

Noise in radio/optical communications

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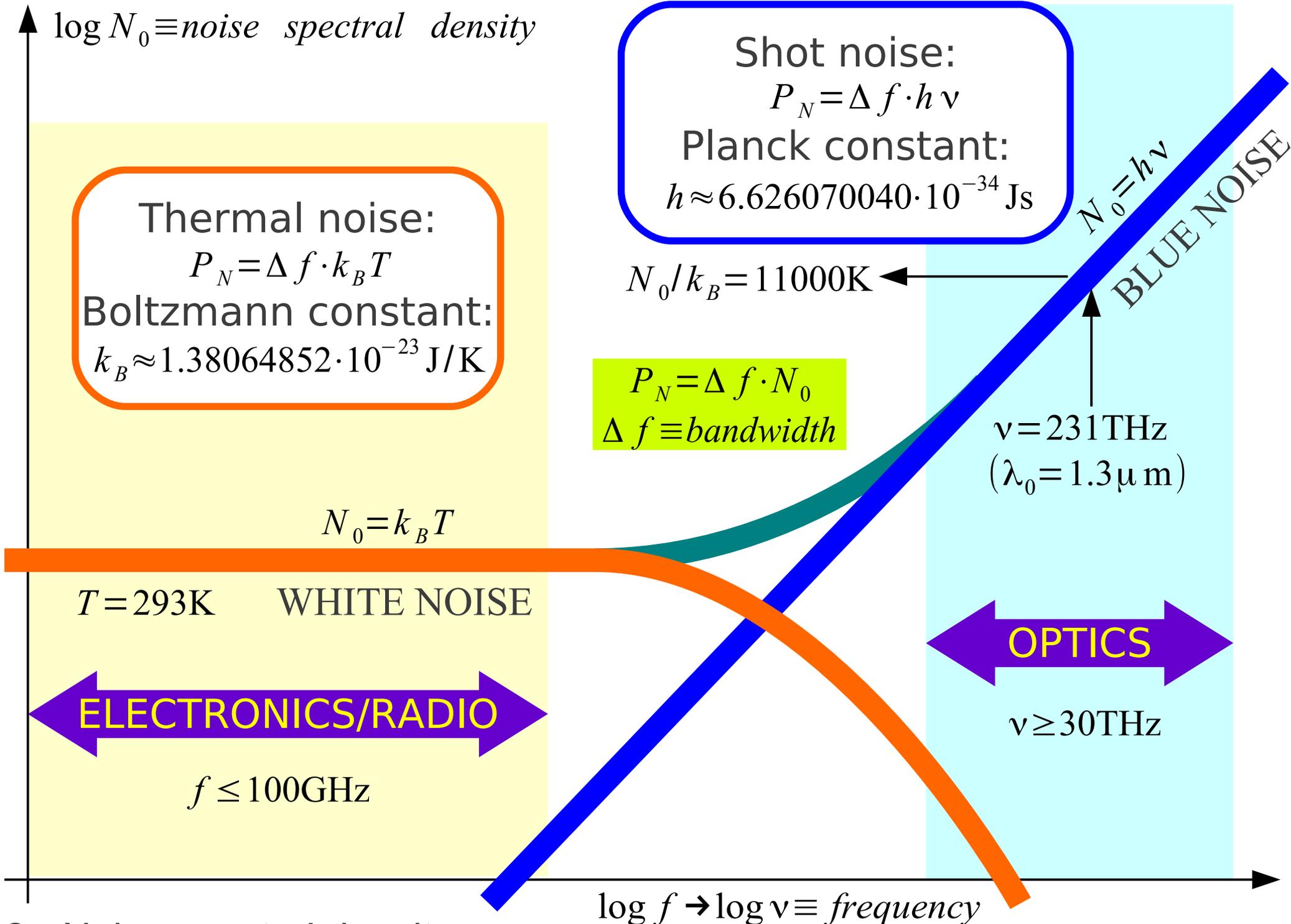
Fifth Solvay International Conference on Electrons and Photons (October 1927). The leading figures Albert Einstein and Niels Bohr disagreed:

Albert Einstein: „God does not play dice!“

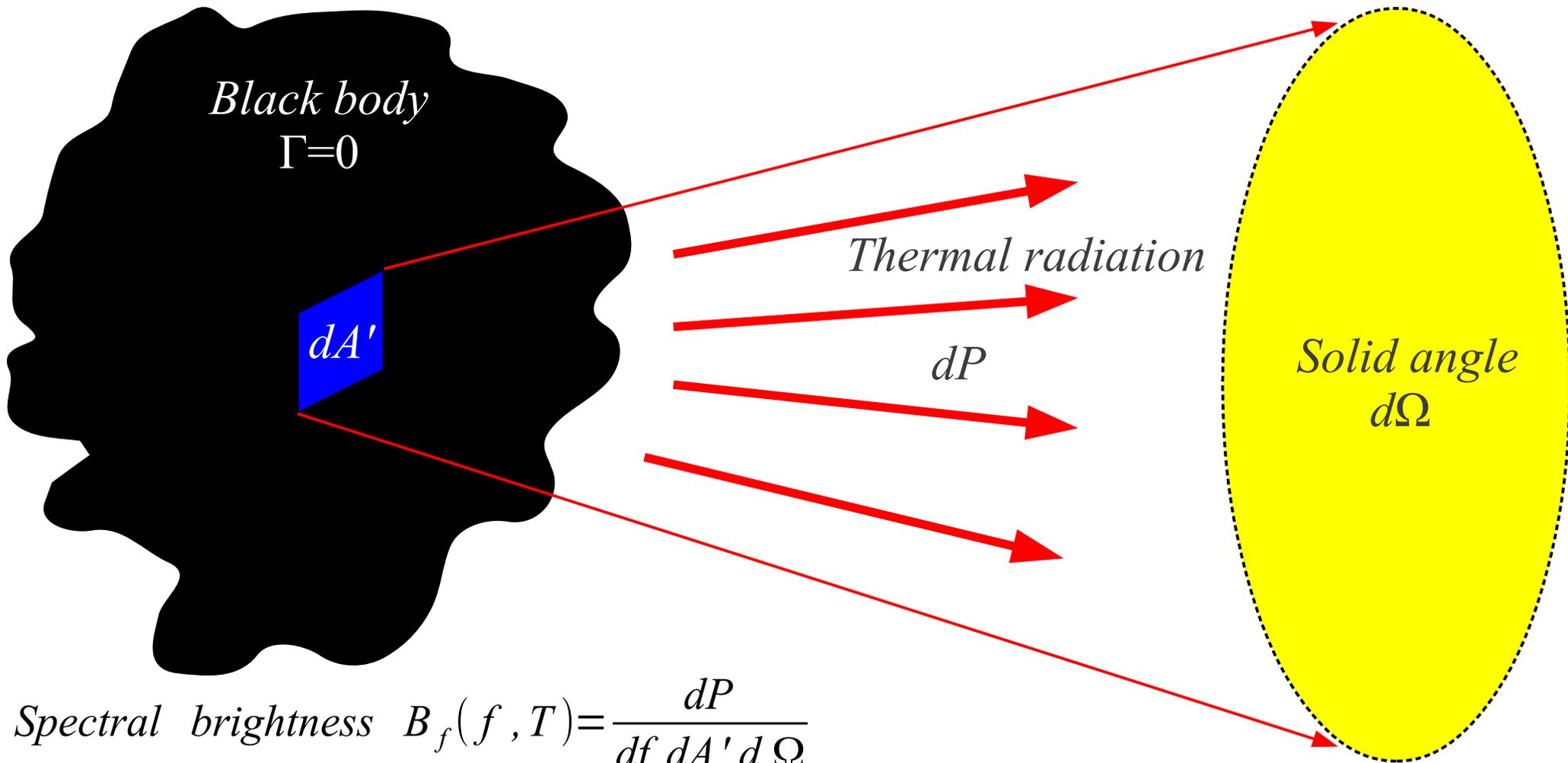
Niels Bohr: „Einstein, stop telling God what to do!“

In telecommunications random signals are called noise. Noise impairs the performance of any communication link.

Noise is a macroscopic description of quantum effects!



2 - Noise spectral density



Spectral brightness $B_f(f, T) = \frac{dP}{df dA' d\Omega}$

Planck law $B_f(f, T) = \frac{2 h f^3}{c_0^2} \cdot \frac{1}{e^{\frac{hf}{k_B T}} - 1}$

Free space ϵ_0, μ_0
 $c_0 = 299792458 \text{ m/s} \approx 3 \cdot 10^8 \text{ m/s}$

Radio $hf \ll k_B T \rightarrow$ *Rayleigh-Jeans approximation* $B_f(f, T) \approx \frac{2 k_B T f^2}{c_0^2} = \frac{2 k_B T}{\lambda^2}$

3 – Black-body thermal radiation

Free space ϵ_0, μ_0

Black body
 $\Gamma=0$

Single polarization

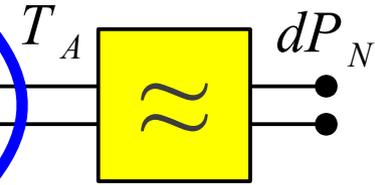
Lossless antenna
 $\eta=1$

$$B_f = \frac{2k_B T}{\lambda^2}$$

dA'

$$dP_N = \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta \Omega$$

$A_{eff}(\Theta, \Phi)$



Bandpass filter Δf

r

$$\Delta \Omega = \frac{A_{eff}(\Theta, \Phi)}{r^2} = \frac{\lambda^2 D(\Theta, \Phi)}{4\pi r^2} = \frac{\lambda^2 |F(\Theta, \Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*, \Phi^*)|^2 d\Omega^*}$$

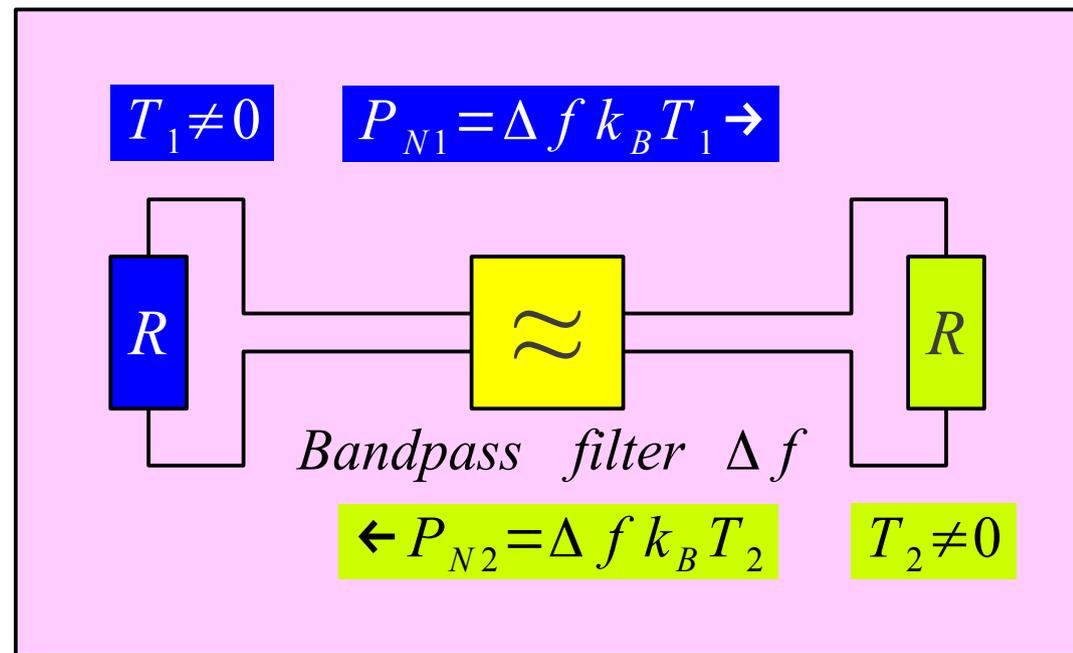
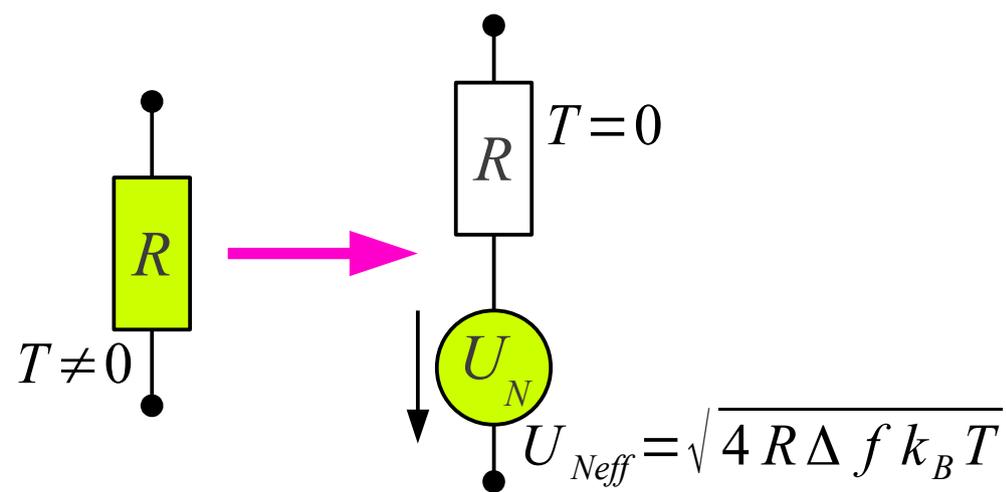
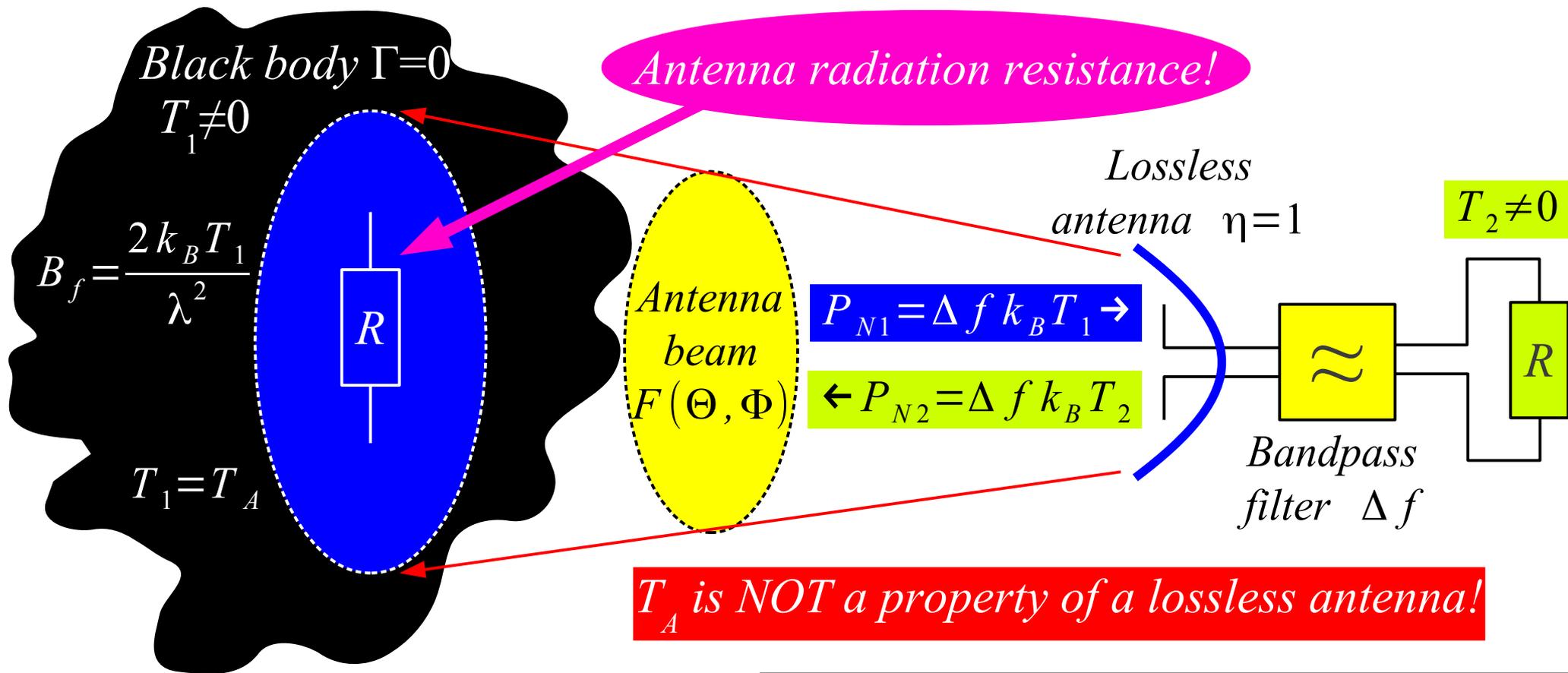
$$dA' = r^2 d\Omega$$

