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HYBRID MODE ANALYSIS OF MICROSTRIP LINES BY THE METHOD OF LINES WITH PSEUDOSPECTRAL DISCRETIZATION

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ABSTRACT: *In this paper, the method of lines (MoLs), which has been proved to be very efficient for calculating the characteristics of one-dimensional and two-dimensional planar microwave structures, is extended to nonequidistant discretization with pseudospectral technique. This pseudospectral MoLs is developed by combining pseudospectral technique and the MoLs, so that its solution is not only analytical along line direction, but also maintains high accuracy in discrete direction. A pseudospectral-based discretization strategy, distribution of collocation nodes, global power series interpolation of differential quadrature, second-order difference matrix, and decoupling of coupled ordinary differential equations are discussed in detail. The convergence behaviors are provided for line discretization, based on both the pseudospectral strategy and the second-order finite difference (FD), respectively. The calculated results are compared with those published in the previous literature and our numerical results show that accuracy is improved with reduced computational effort for the analysis of microstrip lines.* © 2002 Wiley Periodicals, Inc. Microwave Opt Technol Lett 35: 224–227, 2002; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10563

Key words: *pseudospectral technique; method of lines; microstrip line; hybrid mode*

INTRODUCTION

The method of lines (MOLs) was developed by mathematicians in order to solve partial differential equations. The MOLs has certain similarities with the mode-matching technique and with the finite difference method. It differs from the latter in the fact that for a given system of partial differential equations, all but one of the independent variables are discretized, thereby transforming the original set of partial differential equations into a set of coupled ordinary equations (ODEs). This set of equations can be solved analytically in most cases of interest. Consequently, the computational burden, including CPU time and memory storage requirements, is considerably reduced in comparison with classical finite difference (FD) schemes. Therefore, this semi-analytical procedure saves a lot of computing time and thus finds a wide application [1–8]. However, this method increases the solution accuracy by removing the mesh dependence in the mesh line direction, the discrete error still exists in the other directions. Unfortunately, this error becomes main factor to determine the solution accuracy in method of lines. Hence the measurement is thus urgent to be taken to delete this bottleneck.

Pseudospectral methods, which can be seen as high-accuracy limits of FD methods with special non-equally-spaced grid distributions, provide a useful alternative to classic finite difference and finite element methods for the approximate solution of differential equations. Finite elements may sacrifice computational efficiency in exchange for great versatility at general boundaries. On the other extreme, spectral methods are often most effective in cases where the phenomenon under study occurs in the largely regular domains. Between them, pseudospectral methods are fortunately appropriate in a vast regime and can be adapted to most geometry arising in applications although less flexible than finite elements. Even when they employ high orders of approximation, the implementations for it still tend to be comparatively straightforward. Theoretical studies and numerical experience have confirmed that for problems with smooth solutions, pseudospectral methods converge much faster than classic finite difference or finite element methods [6–8]. In fact, pseudospectral method approximates functions and their derivatives by global arguments and with very smooth basis functions:

$$u(x) = \sum_{k=0}^{N} a_k \phi_k(x)$$

where the $\phi_k(x)$ are, for example, Chebeyshev polynomials or trigonometric functions. This approach has notable strengths as follows: 1. For the analytic functions, the approximating error typically decay (as N increases) at exponential rather than at (much slower) polynomial rates. 2. The method is virtually free of both dissipative and dispersive errors. 3. The approach is surprisingly powerful for many cases in which both solutions and variable coefficients are non-smooth or even discontinuous. 4. Especially in several space dimensions, the relatively coarse grids that suffice for most accuracy requirements allow very time- and memory-efficient calculations. But for irregular domains and certain boundary conditions, it still has some difficulties and inefficiencies [9]. In this paper, pseudospectral method is introduced as a tool in the process of discretization in method of lines. Unlike conventional FD method, which approximates derivatives of a function with local arguments such as $du(x)/dx = [u(x + h) - u(x - h)]/2h$ and is typically designed to be exact for polynomials of low order, pseudospectral method is global, high order polynomial approximation. Therefore, the solution accuracy for pseudospectral-based method of lines is highly improved [6–8]. Because only one dimension needs discretization for two-dimensional boundary values, the difficulties for pseudospectral method to treat irregular domain can also be partly avoided if it is combined with method of lines. The static problem and the eigenvalues of hollow metallic waveguide are ever analyzed [6–7] and the pseudospectral MoLs exhibits the excellent convergence speed. In this paper, it is exploited for the hybrid mode analysis of microstrip lines and our results demonstrate that the pseudospectral MoLs has much faster convergence speed than the conventional MoLs.

