# NOISE IN RADIO/OPTICAL COMMUNICATIONS

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*Abstract*

Noise is a random signal that affects the performance of all electronic and/or optical devices. Although the sources of different kinds of noise have their backgrounds in physics, engineers dealing with noise use different methods and units to specify noise. The intention of this tutorial is to describe the main effects of noise in electronics up to optical frequencies while providing links between the physics and engineering worlds. In particular, noise is considered harmful while degrading the signal-to-noise ratio or broadening the spectrum of signal sources. On the other hand, noise can be itself a useful signal. Finally, artificially generated signals that exhibit many properties of random natural noise are sometimes required.

## NATURAL NOISE

Noise is a broadband signal. Therefore it makes sense to describe its intensity by the noise spectral density $N_0$ or amount of noise power per unit bandwidth (see Fig. 1). In electronics the most important type of noise is thermal noise. Thermal noise adds to any signal. In optics the most important type of noise is shot noise. Shot noise is a property of any signal made from a discrete number of photons.

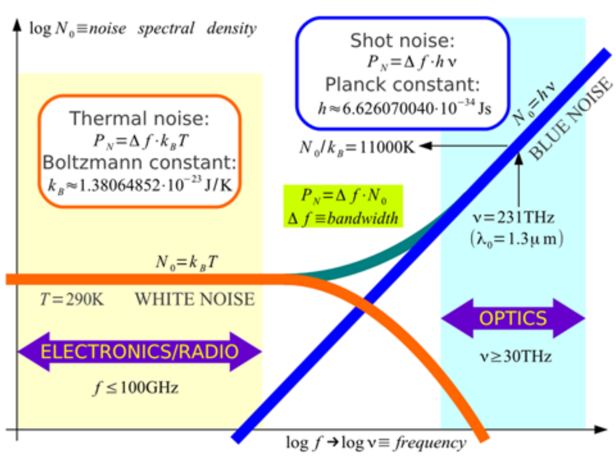


Figure 1: Noise spectral density.

Since the photon energy increases proportional with frequency, the higher the frequency the larger the shot noise spectral density. Such a noise is also called blue noise. Shot noise is unimportant in the radio-frequency range at room temperatures. Shot noise can only be observed at the highest end of the radio-frequency range at cryogenic temperatures.

Thermal noise is caused by thermal radiation. The Planck law (see Fig. 2) describes the spectral brightness $B_f$ or radiated power per unit bandwidth, unit area and unit solid angle of a black body. A black body with zero reflectivity Γ=0 is the most efficient thermal radiator while a perfect mirror |Γ|=1 does not radiate at all.

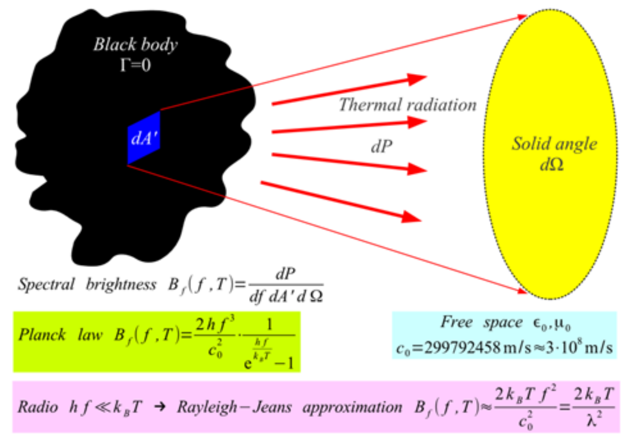


Figure 2: Black-body thermal radiation.

In the radio-frequency range it makes sense to use the Rayleigh-Jeans approximation of the Planck law to calculate the noise power collected by a lossless antenna. An antenna with an electrical connector (see Fig. 3) only collects half of the incident noise power on its effective area $A_{eff}$, the remaining half being orthogonally polarized.

