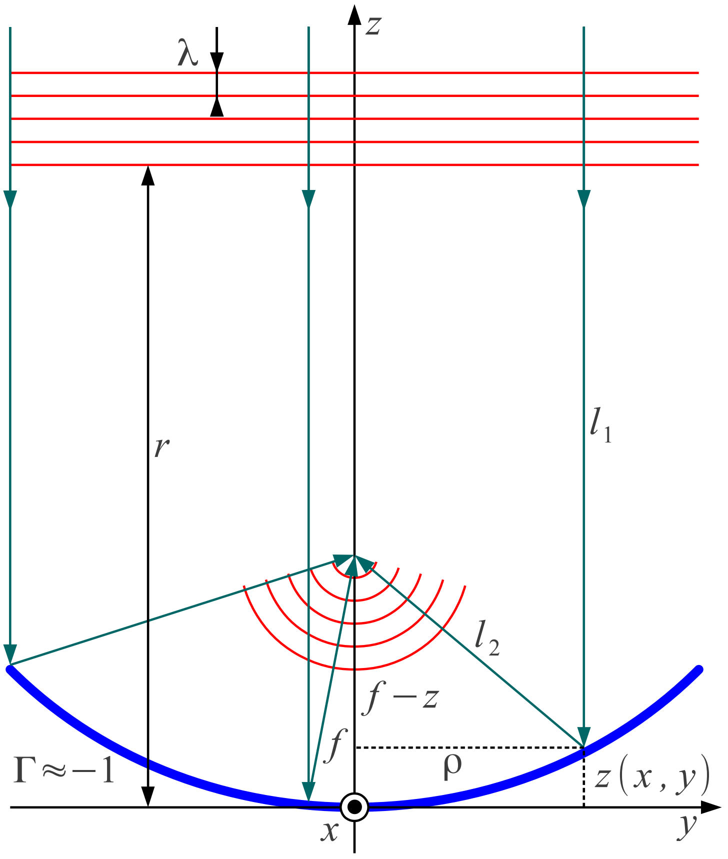
10. Zbiralna zrcala

Omejevanje kvadratne napake faze lahko zahteva nepraktično dolge valovodne lijake. Kvadratno napako faze popravi zbiralna leča oziroma zbiralno zrcalo, ki pretvori krogelne valovne fronte v ravne valovne fronte.

Čeprav so osnove delovanja enake, se praktične izvedbe leč za radijske valove v marsičem razlikujejo od leč za vidno svetlobo. Bistvena razlika je v velikosti leče v primerjavi z valovno dolžino. Leče za vidno svetlobo so običajno dosti večje $d \gg \lambda$ od valovne dolžine. Izmere leč za radijske valove so pogosto primerljive $d \approx \lambda$ z valovno dolžino.



Zbiralno zrcalo

$$l_1 + l_2 = konst. = r + f$$

$$l_1 = r - z$$

$$l_2 = \sqrt{(f-z)^2 + \rho^2}$$

$$f \equiv gori\check{s}\check{c}na \quad razdalja$$

$$r - z + \sqrt{(f-z)^2 + \rho^2} = r + f$$

$$\sqrt{(f-z)^2 + \rho^2} = f + z$$

$$(f-z)^2 + \rho^2 = (f+z)^2$$

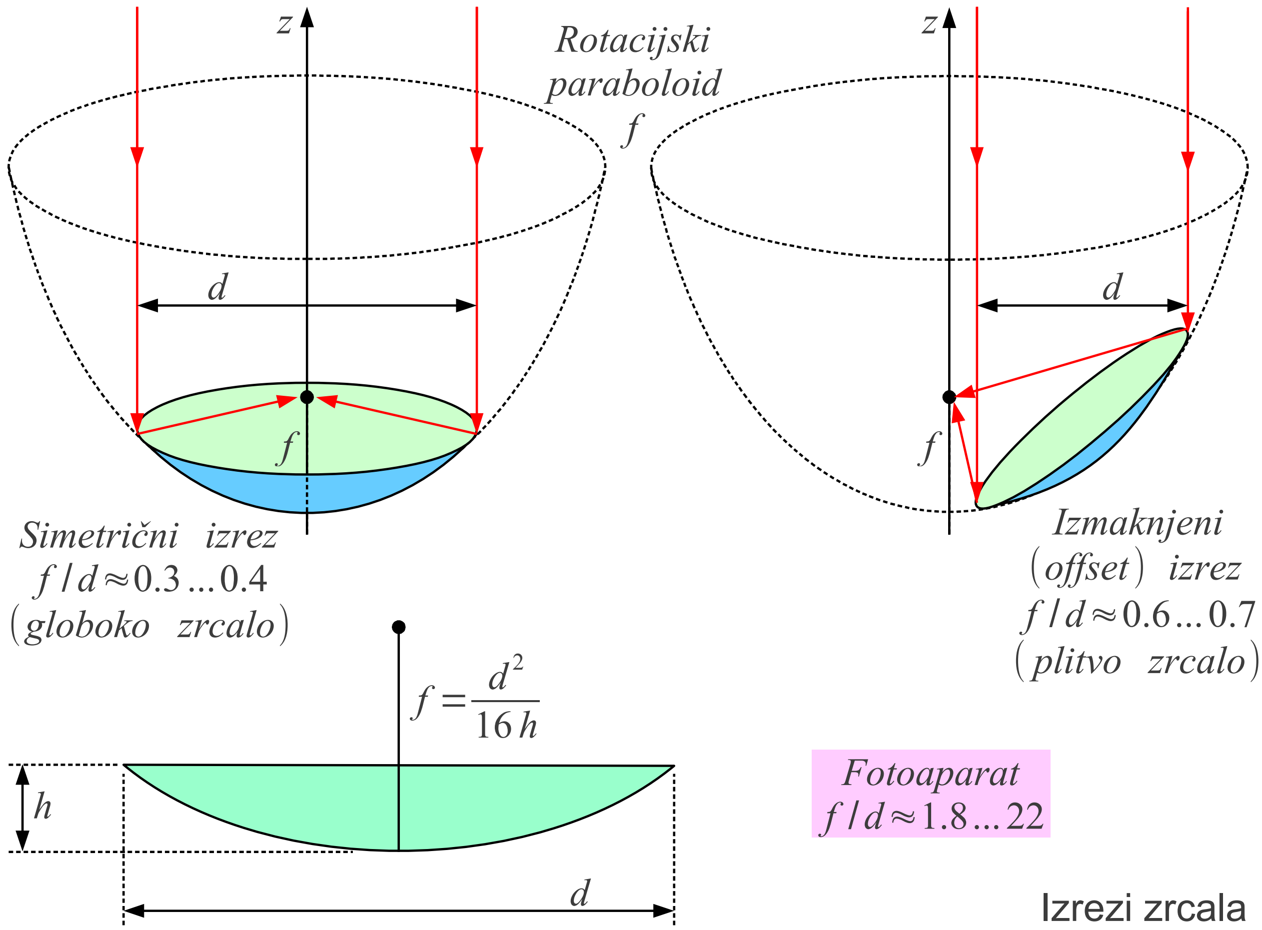
$$f^2 - 2fz + z^2 + \rho^2 = f^2 + 2fz + z^2$$