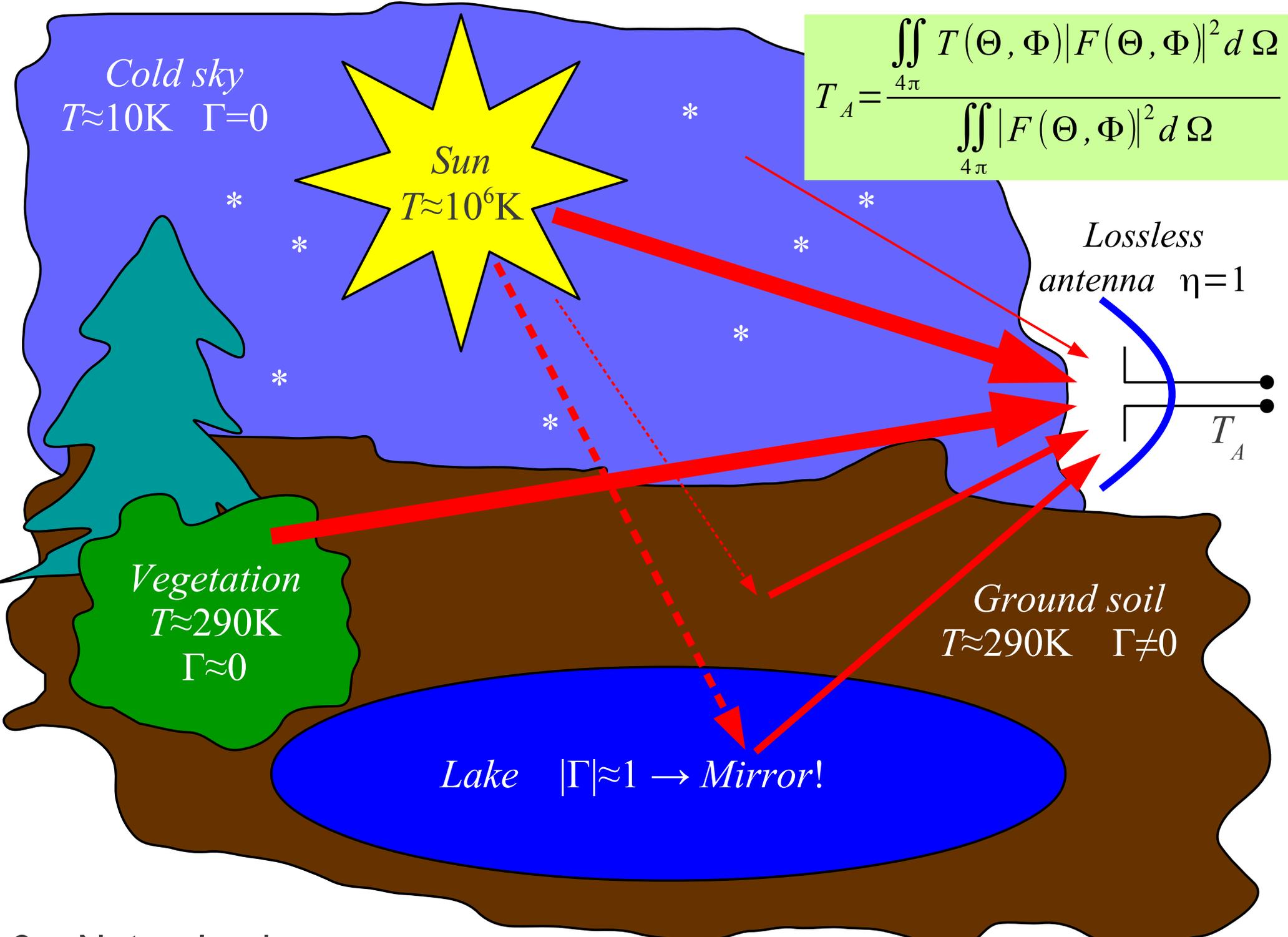
$$P_N = \iint_{A'} \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta \Omega = \iint_{4\pi} \frac{1}{2} \cdot \frac{2k_B T(\Theta, \Phi)}{\lambda^2} \cdot \Delta f \cdot r^2 d\Omega \cdot \frac{\lambda^2 |F(\Theta, \Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*, \Phi^*)|^2 d\Omega^*}$$

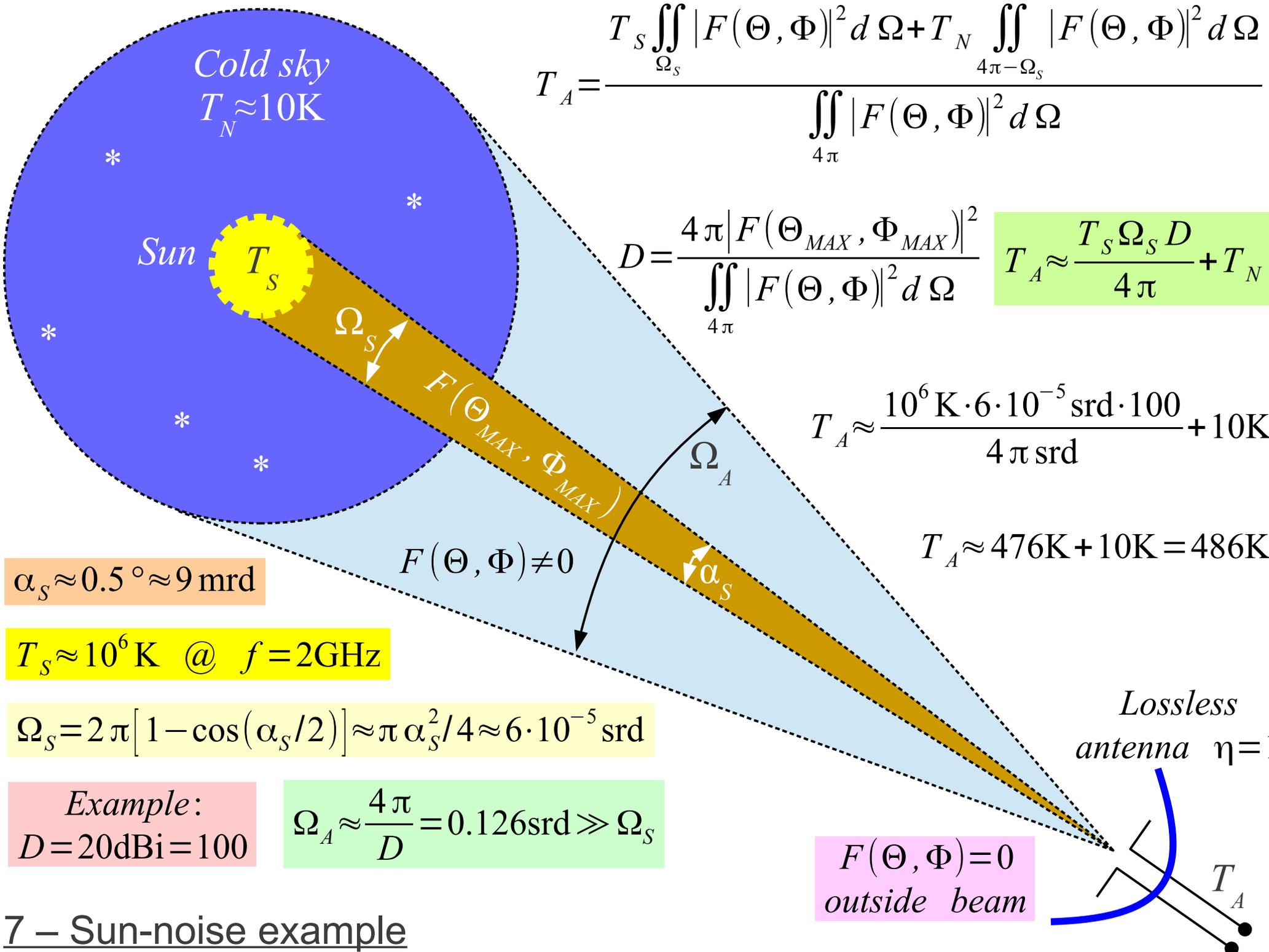
$$P_N = \Delta f k_B \frac{\iint_{4\pi} T(\Theta, \Phi) |F(\Theta, \Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta, \Phi)|^2 d\Omega} = \Delta f k_B T_A$$

$$T_A = \frac{\iint_{4\pi} T(\Theta, \Phi) |F(\Theta, \Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta, \Phi)|^2 d\Omega}$$

4 – Received thermal-noise power







$$T_A = \frac{T_S \iint_{\Omega_S} |F(\Theta, \Phi)|^2 d\Omega + T_N \iint_{4\pi - \Omega_S} |F(\Theta, \Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta, \Phi)|^2 d\Omega}$$

$$D = \frac{4\pi |F(\Theta_{MAX}, \Phi_{MAX})|^2}{\iint_{4\pi} |F(\Theta, \Phi)|^2 d\Omega} \quad T_A \approx \frac{T_S \Omega_S D}{4\pi} + T_N$$

$$T_A \approx \frac{10^6 \text{ K} \cdot 6 \cdot 10^{-5} \text{ srd} \cdot 100}{4\pi \text{ srd}} + 10 \text{ K}$$

$$T_A \approx 476 \text{ K} + 10 \text{ K} = 486 \text{ K}$$

$$\alpha_S \approx 0.5^\circ \approx 9 \text{ mrd}$$

$$T_S \approx 10^6 \text{ K} @ f = 2 \text{ GHz}$$

$$\Omega_S = 2\pi [1 - \cos(\alpha_S/2)] \approx \pi \alpha_S^2 / 4 \approx 6 \cdot 10^{-5} \text{ srd}$$

Example:
 $D = 20 \text{ dBi} = 100$

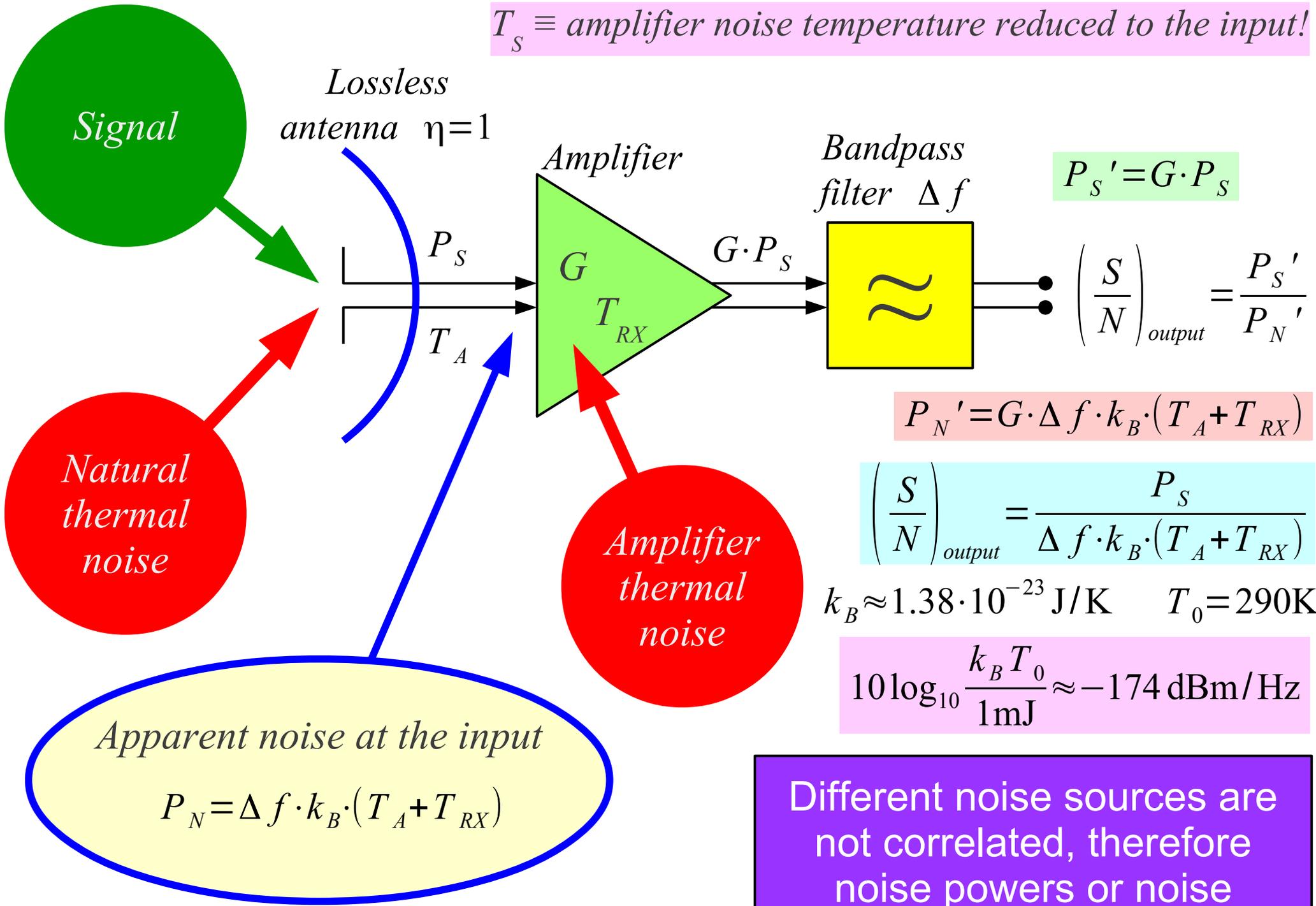
$$\Omega_A \approx \frac{4\pi}{D} = 0.126 \text{ srd} \gg \Omega_S$$

