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NONLINEAR EFFECTS IN OPTICAL FIBERS: ORIGIN, MANAGEMENT AND APPLICATIONS

S. P. Singh † **and N. Singh**

Department of Electronics and Communication
University of Allahabad
Allahabad-211002, India

Abstract—The nonlinear effects in optical fiber occur either due to intensity dependence of refractive index of the medium or due to inelastic-scattering phenomenon. This paper describes various types of nonlinear effects based on first effect such as self-phase modulation, cross-phase modulation and four-wave mixing. Their thresholds, managements and applications are also discussed; and comparative study of these effects is presented.

1. INTRODUCTION

The terms linear and nonlinear (Figure 1), in optics, mean intensity-independent and intensity-dependent phenomena respectively. Nonlinear effects in optical fibers (Table 1) occur due to (1) change in the refractive index of the medium with optical intensity and, (2) inelastic-scattering phenomenon. The power dependence of the refractive index is responsible for the Kerr-effect. Depending upon the type of input signal, the Kerr-nonlinearity manifests itself in three different effects such as Self-Phase Modulation (SPM), Cross-Phase Modulation (CPM) and Four-Wave Mixing (FWM). At high power level, the inelastic scattering phenomenon can induce stimulated effects such as Stimulated Brillouin-Scattering (SBS) and Stimulated Raman-Scattering (SRS). The intensity of scattered light grows exponentially if the incident power exceeds a certain threshold value. The difference between Brillouin and Raman scattering is that the Brillouin generated phonons (acoustic) are coherent and give rise to a macroscopic acoustic wave in the fiber, while in Raman scattering the phonons (optical) are incoherent and no macroscopic wave is generated.

† Also with Physics Department, KNIPSS, Sultanpur, U.P., India.

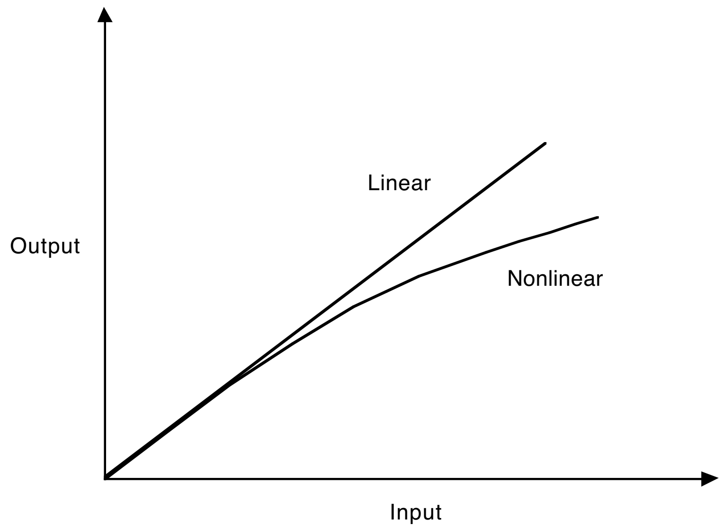
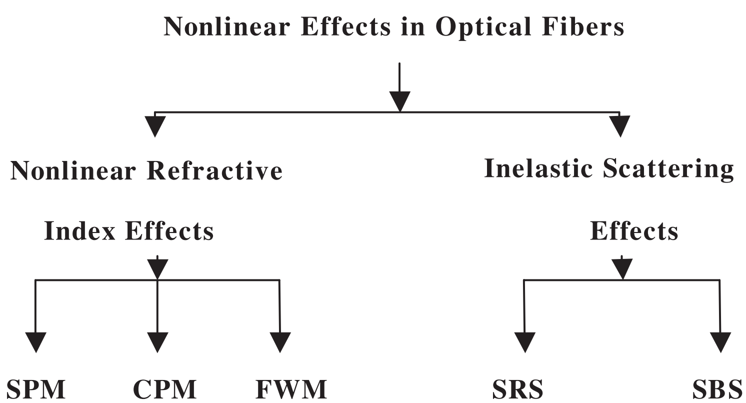


Figure 1. Linear and nonlinear interactions.

Table 1. Nonlinear effects in optical fibers.



Except for SPM and CPM, all nonlinear effects provide gains to some channel at the expense of depleting power from other channels. SPM and CPM affect only the phase of signals and can cause spectral broadening, which leads to increased dispersion. Due to several recent events, the nonlinear effects in optical fibers are an area of academic research [1–4, 15, 17–20].

(i) Use of single mode fiber (SMF) with small cross section of light-carrying area has led to increased power intensity inside the fiber.
(ii) Use of in-line optical amplifiers has resulted in a substantial increase in the absolute value of the power carried by a fiber.
(iii) The deployment of multiwavelength systems together with optical amplifier.
(iv) The deployment of high-bit-rate (>10 Gbits/s per channel) systems.

This paper is organized as follows:

The basics of nonlinear effects are discussed in Section 2. Self-phase modulation, cross-phase modulation and four-wave mixing are described in Sections 3, 4 and 5 respectively. Their thresholds, managements and applications are also given in these sections. These effects are compared in Section 6. Finally, conclusion is presented in Section 7.

2. BASICS

For intense electromagnetic fields, any dielectric medium behaves like a nonlinear medium. Fundamentally, origin of nonlinearity lies in anharmonic motion of bound electrons under the influence of an applied field. Due to this anharmonic motion the total polarization P induced by electric dipoles is not linear but satisfies more general relation as

$$P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E^2 + \varepsilon_0 \chi^{(3)} E^3 + \cdots \tag{1}$$

where ε_0 is the permittivity of vacuum and $\chi^{(k)}$ $(k = 1, 2, \ldots)$ is kth order susceptibility.

The dominant contribution to P is provided by linear susceptibility $\chi^{(1)}$. The second order susceptibility $\chi^{(2)}$ is responsible for second-harmonic generation and sum-frequency generation. A medium, which lacks inversion symmetry at the molecular level, has non-zero second order susceptibility. However for a symmetric molecule, like silica, $\chi^{(2)}$ vanishes. Therefore optical fibers do not exhibit second order nonlinear refractive effects. It is worth to mention here that, the electric-quadrupole and magnetic-dipole moments can generate weak second order nonlinear effects. Defects and color centers inside the fiber core can also contribute to second harmonic generation under certain conditions. Obviously the third order susceptibility $\chi^{(3)}$ is responsible for lowest-order nonlinear effects in fibers [5].

