CALCULATION OF ATTENUATION IN WAVE GUIDES*

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SUMMARY

This paper, based on work carried out in 1941–1942, furnishes tables and curves giving

(*a*) field equations for rectangular and circular wave guides, and

(*b*) attenuation constants of wave-modes likely to be met in practice in these guides.

The text explains the derivation of the tables and curves.

It should be noted that the approach to the problem uses wave-engths to describe both the frequency (λ_e) and the guide itself (λ_{cr}). Wavelengths refer always to the dielectric which fills the guide. The free-space wavelength is not used at all.

Field amplitudes (Table 1) are expressed in terms of power carried by a wave. To obtain comprehensive formulae, the concept of "characteristic density" of energy, or power, is introduced. Other factors determining field amplitudes enter into the equations in the form of field impedances.

A general formula (Table 2) is produced for the attenuation constant α_w caused by losses in the wall-metal. This formula applied to any mode of wave in rectangular or circular guide, and the values of coefficients, which are to be used in a particular case, are given in the table. The curves shown in Fig. 1–4 are of a general character, and may be used for any size of guide and at any frequency. Figs. 1 and 2 deal with the attenuation constant α_w of air-filled copper guides in the transmitting region. Fig. 1 shows attenuation of H_{01}, H_{11} and E_{11}-modes in a rectangular guide, and Fig. 2 shows attenuation of the lowest modes in a circular guide. The curves show the relationship between a quantity ($\alpha_w D^{3/2}$), which is independent of the actual values of linear dimensions, and the ratio λ_e/D, which is the only term influenced by frequency.

Fig. 3 gives the attenuation constant α_d due to loss in the dielectric filling of a guide, and shows the relationship between a quantity ($\alpha_d - \lambda_{cr}$), again independent of the values of linear dimensions, and the ratio λ_e/λ_{cr} through which the frequency affects the attenuation. The curves are drawn for a few loss-angles, δ, of the dielectric and are applicable to any wave-mode in any guide, in both the transmitting and attenuating regions.

Fig. 4 is a counterpart to Fig. 3, showing the values of phase constant β instead of attenuation constant α_d.

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(1) INTRODUCTION

During the year 1941–2, information about wave guides in general, and their attenuation in particular, was scattered throughout various publications (the book by H. R. L. Lamont on wave guides appeared late in 1942), and a need was felt for having a comprehensive picture of phenomena in a wave guide, presented in a form directly applicable to engineering problems. Also, some means were wanted which would enable the attenuation of a guide at any frequency to be obtained readily. Consequently two A.S.E. reports were written by the author, with the intention of supplying this information.

The paper deals with the two common forms of wave guide—rectangular and circular. The Tables give the field equations for any mode of wave propagated in the guide, and the attenuation curves have been calculated for the lowest modes in air-filled copper guides. The curves present relationships between two quantities: one determining the attenuation of a guide and the other depending on frequency and on the dimensions of the guide cross section. These quantities have been chosen so as to obtain curves applicable to any frequency and to a guide of any dimensions. This is why the curves directly give the numerical value of attenuation in any case encountered in practice.

(2) SYSTEMS OF UNITS, QUANTITIES AND NOTATIONS

(2.1) Units

The Gaussian systems of units has been used throughout the calculations. However, the final field equations are presented in a form which would give a correct answer either in the Gaussian or in the M.K.S. rationalized system. To achieve this, the characteristic density P' of energy, or power, which enters into the formulae, has to be obtained differently in each system of units.

(2.2) Frequency

The way of describing the frequency has been chosen after much consideration. Usually, in radio engineering, this essential characteristic of a wave is given either as the frequency itself, or as the wavelength, λ_0, in free space. In wave-guide technique the linear dimensions of the guide play an important role, and hence it is reasonable to use wavelength as the factor describing the frequency of a wave. But, if a guide is filled with a dielectric (κ, μ) and not with air, the wavelength in free space does not appear at all in guide phenomena; the corresponding relevant wavelength is that which would appear in the unbounded dielectric, κ, μ. For these reasons, neither the frequency f nor the wavelength λ_0 in free space is used in the paper; the frequency is described in terms of the wavelength λ_e in the unbounded dielectric (κ, μ) which fills the guide. This wavelength is given by the formula:

$$\lambda_e = \frac{c}{f(\kappa\mu)^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where c is the velocity of light in free space: κ and μ are in the Gaussian system of units.

The only case in which the frequency is explicitly mentioned is that dealing with variations of fields in time; in this case the term ω has the usual meaning: $\omega = 2\pi f$.

(2.3) Wavelength

In the guide, there always exist spatial oscillations of power in the transverse direction of the guide. The actual wavelength of these oscillations for a given wave-mode is entirely determined by the metallic boundaries of the guide, i.e. by the shape and dimensions of the guide cross-section. Hence, if the length of the transversely-directed waves actually existing in the guide dielectric is considered, this length will depend neither on frequency nor on the properties of the dielectric.

For propagation along a guide to be possible, the wavelength λ_e in the unbounded dielectric must be smaller than the wavelength of these transverse waves. Therefore, the wavelength of the "transverse" waves in the dielectric may be called the critical wavelength, λ_{cr}, of the guide.

The wavelength λ_x along the guide depends on the wavelength

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λ_e in the unbounded dielectric and on the critical wavelength λ_{cr} of the guide; through λ_e, it depends on the properties κ, μ of the dielectric and on the frequency used, and through λ_{cr}, on the shape and dimensions of the guide cross-section and on the mode of the wave.

The relationship between these three wavelengths follows from the general solution of Maxwell's equations, viz.,

$$1/\lambda_e^2 = 1/\lambda_{cr}^2 + 1/\lambda_x^2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(2.4) Impedance

The ratios of electric and magnetic fields responsible for the propagation of power are given in terms of the field impedance, Z.

Two main kinds of impedance are used:—

(*a*) The specific (sometimes called intrinsic) impedance, Z_x, of the guide, which is the ratio of transverse components of the electric and magnetic fields, responsible for propagation along the guide, and

(*b*) the transverse impedance, Z_{cr}, of the guide, which is the ratio of those components of fields which are responsible for transverse oscillations.

The specific impedance Z_e of the unbounded dielectric κ, μ is also used. This is the ratio of the electric and magnetic fields of a simple plane wave propagated in the dielectric; Z_e is given by the formula

$$Z_e = (\mu/\kappa)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The relationship between impedances Z_x, Z_{cr} and Z_e for an H-wave is different from that for an E-wave.

In the case of an H-type of wave,

$$Z_x = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}} \frac{\lambda_x}{\lambda_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$Z_{cr} = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}} \frac{\lambda_{cr}}{\lambda_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and hence, by equation (1), we have the relationship,

$$1/Z_e^2 = 1/Z_{cr}^2 + 1/Z_x^2 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In the case of an E-type of wave,

$$Z_x = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}} \frac{\lambda_e}{\lambda_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$Z_{cr} = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}} \frac{\lambda_e}{\lambda_{cr}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and equation (1) gives

$$Z_e^2 = Z_{cr}^2 + Z_x^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the Gaussian system of units is used, where κ and μ are dimensionless and where, for vacuum, $\kappa_0 = \mu_0 = 1$, these impedances are also dimensionless, and the specific impedance of free space becomes:

$$Z_0 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In the M.K.S. rationalized system, κ is expressed in farads/metre and μ in henrys/metre, and hence the impedances in question are obtained in ohms. For vacuum, or air,

$$\kappa_0 = (1/30\pi) \, . \, 10^{-9} \text{ farad/metre}; \mu_0 = 4\pi \, . \, 10^{-7} \text{ henrys/metre} \quad (11)$$

and the specific impedance of free space becomes:

$$Z_0 = 120\pi \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The specific impedance Z_e of a dielectric (κ, μ) can be written in the M.K.S. system as:

$$Z_e = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}} = 120\pi\left(\frac{\mu/\mu_0}{\kappa/\kappa_0}\right)^{\frac{1}{2}} \text{ ohms} \quad . \quad . \quad . \quad (13)$$

(2.5) Power Carried by Wave

The total power P carried by the wave along the guide is taken as the factor determining the actual values of fields in the guide. This has been considered useful for practical purposes, since in practice it is usual to postulate that some power passes through a guide, and then to find the fields, and particularly the maximum values of electric field, appearing in the guide. Or alternatively, when some admissible electric field in the dielectric is assumed, one wishes to know quickly what is the maximum power which could be transmitted in the guide. However, the total power P does not enter into the field equations directly; instead, another quantity, P', is used, which in a given guide and for a given mode of wave is proportional to P. The quantity, P', may be called the "characteristic density" either of energy or of power, depending on the system of units used.

To obtain the characteristic density, P', the total power P is to be divided by the "reduced" cross-section of the guide. The "reduced" cross-section is obtained by multiplying the true cross-sectional area S by a coefficient, k_s, the value of which depends on the shape of the cross-section and on the mode of wave concerned. Values of coefficients k_s are given in Table 1 for each type of wave.

In the ordinary (unrationalized) Gaussian system of units the value of the total power is "irrationalized" by putting the factor 4π in the expression for P'. This quantity then becomes:

$$P' = (4\pi/c)(P/k_sS) \text{ ergs/cm}^3 \quad . \quad . \quad . \quad . \quad (14)$$

and its physical dimension is that of energy-density.

In the M.K.S. rationalized system it becomes

$$P' = P/k_sS \text{ watts/m}^2 \quad . \quad . \quad . \quad . \quad (15)$$

with the dimensions of power-density.

(2.6) Wave-Mode Notation

The name of H-wave is given to a wave in which the longitudinal component of the electric field is everywhere zero. The more recently adopted name for such a wave is the "transverse electric" wave, T.E. Similarly, if the longitudinal component of the magnetic field is everywhere zero, the wave is called an E-wave, the new name for which is "transverse magnetic" wave, T.M.

A particular mode of wave within each of the families is determined by subscripts n and m, added to the letters H and E. The meaning of these subscripts is different in the cases of rectangular and of circular guides. The subscript n denotes the number of half-periods of field variation along the height a of the cross section, and the subscript m, the number of such half-periods along the width b (a and b are the internal dimensions of the wave guide).

In this paper it is assumed that the guide is viewed in such a way that n is not greater than m.

As a result, a may be smaller than, equal to, or greater than b. Both n and m are integral numbers. In the case of an H-wave either one of the subscripts can be equal to zero. The mode H_{nm} then becomes H_{0m}. The lowest and most commonly used mode of an H-wave in a rectangular guide will be denoted by H_{01}, which agrees with the old notations adopted by some writers (e.g. Barrow and Chu), but disagrees with notations used, for instance, by Lamont. It is clear that in the case of this mode the larger side of the cross-section should be taken as the width, b, of the guide.

The similar mode, which would have no variation of the fields along the larger side of the cross-section, would have the same notation, H_{01}, the difference being that the guide should be orientated so as to make the width b smaller than the height a.

Table 1

FIELD EQUATIONS OF TRAVELLING H_{nm} AND E_{nm} WAVES IN RECTANGULAR AND CIRCULAR WAVE GUIDES (WITHOUT LOSSES)

Rectangular Wave Guide

Right-handed Cartesian co-ordinate system

n = an integer.
m = an integer and $m \neq 0$.
$n \leqslant m$.
a and b = inside dimensions of guide in centimetres.