$F(\Theta, \Phi) = 0$
 outside beam

Lossless antenna $\eta = 1$

7 – Sun-noise example

$T_s \equiv$ amplifier noise temperature reduced to the input!



$$P_N' = G \cdot \Delta f \cdot k_B \cdot (T_A + T_{RX})$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_{RX})}$$

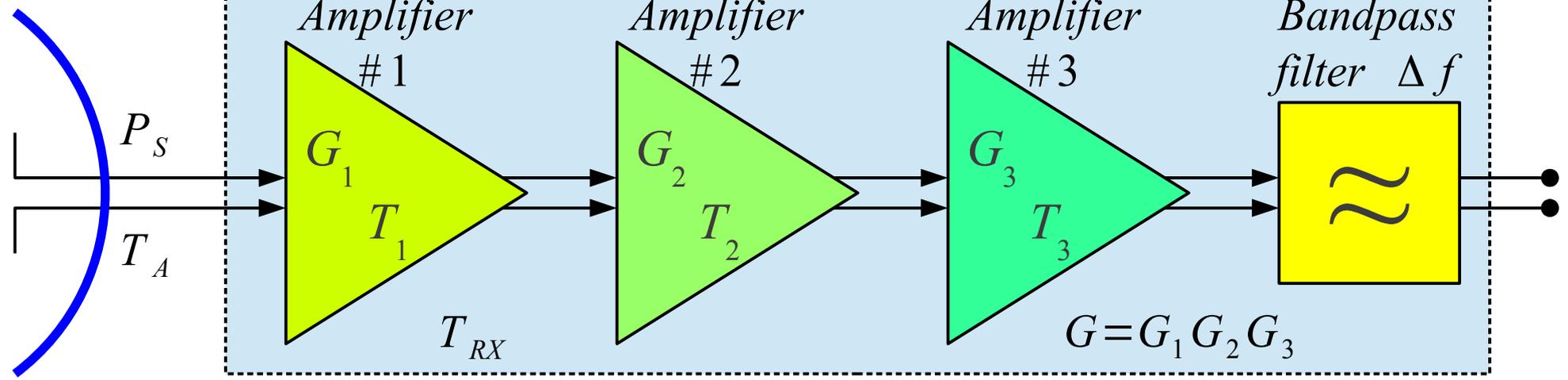
$$k_B \approx 1.38 \cdot 10^{-23} \text{ J/K} \quad T_0 = 290\text{K}$$

$$10 \log_{10} \frac{k_B T_0}{1\text{mJ}} \approx -174 \text{ dBm/Hz}$$

Different noise sources are not correlated, therefore noise powers or noise temperatures are summed!

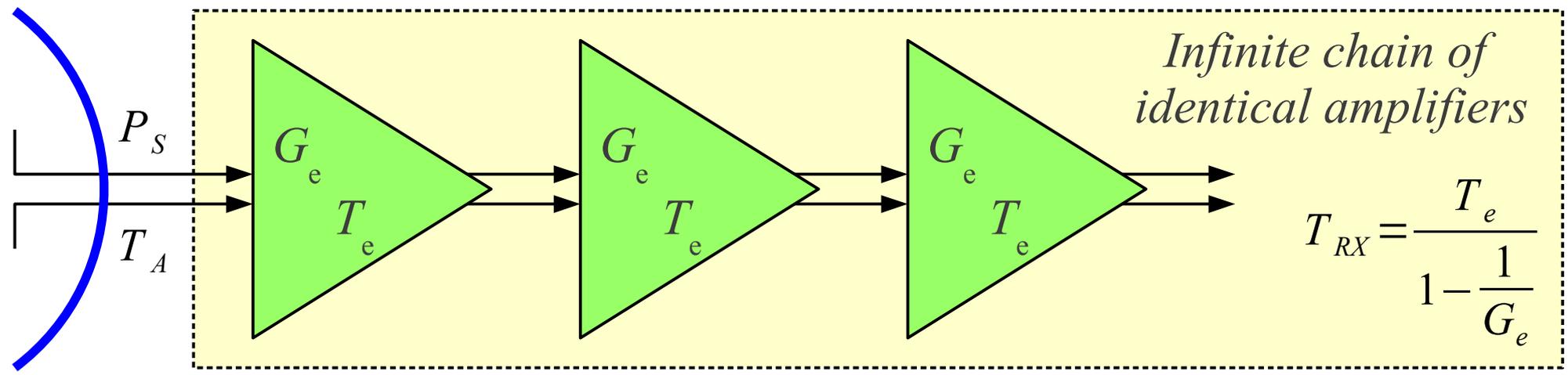
$$P_s' = G_3 G_2 G_1 P_s$$

Lossless antenna $\eta = 1$

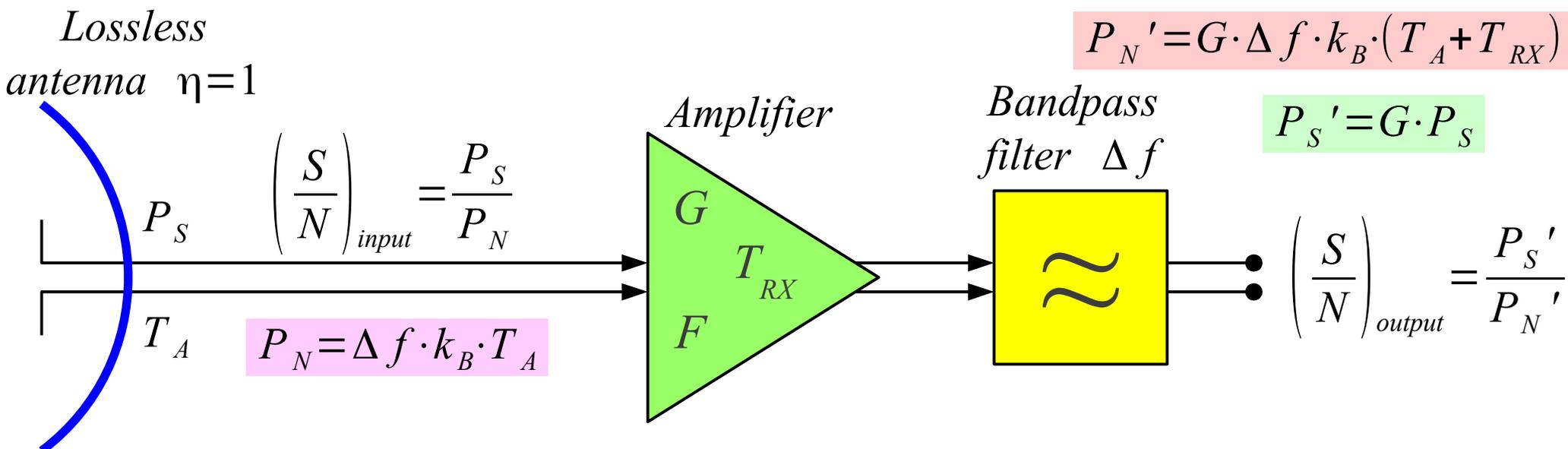


$$P_N' = \Delta f k_B [G_3 G_2 G_1 (T_A + T_1) + G_3 G_2 T_2 + G_3 T_3]$$

$$P_N' = G_3 G_2 G_1 \Delta f k_B (T_A + T_{RX}) \quad \rightarrow \quad T_{RX} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots$$



9 – Chain noise temperature



Nonsense definition of the noise figure:

$$F = \frac{\left(\frac{S}{N}\right)_{input}}{\left(\frac{S}{N}\right)_{output}} = \frac{\frac{P_S}{\Delta f k_B T_A}}{\frac{G P_S}{G \Delta f k_B (T_A + T_{RX})}} = \frac{T_A + T_{RX}}{T_A} = 1 + \frac{T_{RX}}{T_A}$$

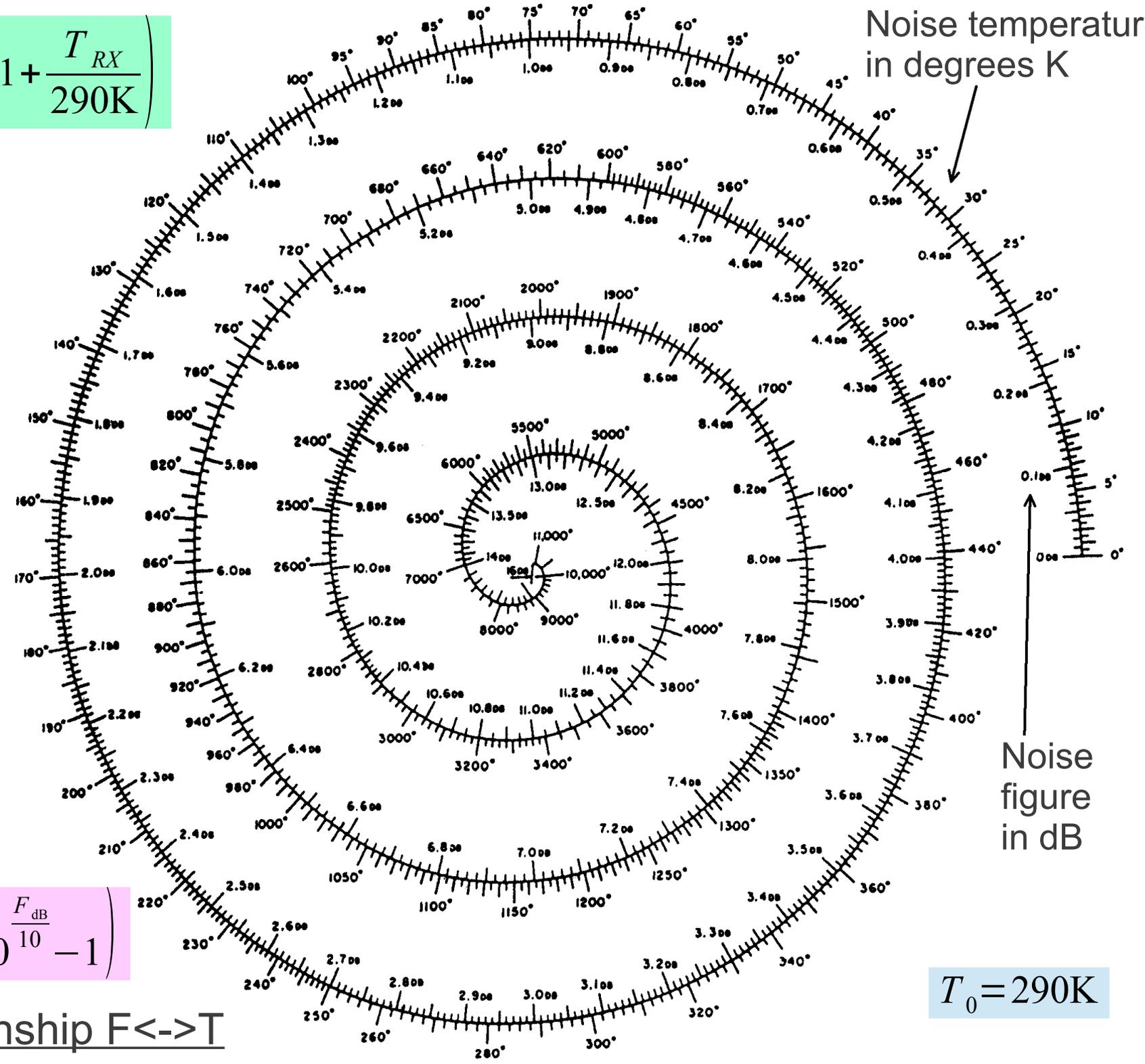
A property of an amplifier can not be a function of T_A !

