Finite-difference time-domain method

Finite-difference time-domain or **Yee's method** (named after the Chinese American applied mathematician Kane S. Yee, born 1934) is a numerical analysis technique used for modeling computational electrodynamics (finding approximate solutions to the associated system of differential equations). Since it is a time-domain method, FDTD solutions can cover a wide frequency range with a single simulation run, and treat nonlinear material properties in a natural way.

The FDTD method belongs in the general class of grid-based differential numerical modeling methods (finite difference methods). The time-dependent Maxwell's equations (in partial differential form) are discretized using central-difference approximations to the space and time partial derivatives. The resulting finite-difference equations are solved in either software or hardware in a leapfrog manner: the electric field vector components in a volume of space are solved at a given instant in time; then the magnetic field vector components in the same spatial volume are solved at the next instant in time; and the process is repeated over and over again until the desired transient or steady-state electromagnetic field behavior is fully evolved.

Contents

History

Finite difference schemes for time-dependent partial differential equations (PDEs) have been

employed for many years in computational fluid dynamics problems,[1] including the idea of using centered finite difference operators on staggered grids in space and time to achieve second-order accuracy.[1] The novelty of Kane Yee's FDTD scheme, presented in his seminal 1966 paper,[2] was to apply centered finite difference operators on staggered grids in space and time for each electric and magnetic vector field component in Maxwell's curl equations. The descriptor "Finite-difference time-domain" and its corresponding "FDTD" acronym were originated by Allen Taflove in 1980.[3] Since about 1990, FDTD techniques have emerged as primary means to computationally model many scientific and engineering problems dealing with electromagnetic wave interactions with material structures. Current FDTD modeling applications range from near-DC (ultralow-frequency geophysics involving the entire Earth-ionosphere waveguide) through microwaves (radar signature technology, antennas, wireless communications devices, digital interconnects, biomedical imaging/treatment) to visible light (photonic crystals, nanoplasmonics, solitons, and biophotonics).[4] In 2006, an estimated 2,000 FDTD-related publications appeared in the science and engineering literature (see Popularity). As of 2013, there are at least 25 commercial/proprietary FDTD software vendors; 13 free-software/open-source-software FDTD projects; and 2 freeware/closed-source FDTD projects, some not for commercial use (see External links).

Development of FDTD and Maxwell's equations

An appreciation of the basis, technical development, and possible future of FDTD numerical techniques for Maxwell's equations can be developed by first considering their history. The following lists some of the key publications in this area.

Partial chronology of FDTD techniques and applications for Maxwell's equations.[5]	
year	event
1928	Courant, Friedrichs, and Lewy (CFL) publish seminal paper with the discovery of conditional stability of explicit time-dependent finite difference schemes, as well as the classic FD scheme for solving second-order wave equation in 1-D and 2-D.[6]
1950	First appearance of von Neumann's method of stability analysis for implicit/explicit time-dependent finite difference methods.[7]
1966	Yee described the FDTD numerical technique for solving Maxwell's curl equations on grids staggered in space and time.[2]
1969	Lam reported the correct numerical CFL stability condition for Yee's algorithm by employing von Neumann stability analysis.[8]
1975	Taflove and Brodwin reported the first sinusoidal steady-state FDTD solutions of two- and three-dimensional electromagnetic wave interactions with material structures;[9] and the first bioelectromagnetics models.[10]
1977	Holland and Kunz & Lee applied Yee's algorithm to EMP problems.[11][12]
1980	Taflove coined the FDTD acronym and published the first validated FDTD models of sinusoidal steady-state electromagnetic wave penetration into a three-dimensional metal cavity.[3]
1981	Mur published the first numerically stable, second-order accurate, absorbing boundary condition (ABC) for Yee's grid.[13]
1982–83	Taflove and Umashankar developed the first FDTD electromagnetic wave scattering models computing sinusoidal steady-state near-fields, far-fields, and radar cross-section for two- and three-dimensional structures.[14][15]
1984	Liao *et al* reported an improved ABC based upon space-time extrapolation of the field adjacent to the outer grid boundary.[16]
1985	Gwarek introduced the lumped equivalent circuit formulation of FDTD.[17]
1986	Choi and Hoefer published the first FDTD simulation of waveguide structures.[18]
1987–88	Kriegsmann *et al* and Moore *et al* published the first articles on ABC theory in *IEEE Transactions on Antennas and Propagation*.[19][20]
1987–88, 1992	Contour-path subcell techniques were introduced by Umashankar *et al* to permit FDTD modeling of thin wires and wire bundles,[21] by Taflove *et al* to model penetration through cracks in conducting screens,[22] and by Jurgens *et al* to conformally model the surface of a smoothly curved scatterer.[23]

1988	Sullivan et al published the first 3-D FDTD model of sinusoidal steady-state electromagnetic wave absorption by a complete human body.[24]
1988	FDTD modeling of microstrips was introduced by Zhang et al.[25]
1990–91	FDTD modeling of frequency-dependent dielectric permittivity was introduced by Kashiwa and Fukai,[26] Luebbers et al,[27] and Joseph et al.[28]
1990–91	FDTD modeling of antennas was introduced by Maloney et al,[29] Katz et al,[30] and Tirkas and Balanis.[31]
1990	FDTD modeling of picosecond optoelectronic switches was introduced by Sano and Shibata,[32] and El-Ghazaly et al.[33]
1992–94	FDTD modeling of the propagation of optical pulses in nonlinear dispersive media was introduced, including the first temporal solitons in one dimension by Goorjian and Taflove;[34] beam self-focusing by Ziolkowski and Judkins;[35] the first temporal solitons in two dimensions by Joseph et al;[36] and the first spatial solitons in two dimensions by Joseph and Taflove.[37]
1992	FDTD modeling of lumped electronic circuit elements was introduced by Sui et al.[38]
1993	Toland et al published the first FDTD models of gain devices (tunnel diodes and Gunn diodes) exciting cavities and antennas.[39]
1993	Aoyagi et al present a hybrid Yee algorithm/scalar-wave equation and demonstrate equivalence of Yee scheme to finite difference scheme for electromagnetic wave equation.[40]
1994	Thomas et al introduced a Norton's equivalent circuit for the FDTD space lattice, which permits the SPICE circuit analysis tool to implement accurate subgrid models of nonlinear electronic components or complete circuits embedded within the lattice.[41]
1994	Berenger introduced the highly effective, perfectly matched layer (PML) ABC for two-dimensional FDTD grids,[42] which was extended to three dimensions by Katz et al,[43] and to dispersive waveguide terminations by Reuter et al.[44]
1994	Chew and Weedon introduced the coordinate stretching PML that is easily extended to three dimensions, other coordinate systems and other physical equations.[45]
1995–96	Sacks et al and Gedney introduced a physically realizable, uniaxial perfectly matched layer (UPML) ABC.[46][47]
1997	Liu introduced the pseudospectral time-domain (PSTD) method, which permits extremely coarse spatial sampling of the electromagnetic field at the Nyquist limit.[48]
1997	Ramahi introduced the complementary operators method (COM) to implement highly effective analytical ABCs.[49]

