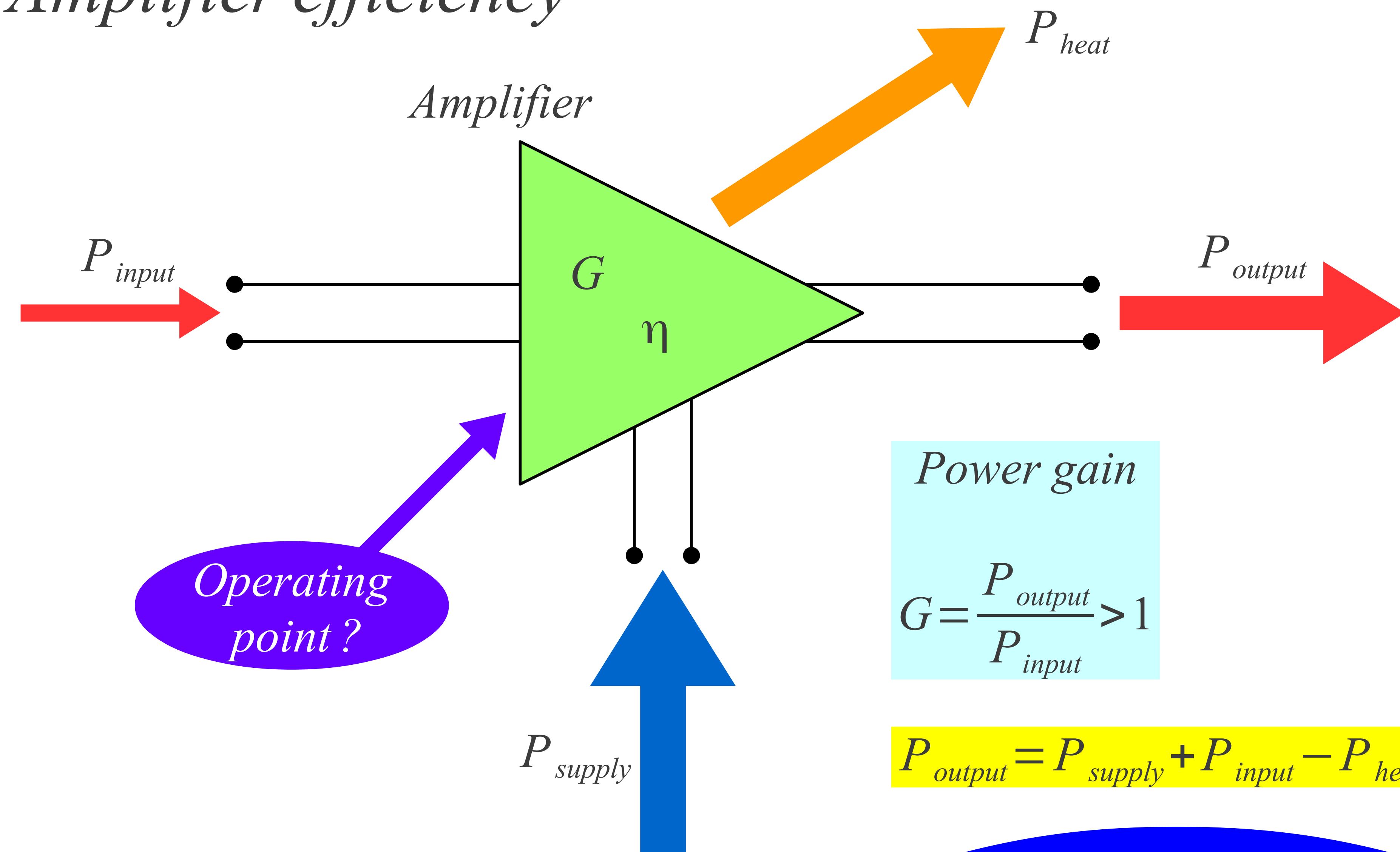
# Communication Electronics

## Lecture 12:

## Intermodulation distortion

# Amplifier efficiency




**Power gain**

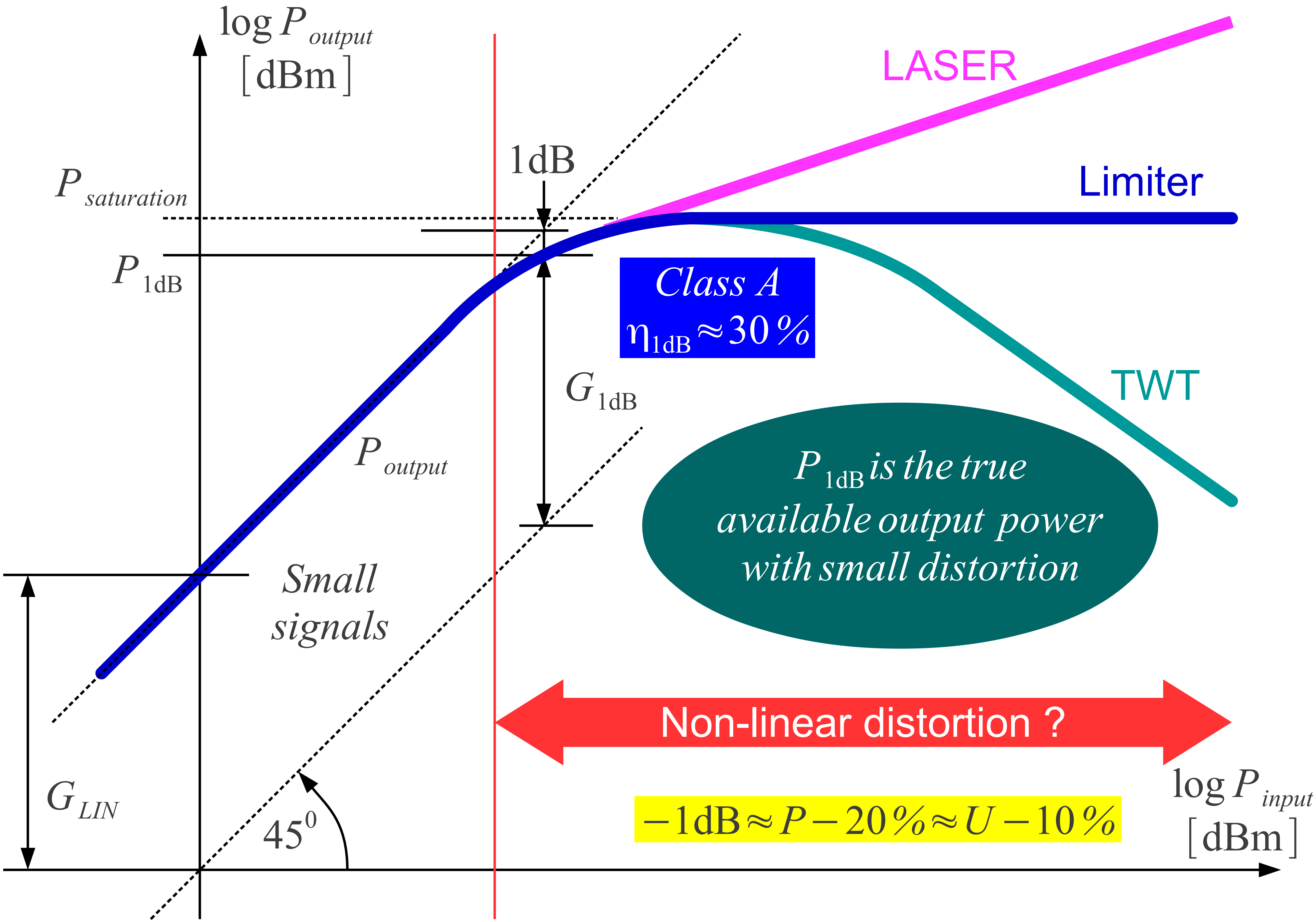
$$G = \frac{P_{output}}{P_{input}} > 1$$