Sensible definition $F = 1 + \frac{T_{RX}}{T_0}$ @ $T_0 = 290K$ $\leftrightarrow T_{RX} = T_0(F - 1)$

Logarithmic units $F_{dB} = 10 \log_{10} F = 10 \log_{10} \left(1 + \frac{T_{RX}}{T_0}\right) \leftrightarrow T_{RX} = T_0 \left(10^{\frac{F_{dB}}{10}} - 1\right)$

10 – Amplifier noise figure

$$F_{dB} = 10 \log_{10} \left(1 + \frac{T_{RX}}{290K} \right)$$



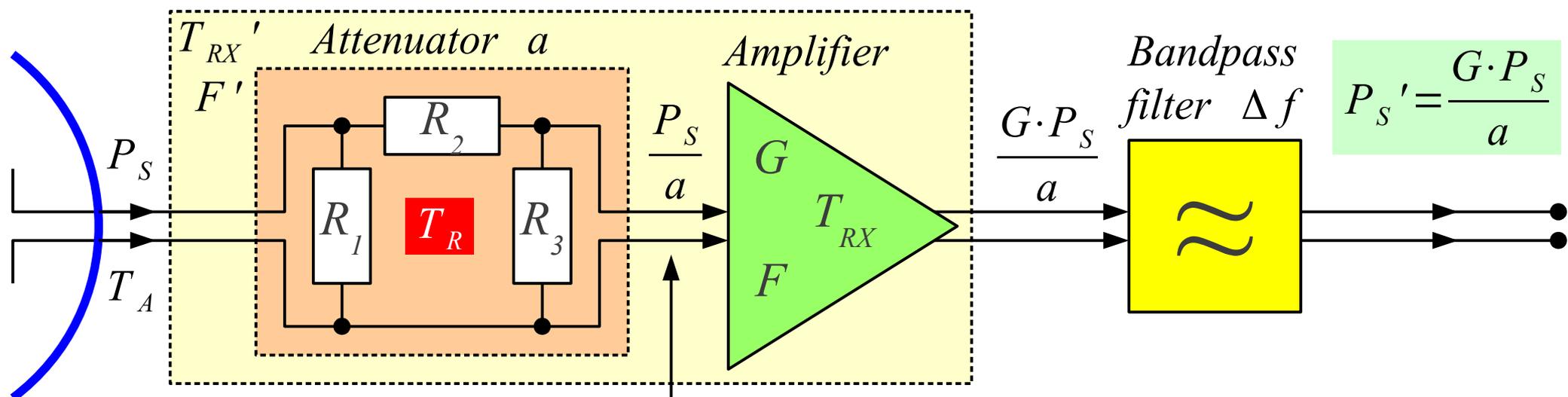
Noise temperature in degrees K

Noise figure in dB

$$T_{RX} = 290K \left(10^{\frac{F_{dB}}{10}} - 1 \right)$$

$$T_0 = 290K$$

11 – Relationship F<->T



Lossless antenna $\eta=1$

$$\frac{T_A}{a} + T_R \left(1 - \frac{1}{a}\right)$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot \left[\frac{T_A}{a} + T_R \left(1 - \frac{1}{a}\right) + T_{RX} \right]$$

$$T_{RX}' = T_R(a-1) + a T_{RX} \quad \left(\frac{S}{N} \right)_{output} = \frac{P_S'}{P_N'} = \frac{P_S}{\Delta f \cdot k_B \cdot [T_A + T_R(a-1) + a T_{RX}]}$$

$$F' = 1 + \frac{T_{RX}'}{T_0} = 1 + \frac{T_R}{T_0}(a-1) + a \frac{T_{RX}}{T_0}$$

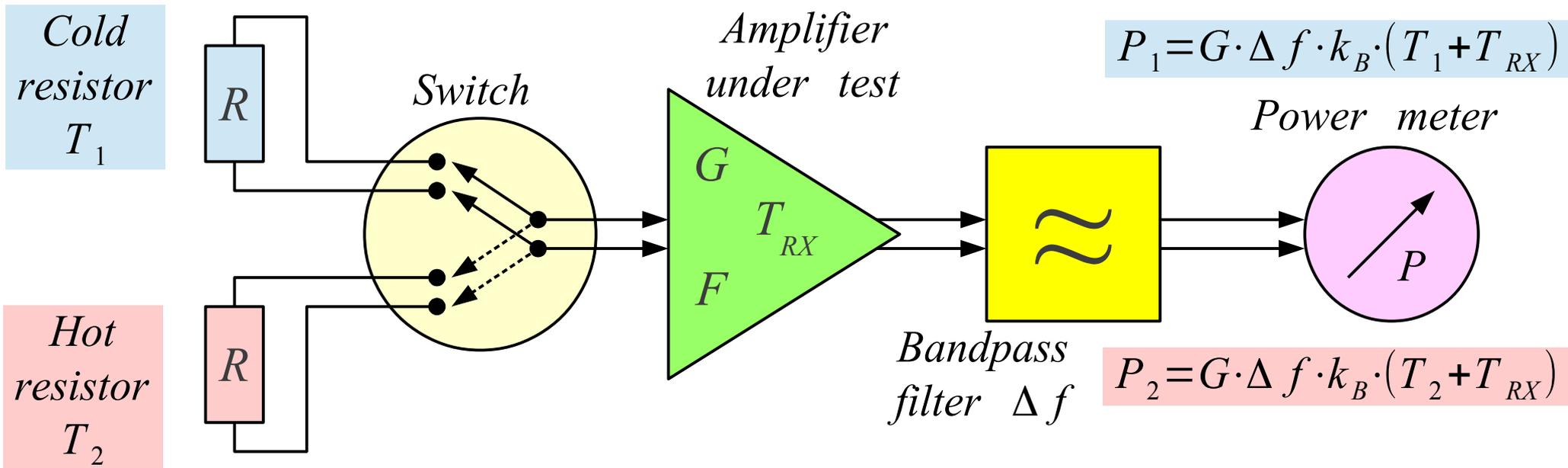
Frequent case $T_R \approx T_0 = 290K$

$$F' \approx a + a \frac{T_{RX}}{T_0} = a \left(1 + \frac{T_{RX}}{T_0} \right) = a \cdot F$$

$$F_{dB}' \approx a_{dB} + F_{dB}$$

- Attenuator examples $T_R \approx T_0 = 290K$
- $$F' \approx a \cdot F \quad F_{dB}' \approx a_{dB} + F_{dB}$$
- (1) lossy antenna $a_{dB} = -10 \log_{10} \eta$
 - (2) lossy transmission line a_{dB}
 - (3) lossy bandpass filter a_{dB}
 - (4) passive-mixer loss a_{dB}

Amplifier device	Gain G [dB]	Noise temperature T_{RX} [K]	Noise figure F_{dB} [dB]
Vacuum tube with control grid (triode, pentode)	10↔20	1600↔9000	8↔15
Vacuum tube with speed modulation (klystron, TWT)	20↔50	3000↔30000	10↔20
Parametric amplifier (room temperature)	10↔15	75↔300	1↔3
Si BJT, JFET or MOSFET (room temperature)	10↔20	75↔300	1↔3
GaAs FET or HEMT (room temperature)	10↔15	20↔120	0.3↔1.5
GaAs FET or HEMT (liquid-nitrogen 77K)	10↔15	7↔35	0.1↔0.5
Si or GaAs MMIC amplifier	10↔25	170↔1600	2↔8
Operational amplifier	40↔100	10^4 ↔ 10^9	16↔66



The unknowns $G \cdot \Delta f \cdot k_B$ cancel in the Y ratio!

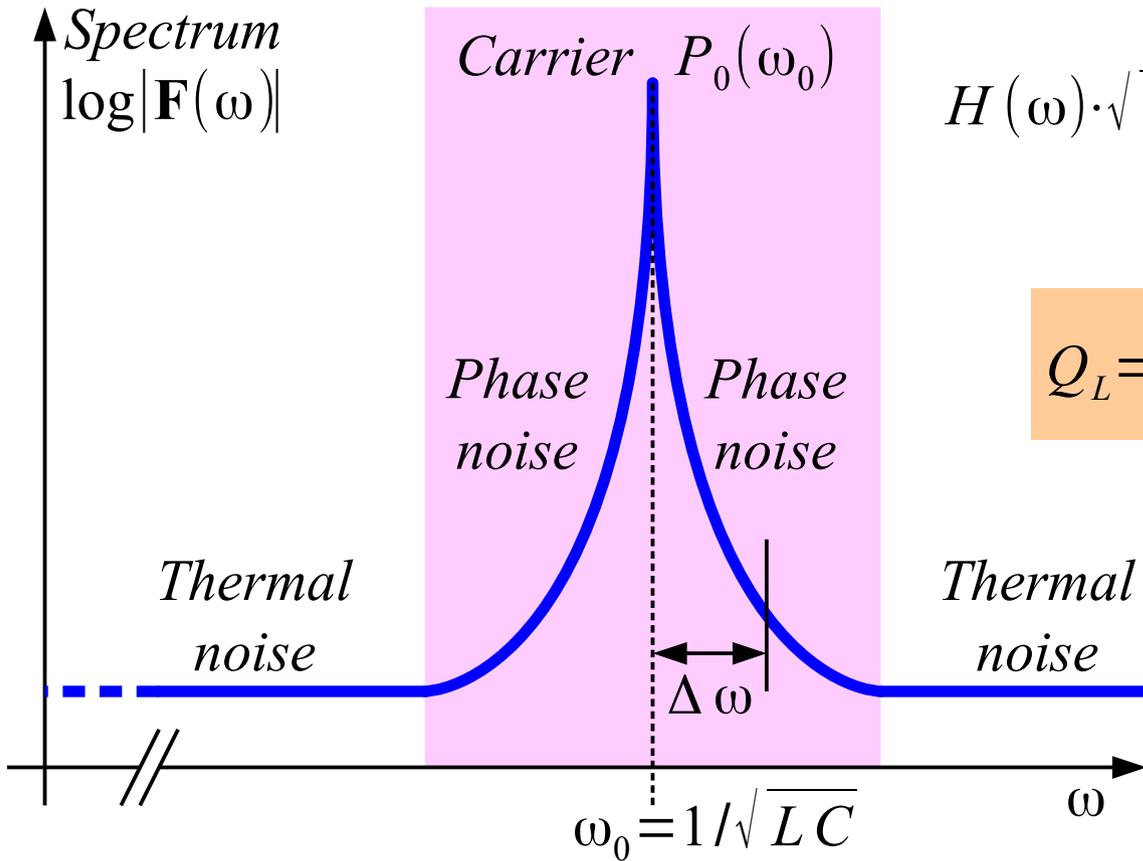
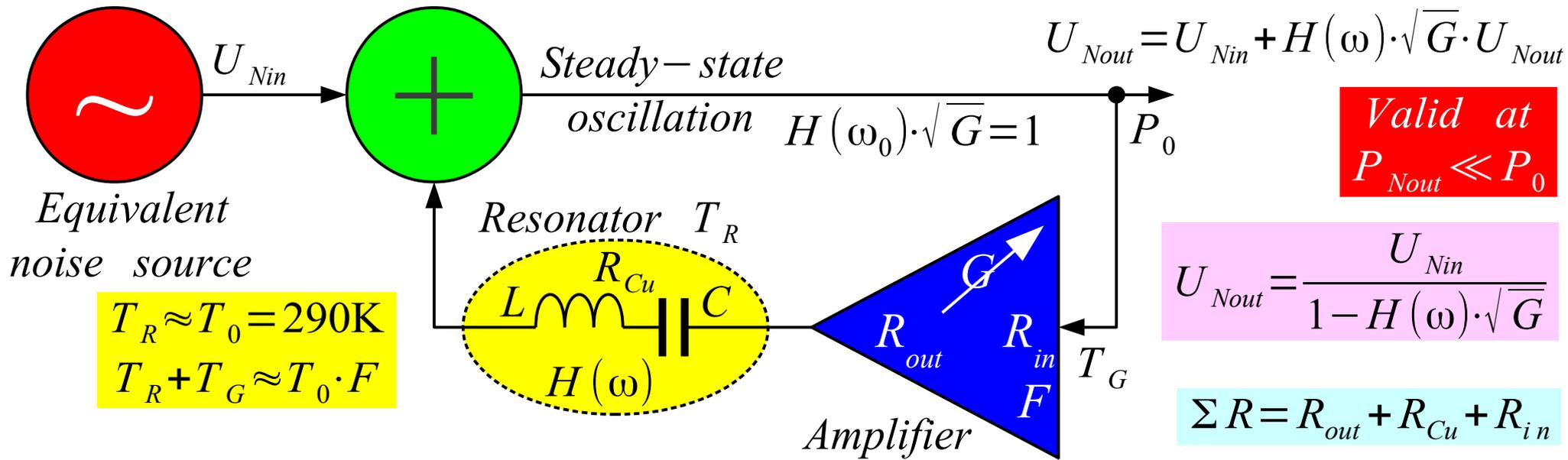
$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_{RX}}{T_1 + T_{RX}}$$

$$T_{RX} = \frac{T_2 - Y \cdot T_1}{Y - 1}$$

$$T_0 = 290\text{K}$$

$$F_{dB} = 10 \log_{10} \left[1 + \frac{T_2 - Y \cdot T_1}{(Y - 1) \cdot T_0} \right]$$

Resistor type	Temperature
Antenna into cold sky	$\sim 20\text{K}$
Liquid N_2 cooled R	$\sim 77\text{K}$
Antenna into absorber	$\sim 290\text{K}$
R at room temperature	$\sim 290\text{K}$
Light-bulb filament as R	$\sim 2000\text{K}$
Ionized gas as R	$\sim 10^4\text{K}$
Avalanche breakdown	$\sim 10^6\text{K}$



$$H(\omega) \cdot \sqrt{G} = \frac{\Sigma R}{\Sigma R + j\omega L + \frac{1}{j\omega C}} \approx \frac{1}{1 + j2Q_L \frac{\Delta\omega}{\omega_0}}$$

$$Q_L = \frac{\omega_0 L}{\Sigma R}$$

$$U_{Nout} \approx U_{Nin} \cdot \left(1 + \frac{\omega_0}{j2Q_L \Delta\omega} \right)$$

$$P_{Nout} \approx P_{Nin} \cdot \left[1 + \left(\frac{\omega_0}{2Q_L \Delta\omega} \right)^2 \right]$$

Amplitude and phase noise

$$P_{Nout} \approx P_{Nin} \cdot \left[1 + \left(\frac{f_0}{2Q_L \Delta f} \right)^2 \right]$$