1998	Maloney and Kesler introduced several novel means to analyze periodic structures in the FDTD space lattice.[50]
1998	Nagra and York introduced a hybrid FDTD-quantum mechanics model of electromagnetic wave interactions with materials having electrons transitioning between multiple energy levels.[51]
1998	Hagness *et al* introduced FDTD modeling of the detection of breast cancer using ultrawideband radar techniques.[52]
1999	Schneider and Wagner introduced a comprehensive analysis of FDTD grid dispersion based upon complex wavenumbers.[53]
2000–01	Zheng, Chen, and Zhang introduced the first three-dimensional alternating-direction implicit (ADI) FDTD algorithm with provable unconditional numerical stability.[54][55]
2000	Roden and Gedney introduced the advanced convolutional PML (CPML) ABC.[56]
2000	Rylander and Bondeson introduced a provably stable FDTD - finite-element time-domain hybrid technique.[57]
2002	Hayakawa *et al* and Simpson and Taflove independently introduced FDTD modeling of the global Earth-ionosphere waveguide for extremely low-frequency geophysical phenomena.[58][59]
2003	DeRaedt introduced the unconditionally stable, "one-step" FDTD technique.[60]
2008	Ahmed, Chua, Li and Chen introduced the three-dimensional locally one-dimensional (LOD)FDTD method and proved unconditional numerical stability.[61]
2008	Taniguchi, Baba, Nagaoka and Ametani introduced a Thin Wire Representation for FDTD Computations for conductive media[62]
2009	Oliveira and Sobrinho applied the FDTD method for simulating lightning strokes in a power substation[63]
2010	Chaudhury and Boeuf demonstrated the numerical procedure to couple FDTD and Plasma fluid model for studying Microwave Plasma Interaction.[64]
2012	Moxley *et al* developed a generalized finite-difference time-domain quantum method for the N-body interacting Hamiltonian.[65]
2013	Moxley *et al* developed a generalized finite-difference time-domain scheme for solving nonlinear Schrödinger equations.[66]
2014	Moxley *et al* developed an implicit generalized finite-difference time-domain scheme for solving nonlinear Schrödinger equations.[67]

FDTD models and methods

When Maxwell's differential equations are examined, it can be seen that the change in the E-field in

time (the time derivative) is dependent on the change in the H-field across space (the curl). This results in the basic FDTD time-stepping relation that, at any point in space, the updated value of the E-field in time is dependent on the stored value of the E-field and the numerical curl of the local distribution of the H-field in space.[2]

The H-field is time-stepped in a similar manner. At any point in space, the updated value of the H-field in time is dependent on the stored value of the H-field and the numerical curl of the local distribution of the E-field in space. Iterating the E-field and H-field updates results in a marching-in-time process wherein sampled-data analogs of the continuous electromagnetic waves under consideration propagate in a numerical grid stored in the computer memory.

This description holds true for 1-D, 2-D, and 3-D FDTD techniques. When multiple dimensions are considered, calculating the numerical curl can become complicated. Kane Yee's seminal 1966 paper proposed spatially staggering the vector components of the E-field and H-field about rectangular unit cells of a Cartesian computational grid so that each E-field vector component is located midway between a pair of H-field vector components, and conversely.[2] This scheme, now known as a **Yee lattice**, has proven to be very robust, and remains at the core of many current FDTD software constructs.
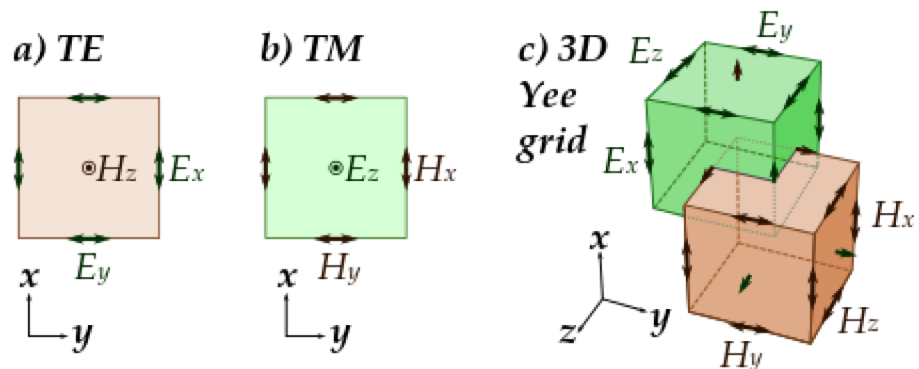

Illustration of a standard Cartesian Yee cell used for FDTD, about which electric and magnetic field vector components are distributed.[2] Visualized as a cubic voxel, the electric field components form the edges of the cube, and the magnetic field components form the normals to the faces of the cube. A three-dimensional space lattice consists of a multiplicity of such Yee cells. An electromagnetic wave interaction structure is mapped into the space lattice by assigning appropriate values of permittivity to each electric field component, and permeability to each magnetic field component.

Furthermore, Yee proposed a leapfrog scheme for marching in time wherein the E-field and H-field updates are staggered so that E-field updates are conducted midway during each time-step between successive H-field updates, and conversely.[2] On the plus side, this explicit time-stepping scheme avoids the need to solve simultaneous equations, and furthermore yields dissipation-free numerical wave propagation. On the minus side, this scheme mandates an upper bound on the time-step to ensure numerical stability.[9] As a result, certain classes of simulations can require many thousands of time-steps for completion.

Using the FDTD method

To implement an FDTD solution of Maxwell's equations, a computational domain must first be established. The computational domain is simply the physical region over which the simulation will be performed. The E and H fields are determined at every point in space within that computational domain. The material of each cell within the computational domain must be specified. Typically, the material is either free-space (air), <u>metal</u>, or <u>dielectric</u>. Any material can be used as long as the <u>permeability</u>, <u>permittivity</u>, and <u>conductivity</u> are specified.

The permittivity of dispersive materials in tabular form cannot be directly substituted into the FDTD scheme. Instead, it can be approximated using multiple Debye, Drude, Lorentz or critical point terms. This approximation can be obtained using open fitting programs[68] and does not necessarily have physical meaning.

Once the computational domain and the grid materials are established, a source is specified. The source can be current on a wire, applied electric field or impinging plane wave. In the last case FDTD can be used to simulate light scattering from arbitrary shaped objects, planar periodic structures at various incident angles,[69][70] and photonic band structure of infinite periodic structures.[71][72]

Since the E and H fields are determined directly, the output of the simulation is usually the E or H field at a point or a series of points within the computational domain. The simulation evolves the E and H fields forward in time.

Processing may be done on the E and H fields returned by the simulation. Data processing may also occur while the simulation is ongoing.

While the FDTD technique computes electromagnetic fields within a compact spatial region, scattered and/or radiated far fields can be obtained via near-to-far-field transformations.[14]

Strengths of FDTD modeling

Every modeling technique has strengths and weaknesses, and the FDTD method is no different.

- FDTD is a versatile modeling technique used to solve Maxwell's equations. It is intuitive, so users can easily understand how to use it and know what to expect from a given model.
- FDTD is a time-domain technique, and when a broadband pulse (such as a Gaussian pulse) is used as the source, then the response of the system over a wide range of frequencies can be obtained with a single simulation. This is useful in applications where resonant frequencies are not exactly known, or anytime that a broadband result is desired.
- Since FDTD calculates the E and H fields everywhere in the computational domain as they evolve in time, it lends itself to providing animated displays

of the electromagnetic field movement through the model. This type of display is useful in understanding what is going on in the model, and to help ensure that the model is working correctly.

