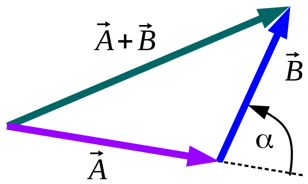
6. Vektorji in koordinate

Kako opisati elektrodinamiko na čimbolj preprost, ampak uporaben način, ki daje zadovoljivo natančnost rezultatov? Naloge z eno veliko izmero, to se pravi eno-dimenzijske naloge lahko opišemo s porazdeljenimi gradniki, torej z vezji z neskončnim številom gradnikov. Kljub izhodišču iz preprostih osnov elektrotehnike in izogibanju relativistiki, rešitve nalog takoj pokažejo na ključno veličino, to je hitrost svetlobe.

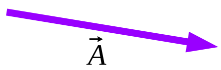
Tri-dimenzijske naloge potrebujejo zahtevnejši pristop. Elektrotehnikom so Maxwell-ove enačbe vsekakor preprostejše za razumevanje od zahtevne relativistike. Maxwell-ove enačbe je treba pretvoriti v diferencialno obliko, da postane naloga zadosti majhna, torej diferencialno majhna. V diferencialno majhni nalogi so zakasnitve diferencialno majhne, torej relativistika ne nagaja. Tu žal brez zahtevne matematike, diferencialne geometrije v različnih tri-dimenzijskih koordinatnih sistemih v prostoru ne gre.

Seštevanje vektorjev



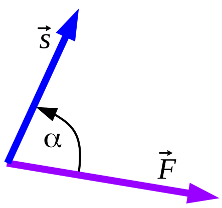
$$|\vec{A}+\vec{B}|=\sqrt{|\vec{A}|^2+|\vec{B}|^2+2|\vec{A}||\vec{B}|\cos\alpha}$$

Velikost vektorja



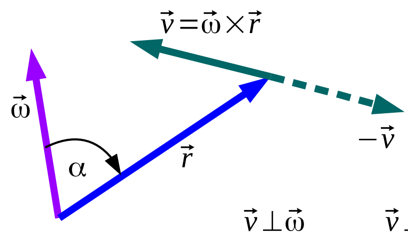
$$|\vec{A}|=\sqrt{\vec{A}\cdot\vec{A}}$$

Skalarni produkt



$$A=\vec{F}\cdot\vec{s}=\vec{s}\cdot\vec{F}=|\vec{F}||\vec{s}|\cos\alpha$$

Vektorski produkt

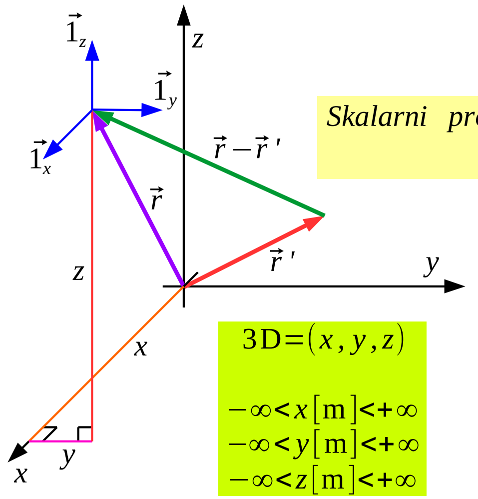


$\vec{v}\perp\vec{\omega}$ $\qquad$ $\vec{v}\perp\vec{r}$

$$|\vec{v}|=|\vec{\omega}\times\vec{r}|=|\vec{\omega}||\vec{r}|\sin\alpha$$

$\vec{r}\times\vec{\omega}=-\vec{v}=-\vec{\omega}\times\vec{r}$ Desni vijak!

Kartezične koordinate



Komponente $\vec{A}=(A_x,A_y,A_z)=\vec{1}_x A_x+\vec{1}_y A_y+\vec{1}_z A_z$

$A_x=\vec{1}_x\cdot\vec{A}\quad A_y=\vec{1}_y\cdot\vec{A}\quad A_z=\vec{1}_z\cdot\vec{A}$

Skalarni produkt $\vec{A}\cdot\vec{B}=(\vec{1}_x A_x+\vec{1}_y A_y+\vec{1}_z A_z)\cdot(\vec{1}_x B_x+\vec{1}_y B_y+\vec{1}_z B_z)$