$$P_{output} = P_{supply} + P_{input} - P_{heat}$$

$$\eta = \frac{P_{output}}{P_{supply} + P_{input}} \approx \frac{P_{output}}{P_{supply}} \leq 100\,\%$$

**Small-signal amplifier** $\quad P_{output} \ll P_{supply}$

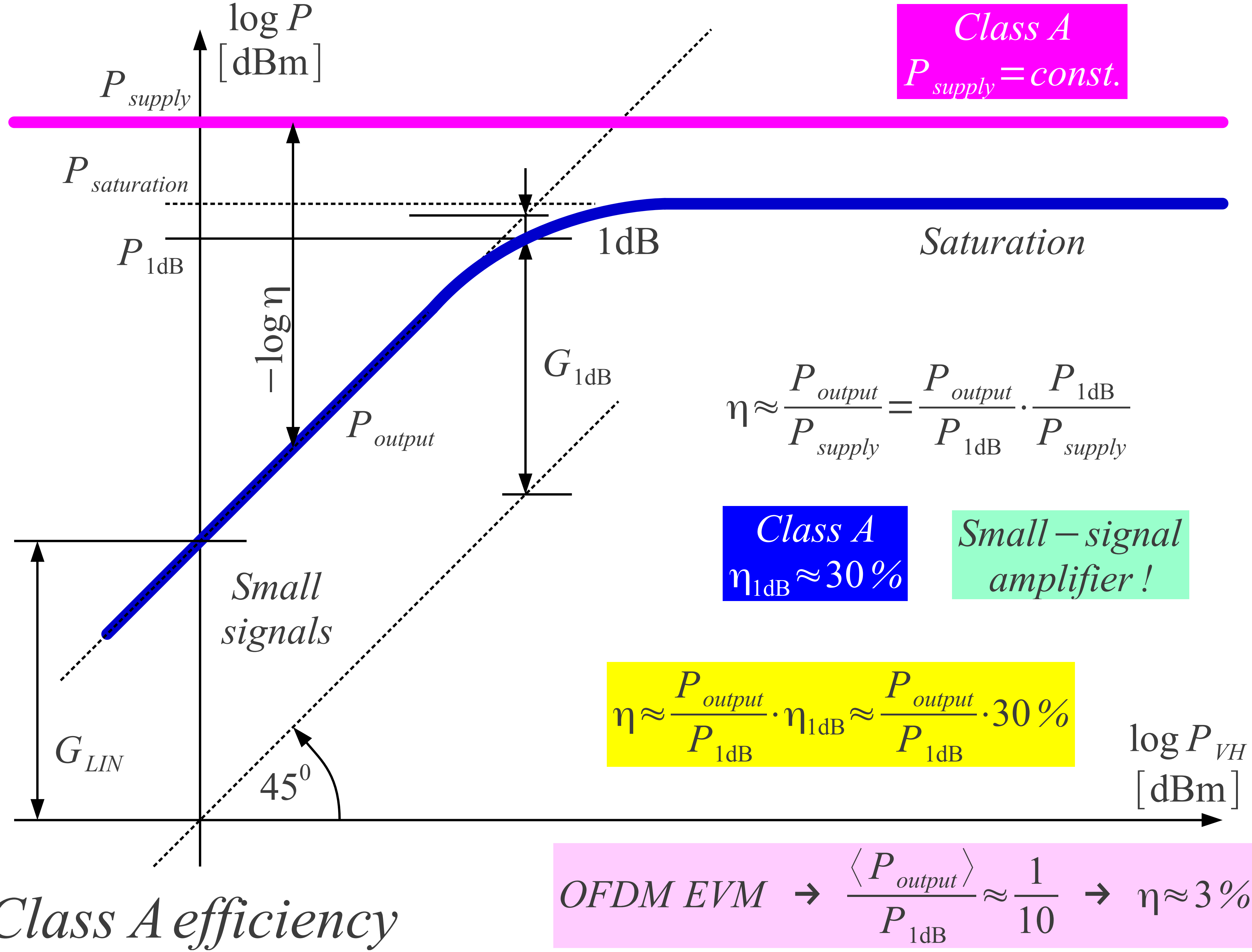
**Power amplifier** $\quad P_{output} \approx P_{heat} \approx \frac{1}{2} P_{supply}$