THEORETICAL ANALYSIS

For convenience, the method is demonstrated on the shielded microstrip line, as shown in Figure 1. Because of symmetry, only the half cross section is considered and the electromagnetic field can be described by two scalar potentials Π^e and Π^h, which have

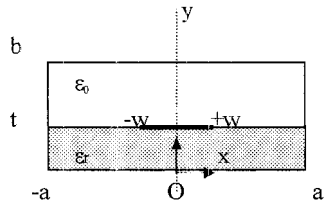

Figure 1 Cross section of a shielded microstrip line

to satisfy Helmholtz' equation and the corresponding boundary condition

$$\frac{\partial^2 \Pi^{e,h}}{\partial x^2} + \frac{\partial^2 \Pi^{e,h}}{\partial y^2} + (k^2 - \beta^2)\Pi^{e,h} = 0 \tag{1}$$

$$\left.\frac{\partial \Pi^e}{\partial x}\right|_{x=0} = 0 \quad \Pi^e|_{x=a} = 0 \quad \left.\frac{\partial \Pi^e}{\partial y}\right|_{y=0,b} = 0$$
$$\Pi^h|_{x=0} = 0 \quad \left.\frac{\partial \Pi^h}{\partial x}\right|_{x=a} = 0 \quad \Pi^h|_{y=0,b} = 0 \tag{2}$$

For the given N distinct nodes $0 = x_1 < x_2 < \cdots < x_{N-1} < x_N = a$ in the segment $[0, a]$ and the global method of differential quadrature, based on the high-order polynomial approximation, is first introduced. Following the idea of integral quadrature, it is assumed that any derivative at a grid point is approximated by a linear summation of all the functional values in the whole computational domain. For example, the first derivatives of $f(x)$ at a point x_i are approximated by

$$f'_x(x_i) = \sum_{j=1}^{N} b_{ij} f(x_j) \quad \text{for } i = 1, 2, \ldots, N \tag{3}$$

where N is the number of grid points, and b_{ij} are the weighting coefficients. To determine the weighting coefficients b_{ij}, pseudospectral method assume that $f(x)$ is approximated by a high-order polynomial,

$$f(x) = \sum_{k=1}^{N} c_k x^{k-1} \tag{4}$$

Then, under the analysis of a linear polynomial vector space, the following explicit formulation is used to compute b_{ij}:

$$b_{ij} = \frac{M^{(1)}(x_i)}{(x_i - x_j) M^{(1)}(x_j)} \quad \text{for } j \neq i \tag{5a}$$

$$b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij} \tag{5b}$$

where

$$M^{(1)}(x_k) = \prod_{j=1, j\neq k}^{N} (x_k - x_j) \tag{6}$$

In the above discussion, the sample points are arbitrary distribution in interval $[0, a]$. In the pseudospectral method, each dependent variable in the differential problem is approximated by a polynomial of finite degree. The discrete approximating equations are then obtained by setting residuals to zero at an appropriate set of collocation points in the solution domain. The proper choice of collocation points is crucial in terms of accuracy, stability, and ease of implementation of boundary conditions. Usually, the equispaced node distribution is adopted but the large deviation is observed near the two endpoints. The easiest way to offset these errors is to concentrate the nodes toward the ends of the interval. It is well known that the Chebyshev distribution has smallest error and minimum node spacing, which decreases as $O(1/N^2)$. The Chebyshev-distribution characteristic of minimum node spacing enhances the ability to treat the irregular domain for discrete technique of pseudospectral method. For a given problem the pair of boundary conditions is either Dirichlet–Neumann, or Neumann–Dirichlet. The difference matrix with the DN lateral boundary condition is denoted by $[a_{DN}]$ and can be obtained as follows:

$$a_{i,j}^{DN} = b_{i+1,j+1} - b_{i+1,N+1} \frac{b_{N+1,j+1}}{b_{N+1,N+1}}$$
$$i = 1, 2, \ldots, N-1, j = 1, 2, \ldots, N-1 \tag{7}$$

and the $[a_{ND}]$ is as follows:

$$a_{i,j}^{ND} = b_{i+1,j+1} - b_{i+1,1} \frac{b_{1,j+1}}{b_{1,1}}$$
$$i = 1, 2, \ldots, N-1, j = 1, 2, \ldots, N-1 \tag{8}$$