For isotropic medium, like optical fiber, polarization vector $\boldsymbol{P}$ will always be in direction of electric field vector $\boldsymbol{E}$. So one may use scalar notations instead of vector notations. For an electric field,

$$E = E_0 \cos(\omega t - kz) \tag{2}$$

the polarization P becomes

$$\begin{aligned} P \;=\; & \varepsilon_0 \chi^{(1)} E_0 \cos(\omega t - kz) + \varepsilon_0 \chi^{(2)} E_0^2 \cos^2(\omega t - kz) \\ & + \varepsilon_0 \chi^{(3)} E_0^3 \cos^3(\omega t - kz) + \cdots \end{aligned} \tag{3}$$

Using some trigonometric relations, equation (3) can be written as

$$\begin{aligned} P \;=\; & \frac{1}{2}\varepsilon_0 \chi^{(2)} E_0^2 + \varepsilon_0 \chi^{(1)} + \frac{3}{4}\chi^{(3)} E_0^2 E_0 \cos(\omega t - kz) \\ & + \frac{1}{2}\varepsilon_0 \chi^{(2)} E_0^2 \cos 2(\omega t - kz) + \frac{1}{4}\varepsilon_0 \chi^{(3)} E_0^3 \cos 3(\omega t - kz) + \cdots \end{aligned} \tag{4}$$

The effect of first term is of little practical importance as it is a constant term and gives a dc field across the medium. The second term oscillating at frequency ω is known as first or fundamental harmonic of polarization. The third term oscillating with frequency 2ω is called the second harmonic of polarization. Similarly fourth term with frequency 3ω is known as third harmonic of polarization. For optical fibers, $\chi^{(2)}$ vanishes, and hence equation (4) becomes

$$P = \varepsilon_0 \chi^{(1)} + \frac{3}{4}\chi^{(3)} E_0^2 E_0 \cos(\omega t - kz) + \frac{1}{4}\varepsilon_0 \chi^{(3)} E_0^3 \cos 3(\omega t - kz) \tag{5}$$

Here higher order terms are neglected because their contribution is negligible. Due to variations in refractive index of the fiber there is lack of phase between frequencies ω and 3ω. Due to this phase mismatch the second term of equation (5) can be neglected and polarization can be written as

$$P = \varepsilon_0 \chi^{(1)} E_0 \cos(\omega t - kz) + \frac{3}{4}\varepsilon_0 \chi^{(3)} E_0^3 \cos(\omega t - kz) \tag{6}$$

This equation contains both linear (first term) and nonlinear (second term) polarizations. For a plane wave represented by equation (2), the intensity (I) is defined as,

$$I = \frac{1}{2} c \varepsilon_0 n_l E_0^2 \tag{7}$$

where c is velocity of light and n_l is linear refractive index of the medium at low fields. Hence,

$$P = \varepsilon_0 \chi^{(1)} + \frac{3}{2}\frac{\chi^{(3)}}{c\varepsilon_0 n_l} I E_0 \cos(\omega t - kz) \tag{8}$$

2.1. Effective Susceptibility and Effective Refractive Index

The effective susceptibility (χ_{eff}) of the medium is defined as,

$$\chi_{eff} = \frac{P}{\varepsilon_0 E} = \chi^{(1)} + \frac{3}{2}\frac{\chi^{(3)}}{c\varepsilon_0 n_l} I \tag{9}$$

Therefore, effective refractive index (n_{eff}) can be written as

$$n_{eff} = (1 + \chi_{eff})^{\frac{1}{2}}$$

or

$$n_{eff} = \left(1 + \chi^{(1)} + \frac{3}{2}\frac{\chi^{(3)}}{c\varepsilon_0 n_l^2} I\right)^{\frac{1}{2}} \tag{10}$$

The last term is usually very small even for very intense light beam. Hence above expression for n_{eff} can be approximated with help of Taylor's series expansion as

$$n_{eff} = n_l + \frac{3}{4}\frac{\chi^{(3)}}{c\varepsilon_0 n_l^2} I \tag{11}$$

or

$$n_{eff} = n_l + n_{nl} I \tag{12}$$

In equation (12) first term $[n_l = (1 + \chi^{(3)})^{\frac{1}{2}}]$ is linear refractive index and second term ($n_{nl} = \frac{3}{4}\frac{\chi^{(3)}}{c\varepsilon_0 n_l^2}$) is nonlinear refractive index. Higher order terms are negligible and hence neglected.

For fused silica fibers $n_l \approx 1.46$ and $n_{nl} \approx 3.2 \times 10^{-20}\,\mathrm{m}^2/\mathrm{W}$. For the propagation of a mode carrying 100 mW of power in a single mode fiber with an effective mode area $\approx 50\,\mu\mathrm{m}^2$, resultant intensity is $2 \times 10^9\,\mathrm{W/m}^2$ and the change in refractive index due to nonlinear effect is,

$$\Delta n = n_{nl} I \approx 6.4 \times 10^{-11}$$

Although, this change in refractive index is very small, but due to very long interaction length (10–10,000 km) of an optical fiber, the accumulated effects (nonlinear) become significant. It is worth to mention that, this nonlinear term is responsible for the formation of solitons.

2.2. Effective Transmission Length

The nonlinear effects depend on transmission length. The longer the fiber link length, the more the light interaction and greater the nonlinear effect. As the optical beam propagates along the link length, its power decreases because of fiber attenuation. The effective length (L_{eff}) is that length, up to which power is assumed to be constant [6]. The optical power at a distance z along link is given as,

$$P(z) = P_{in} \exp(-\alpha z) \tag{13}$$

where P_{in} is the input power (power at $z = 0$) and α is coefficient of attenuation. For a actual link length (L), effective length is defined as, (Figure 2)

$$P_{in} L_{eff} = \int_{z=0}^{L} P(z) dz \tag{14}$$

Using equations (13) and (14), effective link length is obtained as,

$$L_{eff} = \frac{(1 - \exp(-\alpha z))}{\alpha} \tag{15}$$

Since communication fibers are long enough so that $L \gg 1/\alpha$. This results in $L_{eff} \approx 1/\alpha$.

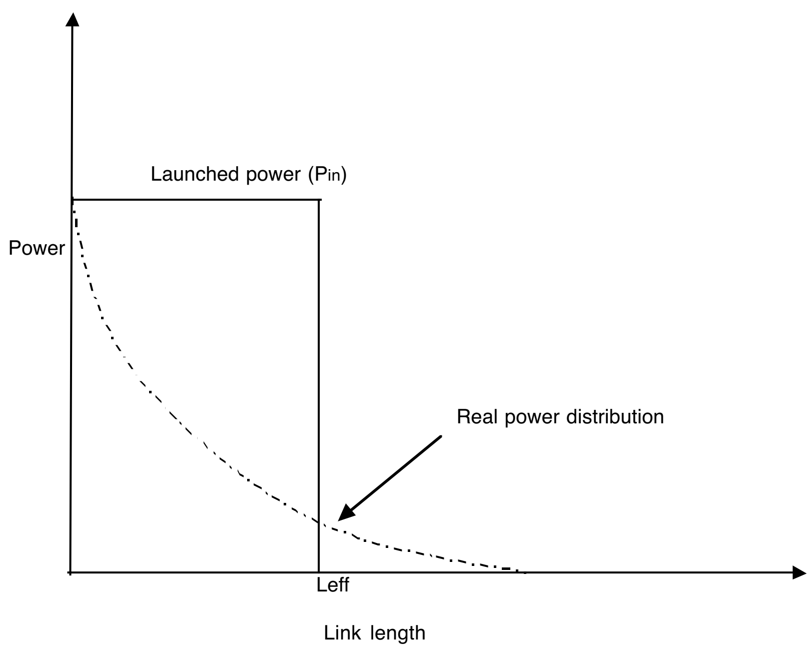


Figure 2. Definition of the effective length.