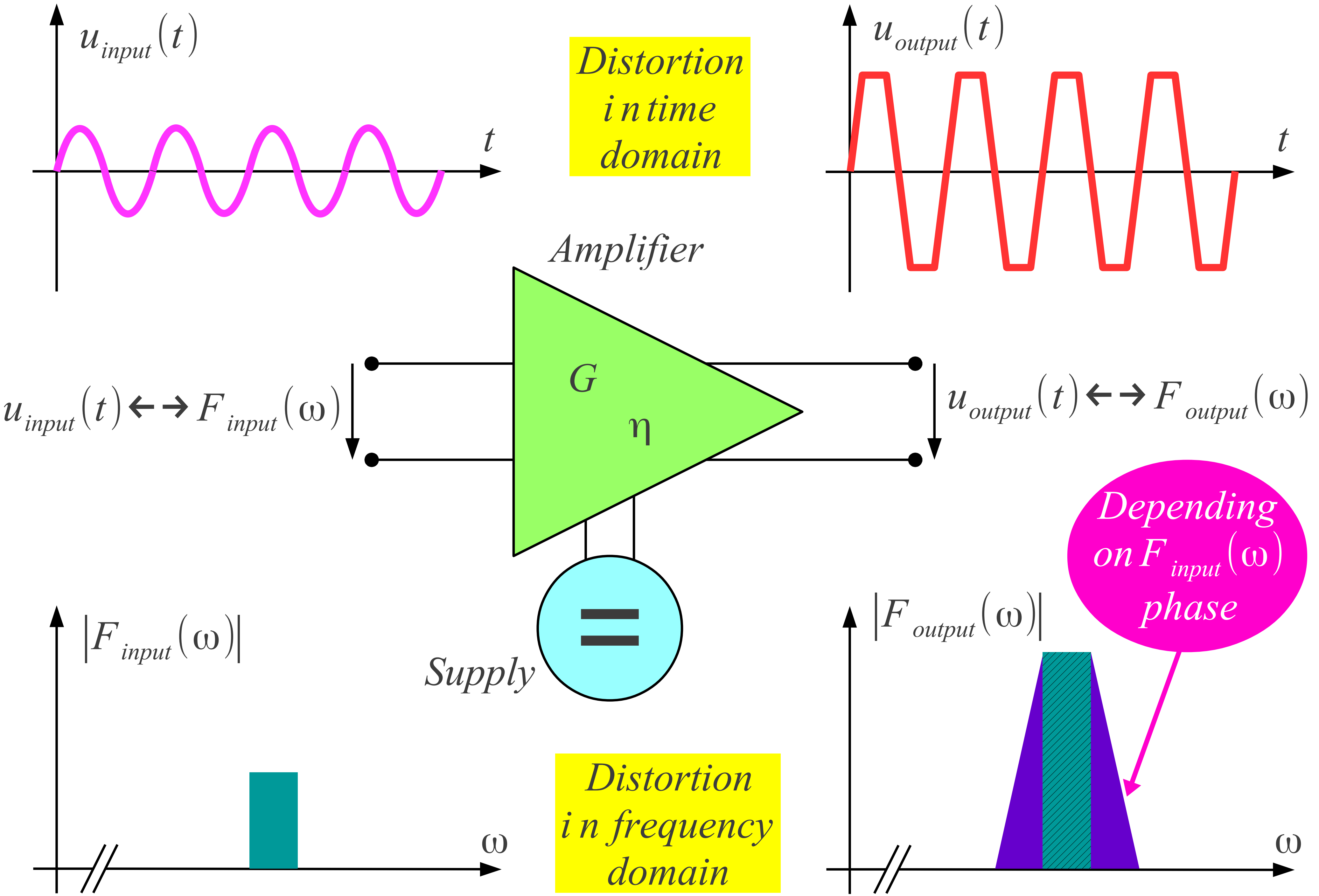



# *Amplifier saturation*

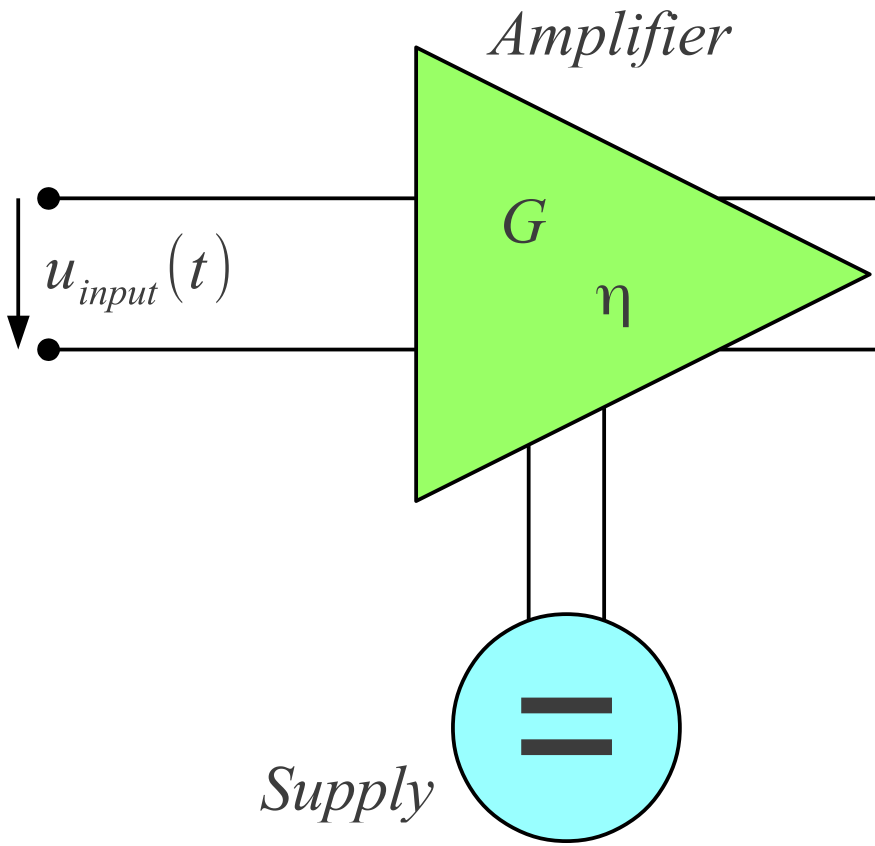
*Multistage amplifier* → $P_{3dB}$ *makes sense*