$$\rho = \sqrt{x^2 + y^2}$$

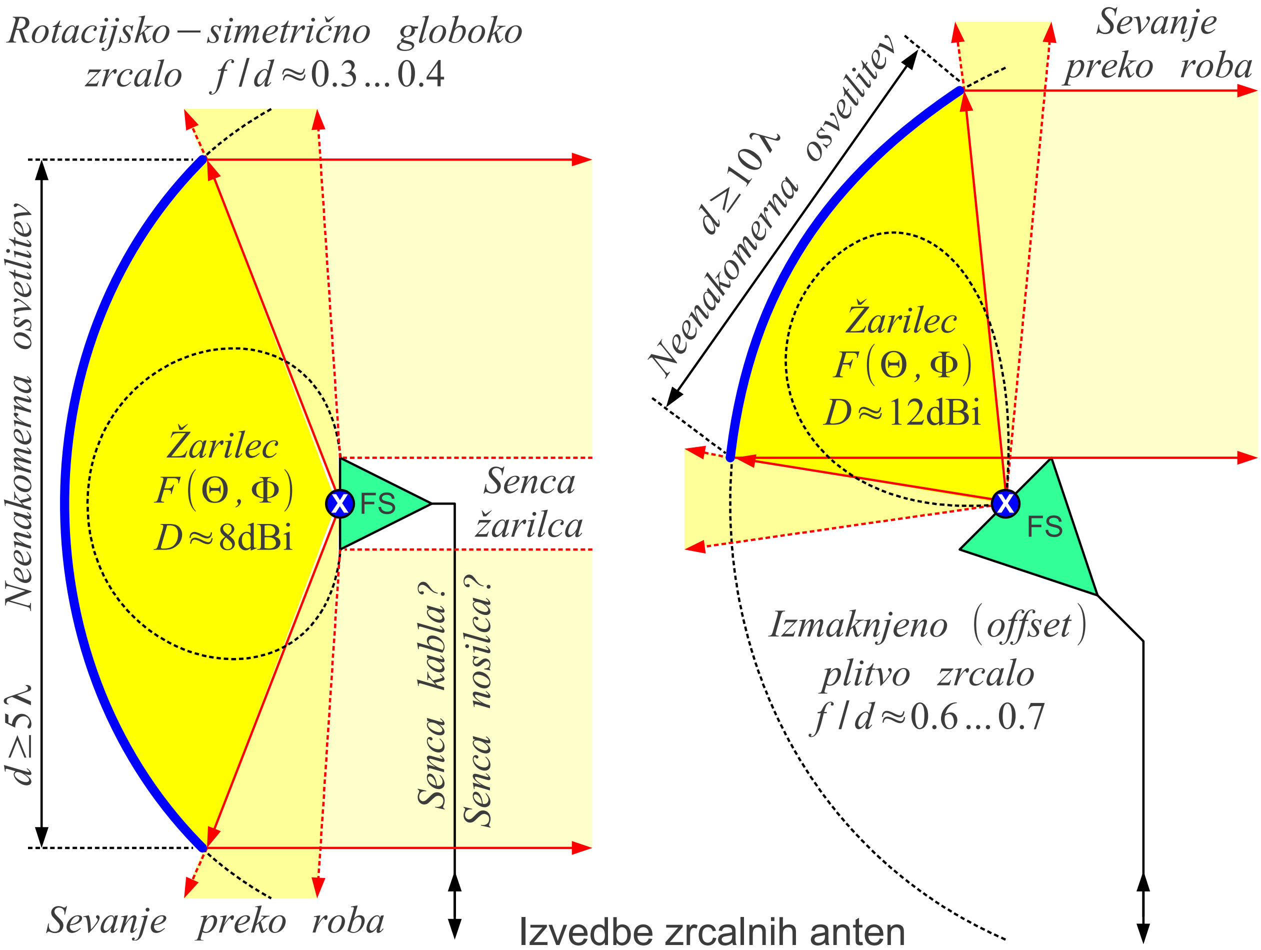
$$\rho^2 = 4fz = x^2 + y^2$$

Rotacijski paraboloid:

