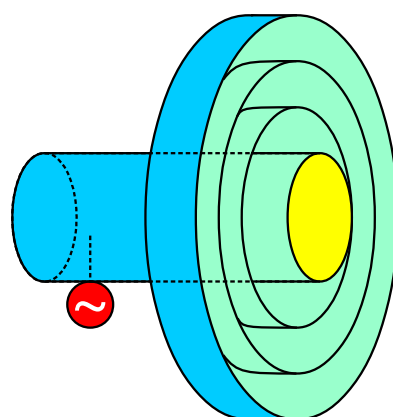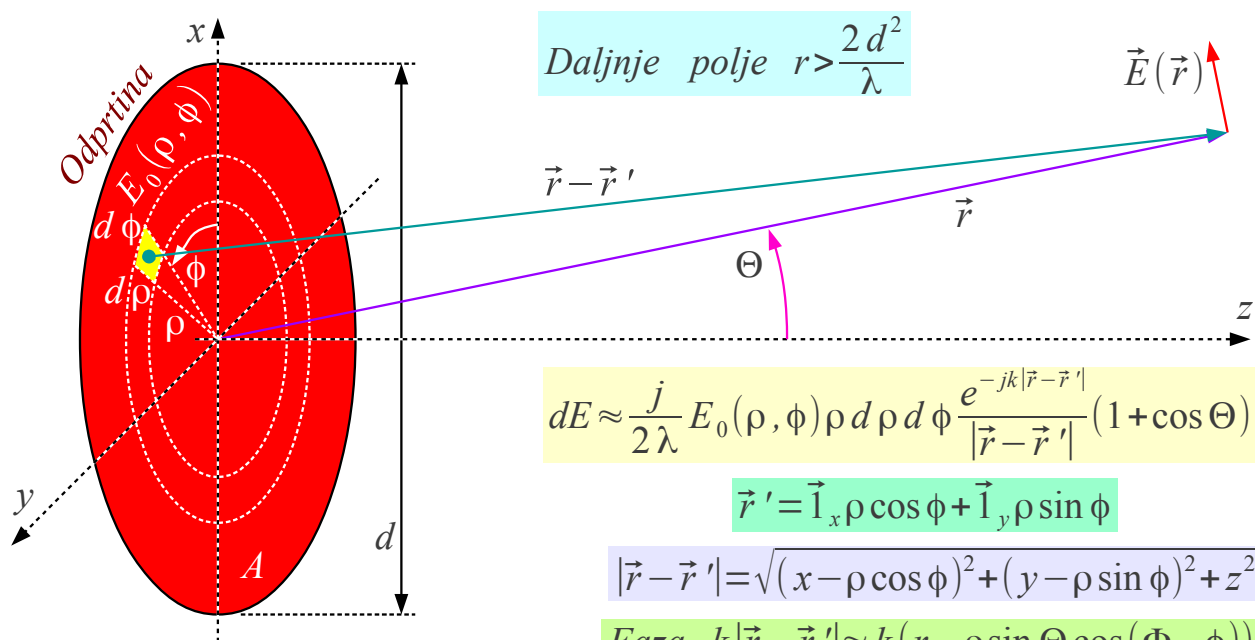
$$dE \approx \frac{j}{2\lambda} E_0(\rho, \phi) \rho \, d\rho \, d\phi \frac{e^{-jkr}}{r}(1 + \cos\Theta) e^{jk\rho\sin\Theta\cos(\Phi - \phi)}$$

$$E = \iint_A dE \approx \frac{j}{2\lambda} \frac{e^{-jkr}}{r}(1 + \cos\Theta) \int_0^{d/2} \int_0^{2\pi} E_0(\rho, \phi) e^{jk\rho\sin\Theta\cos(\Phi - \phi)} \rho \, d\rho \, d\phi$$