$\vec{A}\cdot\vec{B}=A_x B_x+A_y B_y+A_z B_z$

Velikost $|\vec{A}|=\sqrt{\vec{A}\cdot\vec{A}}=\sqrt{A_x^1+A_y^2+A_z^2}$

$|\vec{r}|=r=\sqrt{x^2+y^2+z^2}$

$3\mathrm{D}=(x,y,z)$

$-\infty<x[\mathrm{m}]<+\infty$
$-\infty<y[\mathrm{m}]<+\infty$
$-\infty<z[\mathrm{m}]<+\infty$

Razdalja $|\vec{r}-\vec{r}\,'|=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$

Vektorski produkt

$\vec{A}\times\vec{B}=(\vec{1}_x A_x+\vec{1}_y A_y+\vec{1}_z A_z)\times(\vec{1}_x B_x+\vec{1}_y B_y+\vec{1}_z B_z)$

$\vec{A}\times\vec{B}=\vec{1}_x(A_y B_z-A_z B_y)+$
$+\vec{1}_y(A_z B_x-A_x B_z)+\vec{1}_z(A_x B_y-A_y B_x)$

Enotni vektorji $1=\vec{1}_x\cdot\vec{1}_x=\vec{1}_y\cdot\vec{1}_y=\vec{1}_z\cdot\vec{1}_z$

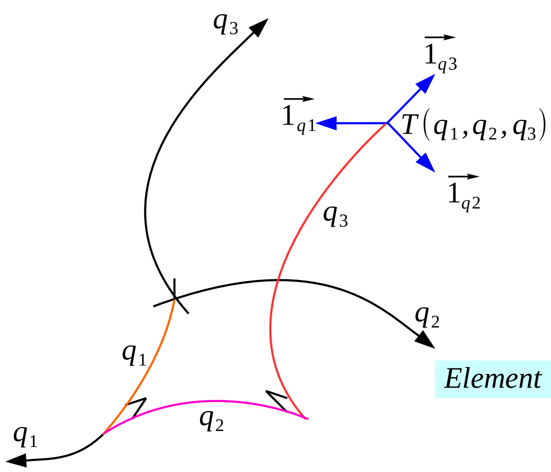
Pravokotni $\vec{1}_x\perp\vec{1}_y\perp\vec{1}_z\perp\vec{1}_x\quad 0=\vec{1}_x\cdot\vec{1}_y=\vec{1}_y\cdot\vec{1}_z=\vec{1}_z\cdot\vec{1}_x$

Desnoročni $\vec{1}_z=\vec{1}_x\times\vec{1}_y\quad \vec{1}_x=\vec{1}_y\times\vec{1}_z\quad \vec{1}_y=\vec{1}_z\times\vec{1}_x$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Krivočrtne koordinate

Element dolžine $dl=\sqrt{dx^2+dy^2+dz^2}$



Povezava s kartezičnimi
$x=x(q_1,q_2,q_3)$
$y=y(q_1,q_2,q_3)$
$z=z(q_1,q_2,q_3)$

$$dl_1=\left(\sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2+\left(\frac{\partial y}{\partial q_1}\right)^2+\left(\frac{\partial z}{\partial q_1}\right)^2}\right)dq_1=h_1\,dq_1$$

Element površine $dA=dl_1 dl_2=h_1 h_2 dq_1 dq_2$

Element prostornine $dV=dl_1 dl_2 dl_3=h_1 h_2 h_3 dq_1 dq_2 dq_3$

$3\mathrm{D}=(q_1,q_2,q_3)$

$$h_1=\sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2+\left(\frac{\partial y}{\partial q_1}\right)^2+\left(\frac{\partial z}{\partial q_1}\right)^2}$$

$$h_2=\sqrt{\left(\frac{\partial x}{\partial q_2}\right)^2+\left(\frac{\partial y}{\partial q_2}\right)^2+\left(\frac{\partial z}{\partial q_2}\right)^2}$$