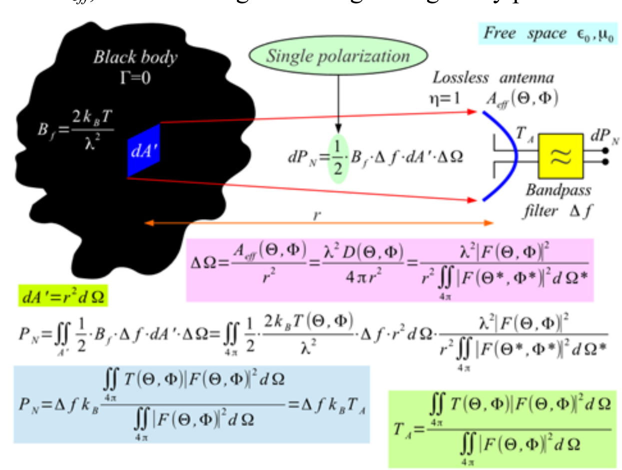


Figure 3: Received thermal-noise power.

In the radio-frequency range the noise spectral density is frequency independent. Thermal noise therefore behaves as white noise. Thermal noise spectral density is simply described by the black-body temperature $T_A$ as observed by the radiation pattern $F(\Theta,\Phi)$ of a lossless antenna (η=1).

Above a certain frequency the thermal noise power begins decreasing when the complete Planck law applies. However, the sum of both noise spectral densities, thermal noise and shot noise, remains a monotonic function of

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frequency. The white-noise behaviour is smoothly replaced by the blue-noise behaviour.

Expressing the noise spectral density with the noise temperature is so popular that the noise temperature is also used in cases when the noise is not of thermal origin.

## SIGNAL-TO-NOISE RATIO

Any receiver (amplifier) adds its own noise therefore further degrading the signal-to-noise ratio (see Fig. 4). The additional receiver noise is usually specified as an equivalent noise temperature $T_{RX}$ at the receiver input even if it is not of thermal origin. In the case of thermal noise, $T_{RX}$ is usually of the same order of magnitude as the physical temperature of the amplifying device.

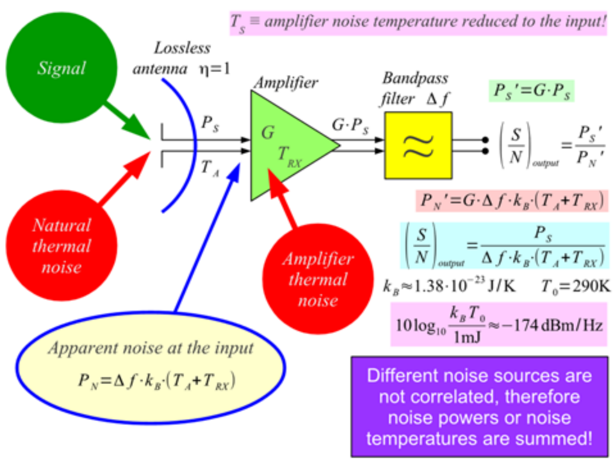


Figure 4: Receiver signal-to-noise ratio.

Alternatively the noise figure $F$ of a receiver can be specified. The noise figure has to be used carefully due to its unfortunate definition. In the case of thermal noise (see Fig. 5), selecting a reference temperature $T_0$=290 K allows a sensible definition of the noise figure $F$ (in linear units) or $F_{dB}$ (in logarithmic units) and a simple conversion from/to the receiver noise temperature $T_{RX}$.

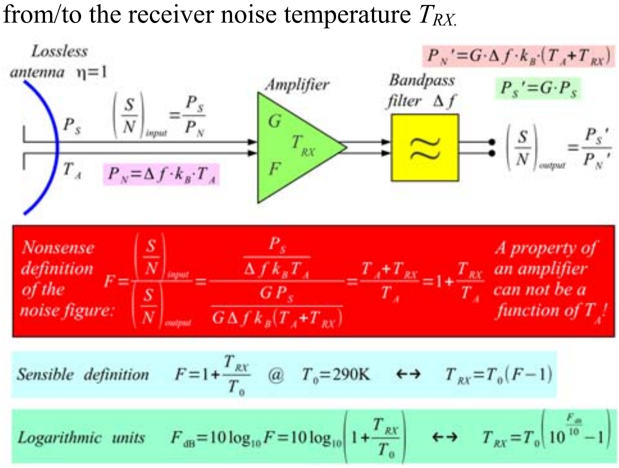


Figure 5: Amplifier noise figure.