- The FDTD technique allows the user to specify the material at all points within the computational domain. A wide variety of linear and nonlinear dielectric and magnetic materials can be naturally and easily modeled.
- FDTD allows the effects of apertures to be determined directly. Shielding effects can be found, and the fields both inside and outside a structure can be found directly or indirectly.
- FDTD uses the E and H fields directly. Since most EMI/EMC modeling applications are interested in the E and H fields, it is convenient that no conversions must be made after the simulation has run to get these values.

Weaknesses of FDTD modeling

- Since FDTD requires that the entire computational domain be gridded, and the grid spatial discretization must be sufficiently fine to resolve both the smallest electromagnetic wavelength and the smallest geometrical feature in the model, very large computational domains can be developed, which results in very long solution times. Models with long, thin features, (like wires) are difficult to model in FDTD because of the excessively large computational domain required. Methods such as Eigenmode Expansion can offer a more efficient alternative as they do not require a fine grid along the z-direction.[73]
- There is no way to determine unique values for permittivity and permeability at a material interface.
- Space and time steps must satisfy the CFL condition, or the leapfrog integration used to solve the partial differential equation is likely to become unstable.
- FDTD finds the E/H fields directly everywhere in the computational domain. If the field values at some distance are desired, it is likely that this distance will force the computational domain to be excessively large. Far-field extensions are available for FDTD, but require some amount of postprocessing.[4]
- Since FDTD simulations calculate the E and H fields at all points within the computational domain, the computational domain must be finite to permit its residence in the computer memory. In many cases this is achieved by inserting artificial boundaries into the simulation space. Care must be taken to minimize errors introduced by such boundaries. There are a number of available highly effective absorbing boundary conditions (ABCs) to simulate an infinite unbounded computational domain.[4] Most modern FDTD implementations instead use a special absorbing "material", called a perfectly matched layer (PML) to implement absorbing boundaries.[42][46]
- Because FDTD is solved by propagating the fields forward in the time domain, the electromagnetic time response of the medium must be modeled

explicitly. For an arbitrary response, this involves a computationally expensive time convolution, although in most cases the time response of the medium (or Dispersion (optics)) can be adequately and simply modeled using either the recursive convolution (RC) technique, the auxiliary differential equation (ADE) technique, or the Z-transform technique. An alternative way of solving Maxwell's equations that can treat arbitrary dispersion easily is the Pseudospectral Spatial-Domain method (PSSD), which instead propagates the fields forward in space.

Grid truncation techniques

The most commonly used grid truncation techniques for open-region FDTD modeling problems are the Mur absorbing boundary condition (ABC),[13] the Liao ABC,[16] and various perfectly matched layer (PML) formulations.[4][42][46] The Mur and Liao techniques are simpler than PML. However, PML (which is technically an absorbing region rather than a boundary condition *per se*) can provide orders-of-magnitude lower reflections. The PML concept was introduced by J.-P. Berenger in a seminal 1994 paper in the Journal of Computational Physics.[42] Since 1994, Berenger's original split-field implementation has been modified and extended to the uniaxial PML (UPML), the convolutional PML (CPML), and the higher-order PML. The latter two PML formulations have increased ability to absorb evanescent waves, and therefore can in principle be placed closer to a simulated scattering or radiating structure than Berenger's original formulation.

To reduce undesired numerical reflection from the PML additional back absorbing layers technique can be used.[74]

Popularity

Notwithstanding both the general increase in academic publication throughput during the same period and the overall expansion of interest in all Computational electromagnetics (CEM) techniques, there are seven primary reasons for the tremendous expansion of interest in FDTD computational solution approaches for Maxwell's equations:

1. FDTD does not require a matrix inversion. Being a fully explicit computation, FDTD avoids the difficulties with matrix inversions that limit the size of frequency-domain integral-equation and finite-element electromagnetics models to generally fewer than 10^9 electromagnetic field unknowns.[4] FDTD models with as many as 10^9 field unknowns have been run; there is no intrinsic upper bound to this number.[4]
2. FDTD is accurate and robust. The sources of error in FDTD calculations are well understood, and can be bounded to permit accurate models for a very large variety of electromagnetic wave interaction problems.[4]
3. FDTD treats impulsive behavior naturally. Being a time-domain technique,

FDTD directly calculates the impulse response of an electromagnetic system. Therefore, a single FDTD simulation can provide either ultrawideband temporal waveforms or the sinusoidal steady-state response at any frequency within the excitation spectrum.[4]

4. FDTD treats nonlinear behavior naturally. Being a time-domain technique, FDTD directly calculates the nonlinear response of an electromagnetic system. This allows natural hybriding of FDTD with sets of auxiliary differential equations that describe nonlinearities from either the classical or semi-classical standpoint.[4] One research frontier is the development of hybrid algorithms which join FDTD classical electrodynamics models with phenomena arising from quantum electrodynamics, especially vacuum fluctuations, such as the Casimir effect.[4][75]

5. FDTD is a systematic approach. With FDTD, specifying a new structure to be modeled is reduced to a problem of mesh generation rather than the potentially complex reformulation of an integral equation. For example, FDTD requires no calculation of structure-dependent Green functions.[4]

6. Parallel-processing computer architectures have come to dominate supercomputing. FDTD scales with high efficiency on parallel-processing CPU-based computers, and extremely well on recently developed GPU-based accelerator technology.[4]

7. Computer visualization capabilities are increasing rapidly. While this trend positively influences all numerical techniques, it is of particular advantage to FDTD methods, which generate time-marched arrays of field quantities suitable for use in color videos to illustrate the field dynamics.[4]

Taflove has argued that these factors combine to suggest that FDTD will remain one of the dominant computational electrodynamics techniques (as well as potentially other multiphysics problems).[4]

Implementations

There are hundreds of simulation tools (e.g. XFdtd, Lumerical, CST studio suite, OptiFDTD etc.) that implement FDTD algorithms, many optimized to run on parallel-processing clusters.

Frederick Moxley suggests further applications with computational quantum mechanics and simulations.[76]

See also

- Computational electromagnetics
- Eigenmode expansion
- Beam propagation method
- Finite-difference frequency-domain

- Finite element method
- Scattering Matrix Method
- Discrete dipole approximation

References

1. J. von Neumann; RD Richtmyer (March 1950). "A method for the numerical calculation of hydrodynamic shocks". *Journal of Applied Physics*. **21** (3): 232–237. Bibcode:1950JAP....21..232V (https://ui.adsabs.harvard.edu /abs/1950JAP....21..232V). doi:10.1063/1.1699639 (https://doi.org /10.1063%2F1.1699639).

2. Kane Yee (1966). "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media". *IEEE Transactions on Antennas and Propagation*. **14** (3): 302–307. Bibcode:1966ITAP...14..302Y (https://ui.adsabs.harvard.edu/abs/1966ITAP...14..302Y). doi:10.1109/TAP.1966.1138693 (https://doi.org /10.1109%2FTAP.1966.1138693).