Normalized
phase-noise
spectral density

Saturation removes amplitude
noise $P_\phi = P_{Nout}/2$

$$\frac{dP_{Nin}}{df} = N_0 = k_B(T_R + T_G) \approx k_B T_0 F$$

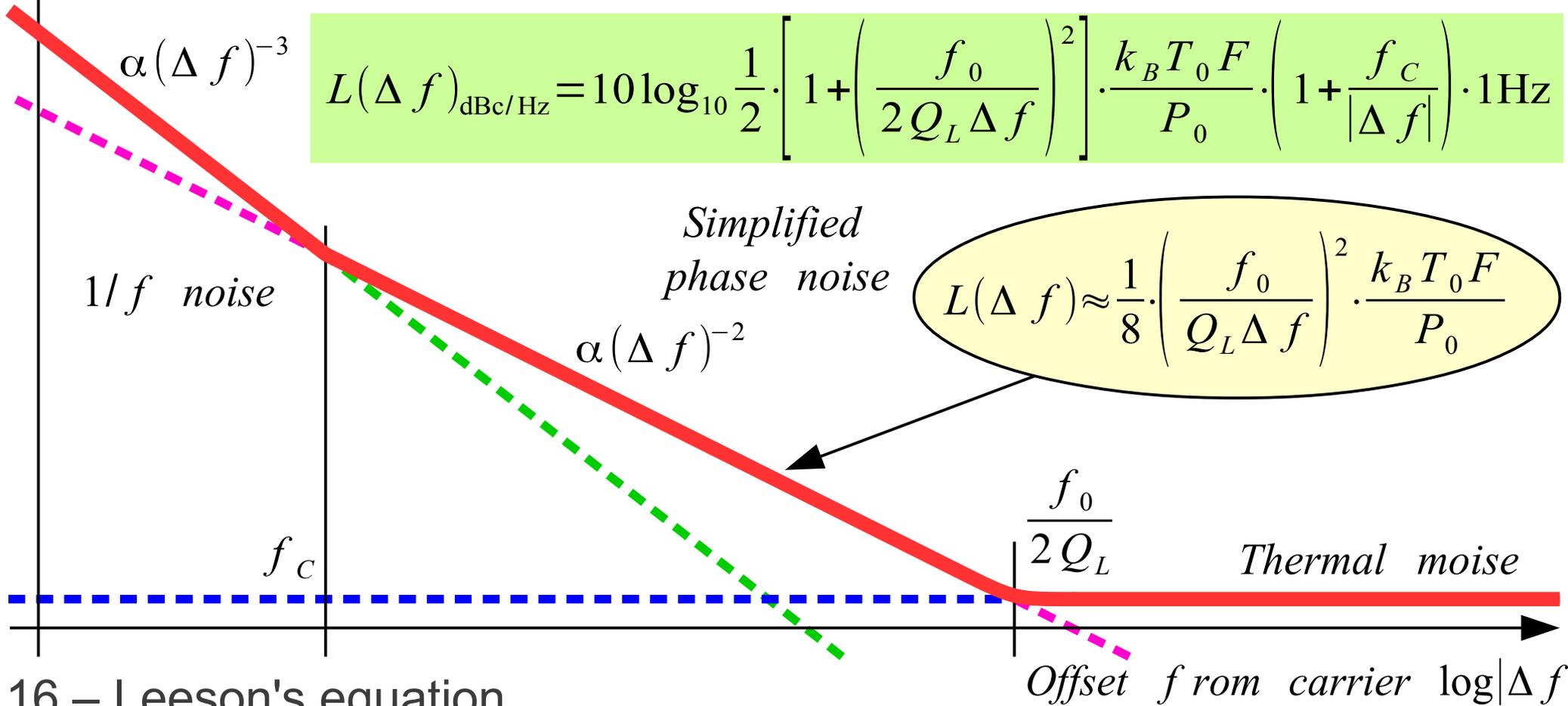
$$L(\Delta f) = \frac{1}{P_0} \cdot \frac{dP_\phi}{df} = \frac{1}{2} \cdot \left[1 + \left(\frac{f_0}{2Q_L \Delta f} \right)^2 \right] \cdot \frac{k_B T_0 F}{P_0} \cdot \left(1 + \frac{f_c}{|\Delta f|} \right) \quad [\text{Hz}^{-1}]$$

Valid at
 $L(\Delta f) \cdot \Delta f \ll 1$

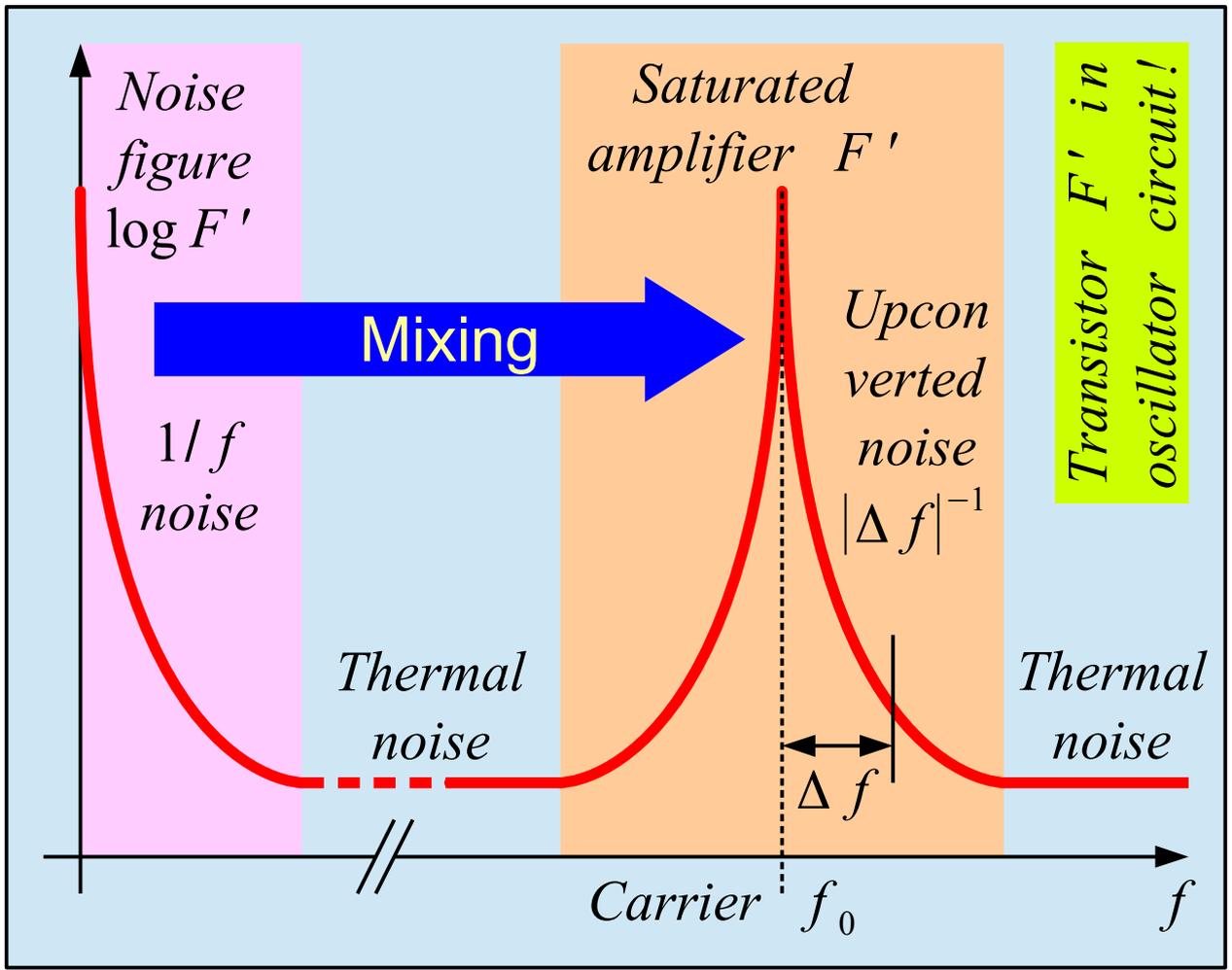
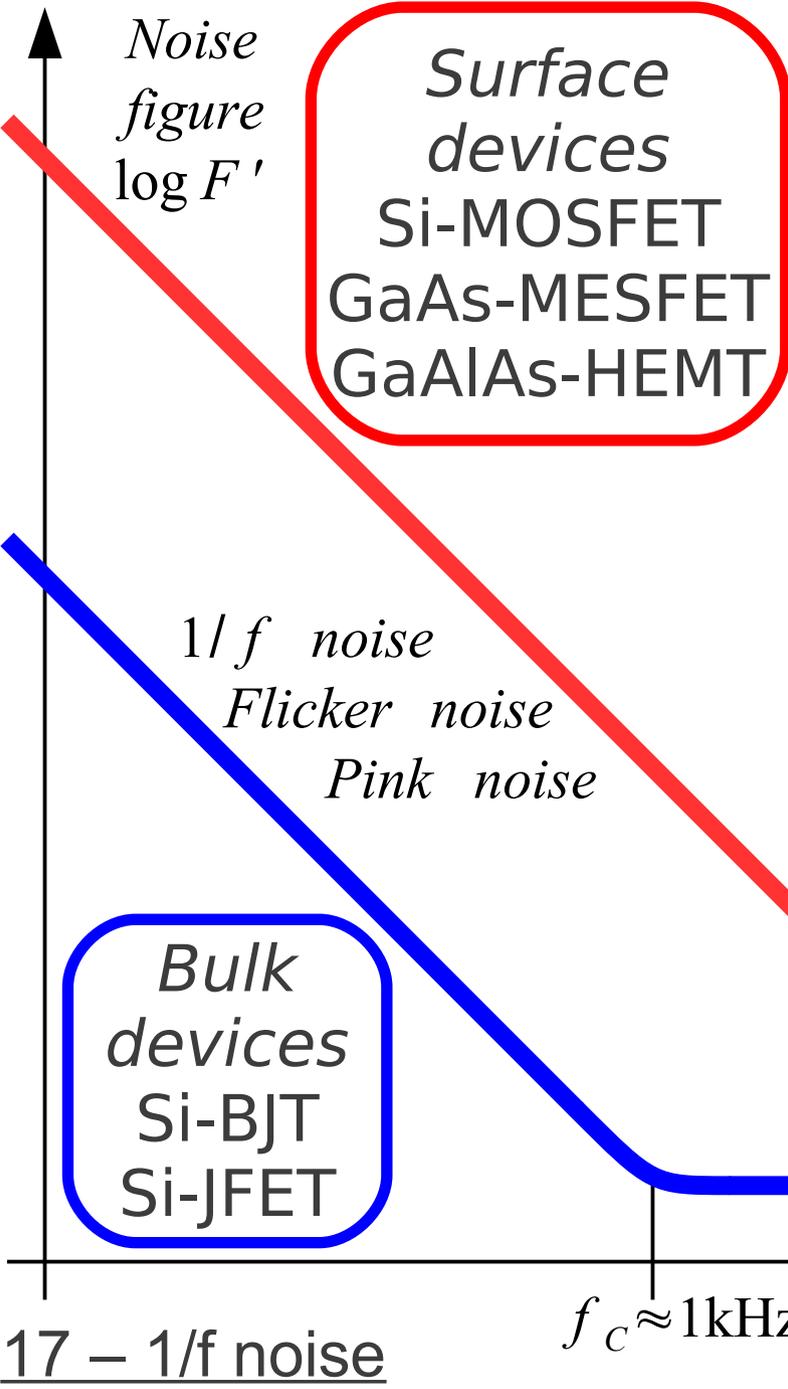
Phase noise only

$P_0 \equiv$ carrier power

1/f noise



1/f noise usually does not have a clear explanation!



$$F' = F \left(1 + \frac{f_c}{f} \right) \equiv \text{increased LF noise!}$$

The loaded resonator quality Q_L defines the oscillator phase noise!

$$L(\Delta f) = \frac{1}{2} \cdot \left[1 + \left(\frac{f_0}{2 Q_L \Delta f} \right)^2 \right] \cdot \frac{k_B T_0 F}{P_0} \cdot \left(1 + \frac{f_c}{|\Delta f|} \right)$$

Variable-frequency oscillators

Q_L

RC VCO

~ 1

BWO tube

~ 1

Varactor-tuned LC VCO

$10 \leftrightarrow 30$

YIG ($Y_3Fe_5O_{12}$) oscillator

$300 \leftrightarrow 1000$

Fixed-frequency oscillators

Q_L

RC multivibrator

~ 1

LC resonator

$30 \leftrightarrow 100$

Cavity resonator

$1000 \leftrightarrow 3000$

Ceramic dielectric resonator

$1000 \leftrightarrow 3000$

AT-cut quartz crystal (fundamental mode)

$3000 \leftrightarrow 10000$

AT-cut quartz crystal (third/fifth overtone)

$10000 \leftrightarrow 30000$

Electro-optical delay line (\$)

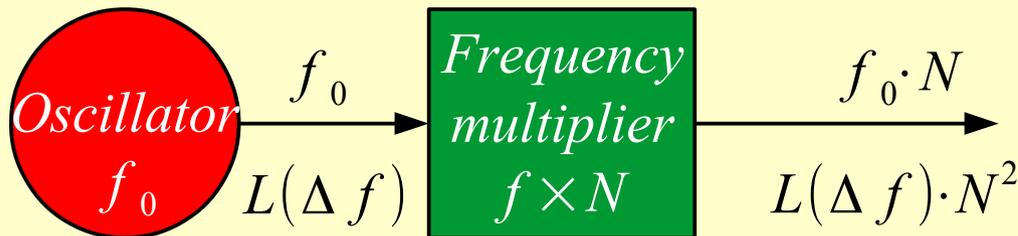
$\sim 10^5$ (noisy!)

Sapphire dielectric resonator (\$\$\$)