H_{nm}-wave	E_{nm}-wave
n an integer, or $n = 0$ $\left.\begin{aligned} H_x &= \frac{1}{Z_{cr}}(2P'Z_x)^{\frac{1}{2}} \cos\frac{2\pi}{\lambda_a}y \cos\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ H_y &= j\frac{Z_{cr}}{Z_a}\frac{1}{Z_x}(2P'Z_x)^{\frac{1}{2}} \sin\frac{2\pi}{\lambda_a}y \cos\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ H_z &= j\frac{Z_{cr}}{Z_b}\frac{1}{Z_x}(2P'Z_x)^{\frac{1}{2}} \cos\frac{2\pi}{\lambda_a}y \sin\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ \mathscr{E}_x &= 0 \\ \mathscr{E}_y &= j\frac{Z_{cr}}{Z_b}(2P'Z_x)^{\frac{1}{2}} \cos\frac{2\pi}{\lambda_a}y \sin\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ \mathscr{E}_z &= -j\frac{Z_{cr}}{Z_a}(2P'Z_x)^{\frac{1}{2}} \sin\frac{2\pi}{\lambda_a}y \cos\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \end{aligned}\right\}$ Real parts	n an integer and $n \neq 0$ $\left.\begin{aligned} H_x &= 0 \\ H_y &= -\frac{Z_b}{Z_{cr}}\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \sin\frac{2\pi}{\lambda_a}y \cos\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ H_z &= \frac{Z_a}{Z_{cr}}\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \cos\frac{2\pi}{\lambda_a}y \sin\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ \mathscr{E}_x &= jZ_{cr}\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \sin\frac{2\pi}{\lambda_a}y \sin\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ \mathscr{E}_y &= \frac{Z_a}{Z_{cr}} Z_x\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \cos\frac{2\pi}{\lambda_a}y \sin\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \\ \mathscr{E}_z &= \frac{Z_b}{Z_{cr}} Z_x\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \sin\frac{2\pi}{\lambda_a}y \cos\frac{2\pi}{\lambda_b}z\varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x+j\omega t\right)} \end{aligned}\right\}$ Real parts
$\lambda_a = \frac{2a}{n} \quad \lambda_b = \frac{2b}{m} \quad \frac{1}{\lambda_a^2} + \frac{1}{\lambda_b^2} = \frac{1}{\lambda_{cr}^2}$ $Z_a = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_a}{\lambda_e} \quad Z_b = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_b}{\lambda_e} \quad Z_{cr} = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_{cr}}{\lambda_e}$ $\frac{1}{Z_a^2} + \frac{1}{Z_b^2} = \frac{1}{Z_{cr}^2}$ $Z_x = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_x}{\lambda_e} \quad \frac{1}{Z_{cr}^2} + \frac{1}{Z_x^2} = \frac{1}{Z_e^2} = \frac{1}{\mu/\kappa}$ $k_s \begin{cases} \text{for } n = 0 & k_s = \frac{1}{2} \\ \text{for } n \neq 0 & k_s = \frac{1}{4} \end{cases}$	$\lambda_a = \frac{2a}{n} \quad \lambda_b = \frac{2b}{m} \quad \frac{1}{\lambda_a^2} + \frac{1}{\lambda_b^2} = \frac{1}{\lambda_{cr}^2}$ $Z_a = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_e}{\lambda_a} \quad Z_b = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_e}{\lambda_b} \quad Z_{cr} = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_e}{\lambda_{cr}}$ $Z_a^2 + Z_b^2 = Z_{cr}^2$ $Z_x = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_e}{\lambda_x} \quad Z_{cr}^2 + Z_x^2 = Z_e^2 = \mu/\kappa$ $k_s = \frac{1}{4}$

Table 1 *(continued)*

Circular Wave Guide

Right-handed cylindrical co-ordinate system.

d internal guide diameter, centimetres.
n along the φ co-ordinate.
m — along the r co-ordinate.
n — an integer, or $n = 0$.
m an integer, and $m \neq 0$.

$J_n(p)$ Bessel function of the first kind of order n.
$g_{n,m}$ m-th root of equation: $J_n(p) = 0$.
$J'(p)$ = first derivative of $J_n(p)$: $J'_n(p) = \frac{dJ_n(p)}{dp}$
$g'_{n,m}$ = m-th root of equation: $J'_n(p) = 0$.

H_{nm}-wave

Real parts:

$$H_x = \frac{1}{Z_{cr}}(2P'Z_x)^{\frac{1}{2}} J_n\left(g'_{n,m}\frac{2r}{d}\right) \cos n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$H_r = -j\frac{1}{Z_x}(2P'Z_x)^{\frac{1}{2}} \frac{J_{n-1}\left(g'_{n,m}\frac{2r}{d}\right) - J_{n+1}\left(g'_{n,m}\frac{2r}{d}\right)}{2} \cos n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$H_\varphi = j\frac{1}{Z_x}(2P'Z_x)^{\frac{1}{2}} \frac{J_{n-1}\left(g'_{n,m}\frac{2r}{d}\right) + J_{n+1}\left(g'_{n,m}\frac{2r}{d}\right)}{2} \sin n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$\mathscr{E}_x = 0$$

$$\mathscr{E}_r = j(2P'Z_x)^{\frac{1}{2}} \frac{J_{n-1}\left(g'_{n,m}\frac{2r}{d}\right) + J_{n+1}\left(g'_{n,m}\frac{2r}{d}\right)}{2} \sin n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$\mathscr{E}_\varphi = j(2P'Z_x)^{\frac{1}{2}} \frac{J_{n-1}\left(g'_{n,m}\frac{2r}{d}\right) - J_{n+1}\left(g'_{n,m}\frac{2r}{d}\right)}{2} \cos n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$\lambda_{cr} = \frac{\pi}{g'_{n,m}}d \quad Z_{cr} = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_{cr}}{\lambda_e} \quad Z_x = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_x}{\lambda_e}$$

$$\frac{1}{Z_{cr}^2} + \frac{1}{Z_x^2} = \frac{1}{Z_e^2} = \frac{1}{\mu/\kappa}$$

$$k_s \begin{cases} \text{for } n = 0 & k_s = J_0^2(g'_{0,m}) \\ \text{for } n \neq 0 & k_s = \frac{J_n^2(g'_{n,m})}{2}\left[1 - \left(\frac{n}{g'_{n,m}}\right)^2\right] \end{cases}$$

Values of $g'_{n,m}$

m \ n	0	1	2	3	4
1	3·8317	1·842	3·054	4·201	5·33
2	7·0156	5·330	6·705	8·015	9·3
3	10·1735	8·536	9·965	11·344	12·7
4	13·3237	11·70	13·1	14·6	15·9
5	16·4706	14·86	16·2	17·8	

E_{nm}-wave

Real parts:

$$H_x = 0$$

$$H_r = -j\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \frac{J_{n-1}\left(g_{n,m}\frac{2r}{d}\right) + J_{n+1}\left(g_{n,m}\frac{2r}{d}\right)}{2} \sin n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$H_\varphi = -j\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \frac{J_{n-1}\left(g_{n,m}\frac{2r}{d}\right) - J_{n+1}\left(g_{n,m}\frac{2r}{d}\right)}{2} \cos n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$\mathscr{E}_x = Z_{cr}\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} J_n\left(g_{n,m}\frac{2r}{d}\right) \cos n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$\mathscr{E}_r = -jZ_x\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \frac{J_{n-1}\left(g_{n,m}\frac{2r}{d}\right) - J_{n+1}\left(g_{n,m}\frac{2r}{d}\right)}{2} \cos n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$\mathscr{E}_\varphi = jZ_x\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} \frac{J_{n-1}\left(g_{n,m}\frac{2r}{d}\right) + J_{n+1}\left(g_{n,m}\frac{2r}{n}\right)}{2} \sin n\phi\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)}$$

$$\lambda_{cr} = \frac{\pi}{g_{n,m}}d \quad Z_{cr} = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_e}{\lambda_{cr}} \quad Z_x = \left(\frac{\mu}{\kappa}\right)^{\frac{1}{2}}\frac{\lambda_e}{\lambda_x}$$

$$Z_{cr}^2 = Z_x^2 - Z_e^2 = \mu/\kappa$$

$$k_s \begin{cases} \text{for } n = 0 & k_s = J_1^2(g_{0,m}) \\ \text{for } n \neq 0 & k_s = \frac{J_{n-1}^2(g_{n,m})}{2} \end{cases}$$

Values of $g_{n,m}$

m \ n	0	1	2	3	4
1	2·4048	3·8317	5·135	6·379	7·586
2	5·5201	7·0156	8·417	9·760	11·064
3	8·6537	10·1735	11·620	13·017	14·373
4	11·7915	13·3237	14·796	16·224	17·616
5	14·9309	16·4706	17·960	19·410	20·827

In the case of an E-wave neither of the subscripts can be equal to zero.

Field equations are given in right-handed Cartesian co-ordinates; the origin of the system is placed in one of the corners of the guide cross-section so that the width b falls in the positive direction of the z-axis, the height a in the positive direction of the y-axis, and the axis of the guide extends in the direction of the x-axis. The signs of fields imply the propagation of power along the guide in the positive x-direction.

Transverse oscillations occurring in a rectangular guide are considered separately in the y- and in the z-directions. The corresponding wavelengths, measured in the dielectric filling the guide, are called λ_a and λ_b. The values of these wavelengths are determined by the guide dimensions, a and b, and by the indices n, m, of the mode concerned. The formulae are:

$$\lambda_a = 2a/n \text{ and } \lambda_b = 2b/m \quad . \quad . \quad . \quad . \quad (16)$$

The relationship between λ_a, λ_b and the critical wavelength λ_{cr} of the guide follows from the general solution of Maxwell's equations, viz.:

$$1/\lambda_{cr}^2 = 1/\lambda_a^2 + 1\lambda_b^2 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

As a logical consequence of this treatment of transverse oscillations, two kinds of transverse impedance of the guide are introduced; one, called Z_a, gives the ratio of fields responsible for oscillations in the y-direction, and the other, Z_b, gives the similar ratio for oscillations in the z-direction.

The values of these impedances differ for an H-type wave from those for an E-type wave.

For an H-type of wave, Z_a and Z_b are given by

$$Z_a = (\mu/\kappa)^{\frac{1}{2}}(\lambda_a/\lambda_e); \; Z_b = (\mu/\kappa)^{\frac{1}{2}}(\lambda_b/\lambda_e) \quad . \quad . \quad (18)$$

and the relationship between them is:

$$1/Z_{cr}^2 = 1/Z_a^2 + 1/Z_b^2 \quad . \quad . \quad . \quad . \quad . \quad (19)$$

However, for an E-type of wave, they are given by

$$Z_a = (\mu/\kappa)^{\frac{1}{2}}(\lambda_e/\lambda_a); \; Z_b = (\mu/\kappa)^{\frac{1}{2}}(\lambda_e/\lambda_b) \quad . \quad . \quad (20)$$

with the following relationship:

$$Z_{cr}^2 = Z_a^2 + Z_b^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The internal diameter of a circular guide is designated d.

Subscripts n and m describing a mode within the H- or E-families of waves have here the following meanings. The subscript n denotes the number of full periods of field variation, counted around the circumference of the guide cross-section. The subscript m characterizes field variations along the radius of the cross-section. It is equal to the number of field maxima and minima between the centre and the circumference of the cross-section for H- and E-type waves, respectively. Both n and m are integral numbers; the lowest possible value of n and m being zero and one, respectively. It is usual to omit the subscript m altogether, if its value is equal to 1; however, in the present paper, the subscript m is always indicated.

Field equations are given in right-handed cylindrical co-ordinates x, r, ϕ, with the origin of the system at the centre of the guide cross-section. The signs of field imply the propagation of power along the guide in the positive x-direction.