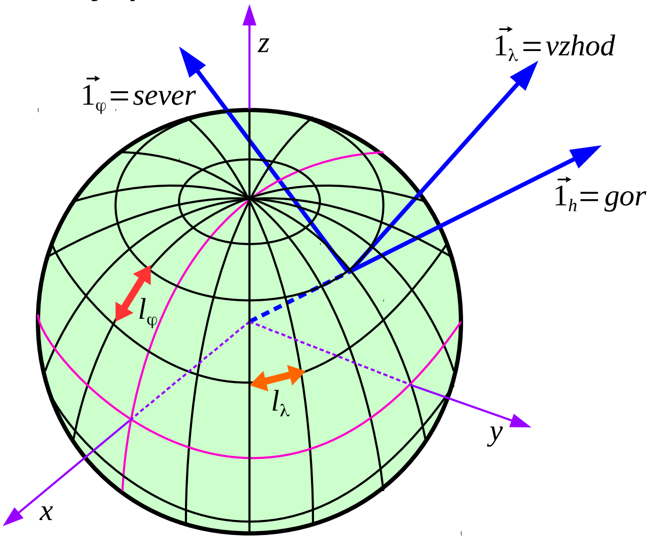
$$h_3=\sqrt{\left(\frac{\partial x}{\partial q_3}\right)^2+\left(\frac{\partial y}{\partial q_3}\right)^2+\left(\frac{\partial z}{\partial q_3}\right)^2}$$

Enotni vektorji $1=\vec{1}_{q1}\cdot\vec{1}_{q1}=\vec{1}_{q2}\cdot\vec{1}_{q2}=\vec{1}_{q3}\cdot\vec{1}_{q3}$

Pravokotni $\vec{1}_{q1}\perp\vec{1}_{q2}\perp\vec{1}_{q3}\perp\vec{1}_{q1}\quad 0=\vec{1}_{q1}\cdot\vec{1}_{q2}=\vec{1}_{q2}\cdot\vec{1}_{q3}=\vec{1}_{q3}\cdot\vec{1}_{q1}$

Desnoročni $\vec{1}_{q3}=\vec{1}_{q1}\times\vec{1}_{q2}\quad \vec{1}_{q1}=\vec{1}_{q2}\times\vec{1}_{q3}\quad \vec{1}_{q2}=\vec{1}_{q3}\times\vec{1}_{q1}$

Zemljepisne koordinate



$3D = (\lambda, \varphi, h)$

$0° \le \lambda[°] < 360°$
$-90° \le \varphi[°] \le 90°$
$-R_Z \le h[\mathrm{km}] < +\infty$

Pravokotni $\vec{1}_\lambda \perp \vec{1}_\varphi \perp \vec{1}_h \perp \vec{1}_\lambda$

Desnoročni $\vec{1}_h = \vec{1}_\lambda \times \vec{1}_\varphi$

Pretvorba $(\lambda, \varphi, h) \rightarrow (x, y, z)$

$$x = (h + R_Z)\cos\left(\frac{\pi}{180°}\lambda\right)\cos\left(\frac{\pi}{180°}\varphi\right)$$
$$y = (h + R_Z)\sin\left(\frac{\pi}{180°}\lambda\right)\cos\left(\frac{\pi}{180°}\varphi\right)$$
$$z = (h + R_Z)\sin\left(\frac{\pi}{180°}\varphi\right)$$
$$R_Z = 6378\ \mathrm{km}$$

Laméjevi koeficienti

$$h_\lambda = \frac{\pi(h + R_Z)}{180°}\cos\left(\frac{\pi}{180°}\varphi\right)$$
$$h_\varphi = \frac{\pi(h + R_Z)}{180°}$$
$$h_h = 1$$

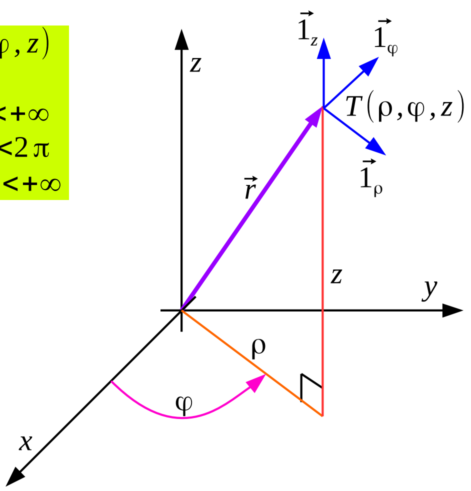
Zgled: $\lambda = 30°$ $\varphi = 45°$ $h = 0\ \mathrm{km}$

$$h_\lambda = 78.7\ \mathrm{km}/°$$
$$h_\varphi = 111.3\ \mathrm{km}/°$$
$$h_h = 1$$
$$\Delta\lambda = 20° \rightarrow l_\lambda = h_\lambda \Delta\lambda = 1574\ \mathrm{km}$$
$$\Delta\varphi = 20° \rightarrow l_\varphi = h_\varphi \Delta\varphi = 2226\ \mathrm{km}$$