For the first derivative of Π^e with respect to the x-direction, one obtains:

$$\frac{d\bar{\Pi}^e}{dx} = [a_{ND}]\bar{\Pi}^e \tag{9}$$

where $\bar{\Pi}^e$ denotes the discretized Π^e. Since Π^e and Π^h have dual boundary conditions, the finite difference expression for the first derivative of Π^h becomes

$$\frac{d\bar{\Pi}^h}{dx} = [a_{DN}]\bar{\Pi}^h \tag{10}$$

Combining Eqs. (9) and (10), one obtains for the second-order derivatives

$$\frac{d^2\bar{\Pi}^e}{dx^2} = [a_{DN}]\frac{d\bar{\Pi}^e}{dx} = [a_{DN}][a_{ND}]\bar{\Pi}^e \tag{11}$$

$$\frac{d^2\bar{\Pi}^h}{dx^2} = [a_{ND}]\frac{d\bar{\Pi}^h}{dx} = [a_{ND}][a_{DN}]\bar{\Pi}^h \tag{12}$$

For a homogeneous layer, the second order pseudospectral difference operators $D_{xx}^{e,h}$ are the products of two different first order operators and their eigenvalues $-\lambda_{e,h}^2$ and the eigenvector matrices $T^{e,h}$ are defined as follows:

$$[D_{xx}^h][T^h] = [a_{ND}][a_{DN}][T^h] = -[T^h][\lambda_h^2] \tag{13}$$

$$[D_{xx}^e][T^e] = [a_{DN}][a_{ND}][T^e] = -[T^e][\lambda_e^2] \tag{14}$$

where $[T^{e,h}]$ are eigenvector matrices of the second-order pseudospectral difference matrices. Although $D_{xx}^{e,h}$ are not sym-

metric, for these real matrices there exist the real matrices $[T^{e,h}]$ such that

$$[T^{e,h}]^{-1}[D_{xx}^{e,h}][T^{e,h}] = -diag[d_{xxn}^2] \tag{15a}$$

$$[T^h]^{-1}[D^e x][T^e] = diag[d_{xxn}]$$

$$[T^e]^{-1}[D^h x][T^h] = -diag[d_{xxn}] \tag{15b}$$

where $[T^h]$ is directly solved from Eq. (13) and the $[T^e]$ is derived as follows [10]:

$$[T^e] = [a_{DN}][T^h][\lambda_h^{-1}]$$

A transformed potential vector $\tilde{U}$ is now introduced,

$$\tilde{U}^{e,h} = [T^{e,h}]^{-1}\bar{\Pi}^{e,h}, \tag{16}$$

so that one can write a system of N ordinary differential equations, which are now uncoupled:

$$\frac{d^2U_n^{e,h}}{dy^2} + (k^2 - \beta^2 - d_{xxn}^2)U_n^{e,h} = 0 \quad n = 1, 2, \ldots, N \tag{17}$$

The general solution to the above equation may be written as follows:

$$U_n^e(y) = \begin{cases} A_n^e \cos(\gamma_{1n}y), & y \in [0, t) \\ B_n^e \cos[\gamma_{2n}(b - y)] & y \in (t, b] \end{cases}$$

$$U_n^h(y) = \begin{cases} A_n^h \sin(\gamma_{1n}y), & y \in [0, t) \\ B_n^h \sin[\gamma_{2n}(b - y)] & y \in (t, b] \end{cases} \tag{18}$$

where

$$\gamma_{1n} = \sqrt{\varepsilon_r k_0^2 - \beta^2 - d_{xxn}^2} \quad \gamma_{2n} = \sqrt{k_0^2 - \beta^2 - d_{xxn}^2}$$