Sevanje krožne odprtine

$$E_0(\rho, \phi) = konst. = C$$

$$J_n(t) = \sum_{m=0}^{\infty} \frac{(-1)^m t^{2m+n}}{2^{2m+n} m!(m+n)!}$$

$$E = \frac{j}{2\lambda} C \frac{e^{-jkr}}{r} (1 + \cos\Theta) \int_0^{d/2} \int_0^{2\pi} e^{jk\rho \sin\Theta \cos(\Phi-\phi)} \rho\, d\rho\, d\phi$$

$$\int_0^{2\pi} e^{jt\cos\phi} d\phi = 2\pi J_0(t)$$

$$E = \frac{j}{\lambda} C \frac{e^{-jkr}}{r} \frac{1+\cos\Theta}{2} \int_0^{d/2} 2\pi J_0(k\rho\sin\Theta)\rho\, d\rho$$

$$\frac{d}{dt}\left[t^n J_n(t)\right] = t^n J_{n-1}(t)$$

$$\int_0^{d/2} 2\pi J_0(k\rho\sin\Theta)\rho\, d\rho = 2\pi\left[\rho\frac{J_1(k\rho\sin\Theta)}{k\sin\Theta}\right]_0^{d/2} = \pi\left(\frac{d}{2}\right)^2 \frac{2J_1\left(\frac{kd}{2}\sin\Theta\right)}{\frac{kd}{2}\sin\Theta}$$

$$A = \pi\left(\frac{d}{2}\right)^2 \qquad \frac{kd}{2} = \frac{\pi d}{\lambda}$$

$$F(\Theta, \Phi) = \frac{1+\cos\Theta}{2} \frac{2J_1\left(\frac{\pi d}{\lambda}\sin\Theta\right)}{\frac{\pi d}{\lambda}\sin\Theta}$$

$$E = \frac{j}{\lambda} C A \frac{e^{-jkr}}{r} \frac{1+\cos\Theta}{2} \frac{2J_1\left(\frac{kd}{2}\sin\Theta\right)}{\frac{kd}{2}\sin\Theta}$$

$$J_1(t) = 0 \;\rightarrow\; t \approx 3.8318$$

Enakomerno osvetljen krog

$$\alpha_{-3dB} \approx \Theta_0 \approx \arcsin\Theta_0 \approx \frac{3.8318\lambda}{\pi d} \approx 1.22\frac{\lambda}{d}$$

$$E_0(\rho,\phi)=C\,e^{-j\Delta\phi(2\rho/d)^2}$$

$$\int_0^{2\pi} e^{jt\cos\phi}\,d\phi=2\pi J_0(t)$$

$$E=\frac{j}{2\lambda}\frac{e^{-jkr}}{r}(1+\cos\Theta)\int_0^{d/2}\int_0^{2\pi} C\,e^{-j\Delta\phi(2\rho/d)^2}e^{jk\rho\sin\Theta\cos(\Phi-\phi)}\rho\,d\rho\,d\phi$$

$$E=\frac{j}{\lambda}\frac{e^{-jkr}}{r}\frac{1+\cos\Theta}{2}\int_0^{d/2} C\,e^{-j\Delta\phi(2\rho/d)^2}2\pi J_0(k\rho\sin\Theta)\rho\,d\rho$$

$$A=\pi\left(\frac{d}{2}\right)^2$$

$$E=\frac{j}{\lambda}C\,A\frac{e^{-jkr}}{r}\frac{1+\cos\Theta}{2}\left[\frac{8}{d^2}\int_0^{d/2} e^{-j\Delta\phi(2\rho/d)^2}J_0(k\rho\sin\Theta)\rho\,d\rho\right]$$

$$F(\Theta,\Phi)=\frac{1+\cos\Theta}{2}\left[\frac{8}{d^2}\int_0^{d/2} e^{-j\Delta\phi(2\rho/d)^2}J_0(k\rho\sin\Theta)\rho\,d\rho\right]$$