A frequent case is an attenuator 0<a<1 between the antenna and receiver. Besides attenuating both the signal $P_S$ and the antenna noise $T_A$, the attenuator adds its own thermal noise $T_R$(1-1/a). The noise figure $F$ or $F_{dB}$ may simplify calculations (see Fig. 6) when the attenuator temperature $T_R \approx T_0$ is close to the reference temperature.

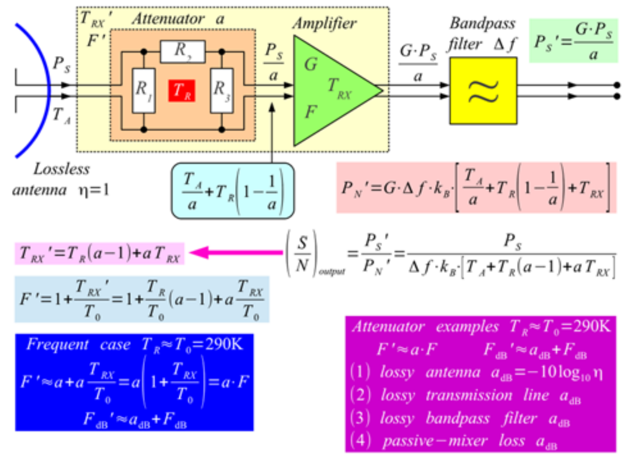


Figure 6: Attenuator noise.

## OSCILLATOR PHASE NOISE

A sine-wave oscillator includes a resonator to define its frequency and an amplifier to compensate for the resonator loss. Initially the amplifier provides excess gain and the oscillation starts out of noise exponentially. In the steady state, saturation reduces the gain of the amplifier and the oscillation amplitude stabilizes.

Unfortunately, in the steady-state oscillation, the noise sources $T_R$ (resonator loss) and $T_G$ (amplifier noise) are still present (see Fig. 7), corrupting the spectrum of the generated signal, broadening the generated-signal line width by adding noise side-bands.

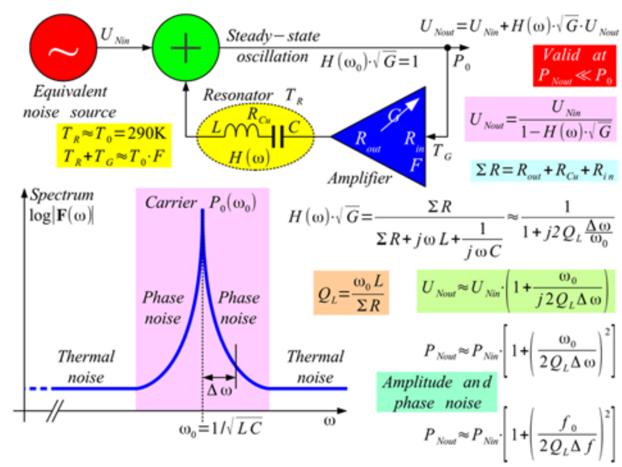


Figure 7: Oscillator phase noise.

The derivation can be simplified assuming that the noise power $P_{Nout}$<<$P_0$ is much smaller than the desired carrier power at $\omega_0$. The amplifier saturation removes most amplitude noise. Noise side-bands therefore include mainly phase noise. The final result (see Fig. 8) is known as the Leeson's equation although today it is usually written in a slightly different way from [1].

Unfortunately, the active devices used in the oscillator generate additional flicker noise ($1/f$ noise or pink noise) at small frequency offsets $|\Delta f|<f_C$. The flicker-noise corner frequency may be as high as $f_C\approx1$ MHz for surface devices like MOSFETs or HEMTs or as low as $f_C\approx1$ kHz for bulk devices like bipolar transistors or junction FETs.