# Class A efficiency

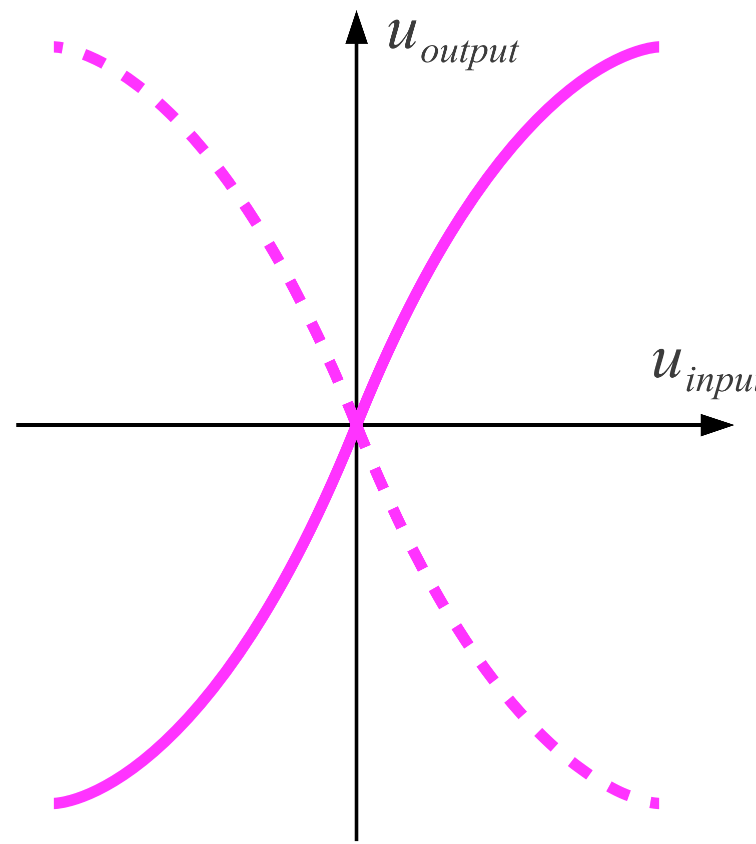





# Amplifier distortion




Saturation
$\alpha_1 \cdot \alpha_3 \leq 0$



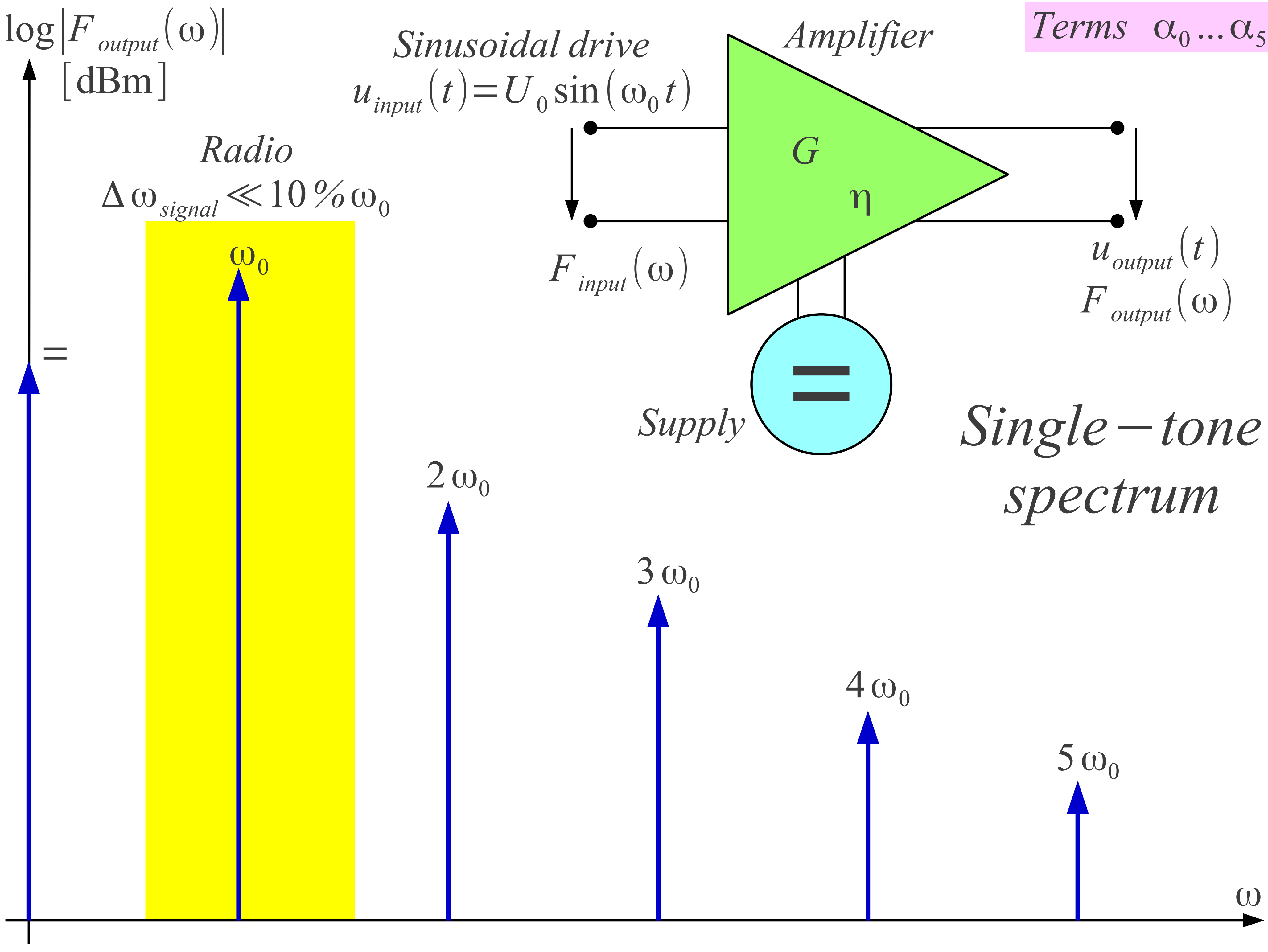

*Amplifier – nonlinearity description with polynomial :*

$$u_{output} = \alpha_0 + \alpha_1 \cdot u_{input} + \alpha_2 \cdot u_{input}^2 + \alpha_3 \cdot u_{input}^3 + \alpha_4 \cdot u_{input}^4 + \alpha_5 \cdot u_{input}^5 + \alpha_6 \cdot u_{input}^6 + \alpha_7 \cdot u_{input}^7 + \ldots$$

