Elektromagnetika 1.kolokvij 9.12.1998, rešitve

1) $\vec{E} = \vec{1}_\theta C \dfrac{e^{-jkr}}{r\sin\theta} \qquad h_r = 1 \qquad h_\theta = r \qquad h_\phi = r\sin\theta$

Prostorske veličine $\rightarrow$ *izračunamo iz Maxwellovih enačb*

$$\vec{H} = -\frac{1}{j\omega\mu_0} rot\vec{E} = -\frac{C}{j\omega\mu_0}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\phi} \\ 0 & \dfrac{e^{-jkr}}{sin\theta} & 0 \end{vmatrix} = \underline{\underline{\vec{1}_\phi \frac{C}{Z_0}\frac{e^{-jkr}}{rsin\theta}}}$$

$$rot\vec{H} = \frac{1}{r^2\sin\theta}\frac{C}{Z_0}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\phi} \\ 0 & 0 & e^{-jkr} \end{vmatrix} = \vec{1}_\theta jC\varepsilon_0\omega\frac{e^{-jkr}}{sin\theta}$$

$$\vec{J} = rot\vec{H} - j\omega\varepsilon_0\vec{E} = \vec{1}_\theta jC\varepsilon_0\omega\frac{e^{-jkr}}{r\,sin\theta} - \vec{1}_\theta jC\varepsilon_0\omega\frac{e^{-jkr}}{r\,sin\theta} = \underline{\underline{0}}$$

$$\rho = div(\varepsilon\vec{E}) = C\varepsilon_0\frac{1}{r^2\sin\theta}\left(\frac{\partial}{\partial\theta} r\sin\theta\frac{e^{-jkr}}{rsin\theta}\right) = \underline{\underline{0}}$$

Ploskovne veličine

$\vec{K} = \underline{\underline{0}}$ *in* $\sigma = -\dfrac{1}{j\varpi} div\vec{K} = \underline{\underline{0}}$ $\rightarrow$ *Ni singularnih ploskev*

3. *Preme veličine, singularnost pri* $\theta = 0, \pi$ *na osi z različno za* $+/-z$

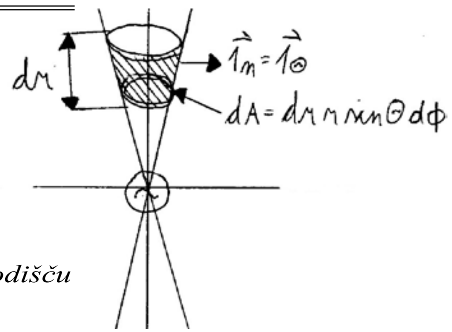
$$I = \oint \vec{H}\,\vec{ds} = \frac{C}{Z_0}\int_0^{2\pi}\frac{e^{-jkr}}{rsin\theta} r\sin\theta\, d\phi = \underline{\underline{\frac{C}{Z_0}2\pi e^{-jkr}}}$$

$$dQ = \oint \vec{D}\,\vec{dA}$$

$$dQ = \int_0^{2\pi}\vec{1}_\theta\varepsilon_0 C\frac{e^{-jkr}}{rsin\theta} dr\,\vec{1}_\theta r\sin\theta d\phi$$

$$dQ = \varepsilon_0 C\frac{e^{-jkr}}{1} dr\, 2\pi = q\, dr$$

$$q = \underline{\underline{2\pi\varepsilon_0 C e^{-jkr}}}$$



4. *Točkasta veličina v koordinatnem izhodišču*

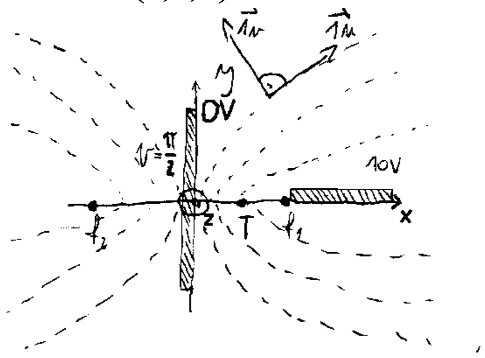
$$Q\big|_{r=0} = \int_{k\rightarrow 0}\vec{D}\,\vec{dA} = \int_v div\vec{D}\,dv = \underline{\underline{0}}$$

2) *Valjni eliptični koordinatni sistem* (u, v, z)

$x = f\cosh u\cos v$

$y = f\sinh u\sin v$

$z = z$



$\Delta V = 0 \qquad \dfrac{\partial}{\partial u} = 0 \qquad \dfrac{\partial}{\partial z} = 0$