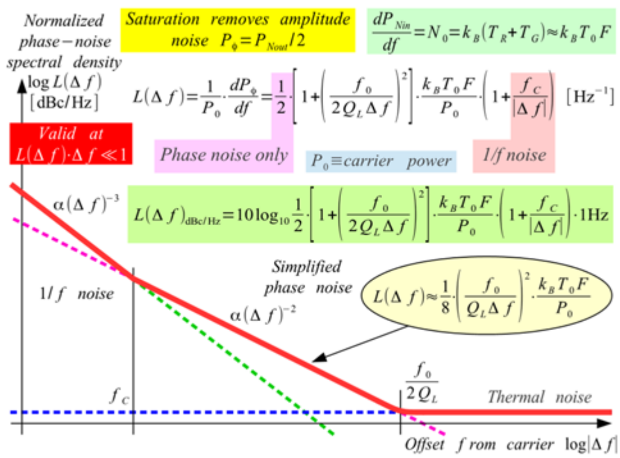


Figure 8: Leeson's equation.

The most important parameter of a good oscillator is the loaded quality $Q_L$ of the resonator. An LC oscillator may achieve a $Q_L\approx100$. A crystal oscillator may achieve a $Q_L\approx10^4$. A HeNe laser may achieve a $Q_L\approx10^8$ but its signal usually can not be used directly. Additional requirements are low-noise devices (low $F$ and low $f_C$) operated at a high carrier power $P_0$.

The oscillator phase noise (see Fig. 9) has many detrimental effects: it increases the bit-error rate due to modulation constellation rotation, it adds residual FM in analog communications, it causes adjacent-channel interference and it causes clock timing jitter.

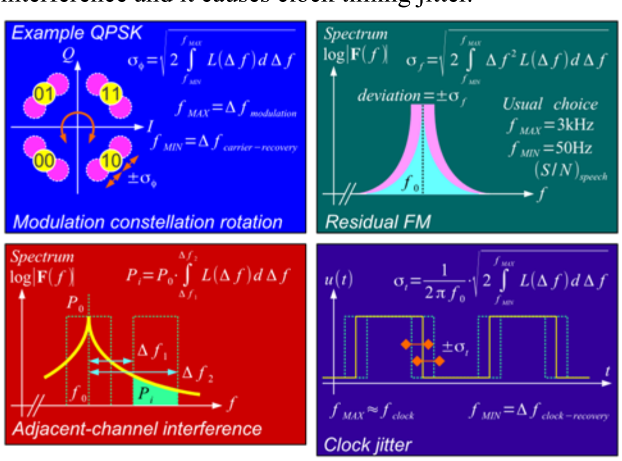


Figure 9: Effects of phase noise.

In all above case, the phase noise is integrated over a finite interval $f_{MIN}<\Delta f<f_{MAX}$ where the Leeson's equation is accurate enough in almost all real-world engineering problems having a finite bandwidth $f_{MAX}=\Delta f_{bandwidth}$ and fast-enough control loops $f_{MIN}=\Delta f_{recovery}$.

All of the above integrals give an infinite result both for $f_{MIN}\rightarrow0$ and/or for $f_{MAX}\rightarrow\infty$. Such an infinite result has no physical background and comes from the approximations used in the derivation of the Leeson's equation. The phase noise without approximations should include additional effects (see Fig. 10). The device $f_{MAX}$ limits the bandwidth even more than the complete Planck law.

At the other end, at very small frequency offsets $f_{MIN}\rightarrow0$ the noise power is no longer small compared to the carrier power as assumed in the simple derivation. The spectrum of any real oscillator is a finite and continuous function with a rounded top at the central frequency $f_0$.

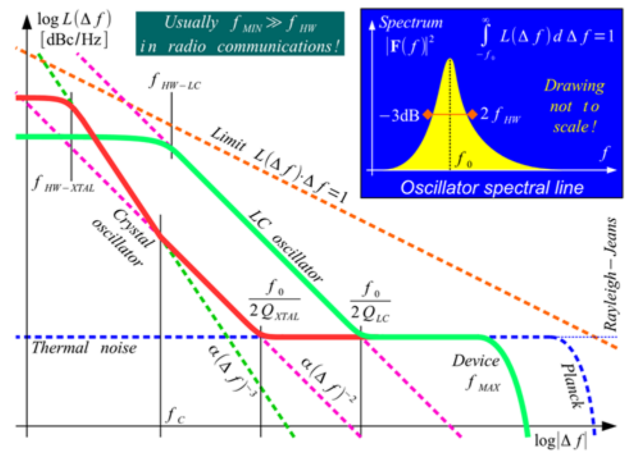


Figure 10: Phase noise without approximations.