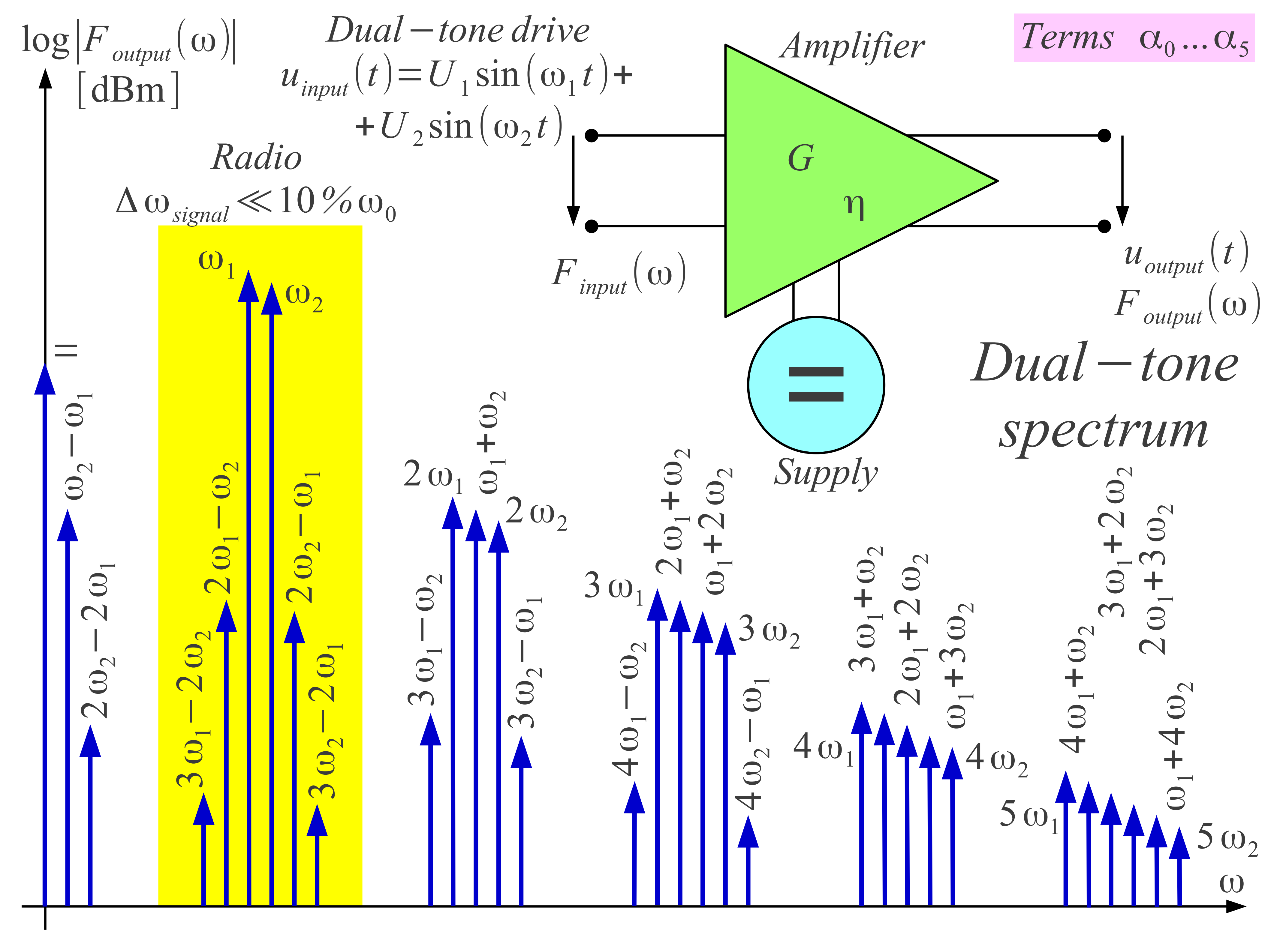
# *Amplifier nonlinearity*

# *Effects of polynomial terms*

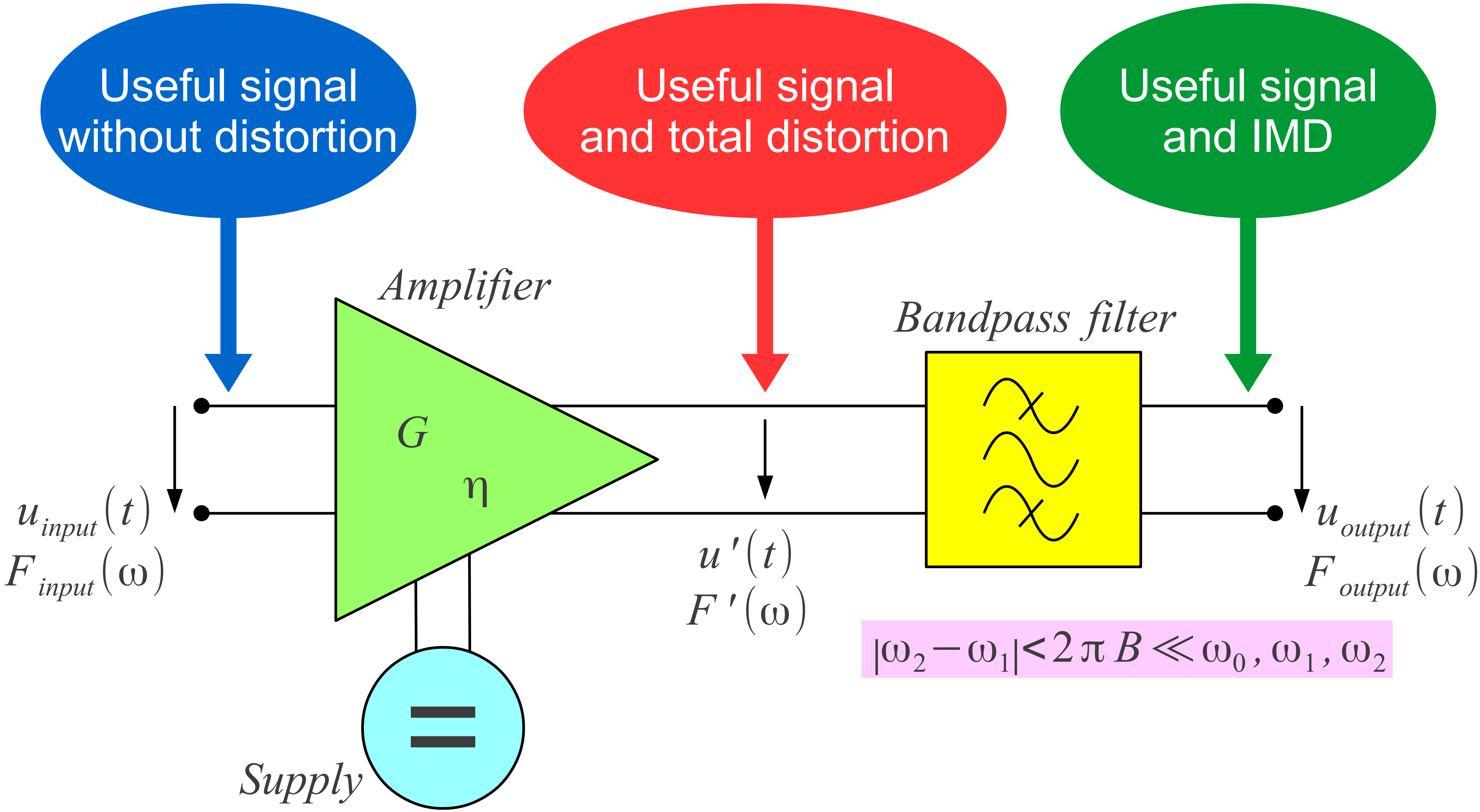
| *Term* | *Sinusoidal drive* $u_{input}(t)=U_0\sin(\omega_0 t)$ | *Two − tone drive* $u_{input}(t)=U_1\sin(\omega_1 t)+U_2\sin(\omega_2 t)$ |
|---|---|---|
| $\alpha_0$ | $=(bias\ point)$ | $=(bias\ point)$ |
| $\alpha_1$ | $\omega_0$ | $\omega_1,\omega_2(linear\ gain)$ |
| $\alpha_2$ | $=(rectifier),2\omega_0$ | $=,2\omega_1,2\omega_2,\omega_2+\omega_1,\omega_2-\omega_1(mixing)$ |
| $\alpha_3$ | $\omega_0(saturation),3\omega_0$ | $\omega_1,\omega_2,3\omega_1,3\omega_2,2\omega_1+\omega_2$<br>$2\omega_1-\omega_2(IMD),\omega_1+2\omega_{2,}2\omega_2-\omega_1(IMD)$ |
| $\alpha_4$ | $=,2\omega_0,4\omega_0$ | $=,2\omega_0,2\omega_2,\omega_2+\omega_1,\omega_2-\omega_1,4\omega_1,4\omega_2,3\omega_1+\omega_2$<br>$2\omega_1+2\omega_2,\omega_1+3\omega_{2,}3\omega_1-\omega_2,2\omega_2-2\omega_1,3\omega_2-\omega_1$ |