$hu = f\sqrt{\sinh u^2 + \sin v^2}$

$hv = f\sqrt{\sinh u^2 + \sin v^2}$

$hz = 1$

$$\Delta V = div(gradV) = \frac{1}{f^2(\sinh u^2 + \sin v^2)}\left[\frac{\partial^2 V}{\partial^2 u} + \frac{\partial^2 V}{\partial^2 v}\right] + \frac{\partial^2 V}{\partial^2 z} = 0$$

$$0 = \frac{1}{f^2(\sinh u^2 + \sin v^2)}\frac{\partial^2 V}{\partial^2 v} \rightarrow V = C_1 v + C_2$$

$$10V = C_1 0 + C_2 \rightarrow C_2 = 10V$$

$$0V = C_1\frac{\pi}{2} + C_2$$

$$C_1 = -\frac{2}{\pi}C_2 = -\frac{20V}{\pi}$$

$$V = -\frac{20V}{\pi}v + 10V$$

Točka T: $u = 0 \quad x = \dfrac{f}{2}$

$$\frac{f}{2} = f * 1 * \cos v \rightarrow v = \frac{\pi}{3}$$

$$V(T) = \frac{20V}{\pi}\frac{\pi}{3} + 10V = \underline{\underline{3,33V}}$$

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3) *Najprej določimo polje v levi polovici korita. V desni polovici je porazdelitev potenciala zrcalno simetrična*

$\Delta V = 0 \qquad V(a-x,y) = -V(x,y)$

$a)\ 0 \le x \le \frac{a}{2}$

$$V(x,y)=\sum_m C_m \sin\left(\frac{m\pi}{a/2}x\right)\sinh\left(\frac{m\pi}{a/2}y\right) \ \rightarrow\ V(x,y=b)=\sum_m C_m \sin\left(\frac{m\pi}{a/2}x\right)\sinh\left(\frac{m\pi}{a/2}b\right)=V_0$$

S Fourierjevo analizo določimo koeficiente C_m

$$\int_0^{a/2} V_0 \sin\left(\frac{n\pi}{a/2}x\right)dx+\int_{a/2}^{a}(-V_0)\sin\left(\frac{n\pi}{a/2}x\right)dx=\int_0^a\sum_m\left(C_m \sin\left(\frac{m\pi}{a/2}x\right)\sinh\left(\frac{m\pi}{a/2}b\right)\right)\sin\left(\frac{n\pi}{a/2}x\right)dx$$

$$\left(\frac{V_0}{2m\pi}\left(2\sin\left(\frac{\pi m}{2}\right)^2-(\cos(m\pi)-\cos(2m\pi))\right)\right)=C_m \sinh\left(\frac{m\pi}{a}b\right)\frac{a}{8}\left(\frac{4m\pi-\sin(4m\pi)}{m\pi}\right)$$

$$\frac{2V_0 a}{m\pi}=\frac{a}{2}C_m \sinh\left(\frac{m\pi}{a/2}b\right) \qquad \rightarrow C_m=\begin{cases}0 & \text{če je } m \text{ sodo}\\ \frac{4V_0}{m\pi}\sinh\left(\frac{m\pi}{a/2}b\right)^{-1} & \text{če je } m \text{ liho}\end{cases}$$

$b)$ *Upoštevamo zrcalno simetrijo, saj je* $\sin\left(\frac{(2k+1)\pi(a-x)}{a/2}\right)=-\sin\left(\frac{(2k+1)\pi x}{a/2}\right)$

$$\underline{\underline{V(x,y)=\sum_{k=0}^{\infty}\frac{4V_0}{(2k+1)\pi}\sinh\left(\frac{(2k+1)\pi}{a/2}b\right)^{-1}\sin\left(\frac{(2k+1)\pi}{a/2}x\right)\sinh\left(\frac{(2k+1)\pi}{a/2}y\right)}}$$

4) $\Delta V(r,\theta,\Phi)=0 \qquad \frac{\partial}{\partial\Phi}=0 \qquad V(\infty)\rightarrow\infty$

$$V(r,\theta)=\sum_{n=0}^{\infty}B_n r^{-(n+1)}P_n(\cos\theta) \qquad za\ \ r>r_0$$

$$B_n=\frac{1}{r_0{}^n}\frac{\int_{-1}^{1}V(\cos\theta)P_n(\cos\theta)\,d\cos\theta}{\int_{-1}^{1}P_n{}^2(\cos\theta)\,d\cos\theta} \qquad V(\cos\theta)=\begin{cases}+V_0;\ 1>\cos\theta>0\\ -V_0;\ 0>\cos\theta\ge-1\end{cases}$$

$$B_n=\frac{1}{r_0{}^n}\frac{-V_0\int_{-1}^{0}P_n(t)\,dt+V_0\int_{0}^{1}P_n(t)\,dt}{\int_{-1}^{1}P_n{}^2(t)\,dt} \quad ; \quad t=\cos\theta \ \rightarrow B_n=\frac{2n+1}{n(n+1)a^n}V_0P_n(0)$$