3. A. Taflove (1980). "Application of the finite-difference time-domain method to sinusoidal steady state electromagnetic penetration problems" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper7.pdf) (PDF). *IEEE Transactions on Electromagnetic Compatibility*. **22** (3): 191–202. Bibcode:1980ITElC..22..191T (https://ui.adsabs.harvard.edu /abs/1980ITElC..22..191T). doi:10.1109/TEMC.1980.303879 (https://doi.org /10.1109%2FTEMC.1980.303879).

4. Allen Taflove and Susan C. Hagness (2005). *Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed* (http://www.artechhouse.com/Detail.aspx?strBookId=1123). Artech House Publishers. ISBN 978-1-58053-832-9.

5. Adapted with permission from Taflove and Hagness (2005).

6. Richard Courant; Kurt Otto Friedrichs; Hans Lewy (1928). "Über die partiellen Differenzengleichungen der mathematischen Physik" (http://resolver.sub.uni-goettingen.de/purl?GDZPPN002272636). *Mathematische Annalen* (in German). **100** (1): 32–74. Bibcode:1928MatAn.100...32C (https://ui.adsabs.harvard.edu/abs/1928MatAn.100...32C). doi:10.1007/BF01448839 (https://doi.org/10.1007%2FBF01448839). JFM 54.0486.01 (https://zbmath.org/?format=complete&q=an:54.0486.01). MR 1512478 (https://www.ams.org/mathscinet-getitem?mr=1512478).

7. G. G. O'Brien, M. A Hyman, and S. Kaplan (1950). "A study of the numerical solution of partial differential equations". *Journal of Mathematical Physics*. **29** (1): 223–251. doi:10.1002/sapm1950291223 (https://doi.org /10.1002%2Fsapm1950291223). MR 0040805 (https://www.ams.org /mathscinet-getitem?mr=0040805).

8. Dong-Hoa Lam (1969). "Finite Difference Methods for Electromagnetic Scattering Problems". *Mississippi State University, Interaction Notes*. **44**.

9. A. Taflove; M. E. Brodwin (1975). "Numerical solution of steady-state electromagnetic scattering problems using the time-dependent Maxwell's equations" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper2.pdf) (PDF). *IEEE Transactions on Microwave Theory and Techniques*. **23** (8): 623–630. Bibcode:1975ITMTT..23..623T (https://ui.adsabs.harvard.edu /abs/1975ITMTT..23..623T). doi:10.1109/TMTT.1975.1128640 (https://doi.org /10.1109%2FTMTT.1975.1128640).

10. A. Taflove; M. E. Brodwin (1975). "Computation of the electromagnetic fields and induced temperatures within a model of the microwave-irradiated human eye" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper3.pdf) (PDF). *IEEE Transactions on Microwave Theory and Techniques*. **23** (11): 888–896. Bibcode:1975ITMTT..23..888T (https://ui.adsabs.harvard.edu /abs/1975ITMTT..23..888T). doi:10.1109/TMTT.1975.1128708 (https://doi.org /10.1109%2FTMTT.1975.1128708).

11. R. Holland (1977). "Threde: A free-field EMP coupling and scattering code". *IEEE Transactions on Nuclear Science*. **24** (6): 2416–2421. Bibcode:1977ITNS...24.2416H (https://ui.adsabs.harvard.edu /abs/1977ITNS...24.2416H). doi:10.1109/TNS.1977.4329229 (https://doi.org /10.1109%2FTNS.1977.4329229).

12. K. S. Kunz; K. M. Lee (1978). "A three-dimensional finite-difference solution of the external response of an aircraft to a complex transient EM environment". *IEEE Transactions on Electromagnetic Compatibility*. **20** (2): 333–341. doi:10.1109/TEMC.1978.303727 (https://doi.org /10.1109%2FTEMC.1978.303727).

13. G. Mur (1981). "Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic field equations". *IEEE Transactions on Electromagnetic Compatibility*. **23** (4): 377–382. doi:10.1109/TEMC.1981.303970 (https://doi.org /10.1109%2FTEMC.1981.303970).

14. K. R. Umashankar; A. Taflove (1982). "A novel method to analyze electromagnetic scattering of complex objects" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper9.pdf) (PDF). *IEEE Transactions on Electromagnetic Compatibility*. **24** (4): 397–405. Bibcode:1982ITElC..24..397U (https://ui.adsabs.harvard.edu /abs/1982ITElC..24..397U). doi:10.1109/TEMC.1982.304054 (https://doi.org /10.1109%2FTEMC.1982.304054).

15. A. Taflove; K. R. Umashankar (1983). "Radar cross section of general three-dimensional scatterers" (http://www.ece.northwestern.edu/ecefaculty/taflove /Paper10.pdf) (PDF). *IEEE Transactions on Electromagnetic Compatibility*. **25** (4): 433–440. doi:10.1109/TEMC.1983.304133 (https://doi.org /10.1109%2FTEMC.1983.304133).

16. Z. P. Liao; H. L. Wong; B. P. Yang; Y. F. Yuan (1984). "A transmitting boundary for transient wave analysis". *Scientia Sinica, Series A*. **27**: 1063–1076.

17. W. Gwarek (1985). "Analysis of an arbitrarily shaped planar circuit — A time-domain approach". *IEEE Transactions on Microwave Theory and Techniques*. **33** (10): 1067–1072. Bibcode:1985ITMTT..33.1067G (https://ui.adsabs.harvard.edu/abs/1985ITMTT..33.1067G). doi:10.1109/TMTT.1985.1133170 (https://doi.org/10.1109%2FTMTT.1985.1133170).

18. D. H. Choi; W. J. Hoefer (1986). "The finite-difference time-domain method and its application to eigenvalue problems". *IEEE Transactions on Microwave Theory and Techniques*. **34** (12): 1464–1470. Bibcode:1986ITMTT..34.1464C (https://ui.adsabs.harvard.edu/abs/1986ITMTT..34.1464C). doi:10.1109/TMTT.1986.1133564 (https://doi.org/10.1109%2FTMTT.1986.1133564).

19. G. A. Kriegsmann; A. Taflove; K. R. Umashankar (1987). "A new formulation of electromagnetic wave scattering using an on-surface radiation boundary condition approach" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper14.pdf) (PDF). *IEEE Transactions on Antennas and Propagation*. **35** (2): 153–161. Bibcode:1987ITAP...35..153K (https://ui.adsabs.harvard.edu/abs/1987ITAP...35..153K). doi:10.1109/TAP.1987.1144062 (https://doi.org/10.1109%2FTAP.1987.1144062).

20. T. G. Moore; J. G. Blaschak; A. Taflove; G. A. Kriegsmann (1988). "Theory and application of radiation boundary operators" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper20.pdf) (PDF). *IEEE Transactions on Antennas and Propagation*. **36** (12): 1797–1812. Bibcode:1988ITAP...36.1797M (https://ui.adsabs.harvard.edu/abs/1988ITAP...36.1797M). doi:10.1109/8.14402 (https://doi.org/10.1109%2F8.14402).