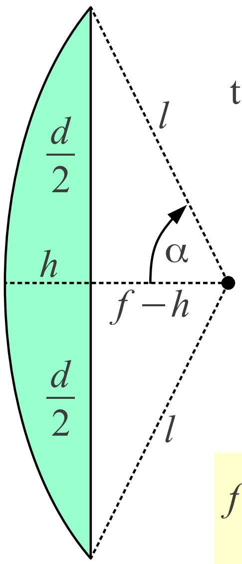
$$z(x,y) = \frac{x^2 + y^2}{4f}$$



Izrezi zrcala



Izvedbe zrcalnih anten



$$\tan\alpha=\frac{\frac{d}{2}}{f-h}=\frac{\frac{d}{2}}{f-\frac{d^2}{16f}}=\frac{2\cdot\frac{d}{4f}}{1-\left(\frac{d}{4f}\right)^2}$$

$$h=\frac{d^2}{16f}$$

$$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\left(\tan\frac{\alpha}{2}\right)^2}$$

$$\tan\frac{\alpha}{2}=\frac{d}{4f}$$

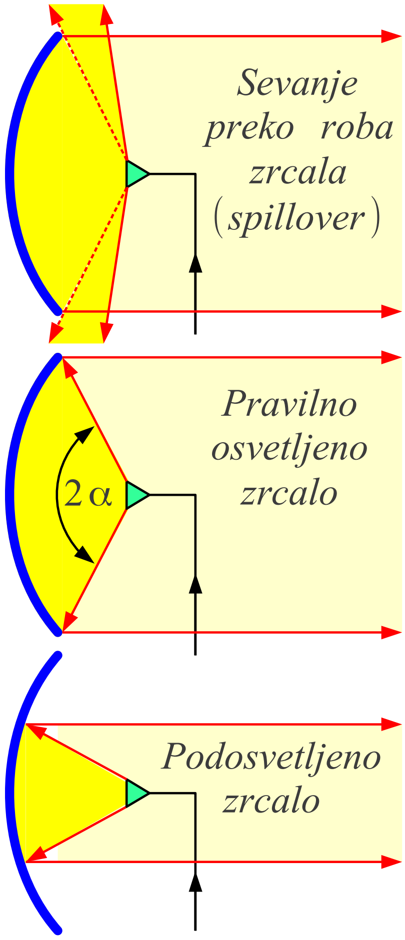
$$f/d=\frac{1}{4\tan\frac{\alpha}{2}}$$

$$\alpha=2\arctan\frac{1}{4(f/d)}$$

$$l=\sqrt{(d/2)^2+(f-h)^2}=f\left[1+\left(\frac{1}{4(f/d)}\right)^2\right]$$

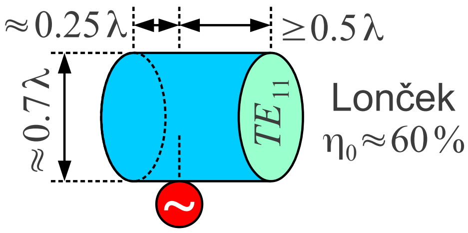
Zgled $f/d=0.4 \rightarrow \alpha\approx 1.12\text{rd}\approx 64^\circ \quad 2\alpha\approx 128^\circ$

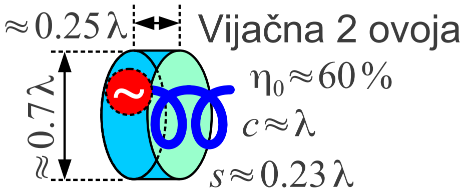
$l\approx 1.39f \rightarrow 20\log_{10}(l/f)\approx 2.86\text{dB}$