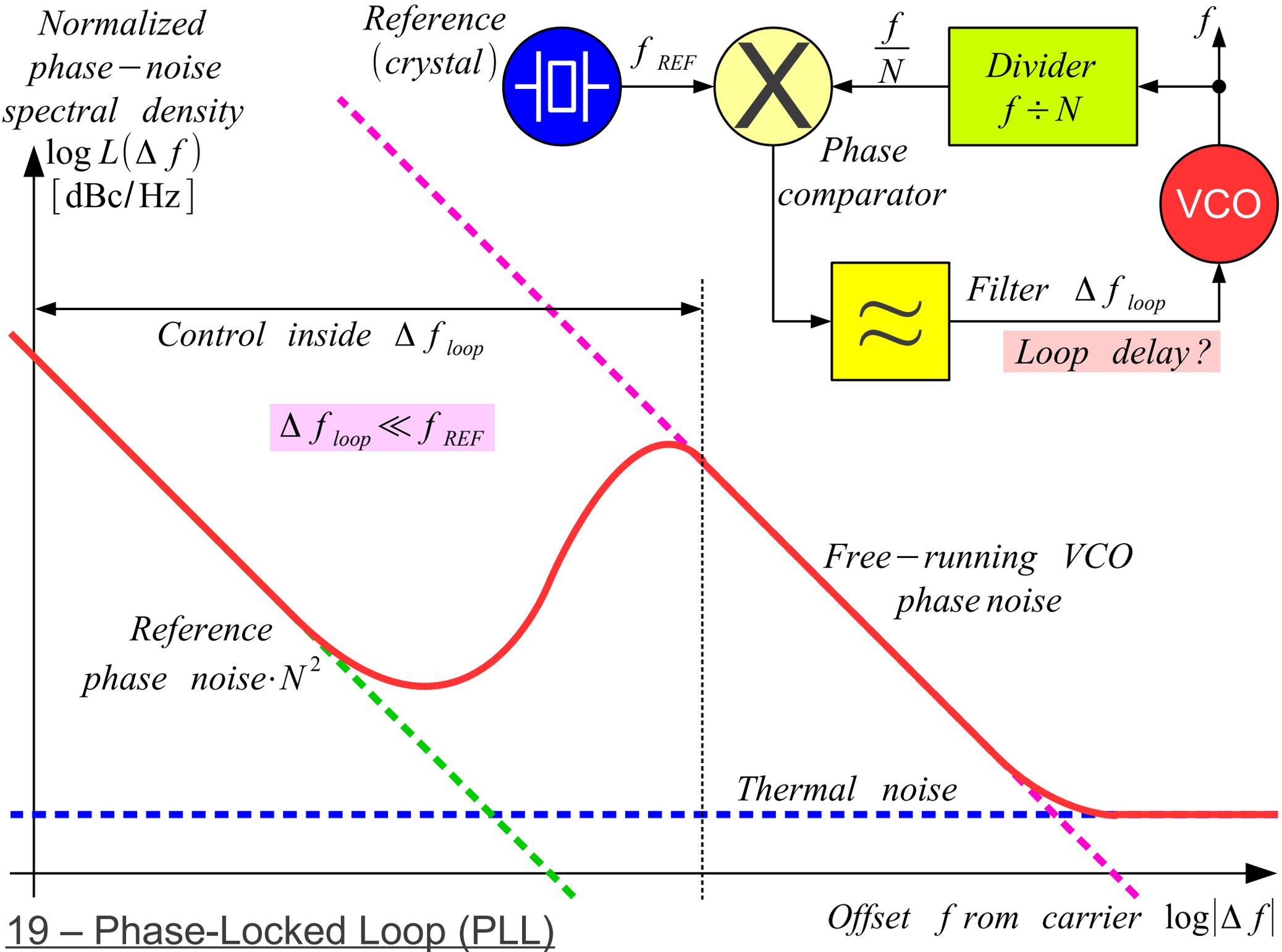
$\sim 3 \cdot 10^5$

Red HeNe LASER

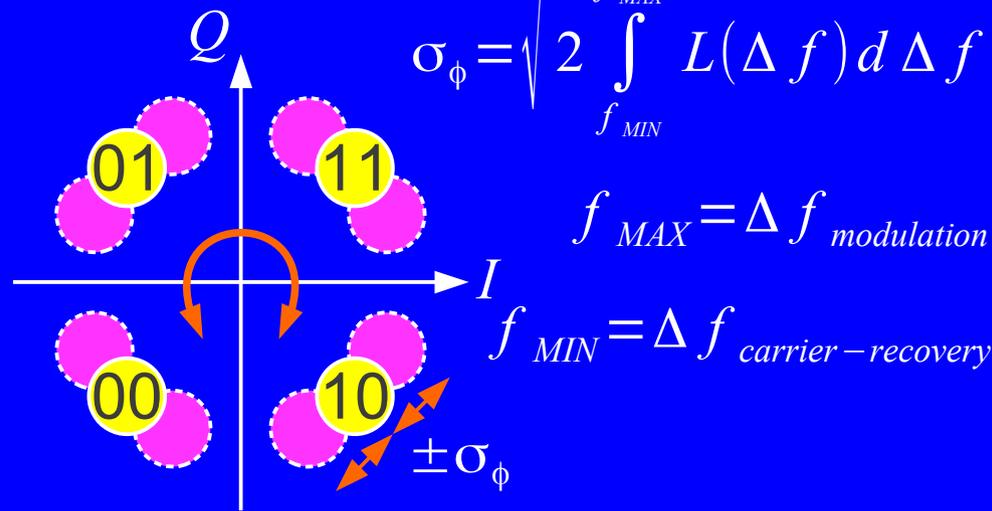
$\sim 10^8$



The phase noise multiplies with the square of the frequency multiplication factor!

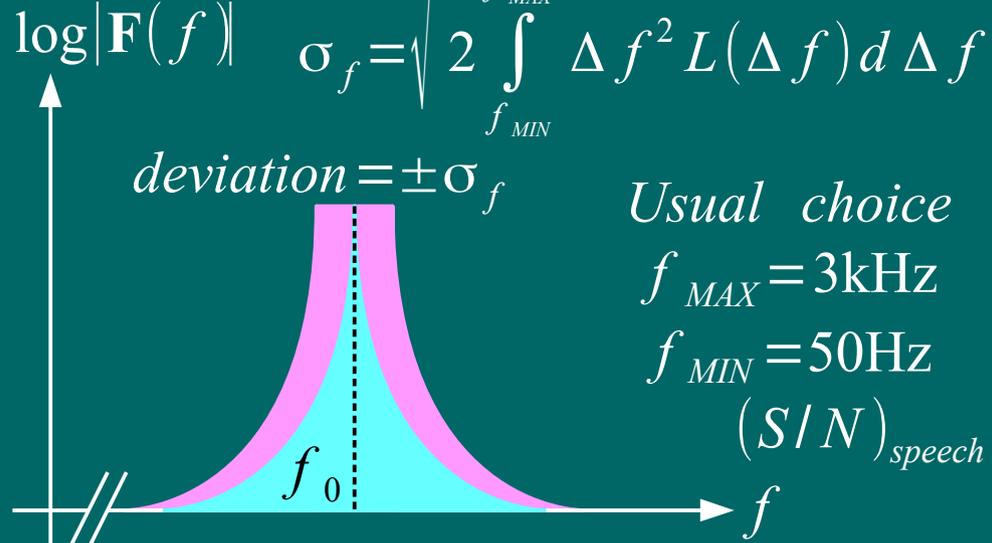


Example QPSK



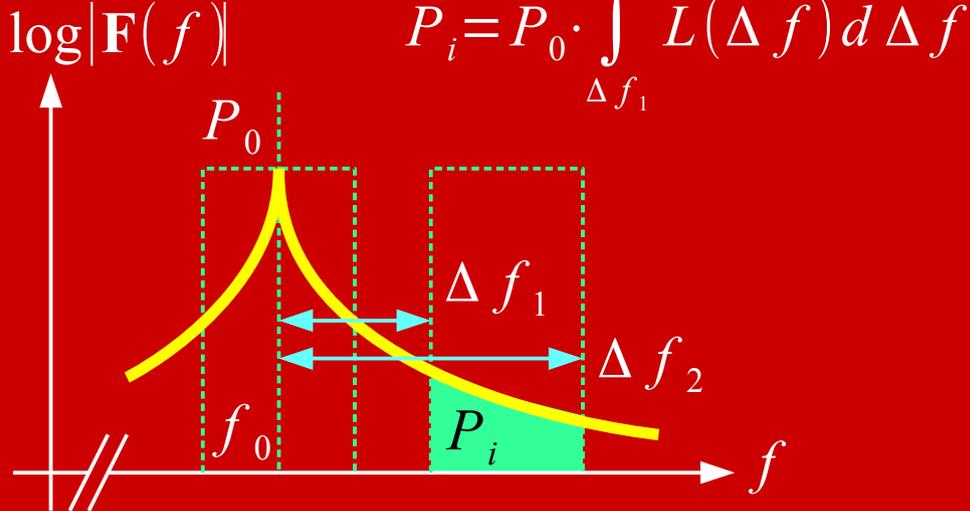
Modulation constellation rotation

Spectrum



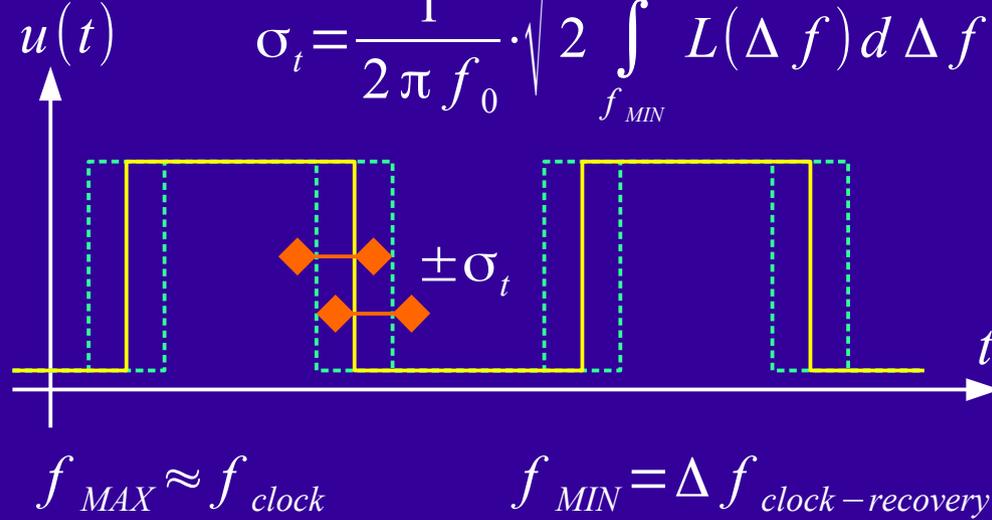
Residual FM

Spectrum

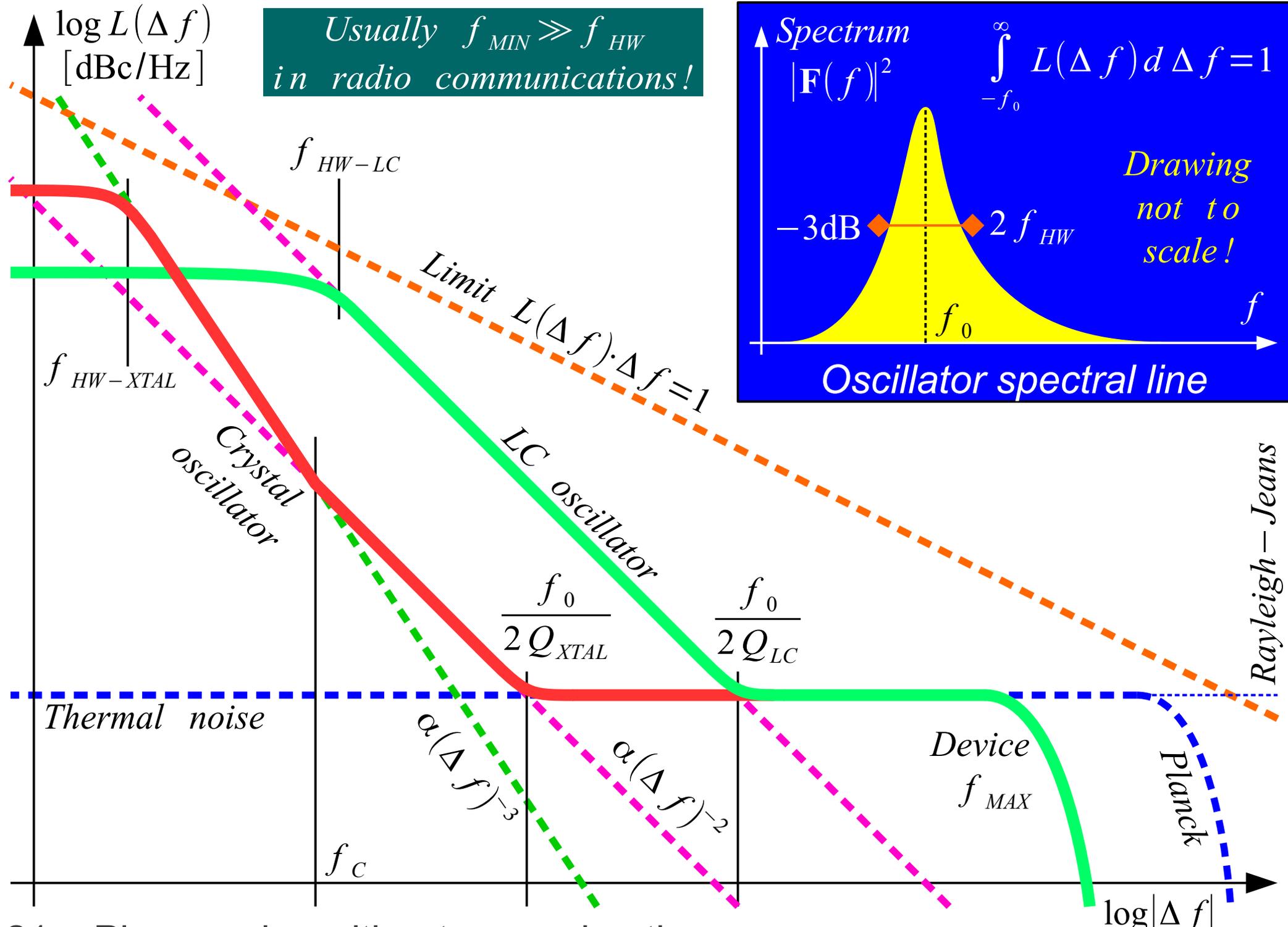


Adjacent-channel interference

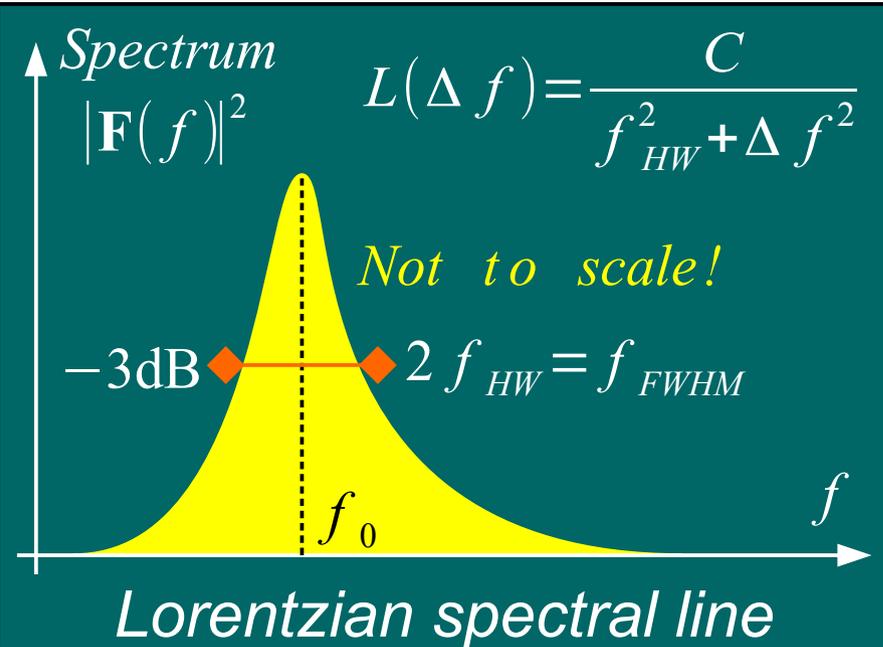
u(t)



Clock jitter



21 – Phase noise without approximations



Flat thermal noise can be neglected: device f_{MAX} or Planck law

LC-oscillator $1/f$ noise can be neglected

$$L(\Delta f) = \frac{1}{8} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{1}{f_{HW}^2 + \Delta f^2} \cdot \frac{k_B T_0 F}{P_0}$$

Lorentzian line in Leeson's equation

$$\int_{-f_0}^{\infty} L(\Delta f) d\Delta f = 1 \approx \int_{-\infty}^{\infty} L(\Delta f) d\Delta f = \frac{1}{8} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{k_B T_0 F}{P_0} \int_{-\infty}^{\infty} \frac{1}{f_{HW}^2 + \Delta f^2} d\Delta f =$$