In optical systems with optical amplifiers, the signal gets amplified at each amplifier stage without resetting the effects due to nonlinearities from previous span. Obviously the effective length in such systems is sum of the effective length of each span. In a link of length L with amplifiers spaced l distance apart, the effective length is approximately given by,

$$L_{eff} = \frac{(1 - \exp(-\alpha z))}{\alpha} \frac{L}{l} \tag{16}$$

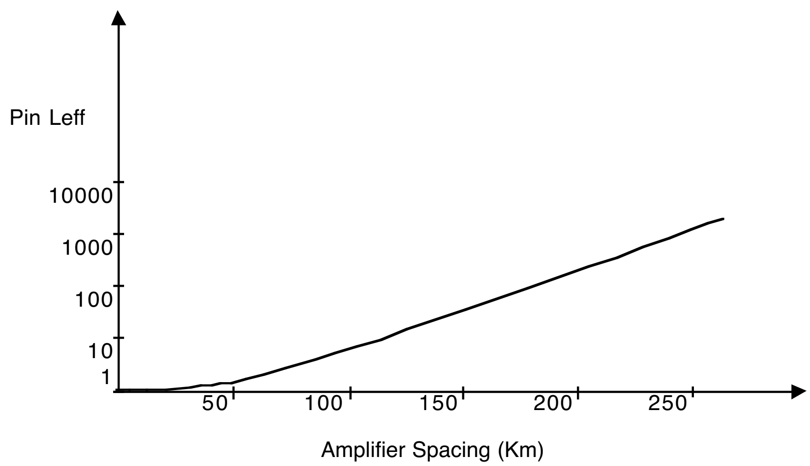


Figure 3. Relative value of $P_{in}L_{eff}$ with respect to amplifier spacing. The ordinate is the value relative to an amplifier spacing of 1 km. And attenuation coefficient $\alpha = 0.22\,\mathrm{dB/km}$.

The Figure 3 shows how $P_{in}L_{eff}$ grows with amplifiers spacing (l). It is clear from this figure that effects of nonlinearities can be reduced by reducing the amplifier spacing.

2.3. Effective Cross-sectional Area

The effect of nonlinearity grows with intensity in fiber and the intensity is inversely proportional to area of the core. Since the power is not uniformly distributed within the cross-section of the fiber, it is reasonable to use effective cross-sectional area (A_{eff}). The A_{eff} is related to the actual area (A) and the cross-sectional distribution of intensity $I(r, \theta)$ in following way [6],

$$A_{eff} = \left[\frac{\int\limits_r \int\limits_\theta r dr d\theta I(r, \theta)}{\int\limits_r \int\limits_\theta r dr d\theta I^2(r, \theta)} \right] \tag{17}$$

where r and θ denote the polar coordinates. Figure 4 provides definition of effective area (A_{eff}).

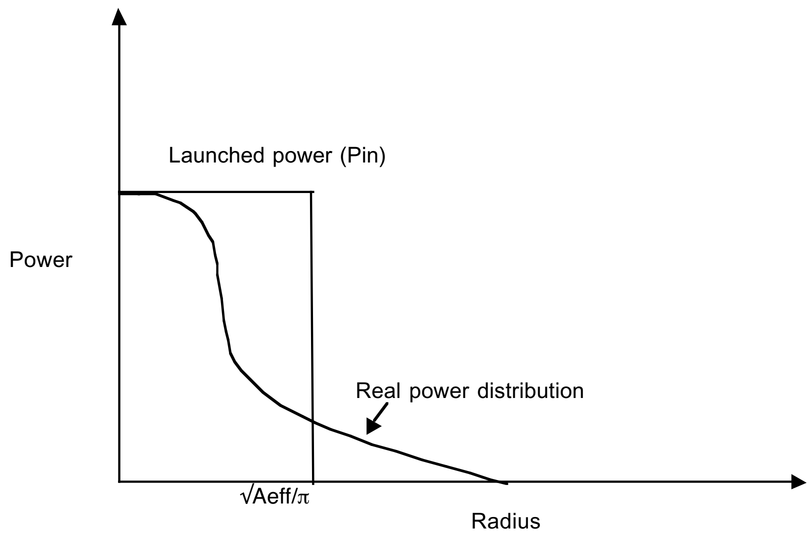


Figure 4. Definition of effective core area.

3. SELF-PHASE MODULATION (SPM)

The higher intensity portions of an optical pulse encounter a higher refractive index of the fiber compared with the lower intensity portions while it travels through the fiber. In fact time varying signal intensity produces a time varying refractive index in a medium that has an intensity-dependant refractive index. The leading edge will experience a positive refractive index gradient (dn/dt) and trailing edge a negative refractive index gradient ($-dn/dt$). This temporally varying index change results in a temporally varying phase change, as shown in Figure 5. The optical phase changes with time in exactly the same way as the optical signal [7]. Since, this nonlinear phase modulation

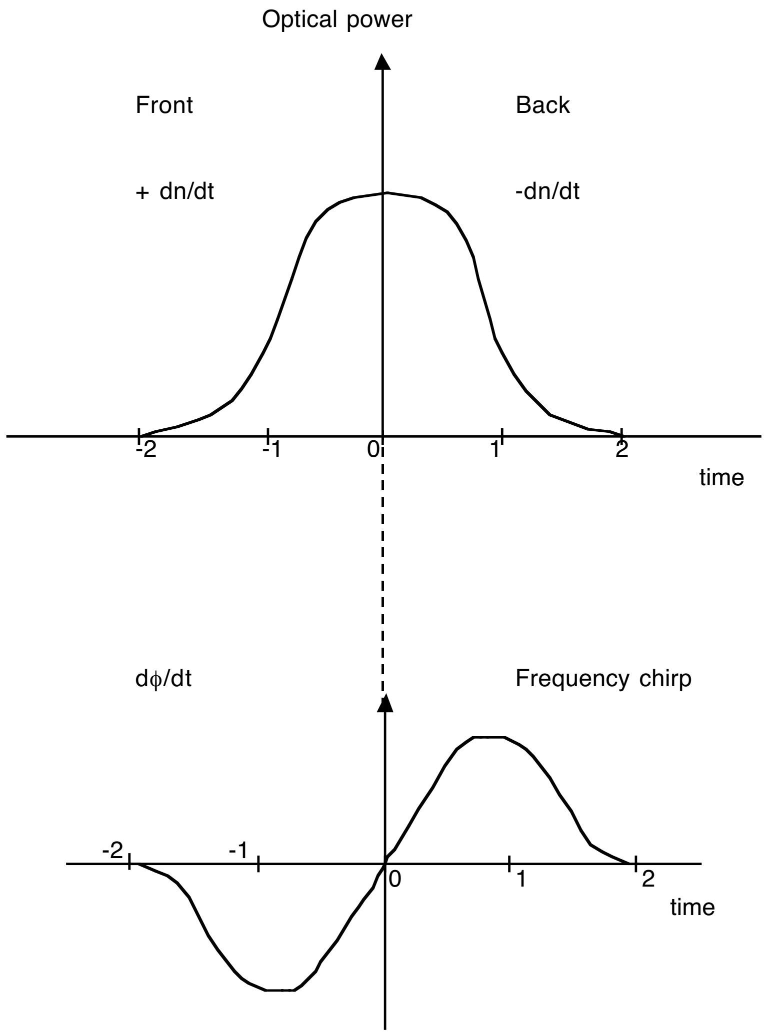


Figure 5. Phenomenological description of spectral broadening of pulse due to SPM.

is self-induced the nonlinear phenomenon responsible for it is called as self-phase modulation.