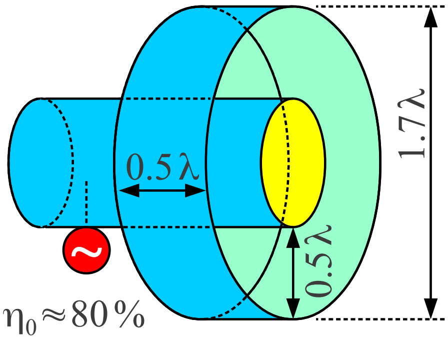


Osvetlitev zrcala

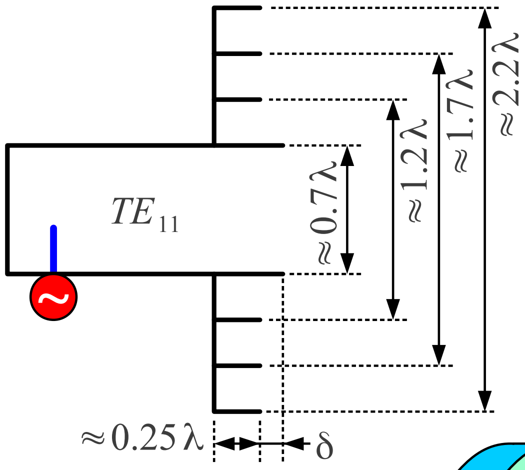
Žarilci za globoka zrcala

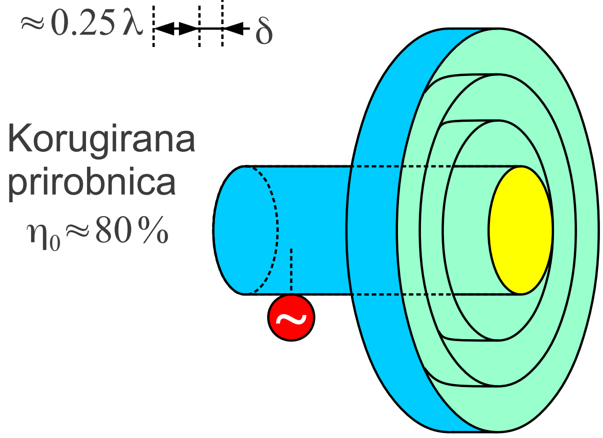


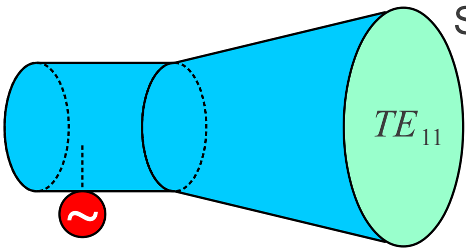


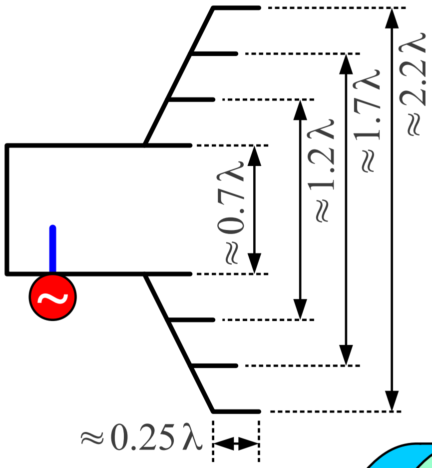


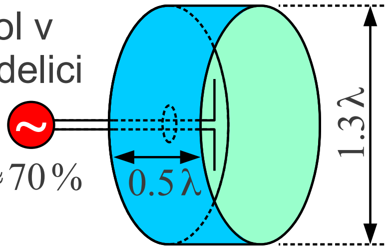
Ovratnik Kumar/VE4MA

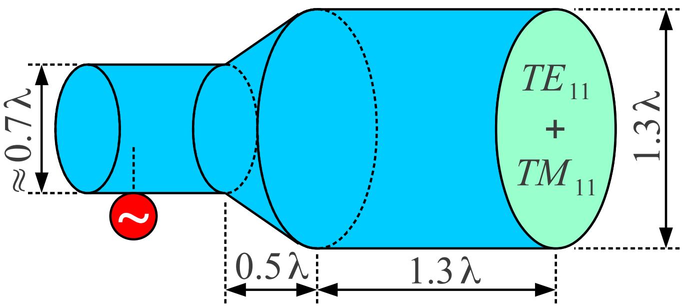


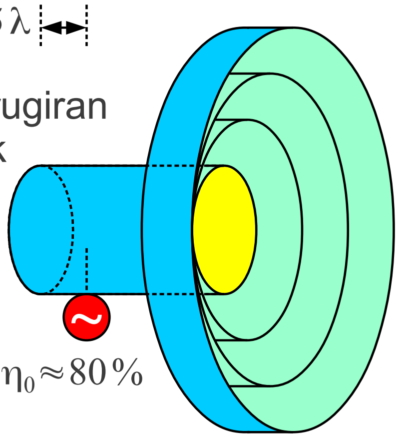


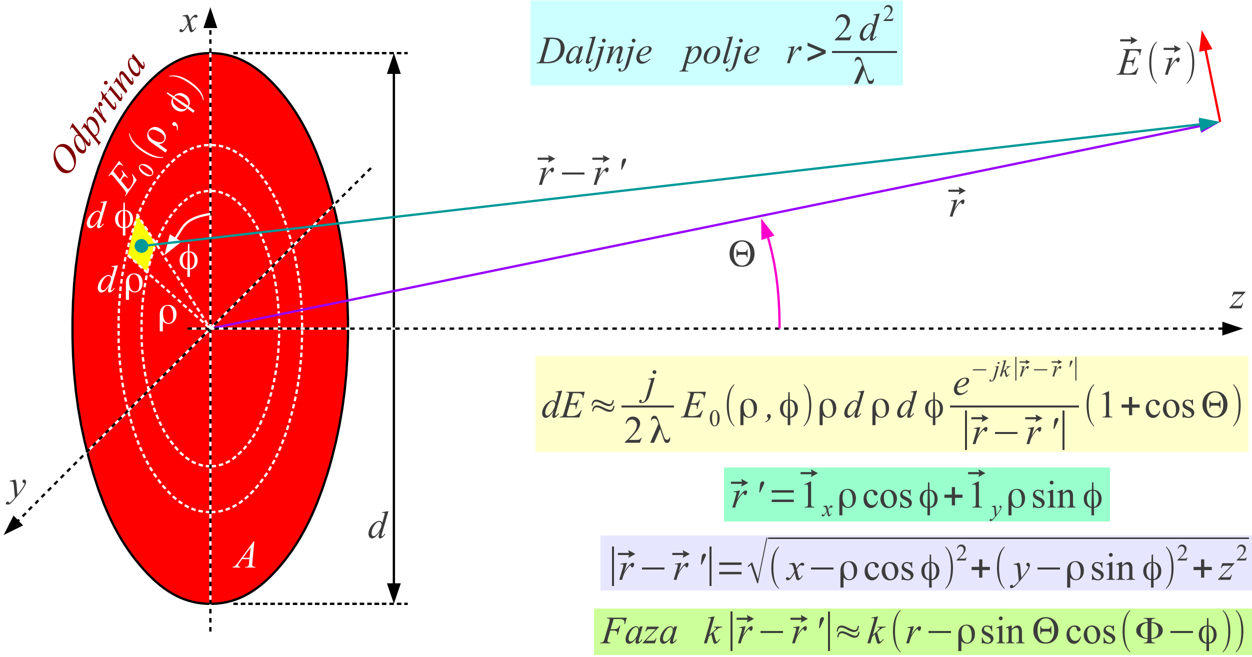

Stožčasti
lijak
$\eta_0 \approx 60\%$
TE_{11}


Žarilci za plitva zrcala
≈2.2λ
≈1.7λ
≈1.2λ
≈0.7λ
≈0.25λ


Dipol v
skodelici
$\eta_0 \approx 70\%$
0.5λ
1.3λ


≈0.7λ
TE_{11}
+
TM_{11}
1.3λ
0.5λ
1.3λ
Dvorodovni žarilec
$\eta_0 \approx 80\%$


Korugiran
lijak
$\eta_0 \approx 80\%$



$$Daljnje\ \ polje\ \ r>\frac{2d^2}{\lambda}$$

$$dE\approx\frac{j}{2\lambda}E_0(\rho,\phi)\rho\,d\rho\,d\phi\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}(1+\cos\Theta)$$