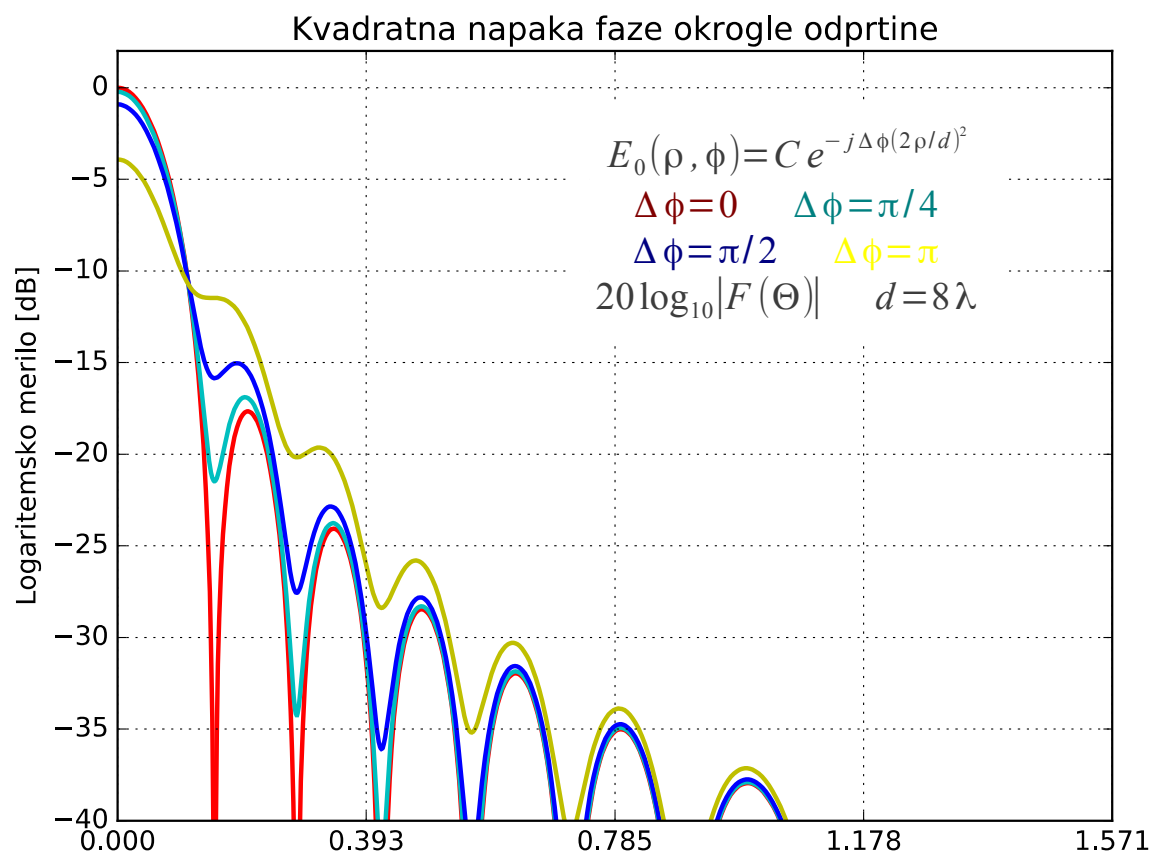$$\Theta=0 \;\rightarrow\; \cos\Theta=1 \qquad J_0(k\rho\sin\Theta)=1 \qquad u=(2\rho/d)^2$$
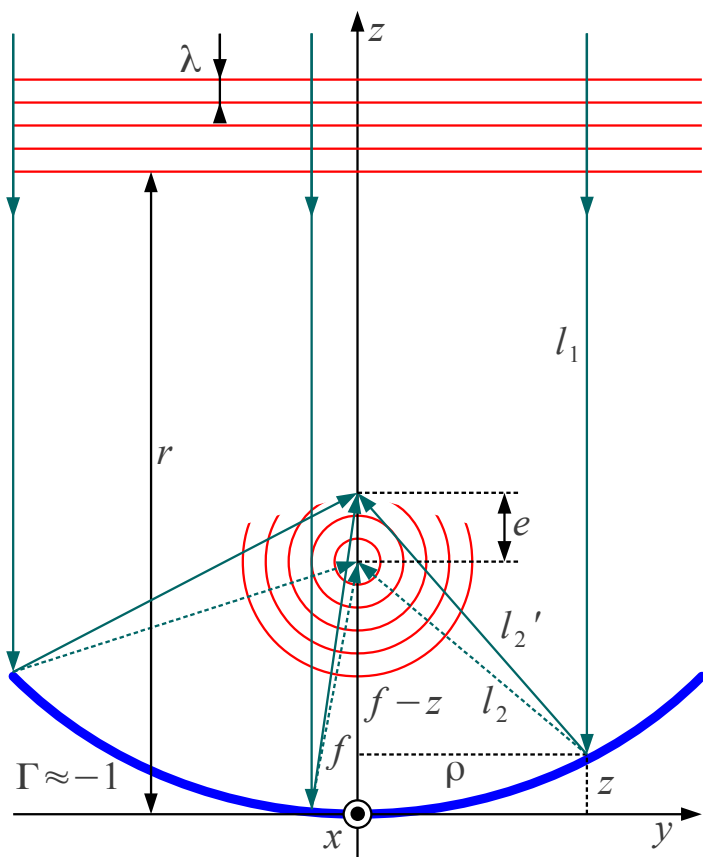
$$F(\Theta=0)=\frac{8}{d^2}\int_0^{d/2} e^{-j\Delta\phi(2\rho/d)^2}\rho\,d\rho=\int_0^1 e^{-j\Delta\phi u}\,du=\frac{e^{-j\Delta\phi}-1}{-j\Delta\phi}=e^{-j\Delta\phi/2}\frac{\sin(\Delta\phi/2)}{\Delta\phi/2}$$