Different parts of the pulse undergo different phase shift because of intensity dependence of phase fluctuations. This results in frequency chirping. The rising edge of the pulse finds frequency shift in upper side whereas the trailing edge experiences shift in lower side. Hence primary effect of SPM is to broaden the spectrum of the pulse [8], keeping the temporal shape unaltered. The SPM effects are more pronounced in systems with high-transmitted power because the chirping effect is proportional to transmitted signal power.

The phase (ϕ) introduced by a field E over a fiber length L is given by

$$\phi = \frac{2\pi}{\lambda} nL \tag{18}$$

where λ is wavelength of optical pulse propagating in fiber of refractive index n, and nL is known as optical path length.

For a fiber containing high-transmitted power n and L can be replaced by n_{eff} and L_{eff} respectively i.e.,

$$\phi = \frac{2\pi}{\lambda} n_{eff} L_{eff}$$

or

$$\phi = \frac{2\pi}{\lambda} (n_l + n_{nl} I) L_{eff} \tag{19}$$

The first term on right hand side refers to linear portion of phase constant (ϕ_l) and second term provides nonlinear phase constant (ϕ_{nl}).

If intensity is time dependent i.e., the wave is temporally modulated then phase (ϕ) will also depend on time [9]. This variation in phase with time is responsible for change in frequency spectrum, which is given by

$$\omega = \frac{d\phi}{dt} \tag{20}$$

In a dispersive medium a change in the spectrum of temporally varying pulse will change the nature of the variation. To observe this, consider a Gaussian pulse, which modulates an optical carrier frequency ω (say) and the new instantaneous frequency becomes,

$$\omega' = \omega_0 + \frac{d\phi}{dt} \tag{21}$$

The sign of the phase shift due to SPM is negative because of the minus sign in the expression for phase, ($\omega t - kz$) i.e.,

$$\phi = -\frac{2\pi}{\lambda} L_{eff} (n_l + n_{nl} I)$$

And therefore ω becomes,

$$\omega' = \omega_0 - \frac{2\pi}{\lambda} L_{eff} n_{nl} \frac{dI}{dt} \tag{22}$$

Clearly at leading edge of the pulse $\frac{dI}{dt} > 0$ hence

$$\omega' = \omega_0 - \omega(t) \tag{23}$$

And at trailing edge $\frac{dI}{dt} < 0$ so,

$$\omega' = \omega_0 + \omega(t) \tag{24}$$

where,

$$\omega(t) = \frac{2\pi}{\lambda} L_{eff} n_{nl} \frac{dI}{dt} \tag{25}$$

This shows that the pulse is chirped i.e., frequency varies across the pulse. This chirping phenomenon is generated due to SPM, which leads to the spectral broadening of the pulse. Figures 6 and 7 show the variation of $I(t)$ and dI/dt for a Gaussian pulse.

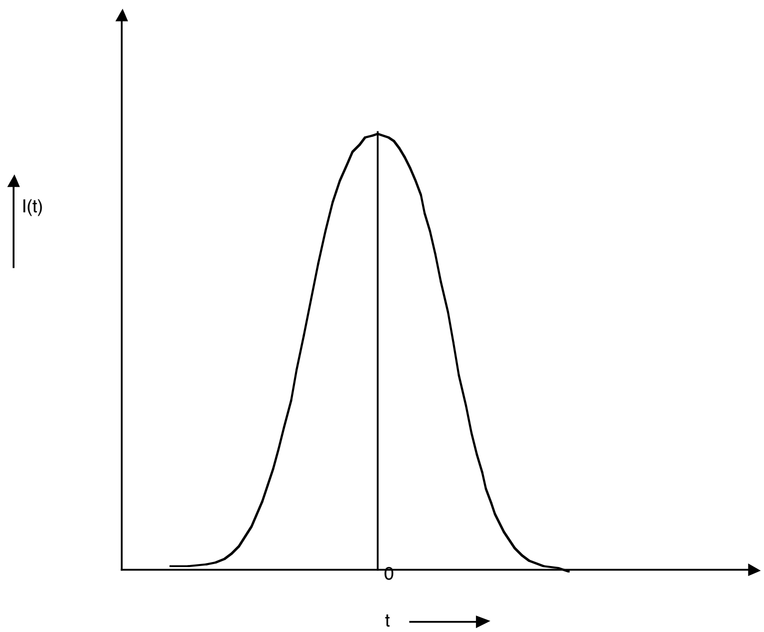


Figure 6. For a pulse with intensity varying as function of time.

There is broadening of the spectrum without any change in temporal distribution in case of self-phase modulation while in case of dispersion, there is broadening of the pulse in time domain and spectral contents are unaltered. In other words, the SPM by itself leads only to chirping, regardless of the pulse shape. It is dispersion that is responsible for pulse broadening. The SPM induced chirp modifies the pulse broadening effects of dispersion.

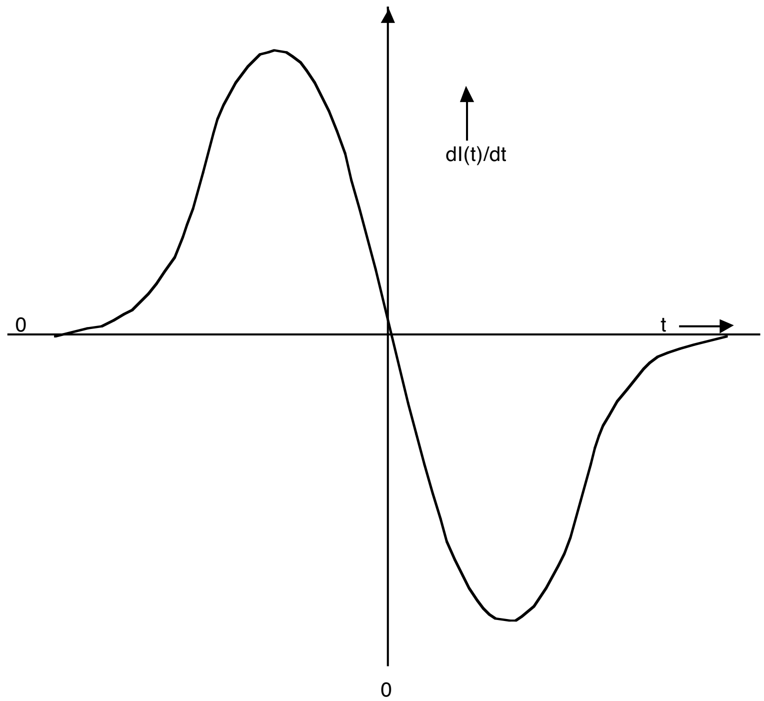


Figure 7. For a pulse with dI/dt varying as function of time.