When the flicker ($1/f$) noise can be neglected, the simplified Leeson's equation evolves into a Lorentzian spectral line (see Fig. 11). The -3 dB spectral-line width of a rather poor ($Q_L=10$) electrical ($f_0=3$ GHz) LC oscillator is just $f_{FWHM}=28$ Hz$\approx10^{-8}f_0$.

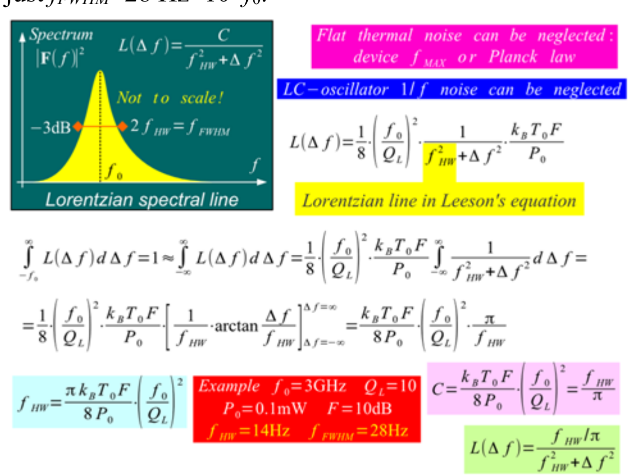


Figure 11: Lorentzian spectral-line width.

While both physicists and engineers observe the same effects, they look at different details. Physicists observe the line width $f_{FWHM}$. Engineers observe the phase-noise spectral density $L(\Delta f)$ at much larger frequency offsets.

## FIBER-OPTICS NOISE

Although shot noise is the largest noise contribution at optical frequencies, other kinds of noise usually limit the performance of optical-fiber links (see Fig. 12). At low power levels in an optical-fiber link, the largest noise contribution is the thermal noise of the electrical amplifier following the photodiode due to impedance mismatch. At higher power levels both the intensity and phase noise of the laser are the largest noise contribution Unwanted reflections (optical connectors etc) form interferometers that transform the laser phase noise into intensity noise.

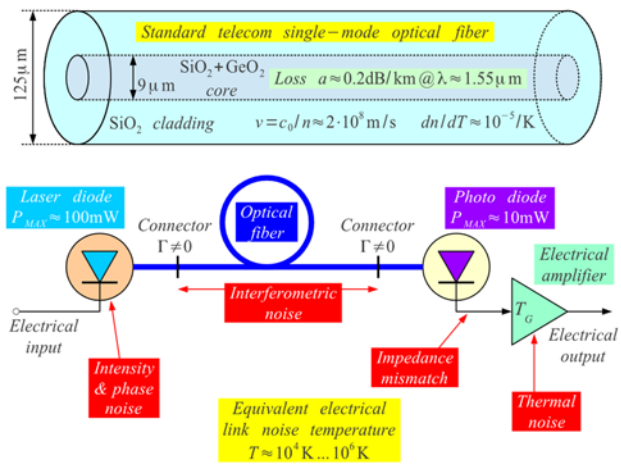


Figure 12: Optical-fiber link.

An electro-optical delay line could replace a very high $Q_O>10^6$ K resonator in an oscillator (see Fig. 13), but it is also very noisy $T_R>>290$ K. Further it requires a mode-select electrical filter with a considerable $Q_M\approx10\%\ Q_O$.

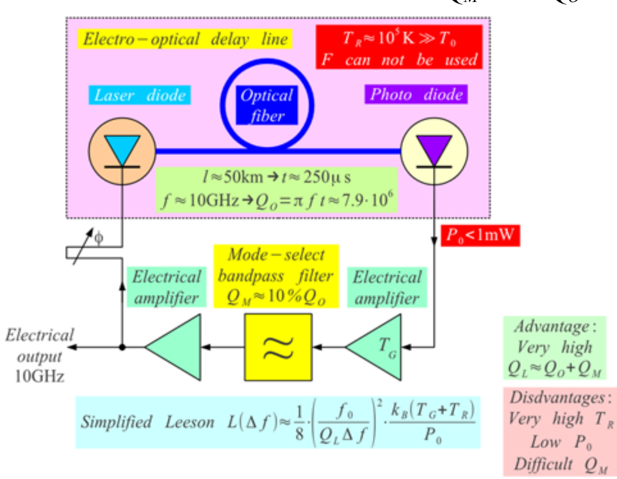


Figure 13: Opto-electronic oscillator.