(2.7) Attenuation

Two very different kinds of attenuation are dealt with in the paper, one due to loss in the walls and the other due to loss in the dielectric filling the guide.

The attenuation constant, α_w, caused by loss in the walls depends on all factors governing propagation in the guide and on the conductivity σ and permeability μ_1 of the wall-metal. In the formulae of Table 2 all quantities are in Gaussian units, e.g. for copper $\sigma = 53 \cdot 10^{16}$ and $\mu_1 = 1$; the units of the resultant α_w are neper/cm. However, the curves of Figs. 1 and 2 give numerical values (for an air-filled copper guide), which correspond to α_w in decibels/metre and to the guide dimensions in centimetres, since these units are considered to be more generally useful. The attenuation constant α_d due to loss in the dielectric depends on all factors governing the propagation, and also on the loss-angle δ of the dielectric. Both the formulae and the curves of Fig. 3 give the values of α_d in decibels/metre, if wavelengths are expressed in metres. The total attenuation constant, α, of a guide made of "lossy" metal and filled with "lossy" dielectric should be taken as the sum of both these attenuation constants:

$$\alpha = \alpha_w + \alpha_d \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

While the curves of Figs. 1 and 2 are restricted to the trans-

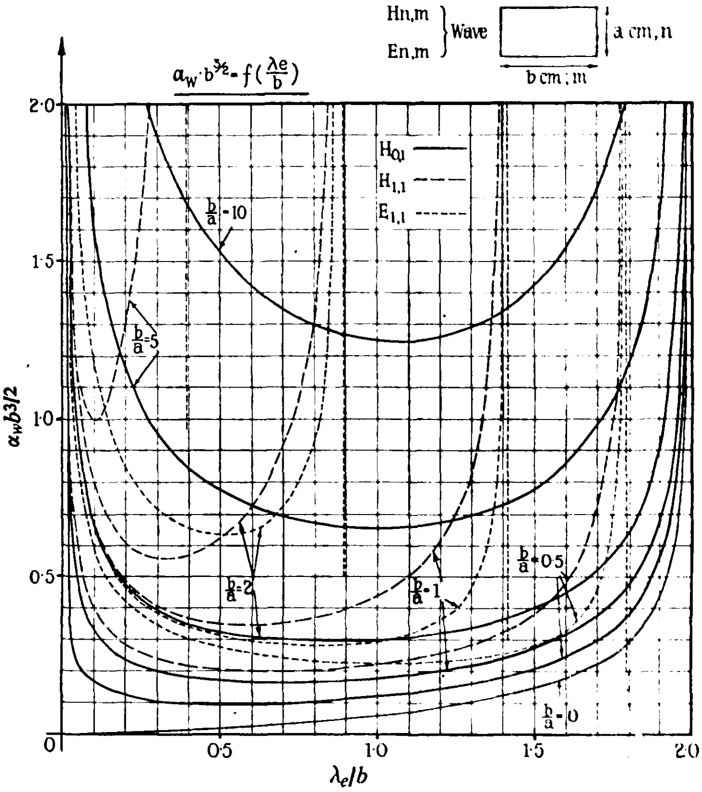

Fig. 1.—Attenuation constant $\alpha_w b^{3/2}$ of a rectangular air-filled copper wave guide.

mitting region of a guide, Fig. 3 shows the attenuation constant α_d for both the transmitting and the attenuating regions. The reason for this is that the attenuation constant, α_w, is obtained from the ratio of the power lost in the walls to the power propagated in the guide, and hence only the transmitting region is of interest, while the constant α_d is obtained as the real part of the propagation constant of a guide filled with lossy dielectric. There is a smooth change of the propagation constant through the threshold between the transmitting and the attenuating regions; in fact, in the case of lossy dielectrics, this threshold is not sharply defined. The curves of Fig. 3 have proved to be convenient for finding the attenuation of a guide attenuator, i.e. of a guide working in its attenuating region.

In addition to α_d, the phase-constant, β, has also been calculated for guides filled with lossy dielectric. The formulae and curves giving this constant are shown in Fig. 4; the values of β are expressed in radians/metre, for wavelengths in metres.

Table 2

General Formula for Attenuation Constant α_w due to Losses in Walls of Rectangular and Circular Wave Guides with H_{nm}- or E_{nm}- Waves

$$\alpha_w = \left[\frac{c}{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{\mathscr{D}}\,\frac{\lambda_x}{\lambda_e(\lambda_e^{\frac{1}{2}})}\,k_r\left(k_x\frac{\lambda_e^2}{\lambda_{cr}^2} + k_t\right)\text{nepers/centimetre} \quad \text{or} \quad \alpha_w = \left[\frac{\ }{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{\mathscr{D}^{3/2}}\,\frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1 - \frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}}\,k_{3/2}k_r\left(k_x\,\frac{\lambda_e^2}{\lambda_{cr}^2} + k_t\right)\text{nepers/centimetre}$$

Values of Coefficients k_r, k_x, k_t and $k_{3/2}$

Mode of wave	Kind of guide	Meaning of indices n, m	Linear dimensions of guide cross-section, cm	k_r	k_x	k_t	$k_{3/2}$
H_{nm}	Rectangular	n = number of half-periods along a	b	$\dfrac{1+\left(\frac{b}{a}\right)^3\left(\frac{n}{m}\right)^2}{1+\left(\frac{b}{a}\right)^2\left(\frac{n}{m}\right)^2}$	1	$\dfrac{1+\frac{b}{a}\left(\frac{n}{m}\right)^2}{1+\left(\frac{b}{a}\right)^3\left(\frac{n}{m}\right)^2}\,\dfrac{b}{a}$	$\left(\frac{m}{2}\right)^{\frac{1}{2}}\sqrt[4]{\left[1+\left(\frac{b}{a}\right)^2\left(\frac{n}{m}\right)^2\right]}$ for $n \neq 0$
		m = number of half-periods along b		1		$\frac{1}{2}\,\frac{b}{a}$	$\left(\frac{m}{2}\right)^{\frac{1}{2}}$ for $n = 0$
	Circular	n = number of full periods along circumference m = number of extreme values of field along radius	d	1		$\dfrac{1}{\left(\frac{g'_{,m}}{n}\right)^2 - 1}$ for $n \neq 0$ 0 for $n = 0$	$\left(\frac{g'_{n,m}}{\pi}\right)^{\frac{1}{2}}$
E_{nm}	Rectangular	As above for rectangular guide	b	$\dfrac{1+\left(\frac{b}{a}\right)^3\left(\frac{n}{m}\right)^2}{1+\left(\frac{a}{b}\right)^2\left(\frac{n}{m}\right)^2}$	0	1	$\left(\frac{m}{2}\right)^{\frac{1}{2}}\sqrt[4]{\left[1+\left(\frac{b}{a}\right)^2\left(\frac{n}{m}\right)^2\right]}$
	Circular	n as above for circular guide m = number of zeros of field along radius	d	1			$\left(\frac{g_{n,m}}{\pi}\right)^{\frac{1}{2}}$

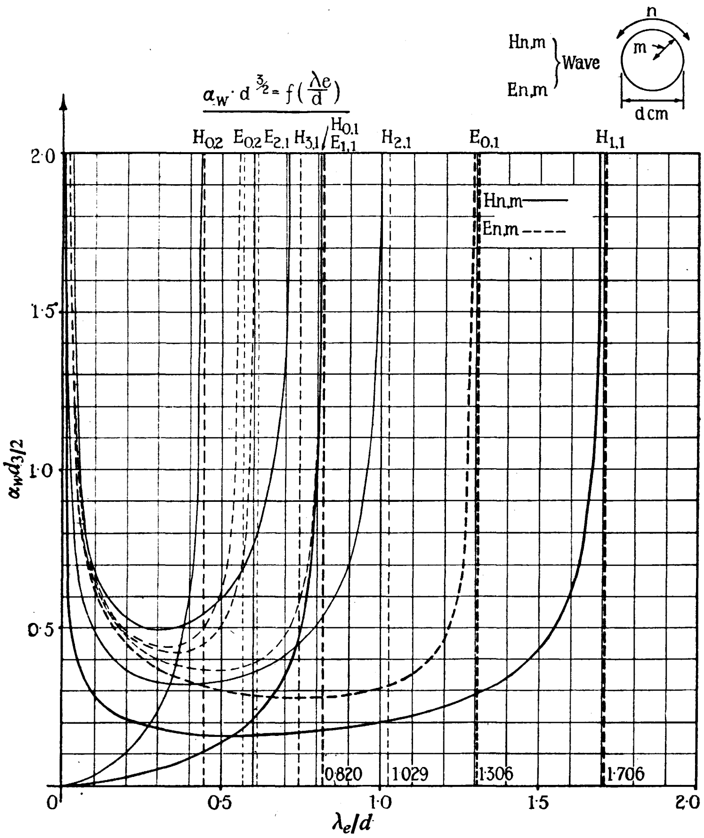

Fig. 2.—Values of $\alpha_w d^{3/2}$ for a circular air-filled copper wave guide.

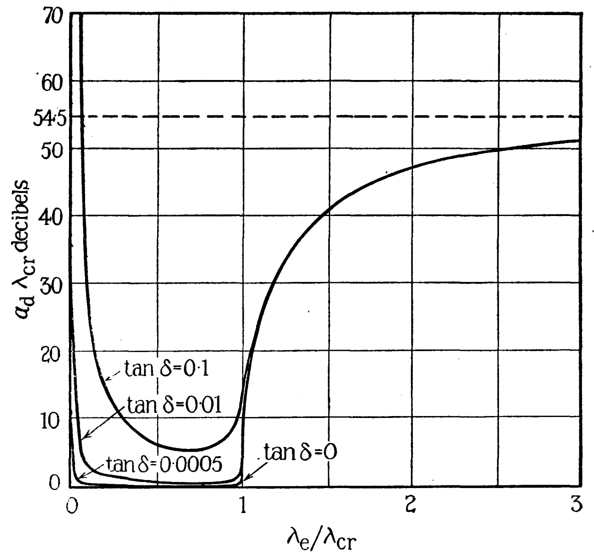

Fig. 3(*a*).—Values of $\alpha_d\lambda_{cr}$.
[See eqns. (108), (109) and (110).]

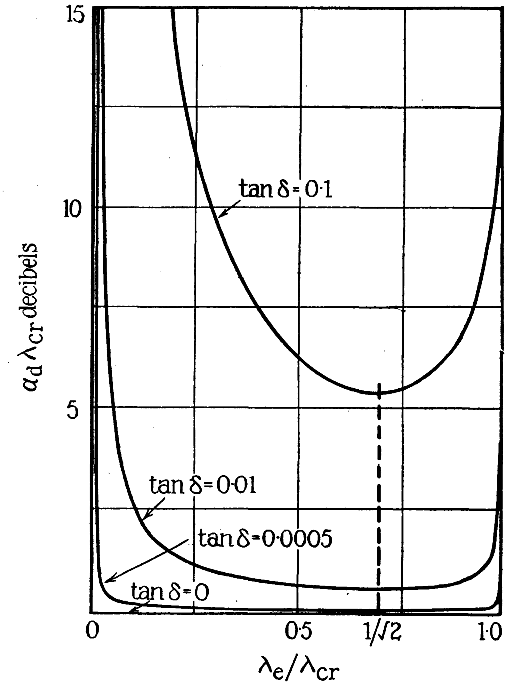

Fig. 3(*b*).—Enlarged portion of Fig. 3(*a*) in the region around $\lambda_e/\lambda_{cr} = 1/\sqrt{2}$.