21. K. R. Umashankar; A. Taflove; B. Beker (1987). "Calculation and experimental validation of induced currents on coupled wires in an arbitrary shaped cavity" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper16.pdf) (PDF). *IEEE Transactions on Antennas and Propagation*. **35** (11): 1248–1257. Bibcode:1987ITAP...35.1248U (https://ui.adsabs.harvard.edu/abs/1987ITAP...35.1248U). doi:10.1109/TAP.1987.1144000 (https://doi.org/10.1109%2FTAP.1987.1144000).

22. A. Taflove; K. R. Umashankar; B. Beker; F. A. Harfoush; K. S. Yee (1988). "Detailed FDTD analysis of electromagnetic fields penetrating narrow slots and lapped joints in thick conducting screens" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper17.pdf) (PDF). *IEEE Transactions on Antennas and Propagation*. **36** (2): 247–257. Bibcode:1988ITAP...36..247T (https://ui.adsabs.harvard.edu/abs/1988ITAP...36..247T). doi:10.1109/8.1102 (https://doi.org/10.1109%2F8.1102).

23. T. G. Jurgens; A. Taflove; K. R. Umashankar; T. G. Moore (1992). "Finite-difference time-domain modeling of curved surfaces" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper39.pdf) (PDF). *IEEE Transactions on Antennas and Propagation*. **40** (4): 357–366. Bibcode:1992ITAP...40..357J (https://ui.adsabs.harvard.edu/abs/1992ITAP...40..357J). doi:10.1109/8.138836 (https://doi.org/10.1109%2F8.138836).

24. D. M. Sullivan; O. P. Gandhi; A. Taflove (1988). "Use of the finite-difference time-domain method in calculating EM absorption in man models" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper18.pdf) (PDF). *IEEE Transactions on Biomedical Engineering*. **35** (3): 179–186. doi:10.1109/10.1360 (https://doi.org/10.1109%2F10.1360). PMID 3350546 (https://pubmed.ncbi.nlm.nih.gov/3350546).

25. X. Zhang; J. Fang; K. K. Mei; Y. Liu (1988). "Calculation of the dispersive characteristics of microstrips by the time-domain finite-difference method". *IEEE Transactions on Microwave Theory and Techniques*. **36** (2): 263–267. Bibcode:1988ITMTT..36..263Z (https://ui.adsabs.harvard.edu/abs/1988ITMTT..36..263Z). doi:10.1109/22.3514 (https://doi.org/10.1109%2F22.3514).

26. T. Kashiwa; I. Fukai (1990). "A treatment by FDTD method of dispersive characteristics associated with electronic polarization". *Microwave and Optical Technology Letters*. **3** (6): 203–205. doi:10.1002/mop.4650030606 (https://doi.org/10.1002%2Fmop.4650030606).

27. R. Luebbers; F. Hunsberger; K. Kunz; R. Standler; M. Schneider (1990). "A frequency-dependent finite-difference time-domain formulation for dispersive materials". *IEEE Transactions on Electromagnetic Compatibility*. **32** (3): 222–227. doi:10.1109/15.57116 (https://doi.org/10.1109%2F15.57116).

28. R. M. Joseph; S. C. Hagness; A. Taflove (1991). "Direct time integration of Maxwell's equations in linear dispersive media with absorption for scattering and propagation of femtosecond electromagnetic pulses" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper35.pdf) (PDF). *Optics Letters*. **16** (18): 1412–4. Bibcode:1991OptL...16.1412J (https://ui.adsabs.harvard.edu/abs/1991OptL...16.1412J). doi:10.1364/OL.16.001412 (https://doi.org/10.1364%2FOL.16.001412). PMID 19776986 (https://pubmed.ncbi.nlm.nih.gov/19776986).

29. J. G. Maloney; G. S. Smith; W. R. Scott Jr. (1990). "Accurate computation of the radiation from simple antennas using the finite-difference time-domain method". *IEEE Transactions on Antennas and Propagation*. **38** (7): 1059–1068. Bibcode:1990ITAP...38.1059M (https://ui.adsabs.harvard.edu/abs/1990ITAP...38.1059M). doi:10.1109/8.55618 (https://doi.org/10.1109%2F8.55618).

30. D. S. Katz; A. Taflove; M. J. Piket-May; K. R. Umashankar (1991). "FDTD analysis of electromagnetic wave radiation from systems containing horn antennas" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper34.pdf) (PDF). *IEEE Transactions on Antennas and Propagation*. **39** (8): 1203–1212. Bibcode:1991ITAP...39.1203K (https://ui.adsabs.harvard.edu /abs/1991ITAP...39.1203K). doi:10.1109/8.97356 (https://doi.org /10.1109%2F8.97356).

31. P. A. Tirkas; C. A. Balanis (1991). *Finite-difference time-domain technique for radiation by horn antennas*. *IEEE Antennas and Propagation Society International Symposium Digest*. **3**. pp. 1750–1753. doi:10.1109/APS.1991.175196 (https://doi.org /10.1109%2FAPS.1991.175196). ISBN 978-0-7803-0144-3.

32. E. Sano; T. Shibata (1990). "Fullwave analysis of picosecond photoconductive switches". *IEEE Journal of Quantum Electronics*. **26** (2): 372–377. Bibcode:1990IJQE...26..372S (https://ui.adsabs.harvard.edu /abs/1990IJQE...26..372S). doi:10.1109/3.44970 (https://doi.org /10.1109%2F3.44970).

33. S. M. El-Ghazaly; R. P. Joshi; R. O. Grondin (1990). "Electromagnetic and transport considerations in subpicosecond photoconductive switch modeling". *IEEE Transactions on Microwave Theory and Techniques*. **38** (5): 629–637. Bibcode:1990ITMTT..38..629E (https://ui.adsabs.harvard.edu /abs/1990ITMTT..38..629E). doi:10.1109/22.54932 (https://doi.org /10.1109%2F22.54932).

34. P. M. Goorjian; A. Taflove (1992). "Direct time integration of Maxwell's equations in nonlinear dispersive media for propagation and scattering of femtosecond electromagnetic solitons" (http://www.ece.northwestern.edu /ecefaculty/taflove/Paper37.pdf) (PDF). *Optics Letters*. **17** (3): 180–182. Bibcode:1992OptL...17..180G (https://ui.adsabs.harvard.edu /abs/1992OptL...17..180G). doi:10.1364/OL.17.000180 (https://doi.org /10.1364%2FOL.17.000180).

35. R. W. Ziolkowski; J. B. Judkins (1993). "Full-wave vector Maxwell's equations modeling of self-focusing of ultra-short optical pulses in a nonlinear Kerr medium exhibiting a finite response time". *Journal of the Optical Society of America B*. **10** (2): 186–198. Bibcode:1993JOSAB..10..186Z (https://ui.adsabs.harvard.edu/abs/1993JOSAB..10..186Z). doi:10.1364/JOSAB.10.000186 (https://doi.org/10.1364%2FJOSAB.10.000186).

36. R. M. Joseph; P. M. Goorjian; A. Taflove (1993). "Direct time integration of Maxwell's equations in 2-D dielectric waveguides for propagation and scattering of femtosecond electromagnetic solitons" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper44.pdf) (PDF). *Optics Letters*. **18** (7): 491–3. Bibcode:1993OptL...18..491J (https://ui.adsabs.harvard.edu/abs/1993OptL...18..491J). doi:10.1364/OL.18.000491 (https://doi.org/10.1364%2FOL.18.000491). PMID 19802177 (https://pubmed.ncbi.nlm.nih.gov/19802177).