Koeficienti $B_n:\ B_{2K}=0 \qquad K=1,2,\ldots$

$B_{2K-1}=V_0r_0{}^{2K}\left[P_{2K}(0)-P_{2K-2}(0)\right]$

Funkcija potenciala za $r>r_0$

$$V(r,\theta)=V_0\sum_{K=1}^{\infty}\frac{P_{2K}(0)-P_{2K-2}(0)}{r^{2K}}r_0{}^{2K}P_{2K-1}(\cos\theta)$$

Poenostavimo za $r>>r_0$

$za\ K=1 \quad V(r,\theta)=V_0\frac{3}{2}\frac{r_0{}^2}{r^2}\cos\theta$

$za\ K=2 \quad V(r,\theta)=V_0\frac{7}{8}\frac{r_0{}^4}{r^4}(5\cos\theta^3-3\cos\theta)$

$za\ K=3 \quad V(r,\theta)=V_0\frac{3}{16}\frac{r_0{}^6}{r^6}(63\cos\theta^5-70\cos\theta^3+15\cos\theta)$

$\underline{\underline{V(r,\theta)=V_0\frac{3}{2}\frac{r_0{}^2}{r^2}\cos\theta}} \qquad za\ r>>r_0$

5) $\vec{E}=-\vec{1}_y\frac{IRp}{a} \qquad V=\frac{IRp}{a}y+V_0$

$P=\gamma E^2 \quad P_1=2P_2 \quad \gamma E_1{}^2=2\gamma E_2{}^2 \qquad E_2=\frac{E_1}{\sqrt{2}}$

Nastavek:

$V=(A\rho+B\rho^{-1})\sin\varphi \rightarrow V=A\rho\sin\varphi\approx\frac{IRp}{a}y \qquad A=\frac{IRp}{a}$

$\rho=r\rightarrow V=0=(A\rho+B\rho^{-1})\sin\varphi\Big|_{\rho=r}$

$B=-Ar^2=\frac{IRp}{a}r^2$

$\vec{K}_0=-\vec{1}_y\frac{I}{W} \qquad E_0=\vec{K}_0\,Rp=\vec{1}_y\frac{IRp}{a} \qquad V=\frac{IRp}{a}(\rho-\frac{r^2}{\rho})\sin\varphi$

$V_1=(-E_0\rho+\frac{B}{\rho})\sin\varphi \qquad V_2=A\rho\sin\varphi$

$\vec{E}_1=-grad\,V_1=\vec{1}_\rho(E_0+\frac{B_1}{\rho^2})\sin\varphi+\vec{1}_\varphi(E_0-\frac{B1}{\rho^2})\cos\varphi$

$\vec{E}_2=-grad\,V_2=-\vec{1}_\rho A\sin\varphi-\vec{1}_\varphi A\cos\varphi=-\vec{1}_y A$

$$\text{Prestopni pogoji}:\begin{cases}E_{\varphi1}=E_{\varphi2}\rightarrow(E_0-\frac{B_1}{r_0{}^2})\cos\varphi=-A_2\cos\varphi\rightarrow E_0-\frac{B_1}{r_0{}^2}=-A\\ K_{\rho1}=K_{\rho2}\rightarrow(E_0+\frac{B_1}{r_0{}^2})\sin\varphi\frac{1}{R_{p1}}=\frac{A_2}{R_{p2}}\sin\varphi\rightarrow E_0+\frac{B_1}{r_0{}^2}=-A\frac{R_{p1}}{R_{p2}}\end{cases}$$

$\vec{E}_1=\vec{1}_\rho(-A\frac{R_{p1}}{R_{p2}})\sin\varphi+\vec{1}_\varphi(-A)\cos\varphi=-\vec{1}_\rho A\frac{R_{p1}}{R_{p2}}\sin\varphi-\vec{1}_\varphi A\cos\varphi$

$\vec{E}_2=-\vec{1}_\rho A\sin\varphi-\vec{1}_\varphi A\cos\varphi$

$2E_0=-A(1+\frac{R_{p1}}{R_{p2}})=E_2(1+\frac{R_{p1}}{R_{p2}})$

$$\frac{2E_0}{E_2}=1+\frac{R_{p1}}{R_{p2}}\rightarrow R_{p2}=R_{p1}\frac{1}{\frac{2E_0}{E_0}-1}=\frac{100\Omega}{\frac{2E_1\sqrt{2}}{E_1}-1}=\underline{\underline{54,69\Omega}}$$