(3) FIELD EQUATIONS

The field equations of any mode of wave for rectangular or circular guides are collected together in Table 1. They refer to the case of zero loss in the guide walls and dielectric; this condition implies that the guide wavelength λ_x and the guide specific impedance Z_x are real.

The equations are essentially the same as those which can be found in any book on wave guides; therefore, their derivation from the general Maxwell equations is not detailed. It is only necessary to indicate how the present formulae can be obtained from any other equivalent form of presentation.

Taking equations of either an H_{nm}- or an E_{nm}-wave in either rectangular or circular guide, write them in the co-ordinate notations used in the present paper and express all coefficients in terms of wavelengths, λ_e, λ_{cr}, λ_x, and of constants κ, μ of the dielectric. If the initial equations do not include a factor determining actual values of field (i.e. when some field is assumed as unity), introduce this factor, calling it A. To obtain the actual instantaneous values of fields, multiply also by the term $\epsilon^{j\omega t}$, if it is not present in the equations; then the instantaneous values of the fields at any point of the guide and at a time t will be equal to the real parts of the expressions.

In the case of a rectangular guide, the wavelengths, λ_a and λ_b, of the transverse oscillations in the a and b directions are also used.

In the case of a circular guide the argument p of the Bessel function is expressed in the form:

$$\left.\begin{array}{l} p = g'_{n,m}2r/d, \text{ for an } H_{nm}\text{-mode, and} \\ p = g_{n,m}2r/d, \text{ for an } E_{nm}\text{-mode.} \end{array}\right\} \quad . \quad . \quad (23)$$

In these expressions, r is the distance of a point from the centre of the guide, d is the guide diameter, $g_{n,m}$ is the m-th root of the equation:

$$J_n(p) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and $g'_{n,m}$ is the m-th root of the equation:

$$J'_n(p) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The values of $g_{n,m}$ and $g'_{n,m}$ are tabulated in Table 1 for several values of n and m.

Further, the following relationships are used:

$$J'_n(p) = \tfrac{1}{2}[J_{n-1}(p) - J_{n+1}(p)] \quad . \quad . \quad . \quad (26)$$

$$\frac{nJ_n(p)}{p} = \frac{1}{2}[J_{n-1}(p) + J_{n+1}(p)] \quad . \quad . \quad . \quad (27)$$

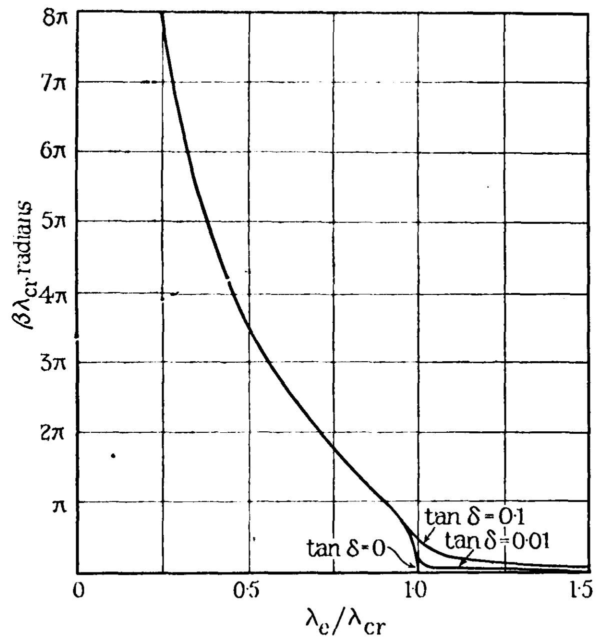

Fig. 4(*a*).—Values of $\beta\lambda_{cr}$.

[See eqns. (111), (112) and (113).]

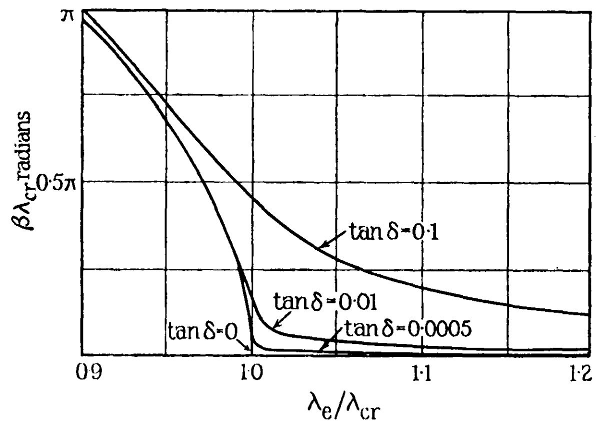

Fig. 4(*b*).—Enlarged portion of Fig. 4(*a*) in the region around $\lambda_c/\lambda_{er} = 1$.

As the next step, the x-component, $\mathbf{N}_x$, of the complex Poynting vector at a chosen point, y, z or r, ϕ, of the guide cross-section is calculated.

Assuming the sign of $\mathbf{N}_x$ as positive in the positive x-direction, the following formulae are used.

For a rectangular guide,

$$\mathbf{N}_x = \frac{c}{8\pi}(\mathscr{E}_y H_z^* - \mathscr{E}_z H_y^*) \quad . \quad . \quad . \quad . \quad (28)$$

and for a circular guide,

$$\mathbf{N}_x = \frac{c}{8\pi}(\mathscr{E}_r H_\varphi^* - \mathscr{E}_\varphi H_r^*) \quad . \quad . \quad . \quad . \quad (29)$$

These formulae are given in the Gaussian system of units.

Since the real part of a complex Poynting vector, multiplied by dS, determines the amount, dP, of the average power passing through an elementary surface, dS, normal to the direction of the Poynting vector, the real part of the integral of $\mathbf{N}_x dS$ taken over the cross-section of the guide gives the average power, P, passing through the whole cross-section, i.e. the power P carried along the guide by the wave considered.

As $\mathbf{N}_x$ turns out to be real for a travelling wave, the simple formula:

$$P = \int_S^W \mathbf{N}_x dS \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

is obtained, where S is the area of the guide cross-section.

The expression for P, obtained from equation (30), will be of the following general form:

$$P = \frac{c}{8\pi}A^2 f_1(\kappa, \mu, \lambda_e, \lambda_{cr}, \lambda_x) f_2(S) \quad . \quad . \quad . \quad (31)$$

where the function $f_2(S)$ is a product of the cross-sectional area, S, and a numerical factor, k_s,

$$f_2(S) = k_s S \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The value of the coefficient, k_s, is different for different wave modes.

In the case of a rectangular guide, for an $\mathrm{H_{nm}}$-mode k_s is given by:

$$\left.\begin{aligned} k_s &= \tfrac{1}{2}, \text{ if } n = 0 \\ k_s &= \tfrac{1}{4}, \text{ if } n \neq 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and for an $\mathrm{E_{nm}}$-mode it becomes:

$$k_s = \tfrac{1}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

In the case of a circular guide, for an $\mathrm{H_{nm}}$-mode:

$$\left.\begin{aligned} k_s &= J_0^2(g'_{0,m}), \text{ if } n = 0, \\ k_s &= \frac{1}{2}J_n^2(g'_{n,m})\left[1 - \left(\frac{n}{(g'_{n,m})}\right)^2\right], \text{ if } n \neq 0 \end{aligned}\right\} \quad . \quad (35)$$

and for an $\mathrm{E_{nm}}$-mode:

$$\left.\begin{aligned} k_s &= J_1^2(g_{0,m}), \text{ if } n = 0 \\ k_s &= \tfrac{1}{2}J_{n-1}^2(g_{n,m}), \text{ if } n \neq 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (36)$$

From equation (31), the value of A^2 is written:

$$A^2 = 2\frac{4\pi}{c}\frac{P}{k_s S}\frac{1}{f(\kappa, \mu, \lambda_e, \lambda_{cr}, \lambda_x)} \quad . \quad . \quad . \quad (37)$$

and the concept of "characteristic density" P' (of energy, in this case) is introduced:

$$P' = \frac{4\pi}{c}\frac{P}{k_s S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Then the value of A becomes:

$$A = \left[\frac{2P'}{f(\kappa, \mu, \lambda_e, \lambda_{cr}, \lambda_x)}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (39)$$

Finally, the factor A in the initial field equations is replaced by the expression (39), and, in the field coefficients, specific and transverse impedances are used instead of κ, μ and λ_s. Then the formulae take the form in which they are shown in Table 1.

The characteristic density, P', appears in these formulae together with the specific guide-impedance, Z_x, being either multiplied or divided by it, according to which way gives the simpler set of coefficients in the equations. The square root of $P'Z_x$ or of P'/Z_x has the dimension of either the electric or the magnetic field in either system of units.

Once one field impedance (Z_x) has been used, it is logical to express other factors of the coefficients in terms of impedances.

The factor 2 emphasizes that the equations give instantaneous values of fields, i.e. that the coefficients refer to amplitudes and not to r.m.s. values, while the power, P, and the characteristic density, P', are taken as time-averages.

The field equations of the modes having $n = 0$ are given in the Table as particular cases of the application of the general formulae.

The table also includes the meanings and units of the symbols used.

(4) ATTENUATION CONSTANT, α_w

By the time the present paper appeared in its original form, several authors[1,2,3,4] had calculated for the particular types of wave the attenuation of wave guides with imperfectly-conducting walls.

The methods used were different. Some of them were based on the calculation of currents in the guide walls, while in others the complex propagation constant was calculated from boundary conditions at the surface of imperfectly-conducting walls. The method of splitting the wave into two plane waves, and calculating the losses in the top, bottom, and side walls of a rectangular guide was also used. In all these methods the assumption was made that the disturbance of fields in a guide, due to losses in walls of relatively high but finite conductivity, was so small that it could be neglected and that, consequently, the field equations for the case of zero loss were still valid. The same assumption is made in the present calculations.

Since wall losses are caused by wall currents, and the latter are determined by magnetic fields, H_w, at the surface of a guide, only these fields enter into the calculations. The influence of electric-field penetration into the wall-metal is neglected, since the energy of the magnetic field in a good conductor is much greater than that of the electric field (at centimetre wavelengths in the case of a copper guide, the ratio of these energies is of the order of 10^7).

The basic formula for the attenuation constant is obtained from the values of the magnetic field, H_w, at the guide surface in the following way.

The magnetic field, H_w, of a wave sliding upon the inner surface of the wall induces a current in it, the density of which, i_w, in the very surface of the wall, can be calculated from the boundary conditions at that surface. Because of the imperfect conductivity σ of the wall, this current-density creates an electric field $\mathscr{E}_w$ along the wall in the direction of the current-density vector, and hence perpendicular to the magnetic field H_w. These fields, $\mathscr{E}_w$ and H_w, together give a Poynting vector $\mathbf{N}_w$ perpendicular to the wall and, therefore, some power P_w is carried into the wall. The amount of this power is the essential factor demanding the value of the attenuation constant, α_w.