37. R. M. Joseph; A. Taflove (1994). "Spatial soliton deflection mechanism indicated by FDTD Maxwell's equations modeling" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper54.pdf) (PDF). *IEEE Photonics Technology Letters*. **2** (10): 1251–1254. Bibcode:1994IPTL....6.1251J (https://ui.adsabs.harvard.edu/abs/1994IPTL....6.1251J). doi:10.1109/68.329654 (https://doi.org/10.1109%2F68.329654).

38. W. Sui; D. A. Christensen; C. H. Durney (1992). "Extending the two-dimensional FDTD method to hybrid electromagnetic systems with active and passive lumped elements". *IEEE Transactions on Microwave Theory and Techniques*. **40** (4): 724–730. Bibcode:1992ITMTT..40..724S (https://ui.adsabs.harvard.edu/abs/1992ITMTT..40..724S). doi:10.1109/22.127522 (https://doi.org/10.1109%2F22.127522).

39. B. Toland; B. Houshmand; T. Itoh (1993). "Modeling of nonlinear active regions with the FDTD method". *IEEE Microwave and Guided Wave Letters*. **3** (9): 333–335. doi:10.1109/75.244870 (https://doi.org/10.1109%2F75.244870).

40. Aoyagi, P.H.; Lee, J.F.; Mittra, R. (1993). "A hybrid Yee algorithm/scalar-wave equation approach". *IEEE Transactions on Microwave Theory and Techniques*. **41** (9): 1593–1600. Bibcode:1993ITMTT..41.1593A (https://ui.adsabs.harvard.edu/abs/1993ITMTT..41.1593A). doi:10.1109/22.245683 (https://doi.org/10.1109%2F22.245683).

41. V. A. Thomas; M. E. Jones; M. J. Piket-May; A. Taflove; E. Harrigan (1994). "The use of SPICE lumped circuits as sub-grid models for FDTD high-speed electronic circuit design" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper49.pdf) (PDF). *IEEE Microwave and Guided Wave Letters*. **4** (5): 141–143. doi:10.1109/75.289516 (https://doi.org/10.1109%2F75.289516).

42. J. Berenger (1994). "A perfectly matched layer for the absorption of electromagnetic waves" (http://www.stanford.edu/class/ee256/Berenger1994.pdf) (PDF). *Journal of Computational Physics*. **114** (2): 185–200. Bibcode:1994JCoPh.114..185B (https://ui.adsabs.harvard.edu/abs/1994JCoPh.114..185B). doi:10.1006/jcph.1994.1159 (https://doi.org/10.1006%2Fjcph.1994.1159).

43. D. S. Katz; E. T. Thiele; A. Taflove (1994). "Validation and extension to three dimensions of the Berenger PML absorbing boundary condition for FDTD meshes" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper51.pdf) (PDF). *IEEE Microwave and Guided Wave Letters*. **4** (8): 268–270. doi:10.1109/75.311494 (https://doi.org/10.1109%2F75.311494).

44. C. E. Reuter; R. M. Joseph; E. T. Thiele; D. S. Katz; A. Taflove (1994). "Ultrawideband absorbing boundary condition for termination of waveguiding structures in FDTD simulations" (http://www.ece.northwestern.edu/ecefaculty/taflove/Paper53.pdf) (PDF). *IEEE Microwave and Guided Wave Letters*. **4** (10): 344–346. doi:10.1109/75.324711 (https://doi.org/10.1109%2F75.324711).

45. W.C. Chew; W.H. Weedon (1994). "A 3D perfectly matched medium from modified Maxwell's equations with stretched coordinates". *Microwave and Optical Technology Letters*. **7** (13): 599–604. Bibcode:1994MiOTL...7..599C (https://ui.adsabs.harvard.edu/abs/1994MiOTL...7..599C). doi:10.1002/mop.4650071304 (https://doi.org /10.1002%2Fmop.4650071304).

46. S. D. Gedney (1996). "An anisotropic perfectly matched layer absorbing media for the truncation of FDTD lattices". *IEEE Transactions on Antennas and Propagation*. **44** (12): 1630–1639. Bibcode:1996ITAP...44.1630G (https://ui.adsabs.harvard.edu/abs/1996ITAP...44.1630G). doi:10.1109/8.546249 (https://doi.org/10.1109%2F8.546249).

47. Z. S. Sacks; D. M. Kingsland; R. Lee; J. F. Lee (1995). "A perfectly matched anisotropic absorber for use as an absorbing boundary condition" (http://ieeexplore.ieee.org/xpl/tocresult.jsp?isnumber=10144). *IEEE Transactions on Antennas and Propagation*. **43** (12): 1460–1463. Bibcode:1995ITAP...43.1460S (https://ui.adsabs.harvard.edu /abs/1995ITAP...43.1460S). doi:10.1109/8.477075 (https://doi.org /10.1109%2F8.477075).

48. Q. H. Liu (1997). *The pseudospectral time-domain (PSTD) method: A new algorithm for solutions of Maxwell's equations* (https://ieeexplore.ieee.org /document/630102). *IEEE Antennas and Propagation Society International Symposium Digest*. **1**. pp. 122–125. doi:10.1109/APS.1997.630102 (https://doi.org/10.1109%2FAPS.1997.630102). ISBN 978-0-7803-4178-4.

49. O. M. Ramahi (1997). "The complementary operators method in FDTD simulations". *IEEE Antennas and Propagation Magazine*. **39** (6): 33–45. Bibcode:1997IAPM...39...33R (https://ui.adsabs.harvard.edu /abs/1997IAPM...39...33R). doi:10.1109/74.646801 (https://doi.org /10.1109%2F74.646801).

50. J. G. Maloney; M. P. Kesler (1998). "Analysis of Periodic Structures". *Chap. 6 in Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method, A. Taflove, Ed., Artech House, Publishers*.

51. A. S. Nagra; R. A. York (1998). "FDTD analysis of wave propagation in nonlinear absorbing and gain media". *IEEE Transactions on Antennas and Propagation*. **46** (3): 334–340. Bibcode:1998ITAP...46..334N (https://ui.adsabs.harvard.edu/abs/1998ITAP...46..334N). doi:10.1109/8.662652 (https://doi.org/10.1109%2F8.662652).

52. S. C. Hagness; A. Taflove; J. E. Bridges (1998). "Two-dimensional FDTD analysis of a pulsed microwave confocal system for breast cancer detection: Fixed-focus and antenna-array sensors" (http://www.ece.northwestern.edu /ecefaculty/taflove/Paper66.pdf) (PDF). *IEEE Transactions on Biomedical Engineering*. **45** (12): 1470–1479. doi:10.1109/10.730440 (https://doi.org /10.1109%2F10.730440). PMID 9835195 (https://pubmed.ncbi.nlm.nih.gov /9835195).

53. J. B. Schneider; C. L. Wagner (1999). "FDTD dispersion revisited: Faster-than-light propagation". *IEEE Microwave and Guided Wave Letters*. **9** (2): 54–56. CiteSeerX 10.1.1.77.9132 (https://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.77.9132). doi:10.1109/75.755044 (https://doi.org/10.1109%2F75.755044).