$$\vec{r}\,'=\vec{1}_x\rho\cos\phi+\vec{1}_y\rho\sin\phi$$

$$|\vec{r}-\vec{r}\,'|=\sqrt{(x-\rho\cos\phi)^2+(y-\rho\sin\phi)^2+z^2}$$

$$Faza\ \ k|\vec{r}-\vec{r}\,'|\approx k(r-\rho\sin\Theta\cos(\Phi-\phi))$$

$$dE\approx\frac{j}{2\lambda}E_0(\rho,\phi)\rho\,d\rho\,d\phi\frac{e^{-jkr}}{r}(1+\cos\Theta)e^{jk\rho\sin\Theta\cos(\Phi-\phi)}$$

$$E=\iint_A dE\approx\frac{j}{2\lambda}\frac{e^{-jkr}}{r}(1+\cos\Theta)\int_0^{d/2}\int_0^{2\pi}E_0(\rho,\phi)e^{jk\rho\sin\Theta\cos(\Phi-\phi)}\rho\,d\rho\,d\phi$$

Sevanje krožne odprtine

$$E_0(\rho,\phi)=konst.=C$$

$$J_n(t)=\sum_{m=0}^{\infty}\frac{(-1)^m t^{2m+n}}{2^{2m+n}m!(m+n)!}$$

$$E=\frac{j}{2\lambda}C\frac{e^{-jkr}}{r}(1+\cos\Theta)\int_0^{d/2}\int_0^{2\pi}e^{jk\rho\sin\Theta\cos(\Phi-\phi)}\rho\, d\rho\, d\phi$$

$$\int_0^{2\pi}e^{jt\cos\phi}d\phi=2\pi J_0(t)$$

$$E=\frac{j}{\lambda}C\frac{e^{-jkr}}{r}\frac{1+\cos\Theta}{2}\int_0^{d/2}2\pi J_0(k\rho\sin\Theta)\rho\, d\rho$$

$$\frac{d}{dt}\left[t^n J_n(t)\right]=t^n J_{n-1}(t)$$

$$\int_0^{d/2}2\pi J_0(k\rho\sin\Theta)\rho\, d\rho=2\pi\left[\rho\frac{J_1(k\rho\sin\Theta)}{k\sin\Theta}\right]_0^{d/2}=\pi\left(\frac{d}{2}\right)^2\frac{2J_1\left(\frac{kd}{2}\sin\Theta\right)}{\frac{kd}{2}\sin\Theta}$$

$$A=\pi\left(\frac{d}{2}\right)^2$$

$$\frac{kd}{2}=\frac{\pi d}{\lambda}$$

$$F(\Theta,\Phi)=\frac{1+\cos\Theta}{2}\frac{2J_1\left(\frac{\pi d}{\lambda}\sin\Theta\right)}{\frac{\pi d}{\lambda}\sin\Theta}$$

$$E=\frac{j}{\lambda}C\,A\frac{e^{-jkr}}{r}\frac{1+\cos\Theta}{2}\frac{2J_1\left(\frac{kd}{2}\sin\Theta\right)}{\frac{kd}{2}\sin\Theta}$$

$$J_1(t)=0\ \rightarrow\ t\approx 3.8318$$

$$\alpha_{-3\mathrm{dB}}\approx\Theta_0\approx\arcsin\Theta_0\approx\frac{3.8318\lambda}{\pi d}\approx 1.22\frac{\lambda}{d}$$

Enakomerno osvetljen krog

$$E_0(\rho,\phi)=C\,e^{-j\Delta\phi(2\rho/d)^2}$$

$$\int_0^{2\pi} e^{jt\cos\phi}\,d\phi=2\pi J_0(t)$$

$$E=\frac{j}{2\lambda}\frac{e^{-jkr}}{r}(1+\cos\Theta)\int_0^{d/2}\int_0^{2\pi}C\,e^{-j\Delta\phi(2\rho/d)^2}e^{jk\rho\sin\Theta\cos(\Phi-\phi)}\rho\,d\rho\,d\phi$$

$$E=\frac{j}{\lambda}\frac{e^{-jkr}}{r}\frac{1+\cos\Theta}{2}\int_0^{d/2}C\,e^{-j\Delta\phi(2\rho/d)^2}2\pi J_0(k\rho\sin\Theta)\rho\,d\rho$$

$$A=\pi\left(\frac{d}{2}\right)^2$$

$$E=\frac{j}{\lambda}C\,A\frac{e^{-jkr}}{r}\frac{1+\cos\Theta}{2}\left[\frac{8}{d^2}\int_0^{d/2}e^{-j\Delta\phi(2\rho/d)^2}J_0(k\rho\sin\Theta)\rho\,d\rho\right]$$

$$F(\Theta,\Phi)=\frac{1+\cos\Theta}{2}\left[\frac{8}{d^2}\int_0^{d/2}e^{-j\Delta\phi(2\rho/d)^2}J_0(k\rho\sin\Theta)\rho\,d\rho\right]$$