| $\alpha_5$ | $\omega_0,3\omega_0,5\omega_0$ | $\omega_1,\omega_2\ldots5\omega_1,5\omega_2\ldots3\omega_3-2\omega_2,3\omega_2-2\omega_1(IMD)\ldots$ |
| $\alpha_6$ | $=,2\omega_0,4\omega_0,6\omega_0$ | $=\ldots6\omega_1,6\omega_2,5\omega_1+\omega_2,5\omega_1-\omega_2,4\omega_1+2\omega_2\ldots$ |
| $\alpha_7$ | $\omega_0,3\omega_0,5\omega_0,7\omega_0$ | $\omega_1,\omega_2\ldots7\omega_1,7\omega_2\ldots4\omega_1-3\omega_2,4\omega_2-3\omega_1(IMD)\ldots$ |



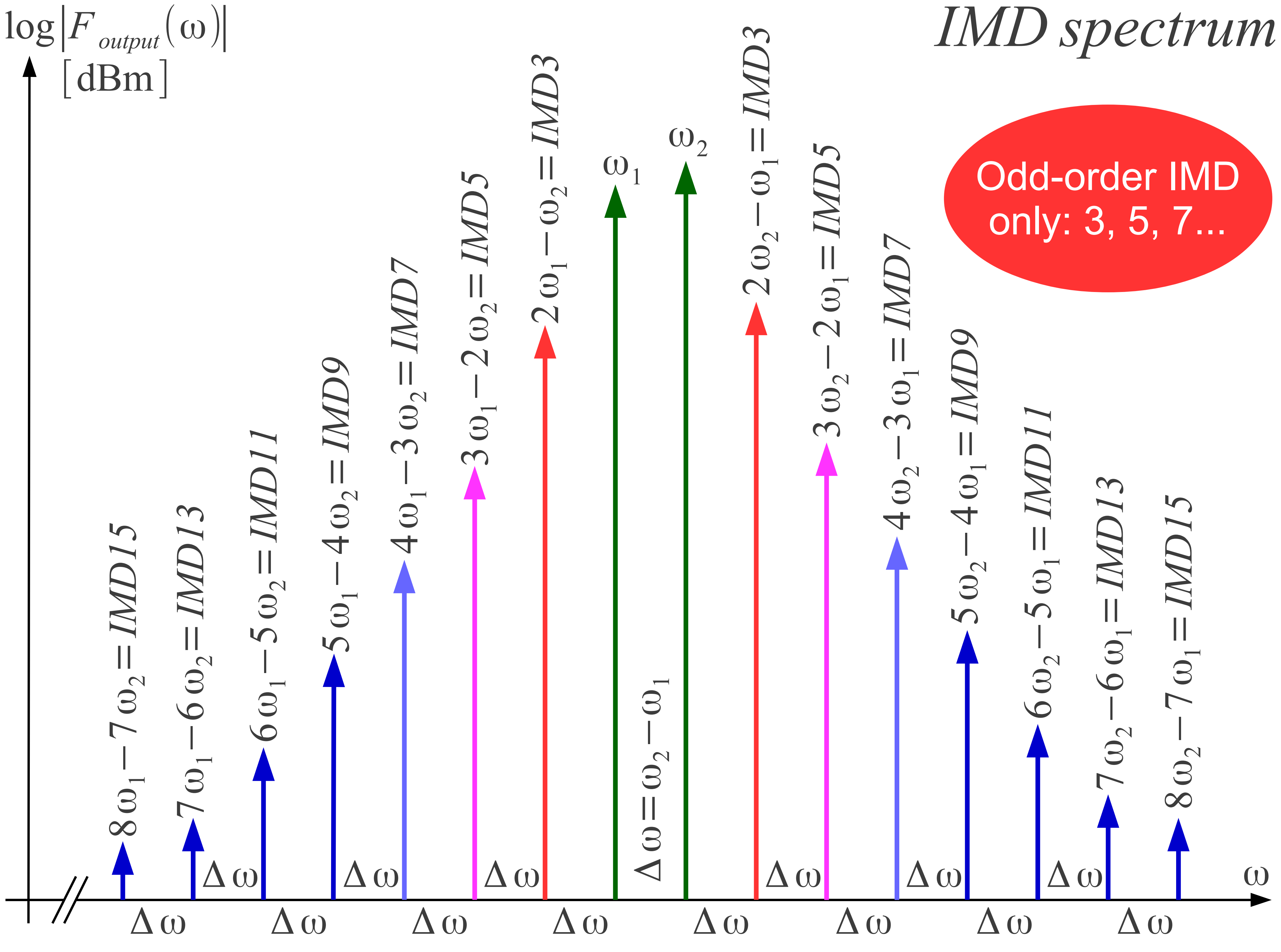
log|F output(ω)| [dBm]
Sinusoidal drive
u input(t) = U 0 sin(ω0 t)
Amplifier
Terms α0 … α5
Radio
Δω signal ≪ 10 % ω0
G
η
F input(ω)
u output(t)
F output(ω)
ω0
=
Supply
Single−tone
spectrum
2 ω0
3 ω0
4 ω0
5 ω0
ω