The magnetic field, H, in the wall is, in the very surface, just equal to H_w, the magnetic field of the wave at the wall surface. If the wall has high conductivity, σ, and permeability μ_1, then at a depth h below the surface, the value of H becomes:[5]

$$H = H_w \epsilon^{-(1+j)\frac{(2\pi\omega\mu_1\sigma)^{\frac{1}{2}}}{c}h} \quad . \quad . \quad . \quad . \quad (40)$$

Since the penetration of the electric field into the wall is neglected, the current-density, i, in the wall is given only by the curl of H:

$$4\pi i/c = \text{curl}\, H \quad . \quad . \quad . \quad . \quad . \quad (41)$$

In view of the assumptions made, the magnetic field, H, is parallel to the wall surface; furthermore, in the presence of very rapid variations in the normal direction, relatively slow variations of H along the surface can be neglected. Equation (4.2) then gives

$$4\pi i/c = -\,\partial H/\partial h \quad . \quad . \quad . \quad . \quad . \quad (42)$$

from which, by using equation (4.1), the current-density i_w, in the very surface of the wall (i.e. for $h = 0$), is obtained:

$$i_w = (1+j)\frac{c}{4\pi}\,\frac{(2\pi\omega\mu_1\sigma)^{\frac{1}{2}}}{c}H_w = (1+j)\left(\frac{\omega\mu_1\sigma}{8\pi}\right)^{\frac{1}{2}}H_w \quad . \quad (43)$$

This current-density is accomplished by an electric field parallel to the wall surface and, because of the continuity of the tangential component of field when passing from one medium to another, the same electric field must appear in the guide on the surface of the wall. Therefore, the latter field, $\mathscr{E}_w$, becomes:

$$\mathscr{E}_w = \frac{i_w}{\sigma} = (1+j)\left(\frac{\omega\mu_1}{8\pi\sigma}\right)^{\frac{1}{2}}H_w \quad . \quad . \quad . \quad (44)$$

The complex Poynting vector $\mathbf{N}_w$, caused by the co-existence of the two fields (H_w and $\mathscr{E}_w$), and directed into the wall, is equal to:

$$\mathbf{N}_w = \frac{c}{8\pi}\mathscr{E}_w H_w^* = (1+j)\frac{c}{8\pi}\left(\frac{\omega\mu_1}{8\pi\sigma}\right)^{\frac{1}{2}}|H_w|^2 \quad . \quad (45)$$

If Δx is a longitudinal element of the wave guide, then an elementary surface of the wall within this length can be written as $\Delta s\Delta x$, where Δs refers to an elementary length of the periphery of the guide cross-section.

The mean power entering the wall through an elementary surface $\Delta s\Delta x$ is equal to the real part of the product $\mathbf{N}_w\Delta s\Delta x$; hence the amount of power ΔP passing into the guide walls in the length Δx of the guide is, by eqn. (45), equal to:

$$\Delta P = \Delta x\frac{c}{8\pi}\left(\frac{\omega\mu_1}{8\pi\sigma}\right)^{\frac{1}{2}}\int_S |H_w|^2 dS \quad . \quad . \quad . \quad (46)$$

where the integration is taken along the whole periphery S of the guide cross-section.

If, for convenience, the total magnetic field, H_w, is divided into two perpendicular components, the sum of corresponding integrals must be used in eqn. (46). Therefore, the final expression for ΔP becomes:

$$\Delta P = \Delta x\frac{c}{8\pi}\left(\frac{\omega\mu_1}{8\pi\sigma}\right)^{\frac{1}{2}}\sum\int_S |H_w|^2 dS\,. \quad . \quad . \quad (47)$$

To obtain the attenuation constant, α_w, of the guide, this loss, ΔP, of power has to be combined with the total power P carried by the wave along the guide through the guide cross-section.

The constant, α_w, (in nepers/centimetre of guide-length) may be defined by the following formula:

$$P = P(0)\epsilon^{-2\alpha_w x} \quad . \quad . \quad . \quad . \quad . \quad (48)$$

where P is the total power passing through the cross-section at a distance x from the origin, $x = 0$, and $P(0)$ is the power at the origin.

By differentiation, the power ΔP lost in the length Δx of the guide at the cross-section, x, is found; viz. the power "gained," $-\Delta P$, is equal to:

$$-\Delta P = -2\alpha_w P\Delta x \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Hence the attenuation constant α_w becomes:

$$\alpha_w = \frac{1}{2}\,\frac{\Delta P}{P\Delta x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

Since α_w refers to attenuation due only to loss in the guide walls, the amount ΔP of power, involved in equation (50), defining x_w, is equal to the amount ΔP lost in the walls and given by equation (17). Therefore the expression for α_w becomes:

$$\alpha_w = \frac{c}{16\pi}\left(\frac{\omega\mu_1}{8\pi\sigma}\right)^{\frac{1}{2}}\frac{1}{P}\sum\int_S |H_w|^2 dS\,. \quad . \quad . \quad (51)$$

By using the characteristic density P' of energy [see eqn. (38)] and writing for ω [eqn. (1)]:

$$\omega = \frac{2\pi c}{(\kappa\mu)^{\frac{1}{2}}\lambda_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

the following expression for α_w is obtained:

$$\alpha_w = \left[\frac{c\mu_1}{\sigma\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{S\lambda_e^{\frac{1}{2}}} \frac{(\mu/\kappa)^{\frac{1}{2}}}{8k_s P'} \sum \int_S^w |H_w|^2 dS \quad . \quad (53)$$

Now this formula is applied to the cases of H- and E-mode waves in rectangular and circular guides.

The most tedious part of the calculation is, of course, the evaluation of the sum of integrals met in eqn. (53). The major steps of this calculation are shown for these four cases.

(4.1) Attenuation of an H_{nm}-wave in Rectangular Wave Guide

In the case of a rectangular guide the top and bottom walls and the side walls are considered separately.

If an H_{nm}-mode is taken (Table 1), then at the top and bottom walls (i.e. for $y = 0$ and $y = a$), only the components H_x and H_z of the magnetic field H_w are found. They vary in the z-direction, with $0 \leqslant z \leqslant b$.

The formulae of Table 1 give the sum of integrals for both top and bottom walls as equal to:

$$2P'Z_x b\left(\frac{1}{Z_{cr}^2} + \frac{Z_{cr}^2}{Z_b^2}\frac{1}{Z_x^2}\right) \quad . \quad . \quad . \quad . \quad (54)$$

Similarly, for the side walls, where $z = 0$ or $z = b$, the components H_x and H_y are generally present, and they vary in the y-direction, with $0 \leqslant y \leqslant a$. The sum of integrals is different in the cases $n = 0$ and $n \neq 0$.

If $n = 0$, the result of integration for both sides of the guide is:

$$4P'Z_x a(1/Z_{cr}^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

and if $n \neq 0$,

$$2P'Z_x a[1/Z_{cr}^2 + (Z_{cr}^2 1/Z_a^2 Z_x^2)] \quad . \quad . \quad . \quad (56)$$

When summing the values of integrals for all four walls of the guide, several algebraic transformations are made.

In the case when $n = 0$, Z_b becomes Z_{cr}, the relationship given by eqn. (6) is used, and λ_s is substituted for Z_s. As a result, the summation gives:

$$4P'\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}} a\frac{\lambda_x}{\lambda_e}\left(\frac{\lambda_e^2}{\lambda_{cr}^2} + \frac{1}{2}\frac{b}{a}\right) \quad . \quad . \quad . \quad . \quad (57)$$

In the case when $n \neq 0$, the corresponding expression is much more complicated. In obtaining it, the relationship given by equations (19) and (6) are used, λ_s substituted instead of Z_s, and by eqn. (16) λ_a and λ_b are expressed in terms of dimensions a and b. Also, the result is arranged to obtain the ratio $\frac{\lambda_e^2}{\lambda_{cr}^2}$ in brackets with the coefficient 1. Finally the following expression is derived:

$$2P'\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}} a\frac{\lambda_x}{\lambda_e}k_r\left(\frac{\lambda_2^e}{\lambda_{cr}^2} + k_t\right) \quad . \quad . \quad . \quad . \quad (58)$$

where
$$k_r = \frac{1 + (b/a)^3(n/m)^2}{1 + (b/a)^2(n/m)^2} \quad . \quad . \quad . \quad . \quad (59)$$

and
$$k_t = \frac{1 + (b/a)(n/m)^2}{1 + (b/a)^3(n/m)^2}\frac{b}{a} \quad . \quad . \quad . \quad . \quad (60)$$

The values of the total sums are now substituted in the equation for α_w [eqn. (53)]. At the same time ab is written for S, and the numerical values for k_s, given by eqn. (33), are used. The values of α_w then become:

For $n = 0$,

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{\lambda_x}{b\lambda_e\lambda_e^{\frac{1}{2}}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2} + \frac{b}{2a}\right) \quad . \quad . \quad . \quad (61)$$

and for $n \neq 0$,

$$\alpha_w = \left[\frac{c\mu}{\sigma\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{\lambda_x}{b\lambda_e\lambda_e^{\frac{1}{2}}}k_r\left(\frac{\lambda_e^2}{\lambda_{cr}^2} + k_t\right) \quad . \quad . \quad . \quad (62)$$

Finally, an alternative form of these formulae is produced, in which the ratio $\frac{\lambda_e}{\lambda_{cr}}$ is the only parameter involving wavelengths. Then for $n = 0$

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{b^{3/2}} \frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1 - \frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}}\left(\frac{m}{2}\right)^{\frac{1}{2}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2} + \frac{1}{2}\frac{b}{a}\right) \quad (63)$$

and for $n \neq 0$,

$$\alpha_w = \left[\frac{c}{\mu}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{b^{3/2}} \frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1 - \frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}} k_{3/2}k_r\left(\frac{\lambda_e^2}{\lambda_{cr}^2} + k_t\right) \quad (64)$$

where the coefficient $k_{3/2}$ is given by:

$$k_{3/2} = \left(\frac{m}{2}\right)^{\frac{1}{2}}\left[1 + \left(\frac{b}{a}\right)^2\left(\frac{n}{m}\right)^2\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad (65)$$

(4.2) Attenuation of an E_{nm}-Wave in Rectangular Wave Guide

For the case of an E_{nm}-mode in a rectangular guide, the calculation runs on similar lines, but is somewhat simpler due to the fact that in this case only the transverse magnetic field is encountered; also, the value of the index n cannot be equal to zero.

After integration, the following values of the sums of integrals are obtained.

For the top and bottom walls,

$$2\frac{P'}{Z_x}b\frac{Z_a^2}{Z_{cr}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

and for both side walls,

$$2\frac{P'}{Z_x}a\frac{Z_b^2}{Z_{cr}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

By summation followed by substitution of λ_s instead of Z_s, the result is obtained in a form similar to that of equation (58).

$$2P'\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}} a\frac{\lambda_x}{\lambda_e}k_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

where the coefficient k_r is the same as that given by eqn. (59).

When this value is substituted in eqn. (53), ab written for S, and $\frac{1}{4}$ for k_s [eqn. (34)] the attenuation constant α_w becomes:

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{b}\frac{\lambda_x}{\lambda_e\lambda_e^{\frac{1}{2}}}k_r \quad . \quad . \quad . \quad (69)$$

Finally, with the ratio $\frac{\lambda_e}{\lambda_{cr}}$ as the only parameter involving wavelengths, the alternative form of this expression is:

$$\alpha_w = \left[\frac{c}{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{b^{3/2}}\,\frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1-\frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}} k_{3/2}k_r \quad . \quad (70)$$

where the coefficient $k_{3/2}$ is given by eqn. (65).

(4.3) Attenuation of an H_{nm}-Wave in Circular Wave Guide

In the case of an H_{nm}-mode in a circular guide, the integration of $|H_w|^2$ is performed around the circumference of the guide cross-section.

Here two perpendicular components of H_w, i.e. H_x and H_φ are found, the values of which are obtained from the formulae of Table 1 by putting $r = \frac{1}{2}d$; the integrals for $|H_x|^2$ and $|H_\varphi|^2$ are calculated separately and the sum is taken.