Valjne koordinate

$3D = (\rho, \varphi, z)$

$0 \le \rho[\mathrm{m}] < +\infty$
$0 \le \varphi[\mathrm{rd}] < 2\pi$
$-\infty < z[\mathrm{m}] < +\infty$



Enotni vektorji $1 = \vec{1}_\rho \cdot \vec{1}_\rho = \vec{1}_\varphi \cdot \vec{1}_\varphi = \vec{1}_z \cdot \vec{1}_z$

Pravokotni $\vec{1}_\rho \perp \vec{1}_\varphi \perp \vec{1}_z \perp \vec{1}_\rho$ $0 = \vec{1}_\rho \cdot \vec{1}_\varphi = \vec{1}_\varphi \cdot \vec{1}_z = \vec{1}_z \cdot \vec{1}_\rho$

Desnoročni $\vec{1}_z = \vec{1}_\rho \times \vec{1}_\varphi$ $\vec{1}_\rho = \vec{1}_\varphi \times \vec{1}_z$ $\vec{1}_\varphi = \vec{1}_z \times \vec{1}_\rho$

Pretvorba $(\rho, \varphi, z) \rightarrow (x, y, z)$

$$x = \rho\cos\varphi$$
$$y = \rho\sin\varphi$$
$$z = z$$
$$\vec{1}_x = \vec{1}_\rho\cos\varphi - \vec{1}_\varphi\sin\varphi$$
$$\vec{1}_y = \vec{1}_\rho\sin\varphi + \vec{1}_\varphi\cos\varphi$$
$$\vec{1}_z = \vec{1}_z$$

Laméjevi koeficienti

$$h_\rho = 1$$
$$h_\varphi = \rho$$
$$h_z = 1$$

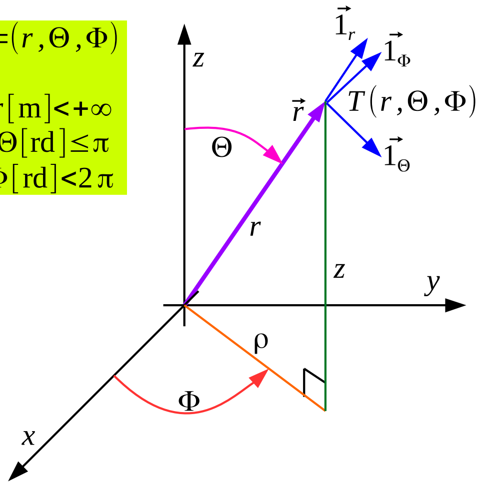
Pretvorba $(x, y, z) \rightarrow (\rho, \varphi, z)$

$$\rho = \sqrt{x^2 + y^2}$$
$$\varphi = \arctan(y/x) \quad (\textit{kvadrant ?})$$
$$z = z$$
$$\vec{1}_\rho = \vec{1}_x\cos\varphi + \vec{1}_y\sin\varphi$$
$$\vec{1}_\varphi = -\vec{1}_x\sin\varphi + \vec{1}_y\cos\varphi$$
$$\vec{1}_z = \vec{1}_z$$

Krogelne koordinate

$3\text{D}=(r,\Theta,\Phi)$

$0\leq r[\text{m}]<+\infty$
$0\leq\Theta[\text{rd}]\leq\pi$
$0<\Phi[\text{rd}]<2\pi$



Pretvorba $(r,\Theta,\Phi)\rightarrow(x,y,z)$

$$x=r\sin\Theta\cos\Phi$$
$$y=r\sin\Theta\sin\Phi$$
$$z=r\cos\Theta$$
$$\vec{1}_x=\vec{1}_r\sin\Theta\cos\Phi+\vec{1}_\Theta\cos\Theta\cos\Phi-\vec{1}_\Phi\sin\Phi$$
$$\vec{1}_y=\vec{1}_r\sin\Theta\sin\Phi+\vec{1}_\Theta\cos\Theta\sin\Phi+\vec{1}_\Phi\cos\Phi$$
$$\vec{1}_z=\vec{1}_r\cos\Theta-\vec{1}_\Theta\sin\Theta$$