3.1. Thresholds and Management

SPM arises due to intensity dependence of refractive index. Fluctuation in signal intensity causes change in phase of the signal. This change in phase induces additional chirp, which leads to dispersion penalty. This penalty will be small if input power is less than certain threshold value. The appropriate chirping of the input pulses can also be beneficial for reducing the SPM effects. For this, chirped RZ or CRZ modulation can be adopted.

The power dependence of nonlinear phase constant (ϕ_{nl}) is responsible for SPM impact on communication systems [5, 20]. To reduce this impact, it is necessary to have $\phi_{nl} \ll 1$. Nonlinear phase constant (ϕ_{nl}) can be written as

$$\phi_{nl} = k_{nl} P_{in} L_{eff}$$

where nonlinear propagation constant $k_{nl} = \frac{2\pi}{\lambda} \frac{n_{nl}}{A_{eff}}$.

So, with $L_{eff} \approx \frac{1}{\alpha}$; one may obtain, $P_{in} \ll \frac{\alpha}{k_{nl}}$.

Therefore, to have $\phi_{nl} \ll 1$ is equivalent to $P_{in} \ll \frac{\alpha}{k_{nl}}$.

Typically $\alpha = 0.2\,\text{dB/km}$ at $\lambda = 1550\,\text{nm}$ and $k_{nl} = 2.35 \times 10^{-3}\,1/\text{mW}$. The input power should be kept below 19.6 mW.

The move to increase the span between in-line optical amplifiers, more power must be launched into each fiber. This increased power increases SPM effect on lightwave systems, which results in pulse spreading. The use of large-effective area fibers (LEAF) reduces intensity inside the fiber and hence SPM impact on the system.

The chirp produced by SPM, which causes broadening [10], depends on the input pulse shape and the instantaneous power level within the pulse. For Gaussian shaped pulse, the chirp is even and gradual, and for a pulse that involves an abrupt change in power level (e.g., square pulse) the amount of chirp is greater. Therefore a suitable input pulse shape may be able to reduce the chirp and hence SPM induced broadening.

In general, all nonlinear effects are weak and depend on long interaction length to build up. So any mechanism that reduces interaction length decreases the effect of non-linearity. The damage due to SPM-induced pulse broadening on system performance depends on the power transmitted and length of the link. An estimation of this is shown in Figure 8, which shows that pulse can be twice as wide at the end of 200 km transmission as it was at the start.

The performance of self-phase modulation-impaired system can

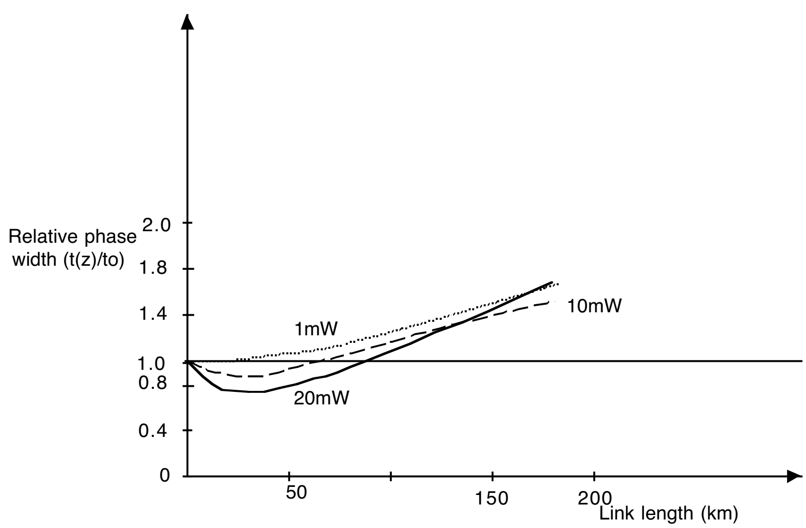


Figure 8. Pulse spreading caused by SPM as a function of distance.

be improved significantly by adjusting the net residual (NRD) of the system. For SPM-impaired system the optimal NRD can be obtained by minimizing the output distortion of signal pulse. The NRD of SPM-impaired dispersion-managed systems can be optimized by a semi analytical expression obtained with help of perturbation theory. This method is verified by numerical simulations for many SPM-impaired systems [21, 22].

3.2. Applications of SPM Phenomenon

Two important applications of SPM concept are in solitons and in pulse compression.

3.2.1. Solitons

SPM leads to chirping with lower frequencies in the leading edge and higher frequencies in the trailing edge. On the other hand the chirping caused by linear dispersion, in the wavelength region above zero dispersion wavelength, is associated with higher frequencies in leading edge and lower frequencies in the trailing edge. Both these effects are opposite. By proper choice of pulse shape (a hyperbolic secant-shape) and the power carried by the pulse, one effect can be compensated with the other. In such situation the pulse would propagate undistorted by mutual compensation of dispersion and SPM. Such a pulse would broaden neither in the time domain (as in linear dispersion) nor in frequency domain (as in SPM) and is called soliton [11, 18]. Since soliton pulse does not broaden during its propagation, it has tremendous potential for applications in super high bandwidth optical communication systems.

3.2.2. Pulse Compression

SPM phenomenon can be used in pulse compression. In the wavelength region where chromatic dispersion is positive, the red-shifted leading edge of the pulse travels slower and moves toward the center of pulse. Similarly, the blue shifted trailing edge travels faster, and also moves toward the center of the pulse. In this situation SPM causes the pulses to narrow.

Another simple pulse compression scheme is based on filtering self-phase modulation-broadened spectrum [23].

3.2.3. Optically Tunable Delays

In ultra-high speed optical communications, the optical /electronic conversion of information puts limit on transmission data rate. Therefore, it is desirable to have all-optical components for buffering and delaying signal pulses. Tunable all-optical delays are important for application in telecommunication, optical coherence tomography and optical sampling. There is a novel technique for all-optical delays which involves spectral broadening via self-phase modulation and wavelength filtering [24, 25]. Tunable delays of more than 4.2 ns for a 3.5-ps input pulse is demonstrated by using this technique.

3.2.4. Optical 40 Gb/s 3R Regenerator

Combined effect of self-phase modulation and cross-absorption modulation is utilized in all optical 3R regenerators [26]. The performance of such regenerators is experimentally verified for 40 Gb/s data rate. The introduction of a predistortion block configuration including a highly non-linear fiber enhances the chromatic dispersion tolerance.

4. CROSS PHASE MODULATION (CPM)

SPM is the major nonlinear limitation in a single channel system. The intensity dependence of refractive index leads to another nonlinear phenomenon known as cross-phase modulation (CPM). When two or more optical pulses propagate simultaneously, the cross-phase modulation is always accompanied by SPM and occurs because the nonlinear refractive index seen by an optical beam depends not only on the intensity of that beam but also on the intensity of the other copropagating beams [13]. In fact CPM converts power fluctuations in a particular wavelength channel to phase fluctuations in other copropagating channels. The result of CPM may be asymmetric spectral broadening and distortion of the pulse shape.