54. F. Zhen; Z. Chen; J. Zhang (2000). "Toward the development of a three-dimensional unconditionally stable finite-difference time-domain method". *IEEE Transactions on Microwave Theory and Techniques*. **48** (9): 1550–1558. Bibcode:2000ITMTT..48.1550Z (https://ui.adsabs.harvard.edu/abs/2000ITMTT..48.1550Z). doi:10.1109/22.869007 (https://doi.org/10.1109%2F22.869007).

55. F. Zheng; Z. Chen (2001). "Numerical dispersion analysis of the unconditionally stable 3-D ADI-FDTD method". *IEEE Transactions on Microwave Theory and Techniques*. **49** (5): 1006–1009. Bibcode:2001ITMTT..49.1006Z (https://ui.adsabs.harvard.edu/abs/2001ITMTT..49.1006Z). doi:10.1109/22.920165 (https://doi.org/10.1109%2F22.920165).

56. J. A. Roden; S. D. Gedney (2000). "Convolution PML (CPML): An efficient FDTD implementation of the CFS-PML for arbitrary media" (http://www3.interscience.wiley.com/journal/73504513/abstract). *Microwave and Optical Technology Letters*. **27** (5): 334–339. doi:10.1002/1098-2760(20001205)27:5<334::AID-MOP14>3.0.CO;2-A (https://doi.org/10.1002%2F1098-2760%2820001205%2927%3A5%3C334%3A%3AAID-MOP14%3E3.0.CO%3B2-A).

57. T. Rylander; A. Bondeson (2000). "Stable FDTD-FEM hybrid method for Maxwell's equations". *Computer Physics Communications*. **125** (1–3): 75–82. doi:10.1016/S0010-4655(99)00463-4 (https://doi.org/10.1016%2FS0010-4655%2899%2900463-4).

58. M. Hayakawa; T. Otsuyama (2002). "FDTD analysis of ELF wave propagation in inhomogeneous subionospheric waveguide models" (http://handle.dtic.mil/100.2/ADP013476). *ACES Journal*. **17**: 239–244.

59. J. J. Simpson; A. Taflove (2002). "Two-dimensional FDTD model of antipodal ELF propagation and Schumann resonance of the Earth" (https://web.archive.org/web/20100617131759/http://www.ece.unm.edu/~simpson/Paper1.pdf) (PDF). *IEEE Antennas and Wireless Propagation Letters*. **1** (2): 53–56. Bibcode:2002IAWPL...1...53S (https://ui.adsabs.harvard.edu/abs/2002IAWPL...1...53S). CiteSeerX 10.1.1.694.4837 (https://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.694.4837). doi:10.1109/LAWP.2002.805123 (https://doi.org/10.1109%2FLAWP.2002.805123). Archived from the original (http://www.ece.unm.edu/~simpson/Paper1.pdf) (PDF) on 2010-06-17.

60. H. De Raedt; K. Michielsen; J. S. Kole; M. T. Figge (2003). "Solving the Maxwell equations by the Chebyshev method: A one-step finite difference time-domain algorithm". *IEEE Transactions on Antennas and Propagation*. **51** (11): 3155–3160. arXiv:physics/0208060 (https://arxiv.org/abs/physics /0208060). Bibcode:2003ITAP...51.3155D (https://ui.adsabs.harvard.edu /abs/2003ITAP...51.3155D). doi:10.1109/TAP.2003.818809 (https://doi.org /10.1109%2FTAP.2003.818809).

61. I. Ahmed; E. K. Chua; E. P. Li; Z. Chen (2008). "Development of the three-dimensional unconditionally stable LOD-FDTD method". *IEEE Transactions on Antennas and Propagation*. **56** (11): 3596–3600. Bibcode:2008ITAP...56.3596A (https://ui.adsabs.harvard.edu /abs/2008ITAP...56.3596A). doi:10.1109/TAP.2008.2005544 (https://doi.org /10.1109%2FTAP.2008.2005544).

62. Taniguchi, Y.; Baba, Y.; N. Nagaoka; A. Ametani (2008). "An Improved Thin Wire Representation for FDTD Computations". *IEEE Transactions on Antennas and Propagation*. **56** (10): 3248–3252. Bibcode:2008ITAP...56.3248T (https://ui.adsabs.harvard.edu/abs/2008ITAP...56.3248T). doi:10.1109/TAP.2008.929447 (https://doi.org/10.1109%2FTAP.2008.929447).

63. R. M. S. de Oliveira; C. L. S. S. Sobrinho (2009). "Computational Environment for Simulating Lightning Strokes in a Power Substation by Finite-Difference Time-Domain Method". *IEEE Transactions on Electromagnetic Compatibility*. **51** (4): 995–1000. doi:10.1109/TEMC.2009.2028879 (https://doi.org /10.1109%2FTEMC.2009.2028879).

64. B. Chaudhury; J. P. Boeuf (2010). "Computational Studies of Filamentary Pattern Formation in a High Power Microwave Breakdown Generated Air Plasma". *IEEE Transactions on Plasma Science*. **38** (9): 2281–2288. Bibcode:2010ITPS...38.2281C (https://ui.adsabs.harvard.edu /abs/2010ITPS...38.2281C). doi:10.1109/TPS.2010.2055893 (https://doi.org /10.1109%2FTPS.2010.2055893).

65. F. I. Moxley III; T. Byrnes; F. Fujiwara; W. Dai (2012). "A generalized finite-difference time-domain quantum method for the N-body interacting Hamiltonian". *Computer Physics Communications*. **183** (11): 2434–2440. Bibcode:2012CoPhC.183.2434M (https://ui.adsabs.harvard.edu /abs/2012CoPhC.183.2434M). doi:10.1016/j.cpc.2012.06.012 (https://doi.org /10.1016%2Fj.cpc.2012.06.012).

66. F. I. Moxley III; D. T. Chuss; W. Dai (2013). "A generalized finite-difference time-domain scheme for solving nonlinear Schrödinger equations". *Computer Physics Communications*. **184** (8): 1834–1841. Bibcode:2013CoPhC.184.1834M (https://ui.adsabs.harvard.edu /abs/2013CoPhC.184.1834M). doi:10.1016/j.cpc.2013.03.006 (https://doi.org /10.1016%2Fj.cpc.2013.03.006).

67. Frederick Moxley; et al. (2014). *Contemporary Mathematics: Mathematics of Continuous and Discrete Dynamical Systems* (http://www.ams.org/bookstore-getitem?item=CONM-618). American Mathematical Society. ISBN 978-0-8218-9862-8.