For a point in the circumference of the guide cross-section, the argument p of the Bessel functions becomes $g'_{n,m}$, and then, in view of the definition of $g'_{n,m}$ (eqn. 23) and of the property of Bessel functions (eqn. 26):

$$J_{n+1}(g'_{n,m}) = J_{n-1}(g'_{n,m}) \quad . \quad . \quad . \quad . \quad (71)$$

For any value of n, the following expression is obtained for the integral of $|H_w|^2$:

$$\int_S |H_w|^2 dS = P'Z_x d\left[\frac{1}{Z_{cr}^2}J_n^2(g'_{n,m})\int_0^{2\pi}\cos^2 n\phi d\phi + \frac{1}{Z_x^2}J_{n-1}^2(g'_{n,m})\int_0^{2\pi}\sin^2 n\phi d\phi\right] \quad . \quad (72)$$

The value of this integral is different in the cases when $n = 0$ and when $n \neq 0$.

If $n = 0$, the second term in eqn. (72) vanishes, and the expression becomes:

$$\int_S |H_w|^2 dS = P'Z_x 2\pi d\frac{J_0^2(g'_{0,m})}{Z_{cr}^2} \quad . \quad . \quad . \quad (73)$$

and if $n \neq 0$, both terms remain, and

$$\int_S |H_w|^2 dS = P'Z_x \pi d\left[\frac{J_n^2(g'_{n,m})}{Z_{cr}^2} + \frac{J_{n-1}^2(g'_{n,m})}{Z_x^2}\right] \quad . \quad (74)$$

The letter Z is now replaced by λ.

For the case $n = 0$ it is seen at once that

$$\int_S |H_w|^2 dS = 2P'\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\pi d\frac{\lambda_x\lambda_e}{\lambda_{cr}}J_0^2(g'_{0,m}) \quad . \quad . \quad (75)$$

In the case when $n \neq 0$, eqn. (6) is used.

In view of equations (27) and (71),

$$\frac{n}{g'_{n,m}}J_n(g'_{n,m}) = J_{m-1}(g'_{n,m}) \quad . \quad . \quad . \quad . \quad (76)$$

so that the integral becomes:

$$\int_S |H_w|^2 dS = P'\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\pi d\frac{\lambda_x}{\lambda_e}J_n^2(g'_{n,m})\left[1-\left(\frac{n}{g'_{n,m}}\right)^2\right]\left[\frac{\lambda_e^2}{\lambda_{cr}^2}+k_t\right] \quad (77)$$

$$\text{where} \quad k_t = 1\Big/\left[\left(\frac{g'_{n,m}}{n}\right)^2 - 1\right] = 1\Big/\left[\left(\frac{g_{n,m}}{n}\right)^2 - 1\right] \quad . \quad (78)$$

The values of integrals are substituted in eqn. (53); at the same time S is replaced by $\frac{1}{4}\pi d^2$ and k_s by its value given in eqn. (35). If $n = 0$, the value of α_w becomes:

$$\alpha_w = \left[\frac{c}{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}\frac{1}{d}\,\frac{\lambda_e\lambda_e^{\frac{1}{2}}}{\lambda_{cr}^2} \quad . \quad . \quad . \quad . \quad (79)$$

This formula is very interesting since it shows immediately that for an H_{0m}-mode in a given circular guide the attenuation constant α_w decreases to zero if the frequency increases to infinity, i.e. if both λ_e and λ_x decrease to zero. This is a well-known property of an H_{0m}-mode of wave in a circular guide.

However, to get uniformity with the formula obtained for other modes of wave, this form of expression for α_w is written in a different way:

$$\alpha_w = \left[\frac{c}{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}\frac{1}{d}\,\frac{\lambda_x}{\lambda_e\lambda_e^{\frac{1}{2}}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2}\right) \quad . \quad . \quad . \quad (80)$$

If $n \neq 0$, the value of α_w becomes:

$$\alpha_w = \left[\frac{c}{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}\frac{1}{d}\,\frac{\lambda_x}{\lambda_e\lambda_e^{\frac{1}{2}}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2}+k_t\right) \quad . \quad . \quad (81)$$

and it is seen that the above-mentioned property of an H_{0m}-mode no longer exists because of the presence of the term k_t in the formula; the value of k_t, given by eqn. (78), cannot be zero except in the case when $n = 0$; for $n \neq 0$, with λ_e decreasing to zero, α_w would be increasing continuously.

As the last step, equations (80) and (81) are modified to contain only the ratio λ_e/λ_{cr}. To do this, the relationship shown in Table 1 is used:

$$\lambda_{cr} = \frac{\pi}{g'_{n,m}}d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

and then:

for $n = 0$,

$$\alpha_w = \left[\frac{c}{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}\frac{1}{d^{3/2}}\,\frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1-\frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}}\left(\frac{g'_{n,m}}{\pi}\right)^{\frac{1}{2}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2}\right) \quad (83)$$

and for $n \neq 0$,

$$\alpha_w = \left[\frac{c}{\sigma}\,\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}\frac{1}{d^{3/2}}\,\frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1-\frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}}\left(\frac{g'_{n,m}}{\pi}\right)^{\frac{1}{2}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2}+k_t\right) \quad (84)$$

Eqn. (84) also includes eqn. (83), since for $n = 0$ the value of k_t becomes zero.

(4.4) Attenuation of an E_{nm}-Wave in Circular Wave Guide

For the last case considered, an E_{nm}-mode in a circular guide, the calculation is slightly shorter than for an H_{nm}-mode.

At the wall surface H_φ is found to be the only component of the magnetic field H_w. The relevant value of H_φ is obtained from the formulae of Table 1 by putting $r = \frac{1}{2}d$. Then the argument p of the Bessel functions becomes $g_{n,m}$, for which value, from equations (24) and (27)

$$J_{n+1}(g_{n,m}) = -J_{n-1}(g_{n,m}) \quad . \quad . \quad . \quad . \quad (85)$$

When this equality is taken into account, the integral of $|H_w|^2$ around the circumference of the guide cross-section becomes:

$$\int_S |H_w|^2 dS = \frac{P'}{Z_x}dJ_{n-1}^2(g_{n,m})\int_0^{2\pi}\cos^2 n\phi d\phi \quad . \quad . \quad . \quad (86)$$

Again the value of this expression is different for $n = 0$ and for $n \neq 0$.

If $n = 0$,

$$\int_S |H_w|^2 dS = \frac{P'}{Z_x} 2\pi d J_1^2(g_{0,m}) \quad . \quad . \quad . \quad . \quad (87)$$

and if $n \neq 0$,

$$\int_S |H_w|^2 dS = \frac{P'}{Z_x} \pi d J_{n-1}^2(g_{n,m}) \quad . \quad . \quad . \quad (88)$$

Z_x is now replaced by λ_s and the values of the integrals are inserted in eqn. (53).

If S is replaced by $\frac{1}{4}\pi d^2$ and k_s by its value given in eqn. (36), the expressions for α_w become identical for both cases, $n = 0$ and $n \neq 0$; i.e.:

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{d} \frac{\lambda_x}{\lambda_e \lambda_c^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad (89)$$

The alternative form of the formula is:

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{d^{3/2}} \frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1 - \frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}} \left(\frac{g_{n,m}}{\pi}\right)^{\frac{1}{2}} \quad . \quad (90)$$

(4.6) General Expression for the Attenuation

When the formulae obtained in the four cases considered are compared, it will be evident that it is possible to derive a general expression for α_w, which could be applied to each of these cases, provided suitable values are given to the coefficients entering the expression.

The two equivalent forms of this general expression are:

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{D} \frac{\lambda_x}{\lambda_c \lambda_e^{\frac{1}{2}}} k_r \left(k_x \frac{\lambda_e^2}{\lambda_{cr}^2} + k_t\right) \quad . \quad (91)$$

and

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{D^{3/2}} \frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1 - \frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{\frac{1}{2}}} k_{3/2} k_r \left(k_x \frac{\lambda_e^2}{\lambda_{cr}^2} + k_t\right) (92)$$

The second form of the expression will be discussed in detail.

The electromagnetic properties of the wall-metal affect the attenuation constant α_w through the factor:

$$\left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

which comprises the ratio of the specific admittance $\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}$ of the unbounded dielectric filling the guide to the conductivity σ of the wall-metal, and the ratio of permeabilities, μ_1 and μ, of the wall-metal and the dielectric.

This factor is directly proportional to the square root of the permeability μ_1 of the wall-metal, and inversely proportional to the square root of its conductivity σ. Thus, having calculated the attenuation constant α_w for a given wave-mode in a given guide for one kind of wall-metal, the corresponding constant α_w for another metal can be easily obtained.

In the case of an air-filled guide with walls made of copper of conductivity $\sigma = 53 \cdot 10^{16}$ and $\mu_1 = 1$, the value of the factor becomes $0{\cdot}238 \cdot 10^{-3}$. This value, with dimensions of the guide taken in centimetres, would give α_w in nepers/centimetre. To obtain α_w in decibels/metre, when the guide dimensions are still kept in centimetres, this value should be multiplied by $8{\cdot}68 \cdot 10^2$, giving $0{\cdot}2065$.

The wave frequency appears in the formula only through the ratio of λ_e to λ_{cr}. This ratio also depends on the dielectric, the shape and dimensions of the guide, and the mode of wave. Nevertheless, all these features form only a kind of background on which the influence of frequency is superposed.

The linear dimensions of the guide cross-section are represented in the formula only by D, meaning either the width b in rectangular guide or the diameter d in circular guide. The attenuation constant α_w is inversely proportional to the three-halves power of the guide dimension. Thus the product $\alpha_w D^{3/2}$ is independent of the actual value of this dimension, and hence the curves, which show the relationship between this product and other quantities influencing the attenuation, are applicable to a guide of any size.

Values of the coefficients k_r, k_x, k_t and $k_{3/2}$ depend on the shape of the guide cross-section and on the mode of wave. k_r is given by eqn. (59) for a rectangular guide as a function of the ratio b/a and of the ratio of subscripts n, m. For a circular guide $k_r = 1$. k_x can be either 1 or 0. It exists, and is equal to 1, when the x-component of the magnetic field exists, i.e. in the case of an H-mode of wave. It becomes zero for any E-mode. k_t exists when there is a transverse component of the magnetic field at the guide wall; hence it vanishes only in the case of an H_{0m}-mode in a circular guide, for which there is only a longitudinal component. In the case of any other H-mode in a circular guide, k_t is given by eqn. (78); for an H-mode in a rectangular guide it is determined by eqn. (60), depending on b/a and n/m; and for all E-modes, whether in rectangular or circular guides, it is unity.

Finally, the coefficient $k_{3/2}$ only enters into the formula (92) in which the guide dimensions appear to the power 3/2, while the three remaining coefficients are present in both formulae (92) and (91). The value of $k_{3/2}$ for rectangular guide is given by eqn. (65), and depends on the ratio b/a and on values of n and m. In the case of circular guides it becomes:

for an H-mode wave,

$$k_{3/2} = \left(\frac{g'_{n,m}}{\pi}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

and for an E-mode,

$$k_{3/2} = \left(\frac{g_{n,m}}{\kappa}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (95)$$

The general formulae for α_w and all relevant information are repeated in Table 2. As particular cases, the formulae are given for the two wave modes most frequently used, i.e. the H_{0m}-mode in a rectangular guide, and the H_{11}-mode in a circular guide. Further, because of its exceptional properties, eqn. (29) is also quoted, giving α_w for an H_{0m}-mode in a circular guide.

The formulae obtained are rather complicated and their application to a particular case is tedious.