Matching the tangential fields in the interfaces can obtain the following equations [11]:

$$\begin{bmatrix} \tilde{Z}_{11} & \tilde{Z}_{12} \\ \tilde{Z}_{21} & \tilde{Z}_{22} \end{bmatrix} \begin{bmatrix} \tilde{J}_{xn} \\ \tilde{J}_{zn} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{xn} \\ \tilde{E}_{zn} \end{bmatrix} \tag{19}$$

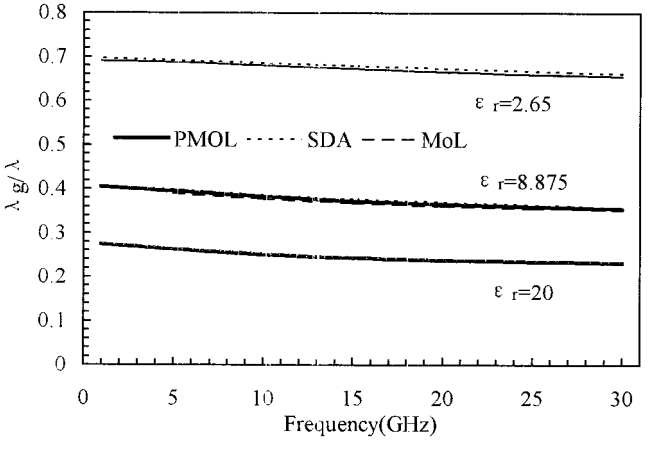

Figure 2 Normalized guide wavelength versus frequency

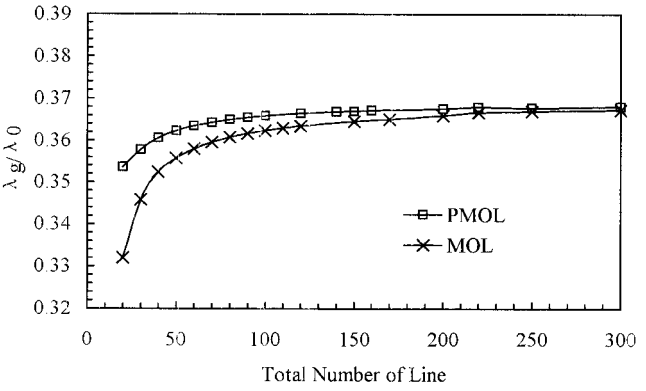

Figure 3 Convergence curves of the normalized guide wavelength for the shielded microstrip line

Since the final boundary condition cannot be applied in the transformed domain, the above equation has to be transformed back to the original domain. For this case, the metallic strip makes up the smaller part of the interface, so that the reverse transformation is performed only for the number of lines that pass through the strip. The following equation results:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} J_{xn} \\ J_{zn} \end{bmatrix} = \begin{bmatrix} E_{xn} \\ E_{zn} \end{bmatrix} = 0 \tag{20}$$

The impedance matrix is now a full matrix and it will have nontrivial solutions for the propagation constants of the structure only, which are found from the determinant equation

$$\det[Z(\beta)] = 0 \tag{21}$$

NUMERICAL RESULTS AND DISCUSSION

As shown in Figure 1, a shielded microstrip line is analyzed with pseudospectral method of lines. The parameters of the structure is as follows: $b - t = 12.7$ mm; $t = 1.27$ mm; $2a = 12.7$ mm; $2w = 1.27$ mm. As given in Figure 2, the normalized guide wavelength is calculated by the pseudospectral MoLs. A comparison is made between the results of the pseudospectral MoLs, conventional MoLs and those from the spectral domain approach [12]. It can be observed that there is a good agreement between them for the normalized guide wavelength. The convergence curves are also plotted in Figure 3 for the normalized guide wavelength of the shielded microstrip line. It can be founded that the pseudospectral MoLs has much faster convergence speed than the conventional MoLs. The new developed pseudospectral MOL adopts high-order interpolation polynomial to approximate the derivatives in controlling equation. As a result, it can approximate the smooth field inside of the domain of interest so that the solution not only is analytical along line direction but also maintains high accuracy in discrete direction. Although the second-order differential matrix for the pseudospectral MoLs is not sparse like that in conventional second-order MoLs, the eigenvalues, eigenvectors and transformation matrix can be numerically solved by matured algorithm and the computation cost is little because of fewer lines needed in the numerical simulation. This method is being extended to analyze three dimension harmonic electromagnetic field boundary value problems.