The effective refractive index of a nonlinear medium can be expressed in terms of the input power (P) and effective core area (A_{eff}) as,

$$n_{eff} = n_l + n_{nl} \frac{P}{A_{eff}} \tag{26}$$

The nonlinear effects depend on ratio of light power to the cross-sectional area of the fiber. If the first-order perturbation theory is applied to investigate how fiber modes are affected by the nonlinear refractive index, it is found that the mode shape does not change but the propagation constant becomes power dependent.

$$k_{eff} = k_l + k_{nl}P \tag{27}$$

where kl is the linear portion of the propagation constant and k_{nl} is nonlinear propagation constant. The phase shift caused by nonlinear propagation constant in traveling a distance L inside fiber is given as

$$\phi_{nl} = \int_0^L (k_{eff} - k_l)dz \tag{28}$$

Using equations (27) and (14) nonlinear phase shift becomes,

$$\phi_{nl} = k_{nl}P_{in}L_{eff} \tag{29}$$

When several optical pulses propagate simultaneously the nonlinear phase shift of first channel ϕ^1_{nl} (say) depends not only on the power of that channel but also on signal power of other channels. For two channels, ϕ^1_{nl} can be given as,

$$\phi^1_{nl} = k_{eff}L_{eff}(P_1 + 2P_2) \tag{30}$$

For N-channel transmission system, the shift for ith channel can be given as [8],

$$\phi^i_{nl} = k_{nl}L_{eff}\left(P_i + 2\sum_{n\neq i}^{N} P_n\right) \tag{31}$$

The factor 2 in above equation has its origin in the form of nonlinear susceptibility [5] and indicates that CPM is twice as effective as SPM for the same amount of power. The first term in above equation represents the contribution of SPM and second term that of CPM. It can be observed that CPM is effective only when the interacting signals superimpose in time.

CPM hinders the system performance through the same mechanism as SPM: chirping frequency and chromatic dispersion, but CPM can damage the system performance even more than SPM. CPM influences the system severely when number of channels is large. Theoretically, for a 100-channels system, CPM imposes a power limit of 0.1 mW per channel.

4.1. Thresholds and Management

The CPM-induced phase shift can occur only when two pulses overlap in time. Due to this overlapping, the intensity-dependent phase shift and consequent chirping is enhanced. Therefore the pulse broadening is also enhanced, which limits the performance of lightwave systems. The effects of CPM can be reduced by increasing the wavelength spacing between individual channels. For increased wavelength spacing, pulse overlaps for such a short time that CPM effects are virtually negligible. In fact, owing to fiber dispersion, the propagation constants of these channels become sufficiently different so that the pulses corresponding to individual channels walk away from each other. Due to this pulse walk-off phenomenon the pulses, which were initially temporally coincident, cease to be so after propagating for some distance and cannot interact further. Thus, effect of CPM is reduced.

In a WDM system, CPM converts power fluctuations in a particular wavelength channel to phase fluctuations in other copropagating channels. This leads to broadening of pulse. It can be greatly mitigated in WDM systems operating over standard non-dispersion shifted single mode fiber [14, 20]. One more advantage of this kind of fiber is its effective core area, which is typically $80\,\mu\mathrm{m}^2$. This large effective area is helpful in reducing nonlinear effects because k_{nl} is inversely proportional to A_{eff}.

Like SPM, the CPM also depends on interaction length of fiber. The long interaction length is always helpful in building up this effect up to a significant level. Keeping inter action length small, one can reduce this kind of nonlinearity.

4.2. Applications of CPM Phenomenon

Optical switching and pulse compression can be done through the CPM phenomenon.

4.2.1. Optical Switching

Phase shift, in an optical pulse, due to CPM phenomenon can be used for optical switching. To take advantage of CPM-induced phase shift for ultra-fast optical switching many interferometric methods have been used [5]. Consider a interferometer designed in such a way that a weak signal pulse, divided equally between its two arms, experiences identical phase shifts in each arm and is transmitted through constructive interference. When a pump pulse at different wavelength is injected into one of the arms, it will change the signal phase through CPM phenomenon in that arm. If the CPM-induced phase shift is large (close to π), this much phase shift results in destructive interference and hence no transmission of signal pulse. Thus an intense pump pulse can switch the signal pulse.

4.2.2. Pulse Compression

Like SPM induced frequency chirp, the CPM induced frequency chirp can also be used for pulse compression. The SPM techniques require the input pulse to be intense and energetic, but the CPM is able to compress even weak input pulses because copropagating intense pump pulse produces the frequency chirp. The CPM induced chirp is affected by pulse walk-off and depends critically on the initial relative pump-signal delay. As a result the use of CPM induced pulse compression requires a careful control of the pump pulse parameters such as its width, peak power, wavelength and initial delay relative to the signal pulse.

4.2.3. Pulse Retiming

In an anomalous-dispersion polarization-maintained fiber ultra-fast optical pulses can be retimed by utilizing cross-phase modulation phenomenon. With help of this phenomenon spectral, temporal and spatial properties of ultra-short pulses can be controlled [27, 28].

5. FOUR-WAVE MIXING (FWM)

The origin of FWM process lies in the nonlinear response of bound electrons of a material to an applied optical field. In fact, the polarization induced in the medium contains not only linear terms but also the nonlinear terms. The magnitude of these terms is governed by the nonlinear susceptibilities of different orders. The FWM process originates from third order nonlinear susceptibility ($\chi^{(3)}$). If three optical fields with carrier frequencies ω_1, ω_2 and ω_3, copropagate inside the fiber simultaneously, ($\chi^{(3)}$) generates a fourth field with frequency ω_4, which is related to other frequencies by a relation, $\omega_4 = \omega_1 \pm \omega_2 \pm \omega_3$.

In quantum-mechanical context, FWM occurs when photons from one or more waves are annihilated and new photons are created at different frequencies such that net energy and momentum are conserved during the interaction.

SPM and CPM are significant mainly for high bit rate systems, but the FWM effect is independent of the bit rate and is critically dependant on the channel spacing and fiber dispersion. Decreasing the channel spacing increases the four-wave mixing effect and so does decreasing the dispersion.

