Elektromagnetika 2.kolokvij 20.1.1999, rešitve:

1) $r_0 = 1m \quad d = 30m \quad I = 10A \quad {}_{\Delta}V = ?$

$A_m = I_{\Delta}\phi = -_{\Delta}W \quad d << r_0$

$M_{12} = \frac{\phi_{12}}{I_L} \qquad {}_{\Delta}\phi = -2\phi_D$

$M = \frac{\pi\mu_0 N_1 N_2 a^2 a^2}{2d^3} = \frac{\pi\, 4\pi 10^{-7} 1H}{2\cdot 30^3} = 7{,}31\cdot 10^{-11} H$

$\Delta W = I\cdot 2\phi_D = I^2 2M = \underline{\underline{1{,}462\cdot 10^{-8} Ws}}$

2) $r_o = 1mm \quad l = 10m \quad f = 1MHz \quad \gamma_{cu} = 56\cdot 10^6 S/m$

$\delta << r_0 \quad in \quad l << \lambda \qquad \frac{R_{\sim}}{R_{=}} = ?$

$R_{=} = \frac{l}{\gamma A} = \frac{l}{\gamma\pi r_0} = \frac{10\Omega}{56\cdot 10^{-6}\pi(1\cdot 10^{-3})^2} = 56{,}8\cdot 10^{-3}\Omega$

$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma}} = 67{,}25\cdot 10^{-6} m$

$R_{\sim} = \frac{l}{\gamma A_{\sim}} = \frac{l}{\gamma 2\pi r\delta} = 422{,}6\cdot 10^{-3}\Omega$

$\frac{R_{\sim}}{R_{=}} = \underline{\underline{7{,}44}}$

3) $W = 20mm \quad d = 5mm \quad f = 100MHz \quad \vec{H} = \vec{1}_x H_0 e^{-jk_0 z} \quad H_0 = 100A/m$

$\vec{E} = ? \quad \vec{S} = ? \quad \vec{P} = ? \quad \vec{U} = ? \quad \vec{I} = ?$

$$\vec{E} = \frac{1}{j\omega\varepsilon_0} rot\vec{H} = \frac{1}{j\omega\varepsilon_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_0 e^{-jk_0 z} & 0 & 0 \end{vmatrix} = \frac{1}{j\omega\varepsilon_0}(\vec{1}_y H_0 e^{-jk_0 z}(-jk)) = -\frac{H_0 k}{\omega\varepsilon_0}\vec{1}_y e^{-jk_0 z} =$$

$$-\vec{1}_y H_0 Z_0 e^{-jk_0 z} = -\vec{1}_y E_0 e^{-jk_0 z} = \underline{\underline{-\vec{1}_y 37699\frac{V}{m}e^{-jk_0 z}}}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 0 & -H_0 Z_0 e^{-jk_0 z} & 0 \\ H_0^* e^{+jk_0 z} & 0 & 0 \end{vmatrix} = \vec{1}_z\frac{Z_0 H_0^2}{2} = \vec{1}_z S_0 = \underline{\underline{\vec{1}_z 1{,}88M\frac{W}{m^2}}}$$

$$P = \int_A S\,dA = \int_0^w\int_0^d \frac{Z_0 H_0^2}{2}dxdy = \frac{Z_0 H_0^2}{2}wd = \underline{\underline{188{,}496W}}$$

$$\vec{K} = \vec{1}_n\times\vec{H}\Big|_{y=0} = \vec{1}_y\times\vec{1}_x H_0 e^{-jk_0 z} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 0 & 1 & 0 \\ H_0 e^{-jkoz} & 0 & 0 \end{vmatrix} = -\vec{1}_z H_0 e^{-jk_0 z} = \underline{\underline{-\vec{1}_z 100\frac{A}{m}e^{-jk_0 z}}}$$

$$I = \int_o^w \vec{K}\vec{ds} = \int_o^w H_0 e^{-jk_0 z}dx = H_0 w e^{-jk_0 z} = I_0 e^{-jk_0 z} = \underline{\underline{\pm 2Ae^{-jk_0 z}}} \quad \to \pm\frac{gornja\ \ plošča}{spodnja\ \ plošča}$$

$$U = -\int\vec{E}\vec{ds} = -\int_0^d(-\vec{1}y)\frac{H_0 k}{\omega\varepsilon_0}e^{-jk_0 z}\vec{1}y\,dy = \frac{H_0 k}{\omega\varepsilon_0}de^{-jk_0 z} = H_0 Z_0 d e^{-jk_0 z} = U_0 e^{-jk_0 z} = \underline{\underline{188{,}4V\,e^{-jk_0 z}}}$$