68. "Fitting of dielectric function" (http://fdtd.kintechlab.com/en/fitting).
69. I. Valuev; A. Deinega; S. Belousov (2008). "Iterative technique for analysis of periodic structures at oblique incidence in the finite-difference time-domain method". *Opt. Lett.* **33** (13): 1491. Bibcode:2008OptL...33.1491V (https://ui.adsabs.harvard.edu/abs/2008OptL...33.1491V). doi:10.1364/ol.33.001491 (https://doi.org/10.1364%2Fol.33.001491).
70. A. Aminian; Y. Rahmat-Samii (2006). "Spectral FDTD: a novel technique for the analysis of oblique incident plane wave on periodic structures". *IEEE Transactions on Antennas and Propagation*. **54** (6): 1818–1825. Bibcode:2006ITAP...54.1818A (https://ui.adsabs.harvard.edu/abs/2006ITAP...54.1818A). doi:10.1109/tap.2006.875484 (https://doi.org/10.1109%2Ftap.2006.875484).
71. A. Deinega; S. Belousov; I. Valuev (2009). "Hybrid transfer-matrix FDTD method for layered periodic structures" (https://semanticscholar.org/paper/86039fea77e66d944ef9ea340b4e7c2e20e2b0d3). *Opt. Lett.* **34** (6): 860. Bibcode:2009OptL...34..860D (https://ui.adsabs.harvard.edu/abs/2009OptL...34..860D). doi:10.1364/ol.34.000860 (https://doi.org/10.1364%2Fol.34.000860).
72. Y. Hao; R. Mittra (2009). *FDTD Modeling of Metamaterials: Theory and Applications* (http://www.artechhouse.com/Detail.aspx?strIsbn=978-1-59693-160-2). Artech House Publishers.
73. D. Gallagher (2008). "Photonics CAD Matures" (http://www.photond.com/files/docs/leos_newsletter_feb08_article.pdf) (PDF). *LEOS Newsletter*.
74. A. Deinega; I. Valuev (2011). "Long-time behavior of PML absorbing boundaries for layered periodic structures". *Comp. Phys. Comm.* **182** (1): 149–151. Bibcode:2011CoPhC.182..149D (https://ui.adsabs.harvard.edu/abs/2011CoPhC.182..149D). doi:10.1016/j.cpc.2010.06.006 (https://doi.org/10.1016%2Fj.cpc.2010.06.006).
75. S. G. Johnson, "Numerical methods for computing Casimir interactions (https://arxiv.org/abs/arXiv:1007.0966)," in Casimir Physics (D. Dalvit, P. Milonni, D. Roberts, and F. da Rosa, eds.), vol. 834 of *Lecture Notes in Physics*, ch. 6, pp. 175–218, Berlin: Springer, June 2011.
76. Hartmut Ruhl; Nils Moschuring; Nina Elkina (2012). "Computational Physics Course 17104 Lecture 9" (http://www.physik.uni-muenchen.de/lehre/vorlesungen/wise_12_13/tvi_mas_compphys/material/lecture9.pdf) (PDF).

Further reading

The following article in *Nature Milestones: Photons* illustrates the historical significance of the FDTD method as related to Maxwell's equations:

- David Pile (May 2010). "Milestone 2 (1861) Maxwell's equations" (http://www.nature.com/milestones/milephotons/full/milephotons02.html). *Nature Milestones: Photons*. doi:10.1038/nmat2639 (https://doi.org

/10.1038%2Fnmat2639). Retrieved 17 June 2010.

Allen Taflove's interview, "Numerical Solution," in the January 2015 focus issue of *Nature Photonics* honoring the 150th anniversary of the publication of Maxwell's equations. This interview touches on how the development of FDTD ties into the century and one-half history of Maxwell's theory of electrodynamics:

- *Nature Photonics interview* (http://www.nature.com/nphoton/focus/maxwell-anniversary/index.html)

The following university-level textbooks provide a good general introduction to the FDTD method:

- Karl S. Kunz; Raymond J. Luebbers (1993). *The Finite Difference Time Domain Method for Electromagnetics* (https://web.archive.org/web/20071210045441/http://www.crcpress.com/shopping_cart/products/product_detail.asp?sku=8657&af=W1129). CRC Press. ISBN 978-0-8493-8657-2. Archived from the original (http://www.crcpress.com/shopping_cart/products/product_detail.asp?sku=8657&af=W1129) on 2007-12-10. Retrieved 2006-08-05.

- Allen Taflove; Susan C. Hagness (2005). *Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed* (http://www.artechhouse.com/Detail.aspx?strBookId=1123). Artech House Publishers. ISBN 978-1-58053-832-9.

- Wenhua Yu; Raj Mittra; Tao Su; Yongjun Liu; Xiaoling Yang (2006). *Parallel Finite-Difference Time-Domain Method* (http://www.artechhouse.com/default.asp?frame=book.asp&book=1-59693-085-3&Country=US&Continent=NO&State=). Artech House Publishers. ISBN 978-1-59693-085-8.

- John B. Schneider (2010). *Understanding the FDTD Method* (http://www.eecs.wsu.edu/~schneidj/ufdtd/index.php). available online.

- EM Lab Poster on FDTD (http://emlab.utep.edu/pdfs/Poster_FDTD.pdf)
- Course Notes on Introduction to FDTD (http://emlab.utep.edu/ee5390fdtd.htm)

External links

Free software/Open-source software FDTD projects:

- FDTD++ (http://www.fdtdxx.com): advanced, fully featured FDTD software, with included C++ source code, along with sophisticated material models and predefined fits as well as discussion/support forums and email support
- openEMS (http://openEMS.de) (Fully 3D Cartesian & Cylindrical graded mesh EC-FDTD Solver, written in C++, using a Matlab/Octave-Interface)
- pFDTD (https://web.archive.org/web/20110517102321/http:

//www.its.caltech.edu/~seheon/FDTD.html) (3D C++ FDTD codes developed by Se-Heon Kim)

- JFDTD (https://web.archive.org/web/20090626051810/http: //www.thecomputationalphysicist.com/) (2D/3D C++ FDTD codes developed for nanophotonics by Jeffrey M. McMahon)
- WOLFSIM (http://www.ece.ncsu.edu/oleg/wiki/WOLFSIM) (NCSU) (2-D)
- Meep (http://ab-initio.mit.edu/meep/) (MIT, 2D/3D/cylindrical parallel FDTD)
- (Geo-) Radar FDTD (http://freshmeat.net/projects/radarfdtd/)
- bigboy (http://sourceforge.net/projects/bigboy) (unmaintained, no release files. must get source from cvs)
- toyFDTD (http://www.cemtach.com/reference/software/toyFDTD/)
- Parallel (MPI&OpenMP) FDTD codes in C++ (http://sourceforge.net/projects /pfdtd/files/) (developed by Zs. Szabó)
- FDTD code in Fortran 90 (https://archive.today/20121217222254/http://cs.tu-berlin.de/~peutetre/sfdtd/)
- FDTD code in C for 2D EM Wave simulation (https://code.google.com /p/emwave2d/)
- Angora (http://angorafdtd.org) (3D parallel FDTD software package, maintained by Ilker R. Capoglu)
- GSvit (http://gsvit.net/) (3D FDTD solver with graphics card computing support, written in C, graphical user interface XSvit available)
- gprMax (http://www.gprmax.com) (Open Source (GPLv3), 3D/2D FDTD modelling code in Python/Cython developed for GPR but can be used for general EM modelling.)

Freeware/Closed source FDTD projects (some not for commercial use):

- EMTL (Electromagnetic Template Library) (http://fdtd.kintechlab.com/en/start) (Free C++ library for electromagnetic simulations. The current version implements mainly the FDTD).

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