It has, therefore, been thought useful to prepare curves which enable one to find quickly the attenuation constant α_w in cases likely to be met in practice. The curves are of a general nature so that they can be applied to a guide of any dimensions, transmitting some simple mode of wave of any desired frequency.

In order to make the families of curves compact but comprehensive, they depict the relationship between the quantity, $\alpha_w D^{3/2}$, involving attenuation but independent of linear dimensions, and a term through which the frequency affects attenuation. For the latter term, the ratio λ_e/D has been chosen instead of λ_e/λ_{cr}, since it has been thought more convenient to relate the wavelength λ_e directly to the guide dimensions instead of to the critical wavelength λ_{cr}. Hence, the curves display the function:

$$\alpha_w D^{3/2} = f\left(\frac{\lambda_e}{D}\right) \quad . \quad . \quad . \quad . \quad . \quad (96)$$

Particular Cases

H_{0m}-wave $\quad n = 0$

$$\left.\begin{aligned}
H_x &= \frac{1}{Z_{cr}}(2P'Z_x)^{\frac{1}{2}} \cos \frac{\pi m}{b} z\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
H_z &= j\frac{1}{Z_x}(2P'Z_x)^{\frac{1}{2}} \sin \frac{\pi m}{b} z\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
\mathscr{E}_y &= j(2P'Z_x)^{\frac{1}{2}} \sin \frac{\pi m}{b} z\, \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
H_y &= \mathscr{E}_x = \mathscr{E}_z = 0
\end{aligned}\right\} \text{Real parts}$$

$$\lambda_{cr} = \frac{2b}{m} \quad \frac{1}{\lambda_x^2} = \frac{1}{\lambda_e^2} - \left(\frac{m}{2b}\right)^2 \quad k_s = \frac{1}{2}$$

H_{0m}-wave $\quad n = 0$

$$\left.\begin{aligned}
H_x &= \frac{1}{Z_{cr}}(2P'Z_y)^{\frac{1}{2}} J_0\left(g'_{o,m}\frac{2r}{d}\right) \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
H_r &= j\frac{1}{Z_x}(2P'Z_x)^{\frac{1}{2}} J_1\left(g'_{o,m}\frac{2r}{d}\right) \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
\mathscr{E}_\varphi &= -j(2P'Z_x)^{\frac{1}{2}} J_1\left(g'_{o,m}\frac{2r}{d}\right) \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
H_\varphi &= \mathscr{E}_x = \mathscr{E}_r = 0
\end{aligned}\right\} \text{Real parts}$$

E_{0m}-wave $\quad n = 0$

$$\left.\begin{aligned}
H_\varphi &= j\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} J_1\left(g_{o,m}\frac{2r}{d}\right) \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
\mathscr{E}_x &= Z_{cr}\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} J_0\left(g_{o,m}\frac{2r}{d}\right) \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
\mathscr{E}_r &= jZ_x\left(\frac{2P'}{Z_x}\right)^{\frac{1}{2}} J_1\left(g_{o,m}\frac{2r}{d}\right) \varepsilon^{\left(-j\frac{2\pi}{\lambda_x}x + j\omega t\right)} \\
H_x &= H_r = \mathscr{E}_\varphi = 0
\end{aligned}\right\} \text{Real parts}$$

1. H_{0m}-wave in rectangular wave guide.

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{b} \frac{\lambda_x}{\lambda_e \lambda_e^{\frac{1}{2}}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2} + \frac{1}{2}\frac{b}{a}\right)$$

2. H_{11}-wave in circular wave guide.

$$n = 1 \quad g'_{1,1} = 1{\cdot}84 \quad k_t = 0{\cdot}42$$

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{d} \frac{\lambda_x}{\lambda_e \lambda_e^{\frac{1}{2}}}\left(\frac{\lambda_e^2}{\lambda_{cr}^2} + 0{\cdot}42\right)$$

3. H_{01}-wave in circular wave guide.

$$\alpha_w = \left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \frac{1}{d} \frac{\lambda_x \lambda_e^{\frac{1}{2}}}{\lambda_{cr}^2}$$

List of Symbols

Gaussian System of Units

Time in seconds.
Length in centimetres.
Energy in ergs.
Power in ergs/sec.
Electric field intensity $\mathscr{E}$ } in electrostatic units.
Permittivity κ }
Magnetic field intensity H in electromagnetic units.
For air or vacuum $\kappa_0 = \mu_0 = 1$.

$j = \sqrt{-1}$.
c = velocity of light in free space = 3×10^{10} cm/sec.
f = frequency, in cycles/second.
$\omega = 2\pi f$; in cycles/second.
λ_e = wavelength in unbounded dielectric $= \frac{2\pi c}{\omega\sqrt{(\varepsilon\mu)}}$ cm $= 2\pi c/[\omega\sqrt{(\kappa\mu)}]$cm.
λ_{cr} = critical wavelength of wave guide, measured in dielectric in centimetres.
λ_x = wavelength along wave guide in centimetres.

$$\frac{1}{\lambda_{cr}^2} + \frac{1}{\lambda_x^2} = \frac{1}{\lambda_e^2}$$

Z_e = specific impedance of unbounded dielectric $= \mathscr{E}/H = \sqrt{(\mu/\kappa)}$ (pure number).
P = mean power, propagated along wave guide, in ergs/second.
S = cross-sectional area of wave guide, in square centimetres.
k_s = cross-sectional coefficient (pure number).
P' = "characteristic energy density," in ergs/cubic centimetre.

$$P' = \frac{4\pi}{c}\frac{P}{k_s S}$$

Remarks

In M.K.S. rationalized system of units:
Electric field intensity $\mathscr{E}$ is expressed in volts/metre.
Magnetic field intensity H is expressed in amp/metre.
Power field intensity P is expressed in watts.
P' is "characteristic power density" in watts/square metre

and $P' = \frac{P(\text{watts})}{k_s S(M^2)}$

The term $120\pi\sqrt{\left(\frac{\mu/\mu_0}{\kappa/\kappa_0}\right)}$ ohms should be used in place of $\sqrt{(\mu/\kappa)}$ to obtain Z, the specific impedance, in ohms.

σ = conductivity of wall-metal, in electrostatic units (e.s.u.).
For copper $\sigma \simeq 53 \times 10^{16}$.

μ_1 = permeability of wall-metal, in electromagnetic units (e.m.u.).
For copper $\mu_1 = 1$.

μ = permeability of guide-filling dielectric, in e.m.u.

For Air and Copper

α_w in nepers/cm

$$\left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} = 0{\cdot}238 \times 10^{-3}$$

α_w in decibels/metre

$$\left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} = 0{\cdot}2065$$

The attenuation constant α_w obtained from these curves refers to an air-filled copper guide. For any other metal with $\mu_1 = 1$, the values of α_w given by the curves should be divided by the square root of the ratio of conductivity σ to that of copper.

In the case of a rectangular wave guide the curves are drawn for various ratios of the guide dimensions, b/a, and for three modes of wave: H_{01}, H_{11} and E_{11}. These curves are shown in Fig. 1. Since the mode H_{01} is that of greatest practical importance, the family of curves for this mode is drawn in full lines.

In the case of a circular guide the curves are drawn for several of the lowest modes of wave (see Fig. 2), with emphasis laid on the three modes: H_{11}, E_{01} and H_{01}. Incidentally, Fig. 2 shows the order of likelihood of appearance of the modes in a circular guide, i.e. H_{11}, E_{01}, H_{21}, H_{01} and E_{11}, H_{31}, E_{21}, E_{02}, H_{02}, and so on.

As an application of the formulae and the curves, the constant a_w is calculated for the H_{01}-mode of wave in an air-filled rectangular copper guide 3 in by 1 in at a frequency $f = 3\,000$ Mc/s.

The various parameters involved in the formula of Table 2 (particular case), then have the following values:

$$\left[\frac{c}{\sigma}\frac{\mu_1}{\mu_0}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} = 0\cdot 2065 \ (\alpha \text{ in db/metre})$$

$$b = 3 \text{ in} = 7\cdot 62 \text{ cm};\ b/a = 3$$

$$\lambda_e = \lambda_0 = 10 \text{ cm}$$

$$\lambda_{cr} = 2b = 15\cdot 24 \text{ cm; hence } \lambda_x = 13\cdot 25 \text{ cm}$$

$$\frac{\lambda_x}{\lambda_e\lambda_e^{\frac{1}{2}}} = 0\cdot 419;\ \frac{\lambda_e^2}{\lambda_{cr}^2} = 0\cdot 431$$

Thus α_w becomes:

$$\alpha_w = 0\cdot 2065\frac{1}{7\cdot 62}0\cdot 419\left(0\cdot 431 + \frac{3}{2}\right) = 0\cdot 022 \text{ db/metre}$$

If the curves of Fig. 1 are used, the value of $\alpha_w b^{3/2}$ for $b/a = 3$ is interpolated from two curves, $b/a = 2$ and $b/a = 5$ at $\lambda_e/b = 10/7\cdot 62 = 1\cdot 131$.

For $b/a = 2,\ \alpha_w b^{3/2} = 0\cdot 34,$

and for $b/a = 5,\ \alpha_w b^{3/2} = 0\cdot 70$

At a given λ_e/b there is a reasonable proportionality of change of $\alpha_w b^{3/2}$ between the different b/a curves; hence for $b/a = 3$ it can be written:

$$\alpha_w b^{3/2} \simeq 0\cdot 34 + \frac{3-2}{5-2}(0\cdot 70 - 0\cdot 34) = 0\cdot 46$$

This gives

$$\alpha_w = \frac{0\cdot 46}{7\cdot 62^{3/2}} \simeq 0\cdot 022 \text{ db/metre}$$

(5) ATTENUATION CONSTANT, α_d

The other kind of attenuation considered in this paper is that due to loss in the dielectric filling a guide.

It is assumed now that the guide walls are perfectly conducting. Then the field equations of Table 1, which show the field distribution in the guide cross-section for a particular mode of wave, remain valid. In particular, the critical wavelength, λ_{cr}, of the guide remains real—as in the case of a lossless dielectric.

From these equations it is seen that the amplitudes of the fields would not vary in the direction x, along the guide, if the term $j2\pi/\lambda_x$ were entirely imaginary, as it is in the transmitting region of a lossless guide.

The dielectric losses cause this term to become complex,

$$j2\pi/\lambda_x = \alpha_d + j\beta \quad . \quad . \quad . \quad . \quad . \quad (97)$$

and then, the real part of it, α_d, shows the rate at which the fields decrease along the guide in the direction of propagation, decreasing, in fact, as $\epsilon^{-\alpha_d x}$. Hence in this case, by definition of attenuation constant, the quantity α_d [as given by eqn. (97)] is the attenuation constant of the guide. Eqn. (97) gives its value in nepers per unit length of guide; to obtain α_d in decibels per unit length, the multiplication factor $8\cdot 68$ is introduced.

The quantity β given by eqn. (97) becomes the phase-constant of the wave along the guide, instead of the term $2\pi/\lambda_x$ as in the case of no loss in the dielectric; β is given in radians per unit length of guide.

To determine how the quantity $2\pi/\lambda_x$ depends on loss in the dielectric, the procedure is as follows. In the case of a lossless dielectric, eqn. (2), derived from Maxwell's equations, gives the relationship:

$$\left(\frac{2\pi}{\lambda_e}\right)^2 = \left(\frac{2\pi}{\lambda_{cr}}\right)^2 + \left(\frac{2\pi}{\lambda_x}\right)^2 \quad . \quad . \quad . \quad . \quad (98)$$

where λ_{cr} depends on dimensions of the guide and on the mode of wave but not on the dielectric, and λ_e, defined by eqn. (1), depends on the frequency and the properties κ, μ of the dielectric. For a lossless dielectric the values of κ and μ are entirely real.