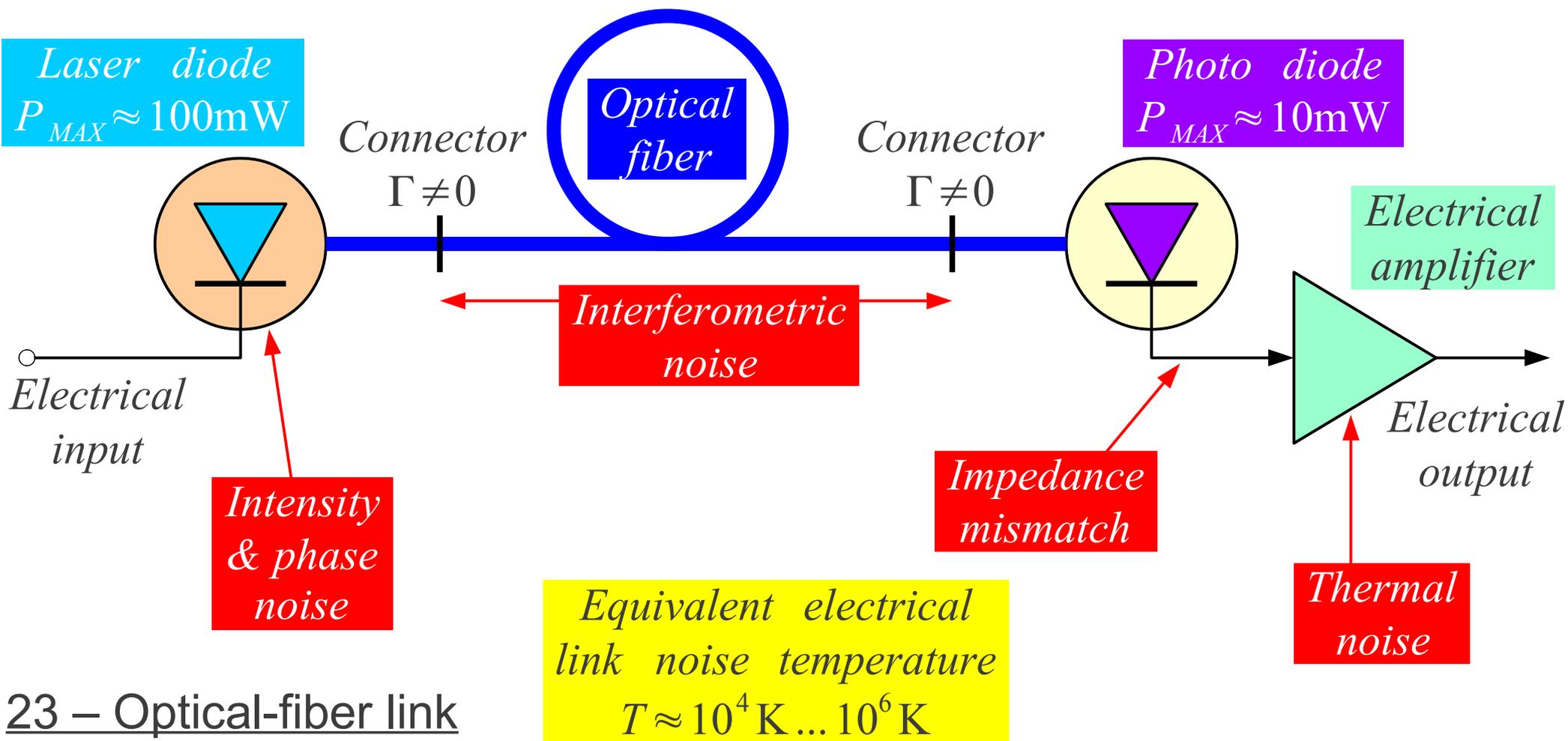
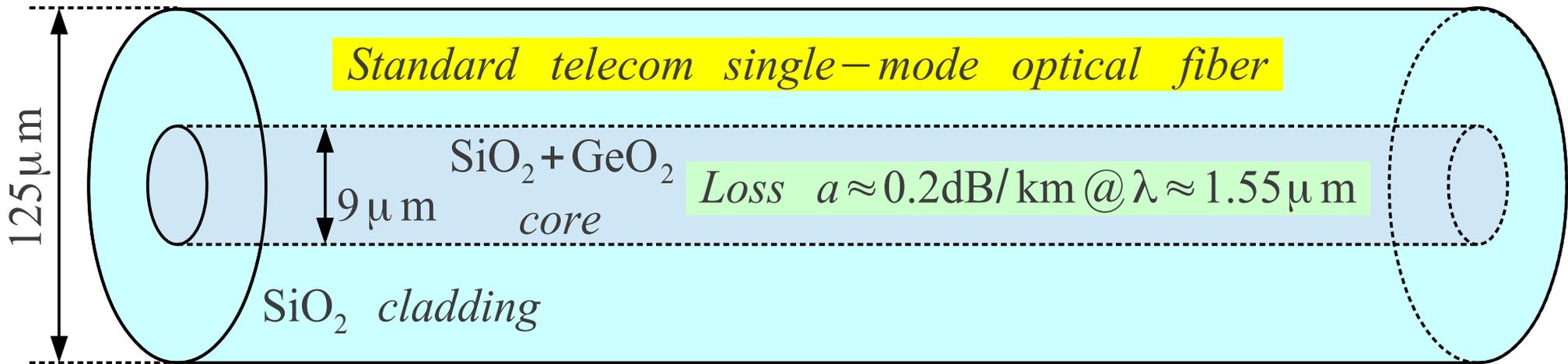
$$= \frac{1}{8} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{k_B T_0 F}{P_0} \cdot \left[\frac{1}{f_{HW}} \cdot \arctan \frac{\Delta f}{f_{HW}} \right]_{\Delta f = -\infty}^{\Delta f = \infty} = \frac{k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{\pi}{f_{HW}}$$

$$f_{HW} = \frac{\pi k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L} \right)^2$$

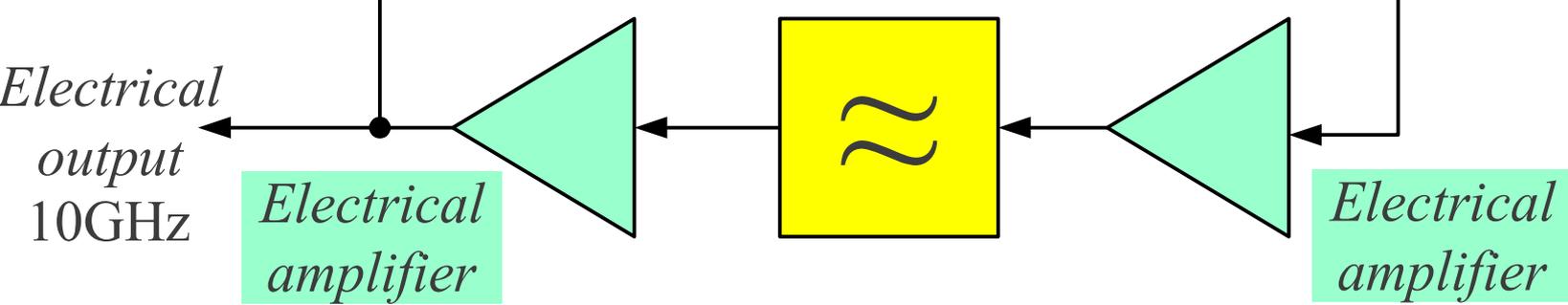
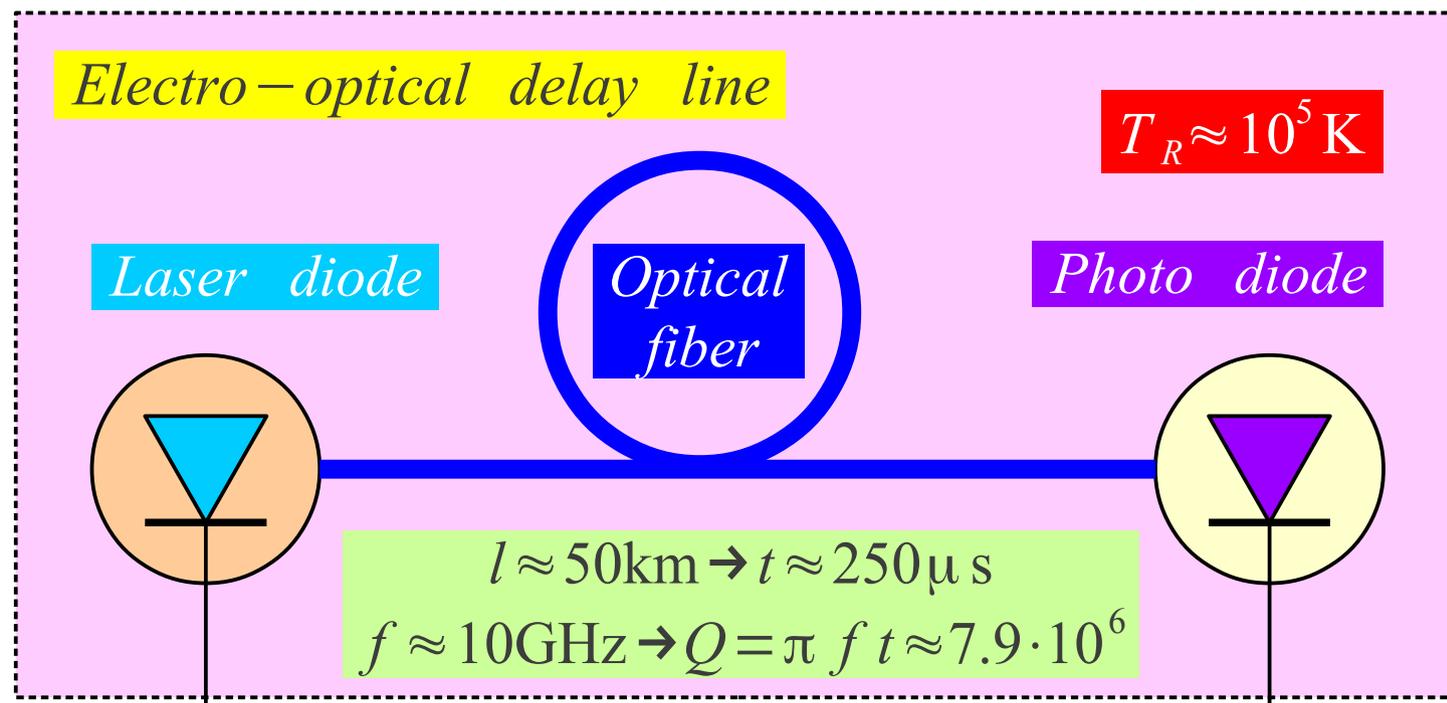
Example $f_0 = 3\text{GHz}$ $Q_L = 10$
 $P_0 = 0.1\text{mW}$ $F = 10\text{dB}$
 $f_{HW} = 14\text{Hz}$ $f_{FWHM} = 28\text{Hz}$

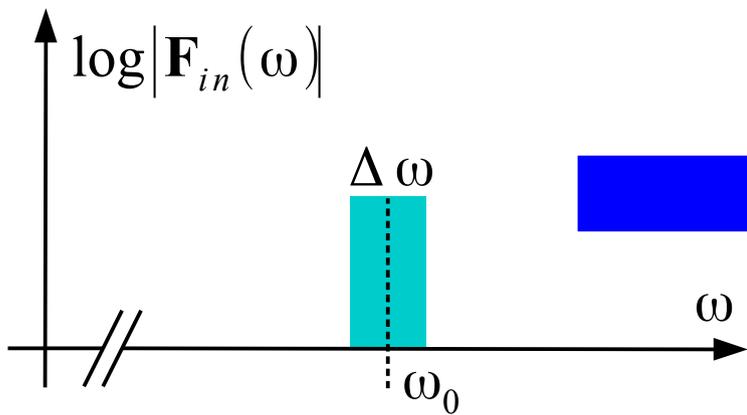
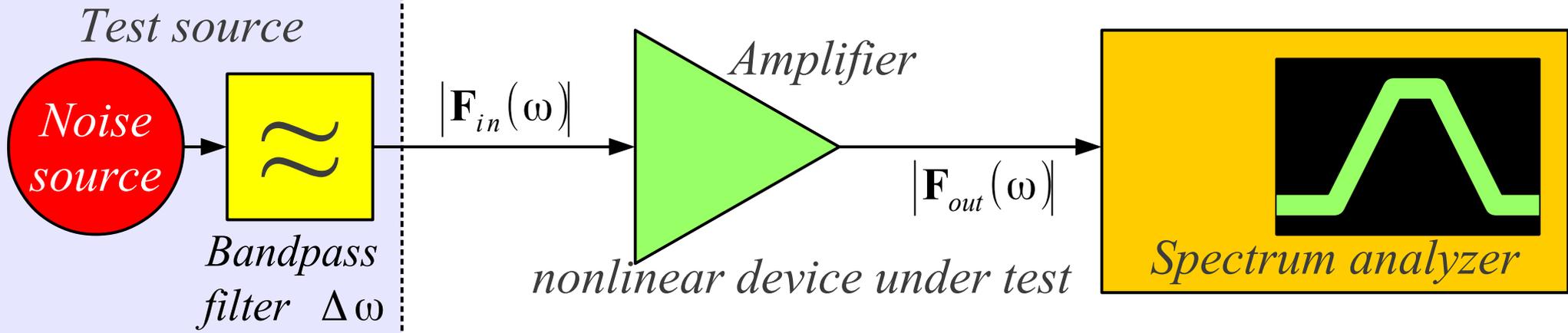
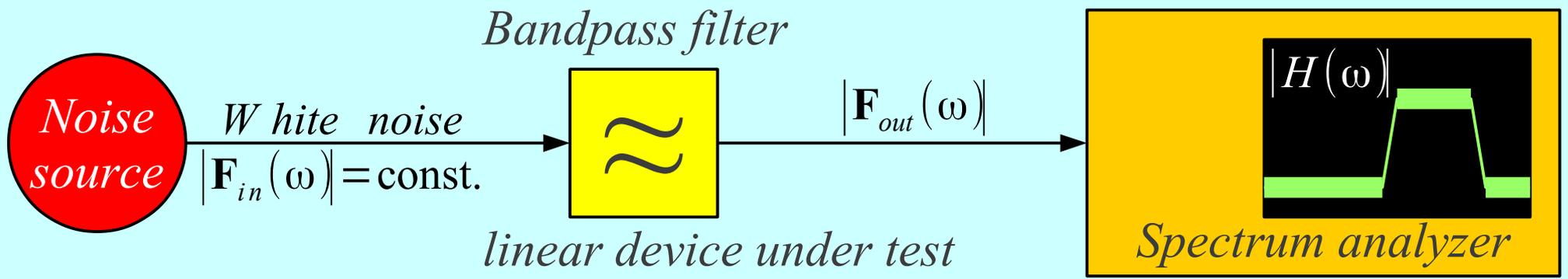
$$C = \frac{k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L} \right)^2 = \frac{f_{HW}}{\pi}$$