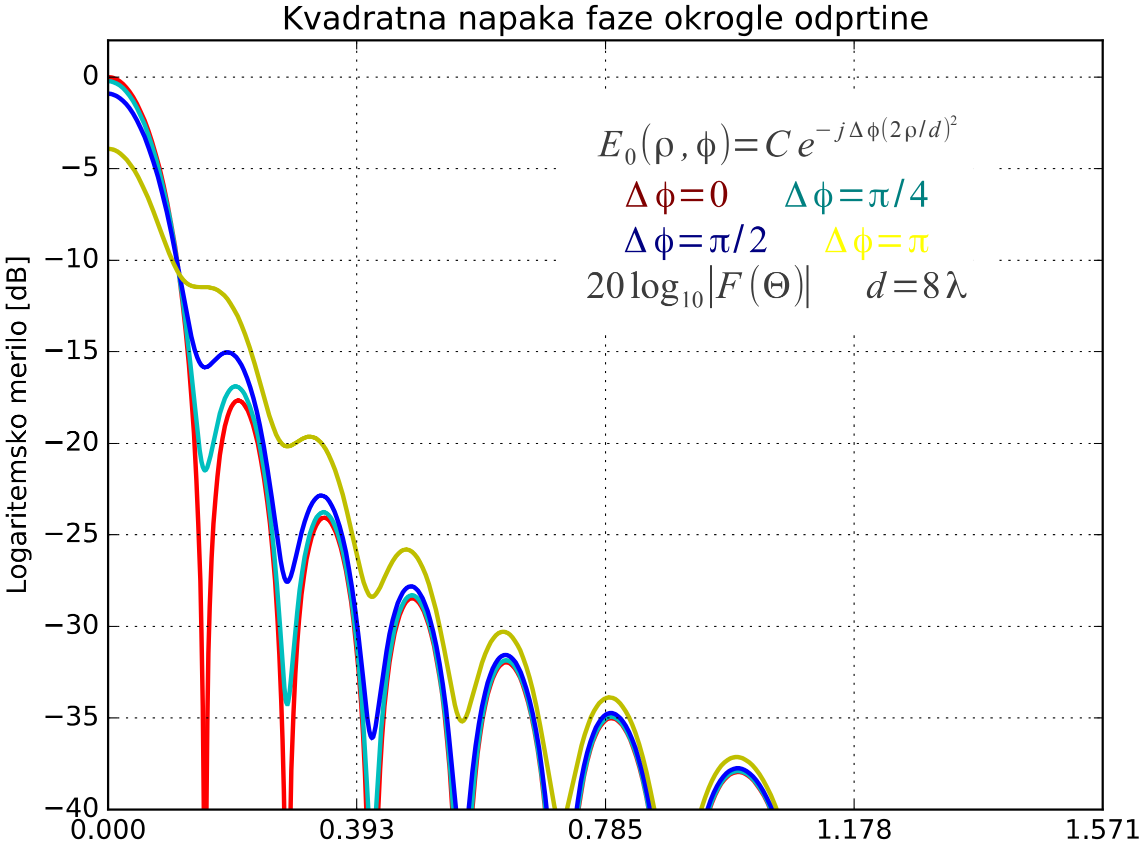
$$\Theta=0\ \rightarrow\ \cos\Theta=1\qquad J_0(k\rho\sin\Theta)=1$$

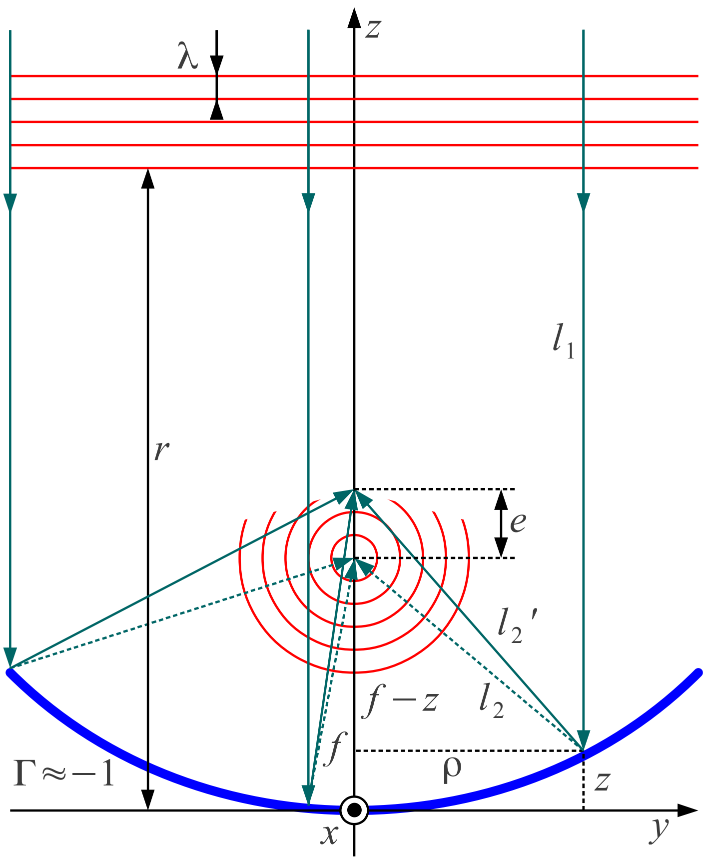
$$u=(2\rho/d)^2$$

$$F(\Theta=0)=\frac{8}{d^2}\int_0^{d/2}e^{-j\Delta\phi(2\rho/d)^2}\rho\,d\rho=\int_0^1 e^{-j\Delta\phi u}du=\frac{e^{-j\Delta\phi}-1}{-j\Delta\phi}=e^{-j\Delta\phi/2}\frac{\sin(\Delta\phi/2)}{\Delta\phi/2}$$

Kvadratna napaka faze

$$\Delta D_{[\mathrm{dB}]}=20\log_{10}|F(\Theta=0)|=20\log_{10}\left|\frac{\sin(\Delta\phi/2)}{\Delta\phi/2}\right|$$


Kvadratna napaka faze okrogle odprtine
$E_0(\rho,\phi)=C\,e^{-j\Delta\phi(2\rho/d)^2}$
$\Delta\phi=0$
$\Delta\phi=\pi/4$
$\Delta\phi=\pi/2$
$\Delta\phi=\pi$
$20\log_{10}|F(\Theta)|$
$d=8\lambda$
Logaritemsko merilo [dB]
0
−5
−10
−15
−20
−25
−30
−35
−40
0.000
0.393
0.785
1.178
1.571