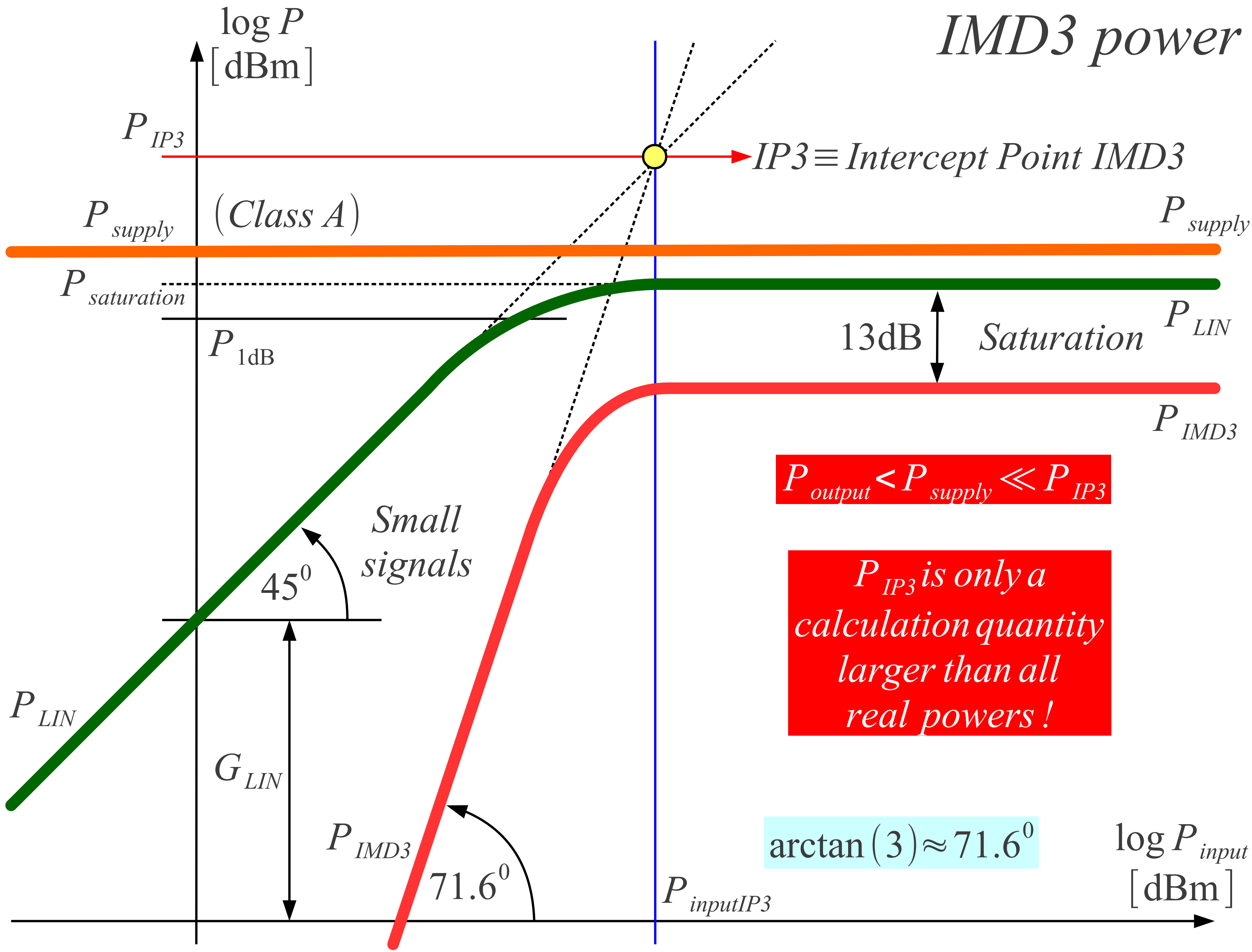
log|F output(ω)| [dBm]
Dual−tone drive
u input(t)=U1 sin(ω1 t)+ +U2 sin(ω2 t)
Amplifier
Terms α0…α5
G
η
F input(ω)
u output(t)
F output(ω)
Supply
Dual−tone spectrum
Radio
Δω signal ≪10% ω0
ω1
ω2
=
ω2−ω1
2ω2−2ω1
3ω1−2ω2
2ω1−ω2
2ω2−ω1
3ω2−2ω1
3ω1−ω2
2ω1
ω1+ω2
2ω2
3ω2−ω1
4ω1−ω2
3ω1
2ω1+ω2
ω1+2ω2
3ω2
4ω2−ω1
4ω1
3ω1+ω2
2ω1+2ω2
ω1+3ω2
4ω2
5ω1
4ω1+ω2
3ω1+2ω2
2ω1+3ω2
ω1+4ω2
5ω2
ω