A noisy electro-optical delay line allows more noise to be generated in the electrical circuits. In particular, an electrical Q multiplier (noisy circuit) can be used to achieve a high $Q_M$ of the mode-select bandpass filter of an opto-electronic oscillator [2] to improve unwanted mode rejection while relatively contributing just a small fraction of the whole-oscillator loop noise.

## NOISE AS USEFUL SIGNAL

White noise can be used as a test signal in electronics exactly in the same was as white light is used as a test signal in optics. In electronics, white noise allows testing both linear and non-linear circuits (see Fig. 14). White noise allows measuring the transfer function $H(\omega)$ of linear circuits. White noise is also useful to measure the intermodulation distortion (IMD) of non-linear circuits.

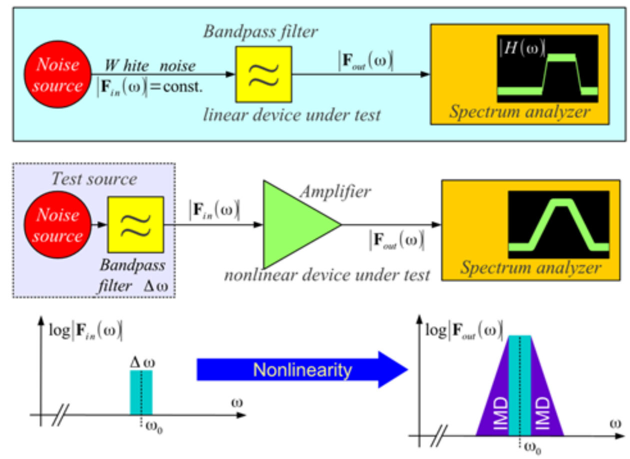


Figure 14: Noise as a test signal.

Truly random noise can also be used to generate secure cryptographic keys of any size. Carefully managing the signal-to-noise ratio in a low-loss communication link with strong forward-error correction (FEC) is the basis of noise (quantum) cryptography.

Finally, some circuits generate signals that are not random, but have many properties of random noise, like autocorrelation function, spectrum, sound or appearance. An *m*-stage binary polynomial divider with a carefully selected feedback (see Fig. 15) can generate such a pseudo-random sequence of the maximum length $N=2^m$-1. While useful for many purposes (synchronization, test signal etc), pseudo-random sequences have no cryptographic value.

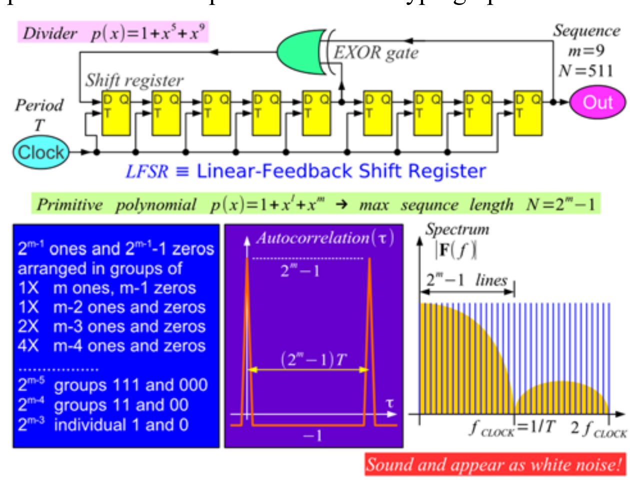


Figure 15: Linear-feedback shift register.

# REFERENCES

[1] D.B. Leeson, “A simple model of feedback oscillator noise spectrum”, in *Proc. IEEE*, vol. 54, no. 2, pp. 329–330, Feb. 1966.

[2] L. Bogataj, M. Vidmar, and B. Batagelj, “Opto-electronic oscillator with quality multiplier”, *IEEE Transactions on Microwave Theory and Techniques*, vol. 64, no. 2, pp. 663-668, Feb. 2016.