In order to understand the FWM effect [6], consider a WDM signal, which is sum of n monochromatic plane waves. The electric field of such signal can be written as

$$E = \sum_{p=1}^{n} E_p \cos(\omega_p t - k_p z) \tag{32}$$

Then the nonlinear polarization is given by

$$P_{nl} = \varepsilon_0 \chi^{(3)} E^3 \tag{33}$$

For this case P_{nl} takes the form as

$$P_{nl} = \varepsilon_0 \chi^{(3)} \sum_{p=1}^{n} \sum_{q=1}^{n} \sum_{r=1}^{n} E_p \cos(\omega_p t - k_p z) E_q \cos(\omega_q t - t_q z) E_r \cos(\omega_r t - k_r z) \tag{34}$$

Expansion of above expression gives,

$$\begin{aligned}
P_{nl} = &\frac{3}{4}\varepsilon_0 \chi^{(3)} \sum_{p=1}^{n} E_p^2 + 2\sum_{q\neq p} E_p E_q E_p \cos(\omega_p t - t_p z) \\
&+\frac{1}{4}\varepsilon_0 \chi^{(3)} \sum_{p=1}^{n} E_p^3 \cos(3\omega_p t - 3k_p z) \\
&+\frac{3}{4}\varepsilon_0 \chi^{(3)} \sum_{p=1}^{n} \sum_{q\neq p} E_p^2 E_q \cos\{(2\omega_p - \omega_q)t - (2k_p - k_q)z\} \\
&+\frac{3}{4}\varepsilon_0 \chi^{(3)} \sum_{p=1}^{n} \sum_{q\neq 1} E_p^2 E_q \cos\{(2\omega_p + \omega_q)t - (2k_p + k_q)z\} \\
&+\frac{6}{4}\varepsilon_0 \chi^{(3)} \sum_{p=1}^{n} \sum_{q>p} \sum_{r>q} E_p E_q E_r \cos\{(2\omega_p + \omega_q + \omega_r)t - (k_p + k_q + k_r)z\} \\
&+\cos\{(\omega_p + \omega_q + \omega_r)t - (k_p + k_q + k_r)z\} \\
&+\cos\{(\omega_p - \omega_q + \omega_r)t - (k_p - k_q + k_r)z\} \\
&+\cos\{(\omega_p - \omega_q - \omega_r)t - (k_p - k_q - k_r)z\}
\end{aligned} \tag{35}$$

The first terms in above equation represents the effect of SPM and CPM. Second, third and fourth terms can be neglected because of phase mismatch. The reason behind this phase mismatch is that, in real fibers $k(3\omega) \neq 3k(\omega)$ so any difference like $(3\omega - 3k)$ is called as phase mismatch. The phase mismatch can also be understood as the mismatch in phase between different signals traveling within the fiber at different group velocities. All these waves can be neglected because they contribute little. The last term represents phenomenon of four-wave mixing. It is this term, which tells that three EM waves propagating in a fiber generate new waves [16] with frequencies $(\omega_p \pm \omega_q \pm \omega_r)$. Four-wave mixing (FWM) is analogous to intermodulation distortion in electrical systems. The last term of polarization expression tells that FWM comes from frequency combinations like $(\omega_p + \omega_q - \omega_r)$. In compact form all these combinations can be written as

$$\omega_{pqr} = \omega_p + \omega_q - \omega_r \quad \text{with } p, q \neq r \tag{36}$$

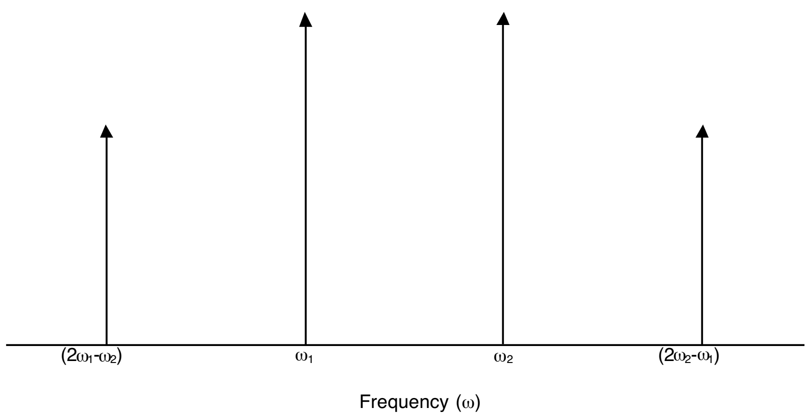


Figure 9. Showing mixing of two waves.

Figure 9 shows a simple example of mixing of two waves at frequency ω_1 and ω_2. When these waves mixed up, they generate sidebands at $(2\omega_1 - \omega_2)$ and $(2\omega_2 - \omega_1)$. Similarly, three copropagating waves will create nine new optical sideband waves at frequencies given by equation (36). These sidebands travel along with original waves and will grow at the expense of signal-strength depletion. In general for N-wavelengths launched into fiber, the number of generated mixed products M is,

$$M = N^2/2 \cdot (N - 1)$$

The efficiency FWM depends on fiber dispersion and the channel spacing. Since the dispersion varies with wavelength, the signal waves and the generated waves have different group velocities. This destroys the phase matching of interacting waves and lowers the efficiency of power transfer to newly generated frequencies. The higher the group velocity mismatch and wider the channel spacing, the lower the four-wave mixing. This is shown in Figure 10. The curves show the frequency-spacing range over which the FWM process is efficient for

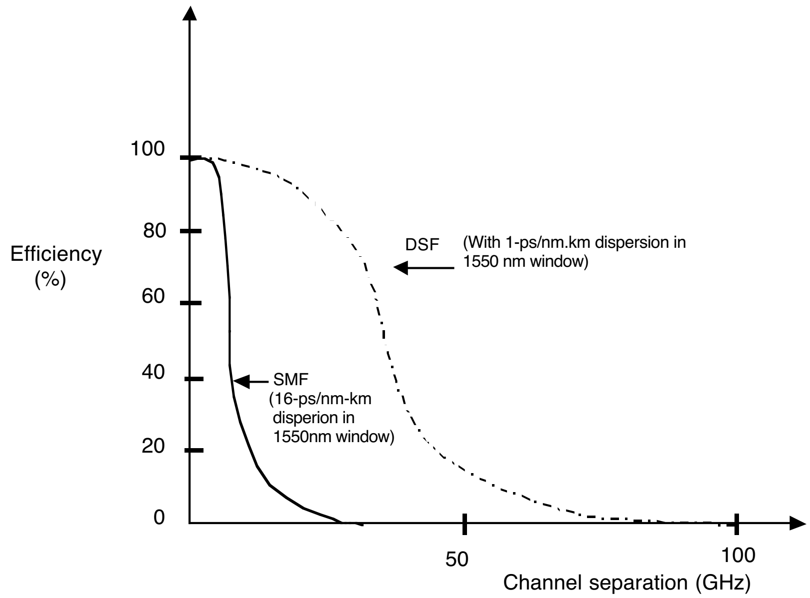


Figure 10. Efficiency of four wave mixing with respect to channel separation.

two dispersion values. It is clear that in conventional SMFs, frequencies with separations less than 20 GHz will mix efficiently. But for DSFs, FWM efficiencies are greater than 20% for separation upto 50 GHz.

5.1. Thresholds and Management

Four-wave mixing process results in power transfer from one channel to other. This phenomenon results in power depletion of the channel, which degrades the performance of that channel (i.e., BER is increased). In order to achieve original BER, some additional power is required which is termed as power penalty. Since, FWM itself is interchannel crosstalk it induces interference of information from one channel with another channel. This interference again degrades the system performance. To reduce this degradation, channel spacing must be increased. This increases the group velocity mismatch between channels and hence FWM penalty is reduced.

Four-wave mixing presents a severe problem in WDM systems using dispersion-shifted fibers (DSF) [12]. Penalty due to FWM can be reduced if a little chromatic dispersion is present in the fiber. Due to chromatic dispersion, different interacting waves travel with different group velocities. This results in reduced efficiency of FWM and hence penalty. A non-zero dispersion-shifted fiber is used for this purpose. The FWM imposes limitations on the maximum transmit power per channel. This limitation for system operating over standard single-mode fiber (SMF) and dispersion-shifted fiber (DSF) is shown in Figure 11.