Osni odmik žarilca

$$l_2=\sqrt{(f-z)^2+\rho^2} \qquad \rho=\sqrt{x^2+y^2}$$

$$l_2'=\sqrt{(e+f-z)^2+\rho^2} \qquad l_1=r-z$$

$$l_1+l_2'=r-z+\sqrt{(e+f-z)^2+\rho^2}=$$
$$=r-z+\sqrt{(f+z)^2+e^2+2ef-2ez}$$

$e\ll f$

$$l_1+l_2'\approx r+f+\frac{(e^2+2ef-2ez)}{2(f+z)}$$

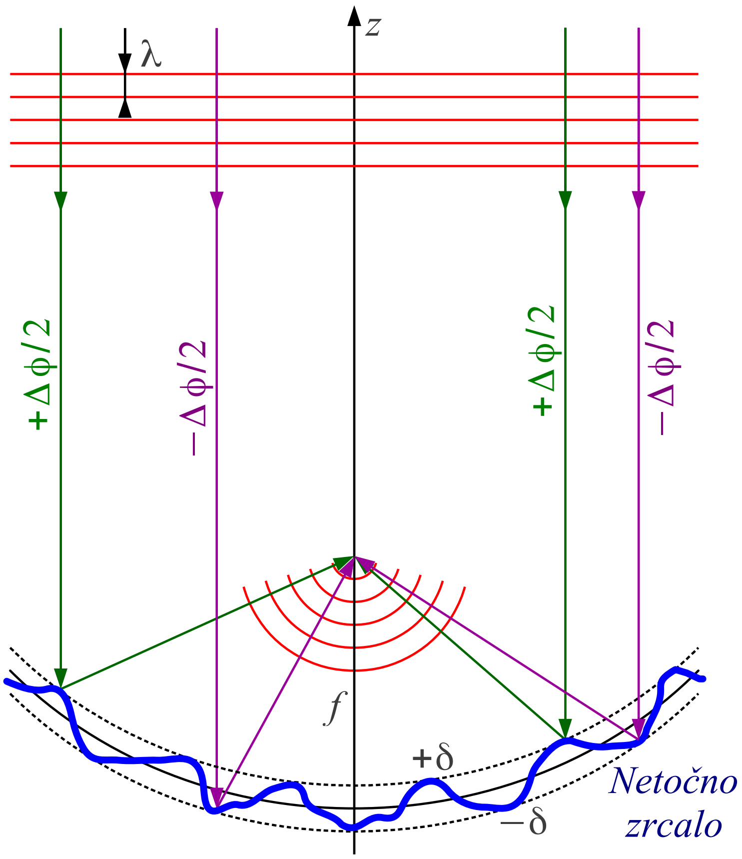
$$\Delta l\approx e\left[\frac{f-z}{f+z}-1\right]=e\frac{-2z}{f+z}$$

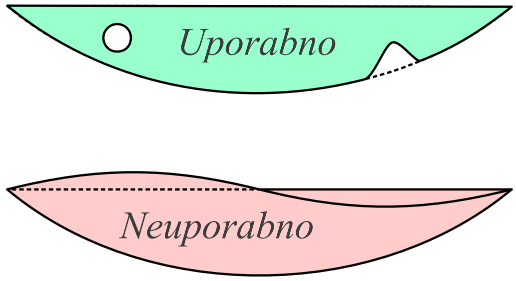
$$\Delta l\approx e\frac{-8\rho^2}{4f^2+\rho^2} \qquad z=\frac{\rho^2}{4f}$$

$$\Delta\phi=k\,\Delta l\approx ke\frac{-8\rho^2}{4f^2+\rho^2}$$

Ob osi zrcala

$\rho\ll f \;\rightarrow\; \Delta\phi\approx-2ke(\rho/f)^2$





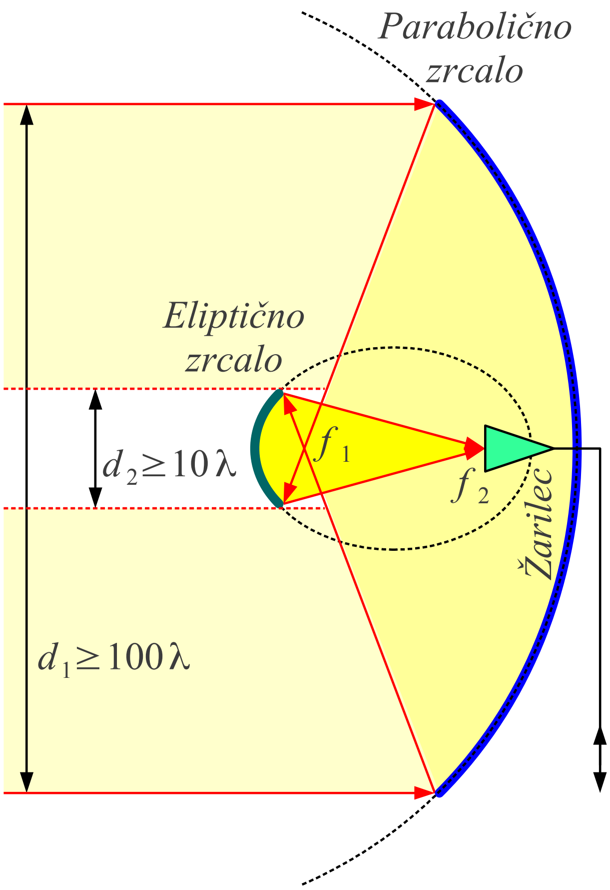
$\Delta\phi \approx 4k\delta$

$$\Delta D_{\mathrm{dB}} \approx 20\log_{10}\left|\frac{\sin(\Delta\phi/2)}{\Delta\phi/2}\right|$$