CONCLUSIONS

In this paper, the novel pseudospectral MoLs is introduced to solve time-harmonic electromagnetic problem and its basic

concept and theory is described in detail through the hybrid mode analysis of microstrip lines. Some numerical results are given to demonstrate the efficiency and accuracy of this method. It can be seen that the pseudospectral MoLs has a fast convergence speed, because it can achieve high accuracy in both analytical and discretized direction.

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ON THE DESIGN OF POLARIZATION-INSENSITIVE SEMICONDUCTOR OPTICAL AMPLIFIERS

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ABSTRACT: *We have used the 4 × 4 Luttinger–Kohn Hamiltonian to analyse the presence of delta-strain in quantum well on the polarization properties of semiconductor optical amplifiers. The analysis is performed for a 1.55 μm InGaAsP/InP lattice matched system grown in the [001] direction with and without the electrostatic effects of the carrier charges. The importance of electrostatic effects is indicated.* © 2002 Wiley Periodicals, Inc. Microwave Opt Technol Lett 35: 227–230, 2002; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10564

Key words: *semiconductor optical amplifiers; quantum wells; delta strain*

INTRODUCTION

In many applications of semiconductor optical amplifiers (SOA) polarization independence is required. The subject has been analysed by several groups [1, 2, 3, 4, 5, 6]. To achieve polarization insensitivity, different approaches have been proposed using combinations of tensile and/or compressive strain in well and/or barrier. It has also been shown that polarization-insensitive optical gain over a wide bandwidth can be realized in a coupled quantum wells [7].

Recently, a new method of achieving polarization insensitivity in MQW modulators was analysed [8], based on introducing a thin layer inside quantum well. An application of this idea to SOA has been published in [9].

In the present work we analyse, for the first time, combined effects of light and heavy hole-band mixing and electrostatic effects on optical modal gain in delta-strained quantum wells. In this analysis, the band-mixing effects are incorporated in the Luttinger–Kohn (LK) effective mass equation [10, 11, 12]. We use a parabolic model for the conduction band electrons and valence band holes in order to model electrostatic effects. For the gain calculations, we will use the LK model instead of the parabolic model for the valence band.

GAIN CALCULATIONS

The eigenfunctions for the parabolic band model are [13]:

$$\Psi_n^\alpha(\mathbf{k}, \mathbf{r}) = F_n^\alpha(z) \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{S}} u_\alpha(\mathbf{r}) \tag{1}$$

where $\mathbf{k}$, $\mathbf{x}$, and S are, respectively, the momentum, position, and area in the plane perpendicular to the wells, $\mathbf{r} = (\mathbf{x}, z)$. The symbol α denotes the conduction (c), heavy-hole (HH), and light-hole (LH) bands. $u_\alpha(\mathbf{r})$ are the Bloch functions for the various bands and $F_n^\alpha(z)$ are the normalized envelope functions in its nth subband which are the solutions of

$$\left[\frac{1}{2}\frac{d}{dz}\left[\frac{1}{m_\alpha^*(z)}\right]\frac{d}{dz} + V_\alpha(z)\right]F_n^\alpha(z) = E_n^\alpha F_n^\alpha(z) \tag{2}$$

$V_\alpha(z)$ and $m_\alpha^*(z)$ are the position dependent conduction band offset and electron effective mass. We assume in Eq. (2) and thereafter that the system of units is $\hbar = 1$. E_n^α is the subband edge energy. For uncoupled subbands, the energy dispersion is parabolic, so that $F_n^\alpha(z)$ has no $\mathbf{k}$ dependence.

For LK model of the valence band, the wavefunctions are

$$\Psi_n^\nu(\mathbf{k}, \mathbf{r}) = \sum_\alpha F_n^\nu(\mathbf{k}, z) \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{S}} \tag{3}$$

where $F_n^\nu(\mathbf{k}, z)$ are the eigenfunctions

$$[[H^{LK}] + [V]]F_n^\nu(\mathbf{k}, z) = E_n^\nu(\mathbf{k}) F_n^\nu(\mathbf{k}, z) \tag{4}$$