It is usual to take account of a small loss in a dielectric by attributing an imaginary component to its dielectric constant κ, writing:

$$\kappa = \kappa' - j\kappa'' \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

and the lossiness of a dielectric is determined by the ratio:

$$\kappa''/\kappa' = \tan\delta \quad . \quad . \quad . \quad . \quad . \quad (100)$$

where δ is called the loss-angle of the dielectric.

Hence, κ can be written:

$$\kappa = \kappa'(1 - j\tan\delta) \quad . \quad . \quad . \quad . \quad (101)$$

When this value of κ is introduced in eqn. (1) and the notations of eqn. (97) are used for $2\pi/\lambda_x$, the basic equation (98) becomes:

$$\left(\frac{2\pi f(\kappa'\mu)^{\frac{1}{2}}}{c}\right)^2(1 - j\tan\delta) = \left(\frac{2\pi}{\lambda_{cr}}\right)^2 - (\alpha_d + j\beta)^2 \ . \quad (102)$$

κ' appears here in the same way, as does κ in eqn. (1) and, therefore, the same symbol λ_e is used for the expression:

$$\lambda_e = \frac{c}{f(\kappa'\mu)^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad (103)$$

However, λ_e, so defined, is not quite equal to the wavelength in the unbounded lossy dielectric κ', δ, μ. The exact value of this wavelength can be obtained from eqn. (102) as equal to $2\pi/\beta$, when λ_{cr} is made infinitely large.

The final form of the basic equation, involving the newly defined symbol λ_e, becomes:

$$-(\alpha_d + j\beta)^2 = \left(\frac{2\pi}{\lambda_e}\right)^2(1 - j\tan\delta) - \left(\frac{2\pi}{\lambda_{cr}}\right)^2 \quad . \quad (104)$$

where all the terms, α_d, β, δ, λ_e and λ_{cr}, are real.

Since in these considerations no particular mode of wave has been assumed, eqn. (104) is valid for any guide and for any mode of wave.

Equation (104) leads to two simultaneous equations:

$$\left.\begin{aligned}\beta^2 - \alpha_d^2 &= \left(\frac{2\pi}{\lambda_e}\right)^2 - \left(\frac{2\pi}{\lambda_{cr}}\right)^2\\ 2\alpha_d\beta &= \left(\frac{2\pi}{\lambda_e}\right)^2 \tan\delta\end{aligned}\right\} \quad . \quad . \quad . \quad (105)$$

These equations, solved for real and positive values of α_d and β, give the following results:

$$\alpha_d = \frac{2\pi}{\lambda_e}\left\{\frac{1}{2}\left\{\left|\left\{\left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]^2 + \tan^2\delta\right\}^{\frac{1}{2}}\right| - \left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]\right\}\right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad (106)$$

$$\beta = \frac{2\pi}{\lambda_e}\left\{\frac{1}{2}\left\{\left|\left\{\left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]^2 + \tan^2\delta\right\}^{\frac{1}{2}}\right| + \left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]\right\}\right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad (107)$$

It is easy to check that for a lossless dielectric ($\delta = 0$) and for $\lambda_e < \lambda_{cr}$, α_d becomes zero, and β becomes $2\pi/\lambda_x$ with λ_x given by eqn. (2).

Both α_d and β are inversely proportional to λ_e, hence the expressions $\alpha_d\lambda_e$ and $\beta\lambda_e$ are entirely independent of any linear dimensions; they depend only on the ratio λ_e/λ_{cr} and on the loss-angle δ.

However, it has been thought more convenient to use the products, $\alpha_d\lambda_{cr}$ and $\beta\lambda_{cr}$, as the quantities independent of linear dimensions, since, for a given mode, λ_{cr} is determined directly by the dimensions of the guide. Therefore, the general curves of Figs. 3 and 4 show the values of these quantities and not those of $\alpha_d\lambda_e$ and $\beta\lambda_e$.

The formulae (106) and (107) are valid for any values of λ_e/λ_{cr} and $\tan\delta$. However, a convenient simplification is possible for small values of δ ($\tan\delta < 0\cdot 1$, say) in cases where the ratio λ_e/λ_{cr} does not approach unity.

The following formulae give the attenuation constant α_d in decibels/metre, if λ_{cr} is expressed in metres.

If $\tan\delta$ is small and λ_e much smaller than λ_{cr}, the square-root term containing $\tan^2\delta$ in eqn. (106) may be expanded in the form of a series and, when higher powers are neglected, the approximate value of $\alpha_d\lambda_{cr}$ becomes:

$$\alpha_d\lambda_{cr} \simeq 27\cdot 26\frac{\lambda_{cr}}{\lambda_e}\frac{\tan\delta}{\left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]^{\frac{1}{2}}} \text{ decibels} \quad . \quad (108)$$

If, with $\tan\delta$ small, λ_e is much larger than λ_{cr}, the approximate formula obtained in the same way becomes:

$$\alpha_d\lambda_{cr} \simeq 54\cdot 51\left[1-\left(\frac{\lambda_{cr}}{\lambda_e}\right)^2\right]^{\frac{1}{2}} \text{ decibels} \quad . \quad . \quad (109)$$

For values of λ_e in the region of λ_{cr} the unsimplified formula must be employed:

$$\alpha_d\lambda_{cr} = 54\cdot 51\frac{\lambda_{cr}}{\lambda_e}\left\{\frac{1}{2}\left\{\left|\left\{\left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]^2 + \tan^2\delta\right\}^{\frac{1}{2}}\right| - \left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]\right\}\right\}^{\frac{1}{2}} \text{ decibels} \quad . \quad (110)$$

Similarly, the following expressions are obtained for the phase-constant, β, with β in radians/metre, if λ_{cr} is in metres.

If $\tan\delta$ is small, and $\lambda_e \ll \lambda_{cr}$,

$$\beta\lambda_{cr} \simeq 2\pi\left[\left(\frac{\lambda_{cr}}{\lambda_e}\right)^2 - 1\right]^{\frac{1}{2}} \text{ radians} \quad . \quad . \quad (111)$$

If $\tan\delta$ is small and $\lambda_e \gg \lambda_{cr}$, then

$$\beta\lambda_{cr} \simeq \pi\frac{\lambda_{cr}}{\lambda_e}\frac{\tan\delta}{\left[\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2 - 1\right]^{\frac{1}{2}}} \text{ radians} \quad . \quad . \quad (112)$$

but when λ_e lies in the region of λ_{cr}, the full formula must be used:

$$\beta\lambda_{cr} = 2\pi\frac{\lambda_{cr}}{\lambda_e}\left\{\frac{1}{2}\left\{\left|\left\{\left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]^2 + \tan^2\delta\right\}\right| + \left[1-\left(\frac{\lambda_e}{\lambda_{cr}}\right)^2\right]\right\}\right\}^{\frac{1}{2}} \text{ radians} \quad . \quad (113)$$

From these expressions the following is noted. In the transmitting region of a guide (i.e. when $\lambda_e < \lambda_{cr}$), if λ_e is not too close to λ_{cr}, the attenuation constant α_d is proportional to the tangent of the loss-angle of the dielectric filling the guide, while the phase-constant β may be said to be independent of it. At very high frequencies, i.e. with λ_e decreasing to zero, α_d becomes proportional to frequency and the wave is quickly attenuated.

In the attenuating region of a guide (i.e. when $\lambda_e > \lambda_{cr}$), if λ_e is not too close to λ_{cr}, the attenuation constant α_d ceases to depend on the loss-angle of the dielectric; instead, the phase-constant β is no longer zero, as in the case of a lossless dielectric, but becomes proportional to $\tan\delta$. At very low frequencies the value of $\alpha_d\lambda_{cr}$ tends to the limit of $54\cdot 5$ db and the phase-constant decreases to zero.

Due to loss in the dielectric the threshold between the two regions becomes less distinct and, when passing it in the direction of increasing λ_e, a gradual increase of α_d and a gradual decrease of β is observed.

In Figs. 3 and 4 curves are drawn depicting the relationships of equations (110) and (113) for a few values of $\tan\delta$, i.e. for 0; $0\cdot 0005$; $0\cdot 01$ and $0\cdot 1$.

In Fig. 3(*b*) the curves for the transmitting region of the guide are presented on an enlarged scale. It is also shown that the minimum value of the attenuation constant α_d occurs when the ratio λ_{cr}/λ_e is equal to $\sqrt{2}$, or, in other words, when $\lambda_x \simeq \lambda_{cr}$, i.e. the wavelength along the guide is equal to the wavelength in the transverse direction.

In Fig. 4(*b*), the region of $\lambda_e = \lambda_{cr}$ is shown on a larger scale. An abrupt change of the curve is apparent for the case of a lossless dielectric, as distinct from the smooth transition when the dielectric is not lossless.

As an example of the application of the formulae obtained, a calculation is made of the attenuation constants α_w and α_d for a copper guide which is filled (in order to reduce its cross-sectional dimensions) with the best obtainable dielectric such as polystyrene ($\kappa = 2\cdot 55$; $\tan\delta = 0\cdot 0006$).

Assume a rectangular guide, the H_{01}-mode of wave, and a frequency $f = 3\,000$ Mc/s. By eqn. (103), λ_e becomes equal to $6\cdot 3$ cm, and a 3-in by 1-in guide can be reduced to the dimensions, $b = 4\cdot 8$ cm and $a = 1\cdot 6$ cm.

Fig. 1, for $\dfrac{\lambda_e}{b} = \dfrac{6\cdot 3}{4\cdot 8} = 1\cdot 31$ and $\dfrac{b}{a} = 3$, gives the value of $\alpha_w b^{3/2}$ as equal to $0\cdot 46$. Since this figure refers to an air-filled guide, it is multiplied by $\sqrt[4]{\kappa'}$ to take into account the change in the numerical factor $\left[\frac{c}{\sigma}\frac{\mu_1}{\mu}\left(\frac{\kappa}{\mu}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$; and the correct value of $\alpha_w b^{3/2}$ becomes:

$$\alpha_w b^{3/2} = 0\cdot 46\sqrt[4]{2\cdot 55} = 0\cdot 58$$

Hence $\alpha_w = \dfrac{0\cdot 58}{4\cdot 8^{3/2}} = 0\cdot 055$ decibel/metre

Since Fig. 3(*a*) would give an inaccurate value of $\alpha_d\lambda_{cr}$ for $\tan\delta = 0\cdot 0006$, the formula (108) is used instead. For $\dfrac{\lambda_e}{\lambda_{cr}} = \dfrac{6\cdot 3}{2(4\cdot 8)} = 0\cdot 656$, $\alpha_d\lambda_{cr}$ becomes

$$\alpha_d\lambda_{cr} = 0\cdot 033 \text{ decibel}$$

and hence $\alpha_d = \dfrac{0\cdot 033}{0\cdot 096} = 0\cdot 344$ decibel/metre

It is seen that the attenuation due to loss in the dielectric is six times greater than that caused by losses in the wall metal, in spite of the fact that the best solid dielectric is used. The total value of the attenuation constant α for this guide would then be:

$$\alpha = \alpha_d + \alpha_w \simeq 0\cdot 4 \text{ decibel/metre}$$

as compared with $0\cdot 022$ decibel/metre for the full-size 3-in by 1-in guide filled with air and carrying the same mode of wave at the same frequency. From this it is seen that, at present, the reduction of size of a wave guide by filling it with a solid dielectric is always accompanied by an enormous increase in the guide attenuation.

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