$$L(\Delta f) = \frac{f_{HW} / \pi}{f_{HW}^2 + \Delta f^2}$$

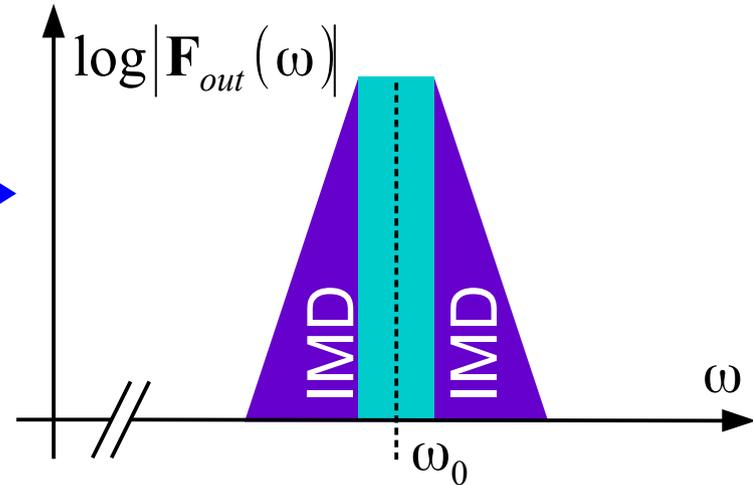


$$t_g = \frac{2Q}{\omega}$$

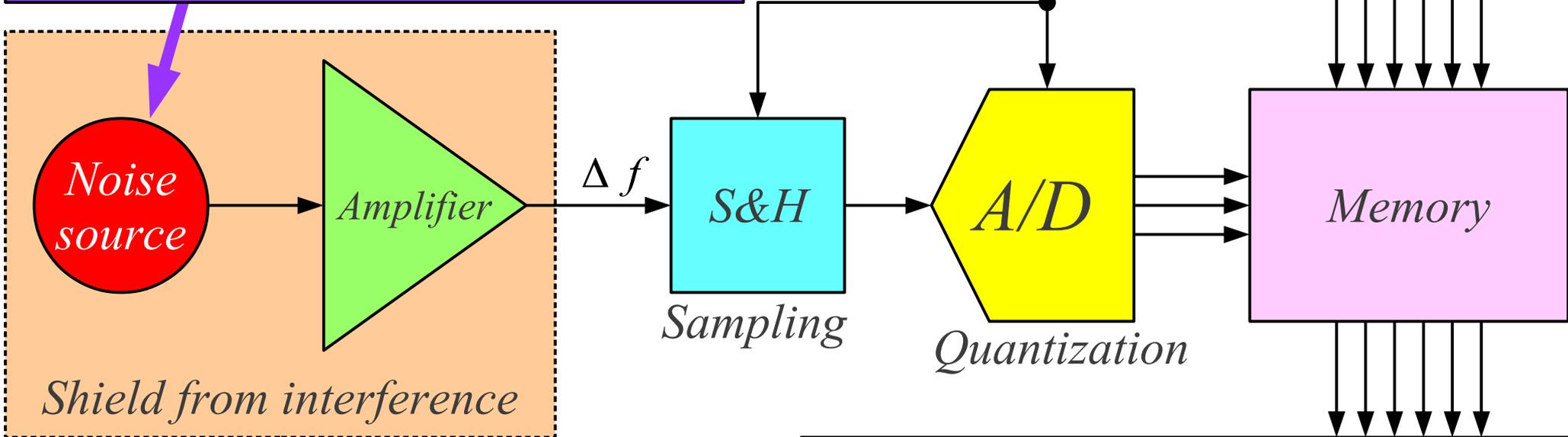




Nonlinearity

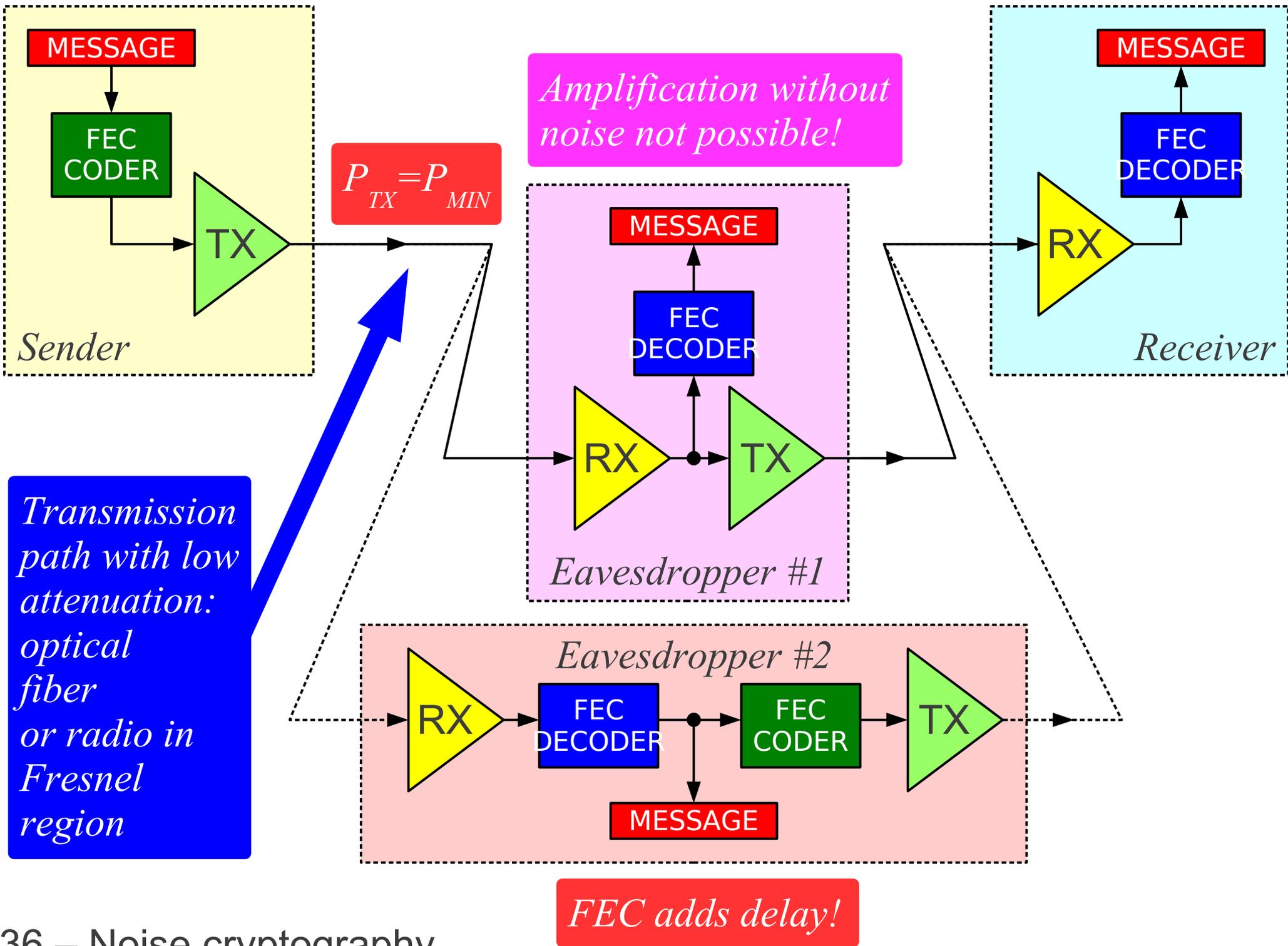


Natural sources of random signals:
Thermal noise
Shot noise
Avalanche breakdown
Radioactive decay
...

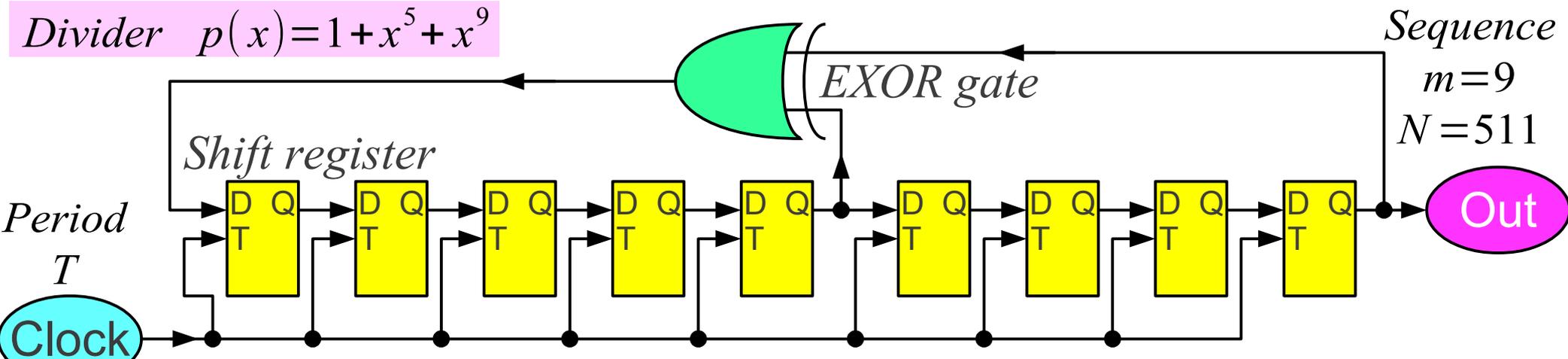


*Interference is not random!
Interference might be intentional!*

Arbitrary-length cryptographic key:
Password (rather short key...)
Keys for DES, AES etc
One-time pad
(very long but unbreakable key!)



Divider $p(x) = 1 + x^5 + x^9$



LFSR \equiv Linear-Feedback Shift Register

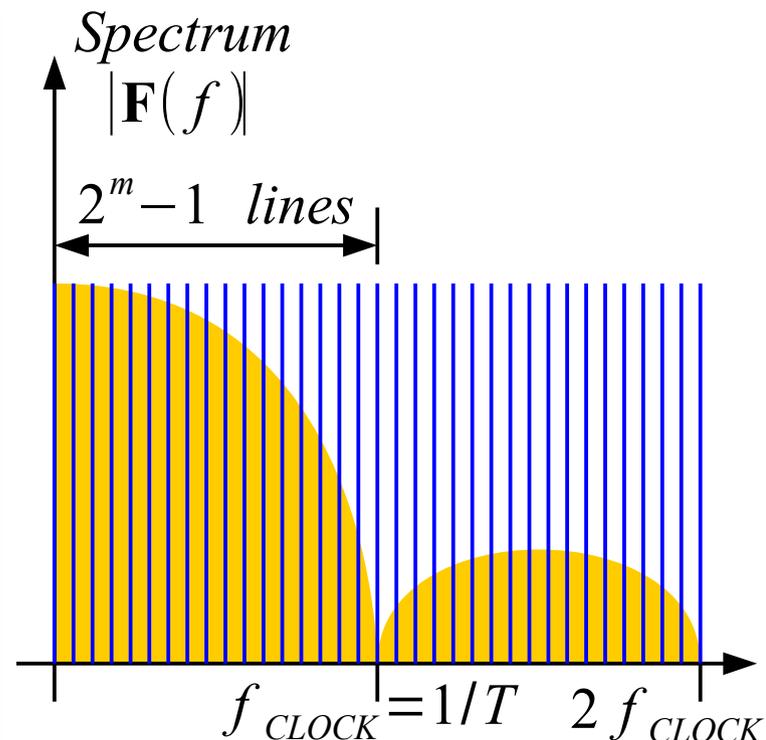
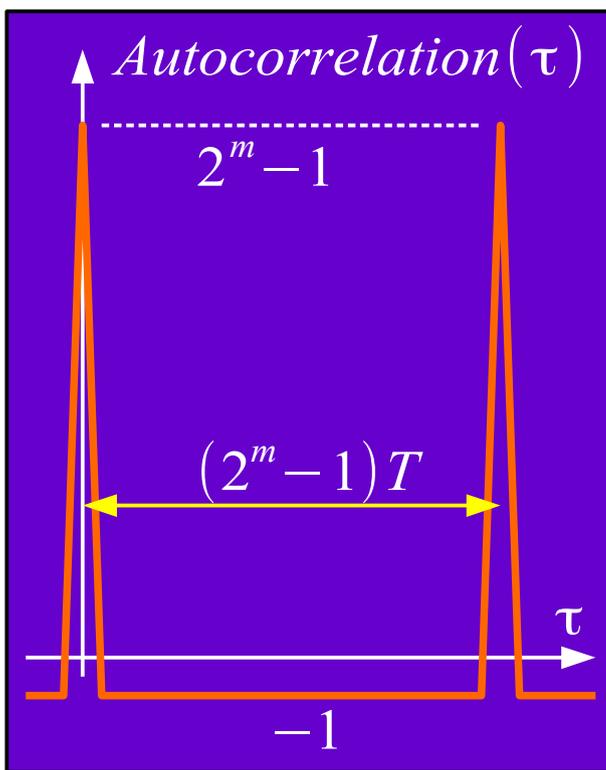
Primitive polynomial $p(x) = 1 + x^l + x^m \rightarrow$ max sequence length $N = 2^m - 1$

2^{m-1} ones and $2^{m-1}-1$ zeros arranged in groups of

- 1X m ones, m-1 zeros
- 1X m-2 ones and zeros
- 2X m-3 ones and zeros
- 4X m-4 ones and zeros

.....

- 2^{m-5} groups 111 and 000
- 2^{m-4} groups 11 and 00
- 2^{m-3} individual 1 and 0



Two-valued autocorrelation with a single very pronounced peak:

- synchronization headers for data frames
- spreading sequences in CDMA
- accurate time transfer in radio navigation (GPS, GLONASS)

Perfect frequency spectrum of uniformly-spaced lines and simple generation/checking:

- test sequences for all kinds of telecommunication links
- data scrambling (randomization) as part of line coding

Peak-to-average power ratio:

$$LFSR: \frac{P_{MAX}}{\langle P \rangle} \approx 1 \quad \text{Noise: } \frac{P_{MAX}}{\langle P \rangle} \rightarrow \infty$$

LFSR pseudo-random sequences are of NO cryptographic value: algorithm Berlekamp-Massey 1969

LFSR sequences are the result of pure mathematics that does not appear anywhere else in nature!

*How to present ourselves to the inhabitants of a neighbor galaxy?
How to find out that they look for us?*