Pretvorba $(x,y,z)\rightarrow(r,\Theta,\Phi)$

$$r=\sqrt{x^2+y^2+z^2}$$
$$\Theta=\arccos\left(z/\sqrt{x^2+y^2+z^2}\right)$$
$$\Phi=\arctan(y/x)\quad(\textit{kvadrant ?})$$
$$\vec{1}_r=\vec{1}_x\sin\Theta\cos\Phi+\vec{1}_y\sin\Theta\sin\Phi+\vec{1}_z\cos\Theta$$
$$\vec{1}_\Theta=\vec{1}_x\cos\Theta\cos\Phi+\vec{1}_y\cos\Theta\sin\Phi-\vec{1}_z\cos\Theta$$
$$\vec{1}_\Phi=-\vec{1}_x\sin\Phi+\vec{1}_y\cos\Phi$$

Enotni vektorji $1=\vec{1}_r\cdot\vec{1}_r=\vec{1}_\Theta\cdot\vec{1}_\Theta=\vec{1}_\Phi\cdot\vec{1}_\Phi$

Pravokotni $\vec{1}_r\perp\vec{1}_\Theta\perp\vec{1}_\Phi\perp\vec{1}_r\quad 0=\vec{1}_r\cdot\vec{1}_\Theta=\vec{1}_\Theta\cdot\vec{1}_\Phi=\vec{1}_\Phi\cdot\vec{1}_r$

Desnoročni $\vec{1}_\Phi=\vec{1}_r\times\vec{1}_\Theta\quad\vec{1}_r=\vec{1}_\Theta\times\vec{1}_\Phi\quad\vec{1}_\Theta=\vec{1}_\Phi\times\vec{1}_r$

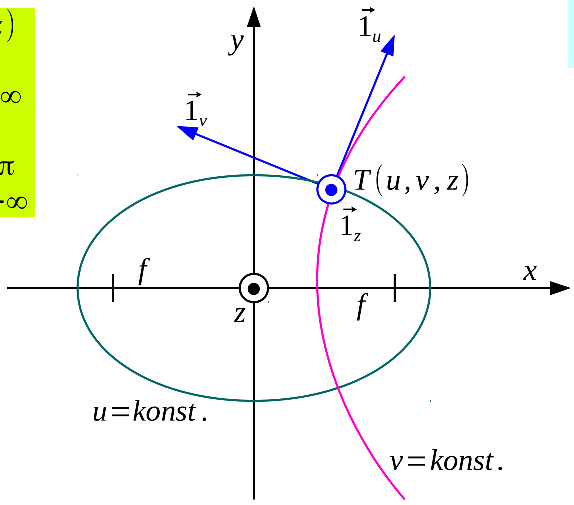
Laméjevi koeficienti

$$h_r=1$$
$$h_\Theta=r$$
$$h_\Phi=r\sin\Theta$$

Valjno-eliptične koordinate

$3\text{D}=(u,v,z)$

$0\leq u[\text{Np}]<+\infty$
$f[\text{m}]$
$0\leq v[\text{rd}]<2\pi$
$-\infty<z[\text{m}]<+\infty$



Pretvorba $(u,v,z)\rightarrow(x,y,z)$

$$x=f\cosh u\cos v$$
$$y=f\sinh u\sin v$$
$$z=z$$

Laméjevi koeficienti

$$h_u=f\sqrt{\sinh^2 u+\sin^2 v}$$
$$h_v=f\sqrt{\sinh^2 u+\sin^2 v}$$
$$h_z=1$$

Enotni vektorji $1=\vec{1}_u\cdot\vec{1}_u=\vec{1}_v\cdot\vec{1}_v=\vec{1}_z\cdot\vec{1}_z$

Pravokotni $\vec{1}_u\perp\vec{1}_v\perp\vec{1}_z\perp\vec{1}_u\quad 0=\vec{1}_u\cdot\vec{1}_v=\vec{1}_v\cdot\vec{1}_z=\vec{1}_z\cdot\vec{1}_u$

Desnoročni $\vec{1}_z=\vec{1}_u\times\vec{1}_v\quad\vec{1}_u=\vec{1}_v\times\vec{1}_z\quad\vec{1}_v=\vec{1}_z\times\vec{1}_u$

Približek $u\gg 1$

$(u,v,z)\rightarrow(\rho,\varphi,z)$

$$\rho\approx\frac{f}{2}e^u$$
$$\varphi\approx v$$
$$z=z$$

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