$$\Delta D_{[dB]}=20\log_{10}|F(\Theta=0)|=20\log_{10}\left|\frac{\sin(\Delta\phi/2)}{\Delta\phi/2}\right|$$

Kvadratna napaka faze

Kvadratna napaka faze okrogle odprtine

$$E_0(\rho, \phi) = C\, e^{-j\Delta\phi(2\rho/d)^2}$$

$$\Delta\phi = 0 \qquad \Delta\phi = \pi/4$$

$$\Delta\phi = \pi/2 \qquad \Delta\phi = \pi$$

$$20\log_{10}\left|F(\Theta)\right| \qquad d = 8\lambda$$

Osni odmik žarilca

$$l_2=\sqrt{(f-z)^2+\rho^2} \qquad \rho=\sqrt{x^2+y^2}$$

$$l_2{'}=\sqrt{(e+f-z)^2+\rho^2} \qquad l_1=r-z$$

$$l_1+l_2{'}=r-z+\sqrt{(e+f-z)^2+\rho^2}=$$
$$=r-z+\sqrt{(f+z)^2+e^2+2ef-2ez}$$

$$e\ll f$$
$$l_1+l_2{'}\approx r+f+\frac{(e^2+2ef-2ez)}{2(f+z)}$$

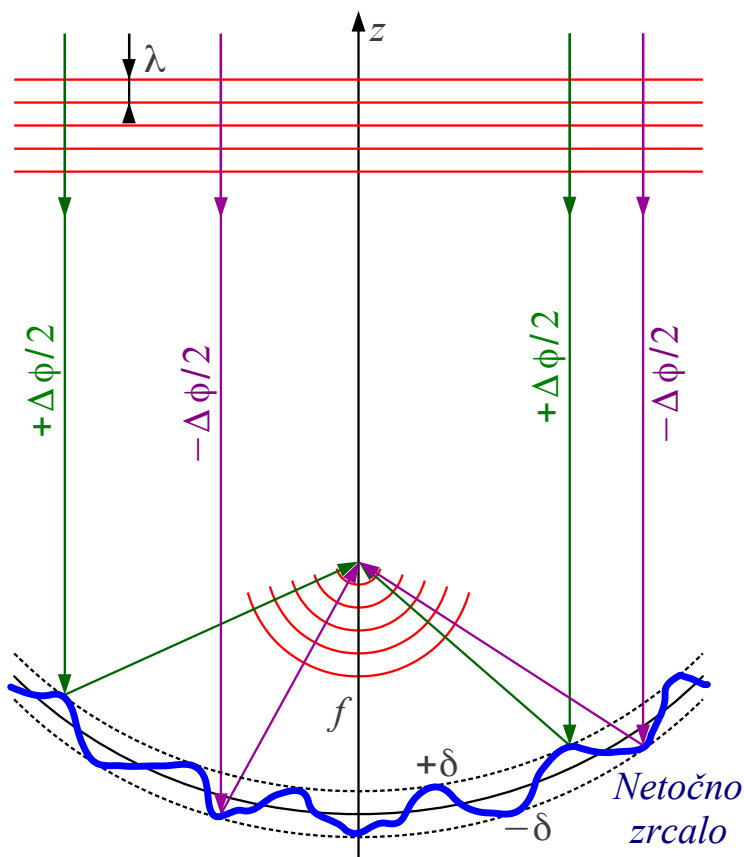$$\Delta l\approx e\left[\frac{f-z}{f+z}-1\right]=e\frac{-2z}{f+z}$$

$$\Delta l\approx e\frac{-8\rho^2}{4f^2+\rho^2} \qquad z=\frac{\rho^2}{4f}$$

$$\Delta\phi=k\,\Delta l\approx ke\frac{-8\rho^2}{4f^2+\rho^2}$$

$$Ob\ \ osi\ \ zrcala$$
$$\rho\ll f \ \rightarrow\ \Delta\phi\approx-2\,ke\,(\rho/f)^2$$