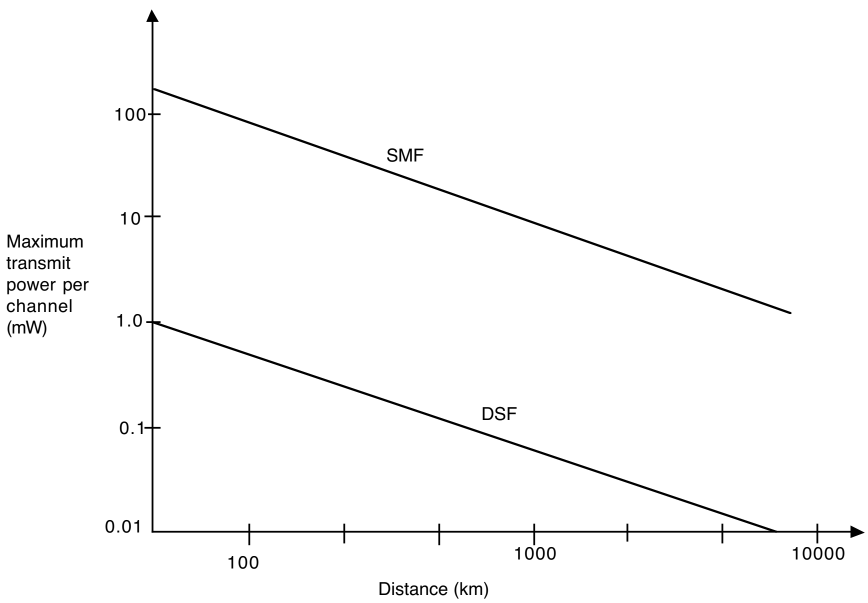


Figure 11. Maximum transmitted power per channel versus distance imposed by FWM.

Like other nonlinear effects limitations of FWM on a communication system depend on the effective area (A_{eff}), effective fiber length (L_{eff}) and, of course, on the intensity of transmitted signal. Using NZ-DSF of large effective area and small effective length with reduced transmitted signal power results in reduction of penalty due to FWM process.

FWM produces severe limitations on performance of WDM all-optical networks. The number of FWM components increases with the increase in number of users. If these generated wavelengths coincide with the original signal wavelength, then it results in interference causing degradation in signal-to-noise ratio (SNR). This effect can be reduced by using modified repeated unequally spaced channel allocation [29].

5.2. Applications of FWM Process

Two important applications of FWM are squeezing and wavelength conversion. These are described below.

5.2.1. Squeezing

The FWM process can be used to reduce quantum noise through a phenomenon called squeezing. In fact squeezing is a process of generating the special states of an electromagnetic field for which noise fluctuations, in some frequency range, are reduced below the quantum-noise level [5]. FWM can be used for squeezing as noise components at the signal and idler frequencies are coupled through the fiber nonlinearity.

Physically, squeezing can be understood as deamplification of signal and idler waves for certain values of relative phase between the two waves. Photons of random phases are generated due to spontaneous emission at the signal and idler frequencies. Four-wave mixing process increases or decreases the number of signal-idler photon pairs depending on their relative phases. Noise is reduced below the quantum-noise level when the phase of the local oscillator is adjusted to match the relative phase corresponding to the photon pair,whose number was reduced as a result of FWM process.

5.2.2. Wavelength Conversion

Four-wave-mixing phenomenon can be used effectively for wavelength conversion too. The function of wavelength converter is to transform information from one wavelength to another. A phenomenological method of wavelength conversion is shown in Figure 12. When a data input (λ_1) and a probe signal (λ_2) are injected into a nonlinear medium, due to mixing process a new signal (λ_3) is generated in association with

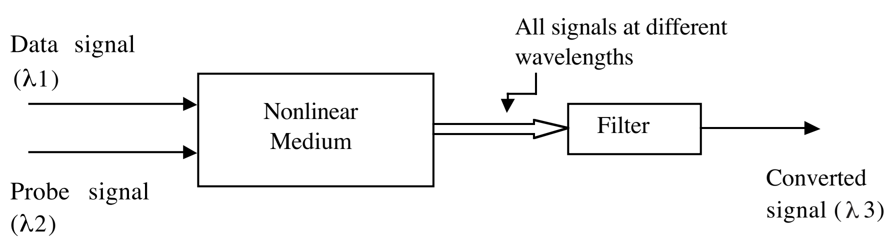


Figure 12. Phenomenological description of wavelength conversion through FWM process.

other signal wavelengths such that;

$$\frac{1}{\lambda_3} = \frac{2}{\lambda_1} - \frac{1}{\lambda_2}$$

or

$$\omega_3 = 2\omega_1 - \omega_2$$

where ω is angular frequency.

The wavelength conversion is an important component in all-optical networks, since the wavelength of incoming signal may already be in use by another information channel residing on the destined outgoing path. Converting the incoming signal to new wavelength will allow both information channels to traverse the same fiber simultaneously.

Four-wave-mixing based wavelength conversion at 1.55 μm in a 2.2 m long dispersion-shifted lead-silicate holy fiber has been investigated [30]. It is shown that highly efficient and broadband wavelength conversion, covering the entire C band, can be achieved for such fibers at reasonable optical pump power.

Table 2. Comparison of nonlinear refractive effects.

Nonlinear Phenomenon → **Characteristics ↓**	**SPM**	**CPM**	**FWM**
1. Bit-rate	Dependent	Dependent	Independent
2. Origin	Nonlinear susceptibility $\chi^{(3)}$	Nonlinear susceptibility $\chi^{(3)}$	Nonlinear susceptibility $\chi^{(3)}$
3. Effects of $\chi^{(3)}$	Phase shift due to pulse itself only	Phase shift is alone due to copropagating signals	New waves are generated
4. Shape of broadening	Symmetrical	May be a symmetrical	—
5. Energy transfer between medium and optical pulse	No	No	No
6. Channel spacing	No effect	Increases on decreasing the spacing	Increases on decreasing the spacing

6. COMPARISON OF DIFFERENT NONLINEAR EFFECTS

Different nonlinear effects based on Kerr-effect are compared in Table 2. The parameters taken are bit-rate, origin, effects of third-order susceptibility, shape of broadening, energy transfer between medium and optical pulse and effect of channel spacing.

7. CONCLUSION

Nonlinear effects such as SPM, CPM, and FWM are discussed. These effects degrade the performance of fiber optic systems. Impact of SPM is negligible if power per channel is below 19.6 mW. FWM has severe effects in WDM systems, which uses dispersion-shifted fiber. If some dispersion is their, then effect of FWM is reduced. That is why non-zero dispersion-shifted fibers are normally used in WDM systems. Though these effects degrade nature, they are also useful for many applications such as SPM in solitons and pulse compression, CPM in optical switching, and FWM in squeezing and wavelength conversion.

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