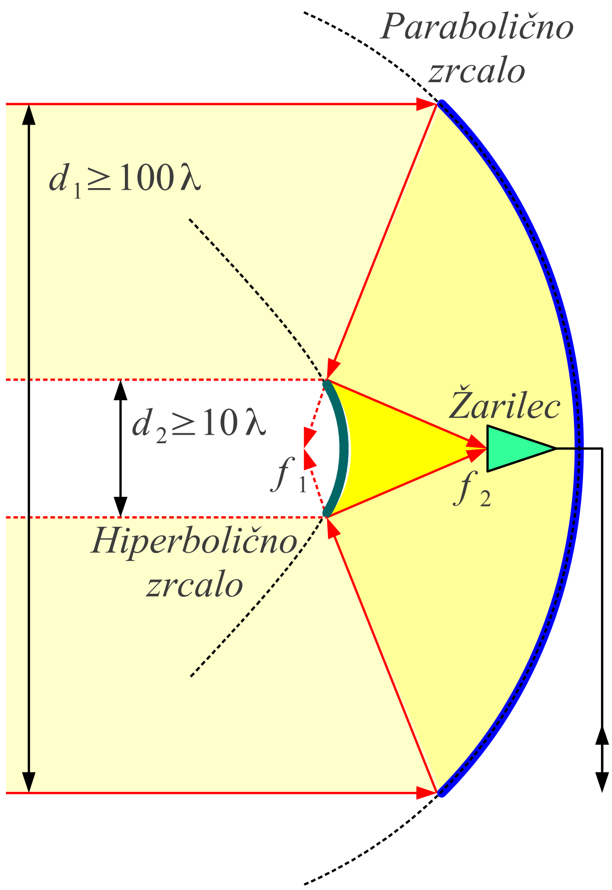
$\pm\delta$	$\Delta\phi$	$\Delta D[\mathrm{dB}]$
$\pm\lambda/4$	2π	$-\infty$
$\pm\lambda/8$	π	-3.922
$\pm\lambda/16$	$\pi/2$	-0.912
$\pm\lambda/32$	$\pi/4$	-0.224

Napake oblike zrcala

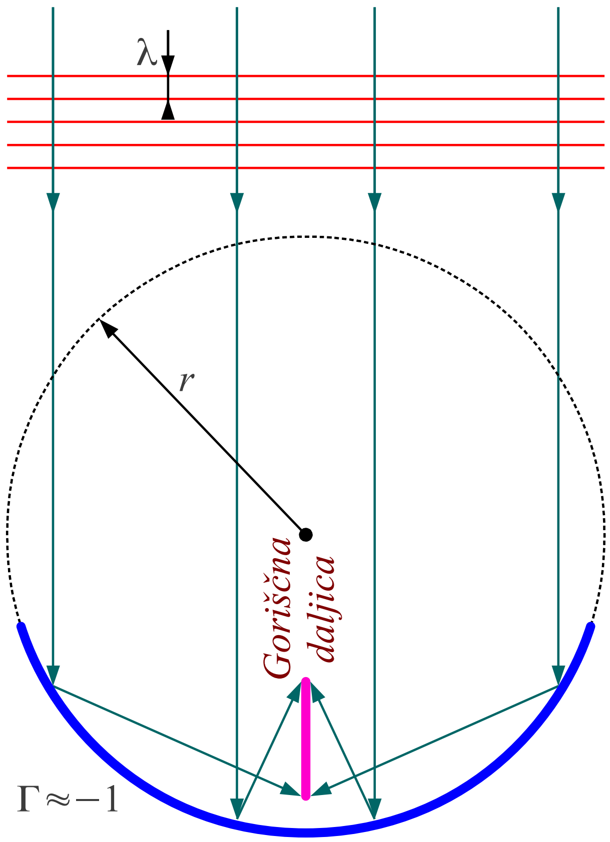
Zgled: $f=12\mathrm{GHz}$ $\lambda=25\mathrm{mm}$ $\pm\delta=\pm0.8\mathrm{mm}$

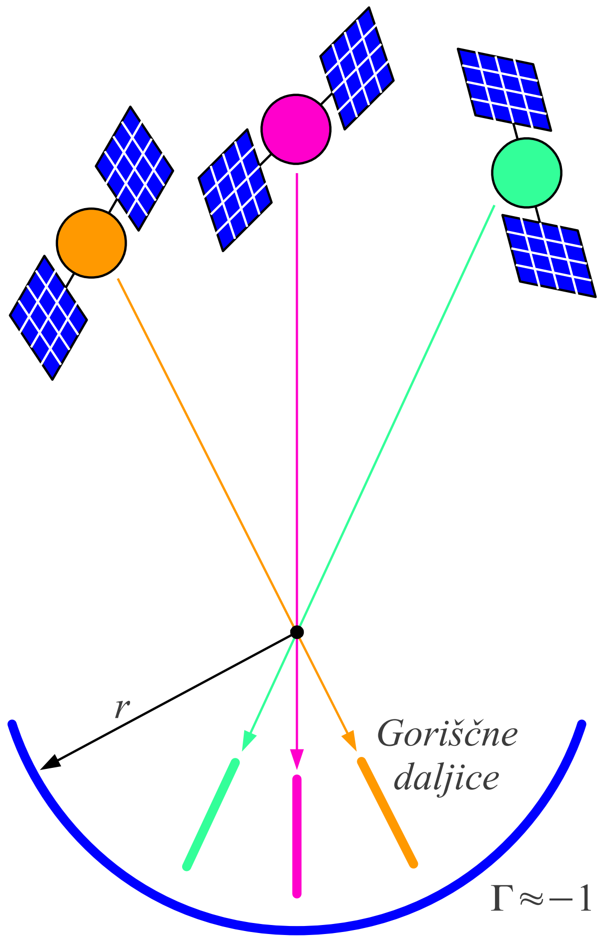


Gregorijanska antena



Cassegrainova antena





Krogelno zrcalo

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