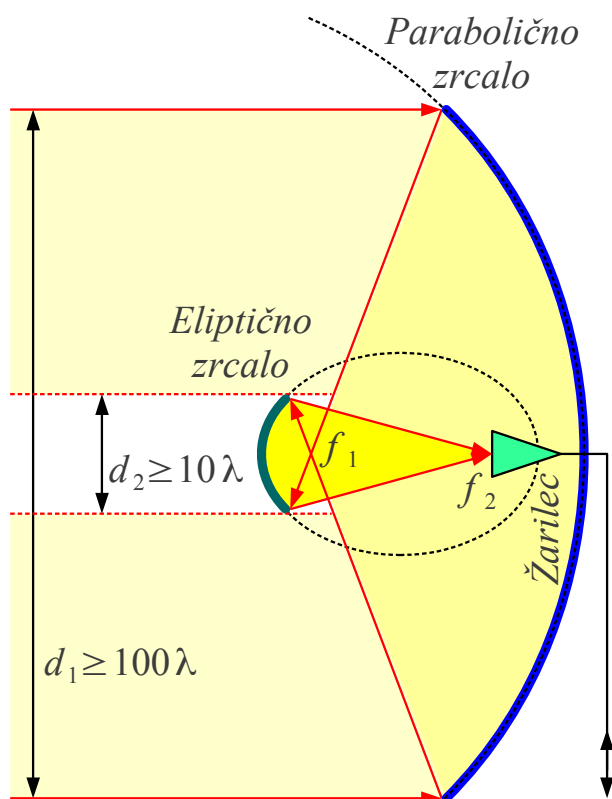Napake oblike zrcala

Uporabno

Neuporabno

$$\Delta\phi \approx 4\,k\,\delta$$

$$\Delta D_{\mathrm{dB}} \approx 20\log_{10}\left|\frac{\sin(\Delta\phi/2)}{\Delta\phi/2}\right|$$

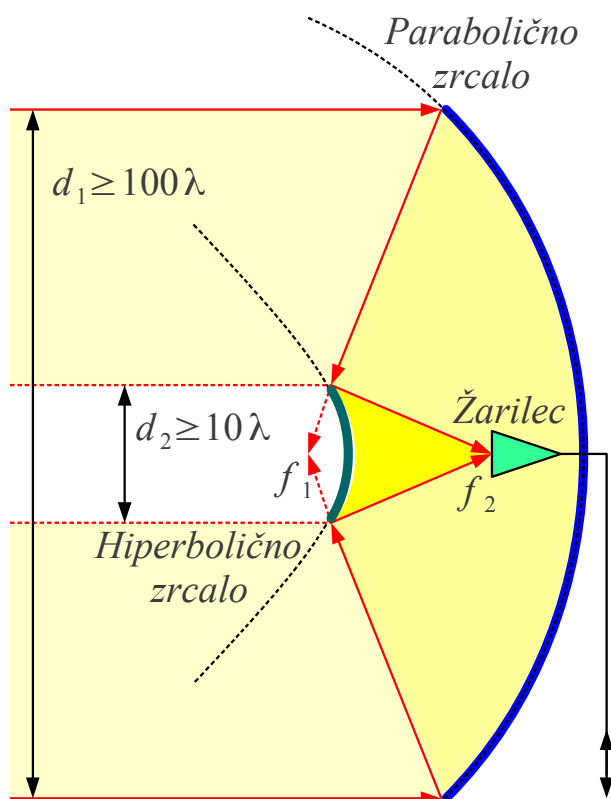| $\pm\delta$	$\Delta\phi$	$\Delta D\,[\mathrm{dB}]$
$\pm\lambda/4$	2π	$-\infty$
$\pm\lambda/8$	π	-3.922
$\pm\lambda/16$	$\pi/2$	-0.912
$\pm\lambda/32$	$\pi/4$	-0.224

Zgled : $f = 12\,\mathrm{GHz}$ $\lambda = 25\,\mathrm{mm}$ $\pm\delta = \pm0.8\,\mathrm{mm}$

Gregorijanska antena

Cassegrainova antena

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