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4) $a = 1cm \quad b = 2cm \quad c = 3cm \quad f_{MIN} = 1GHz \quad \varepsilon_r = ?$

$$f = \frac{1}{2\sqrt{\varepsilon_0 \varepsilon_r \mu_0}} = \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$f_{\min} = f_{011} \quad m, n, p \rightarrow$ *cela števila, vsaj 2 različna od 0*

$$f^2 = \frac{1}{4\varepsilon_0 \varepsilon_r \mu_0}\left[\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2\right]$$

poiščemo 2 največja izmed a, b, c

$$\varepsilon_r = \frac{c_o^2}{4f}\left(\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2\right) =$$

$$\frac{3*10^8}{4(1*10^9)^2}\left[\left(\frac{1}{0,02}\right)^2 + \left(\frac{1}{0,03}\right)^2\right] = \underline{\underline{81,176}}$$

5) $\varepsilon_r = 2 \quad f = 10GHz \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \rightarrow Z = \sqrt{\frac{\mu_0}{\varepsilon}} = \frac{Z_0}{\sqrt{\varepsilon_r}}$

$$\vec{E}_{1+} = \vec{1}_x A_{1+} e^{-jk_0 z} \quad \vec{H}_{1+} = \frac{j}{\omega\mu_0} rot \vec{E}_{1+} = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_{1+}e^{-jk_0 z} & 0 & 0 \end{vmatrix} = \vec{1}_y \frac{A_{1+}}{Z_0} e^{-jk_0 z}$$

$$\vec{E}_{1-} = \vec{1}_x A_{1-} e^{jk_0 z} \quad \vec{H}_{1-} = -\vec{1}_y \frac{A_{1-}}{Z_0} e^{jk_0 z} \quad \vec{E}_{2+} = \vec{1}_x A_{2+} e^{-jk_0 z}$$

$$\vec{H}_{2+} = \vec{1}_y \frac{A_{2+}}{Z} e^{-jk_0 z} \quad \rightarrow \vec{1}_{E_{1-}} \times \vec{1}_{H_{1-}} = -\vec{1}_z \ \textit{odbiti val}$$

pri $z = 0: \ \vec{E}_{1+} + \vec{E}_{1-} = \vec{E}_{2+} \rightarrow A_{1+} + A_{1-} = A_{2+}$

$$\vec{H}_{1+} + \vec{H}_{1-} = \vec{H}_{2+} \rightarrow \frac{A_{1+}}{Z_0} - \frac{A_{1-}}{Z_0} = \frac{A_{2+}}{Z} \rightarrow A_{2+} = \frac{Z}{Z_0}(A_{1+} - A_{1-})$$

$$A_{1+} + A_{1-} = \frac{Z}{Z_0}(A_{1+} - A_{1-}) \rightarrow A_{1-} = \frac{Z - Z_0}{Z + Z_0} A_{1+} = -\frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} A_{1+}$$

$$\Gamma = \frac{A_{1-}}{A_{1+}} = -\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = -0,172 \quad A_{2+} = A_{1+} + A_{1-} = (1 + \Gamma)A_{1+}$$

$$\vec{S}_{2+} = \vec{S}_2 = \frac{1}{2}\vec{E}_{2+} \times \vec{H}_{2+}^* = \frac{1}{2}\vec{1}_x (1+\Gamma) A_{1+} e^{-jkz} \times \vec{1}_y \frac{(1+\Gamma)A_{1+}^*}{Z} e^{jkz} = \vec{1}_z \frac{(1+\Gamma)^2 A_{1+}^2}{2Z} =$$

$$\underline{\underline{\vec{1}_z 0,515 \frac{W}{m^2}}}$$

$$\vec{S}_1 = \frac{1}{2}(\vec{E}_{1+} + \vec{E}_{1-}) \times (\vec{H}_{1+} + \vec{H}_{1-})^* = \frac{1}{2}\vec{1}_x (A_{1+} e^{jk_0 z} + \Gamma A_{1+} e^{jk_0 z}) \times \vec{1}_y \left(\frac{A_{1+}}{Z_0} e^{jk_0 z} - \frac{\Gamma A_{1+}}{Z_0} e^{-jk_0 z}\right) =$$

$$\vec{1}_z \frac{A_{1+}^2}{2Z_0}(e^{-jk_0 z} + \Gamma e^{jk_0 z})(e^{jk_0 z} + \Gamma e^{-jk_0 z}) = \vec{1}_z \frac{|A_{1+}^2|}{Z_0}(1 - |\Gamma|^2 + j2\Gamma \sin 2k_0 z) =$$

$$\underline{\underline{\vec{1}_z 0,531 \frac{W}{m^2}(0,971 + j0,343 \sin 2k_0 z)}}$$