# *InterModulation Distortion (IMD)*



log|F output(ω)|
[dBm]
IMD spectrum
Odd-order IMD
only: 3, 5, 7...
8ω1−7ω2=IMD15
7ω1−6ω2=IMD13
6ω1−5ω2=IMD11
5ω1−4ω2=IMD9
4ω1−3ω2=IMD7
3ω1−2ω2=IMD5
2ω1−ω2=IMD3
ω1
ω2
Δω=ω2−ω1
2ω2−ω1=IMD3
3ω2−2ω1=IMD5
4ω2−3ω1=IMD7
5ω2−4ω1=IMD9
6ω2−5ω1=IMD11
7ω2−6ω1=IMD13
8ω2−7ω1=IMD15
Δω
ω


IMD3 power
log P [dBm]
P IP3
IP3 ≡ Intercept Point IMD3
P supply
(Class A)
P supply
P saturation
P 1dB
P LIN
13dB
Saturation
P IMD3
P output < P supply ≪ P IP3
Small signals
45⁰
P IP3 is only a calculation quantity larger than all real powers !
P LIN
G LIN
P IMD3
71.6⁰
arctan(3) ≈ 71.6⁰
log P input [dBm]
P inputIP3

Small signals $P_{output} < P_{1dB}$

$$P_{LIN} \approx G \cdot P_{input}$$

$$P_{IMD3} \approx G_3 \cdot P_{input}^3$$

IMD3 intercept point (IP3)

$$P_{IP3} = G \cdot P_{inputIP3} = G_3 \cdot P_{inputIP3}^3$$

$$P_{inputIP3} = \frac{P_{IP3}}{G} \rightarrow P_{IP3} = G_3 \cdot \left( \frac{P_{IP3}}{G} \right)^3$$

$$G_3 = \frac{G^3}{P_{IP3}^2} \rightarrow P_{IMD3} = \frac{G^3}{P_{IP3}^2} \cdot P_{input}^3$$

$$P_{IMD3} = \frac{P_{LIN}^3}{P_{IP3}^2}$$

Calculation of $P_{IMD3}$ via $P_{IP3}$

$$P_{IMD3} = \frac{P_{LIN}^3}{P_{IP3}^2}$$

$$\log P_{IMD3} = 3 \log P_{LIN} - 2 \log P_{IP3}$$

Up t o which order $P_{IMDn}$ makes sense ?

Higher − order IMD →

→ intercept points IPn

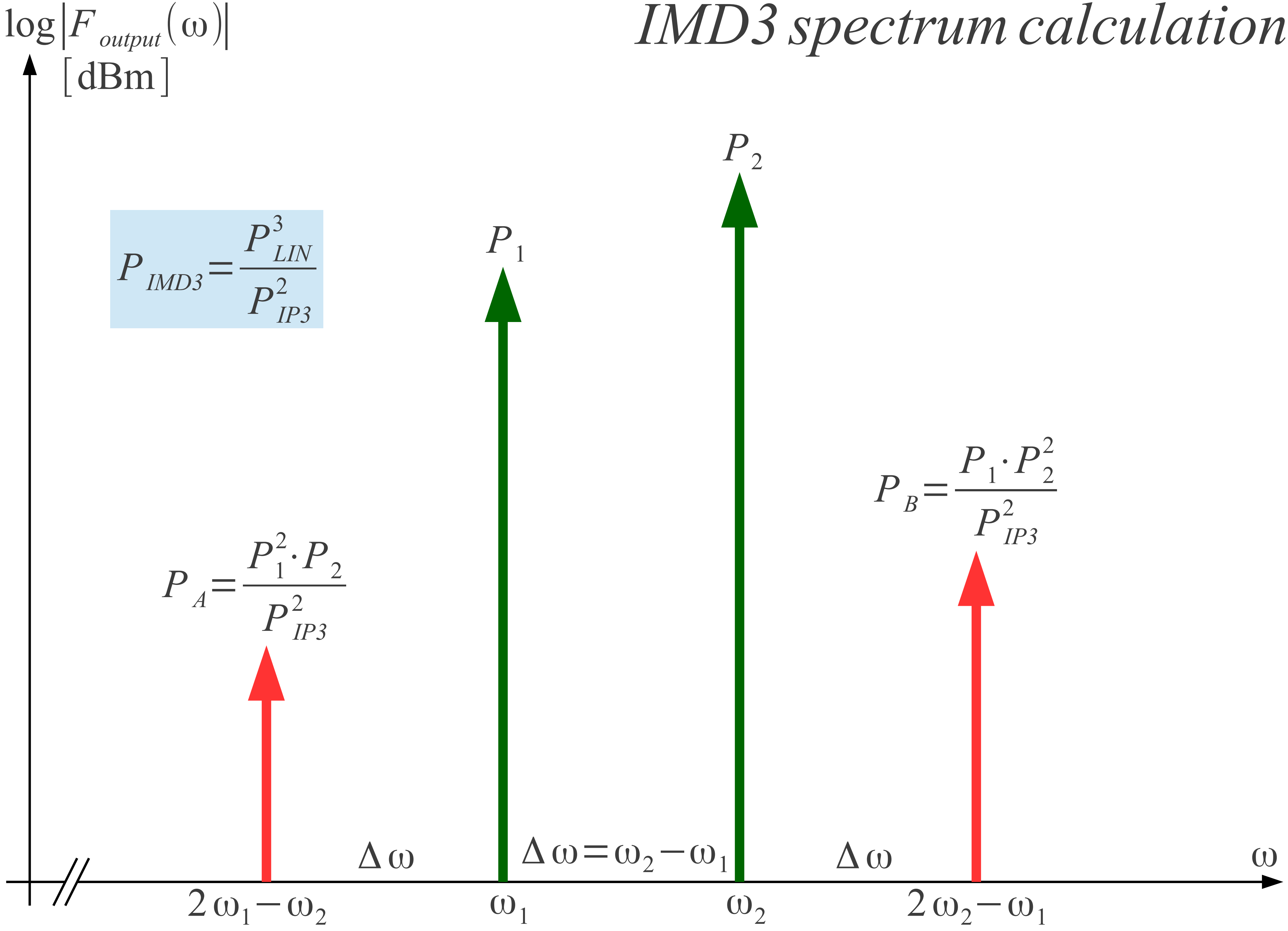
Calculation of $P_{IMDn}$ via $P_{IPn}$

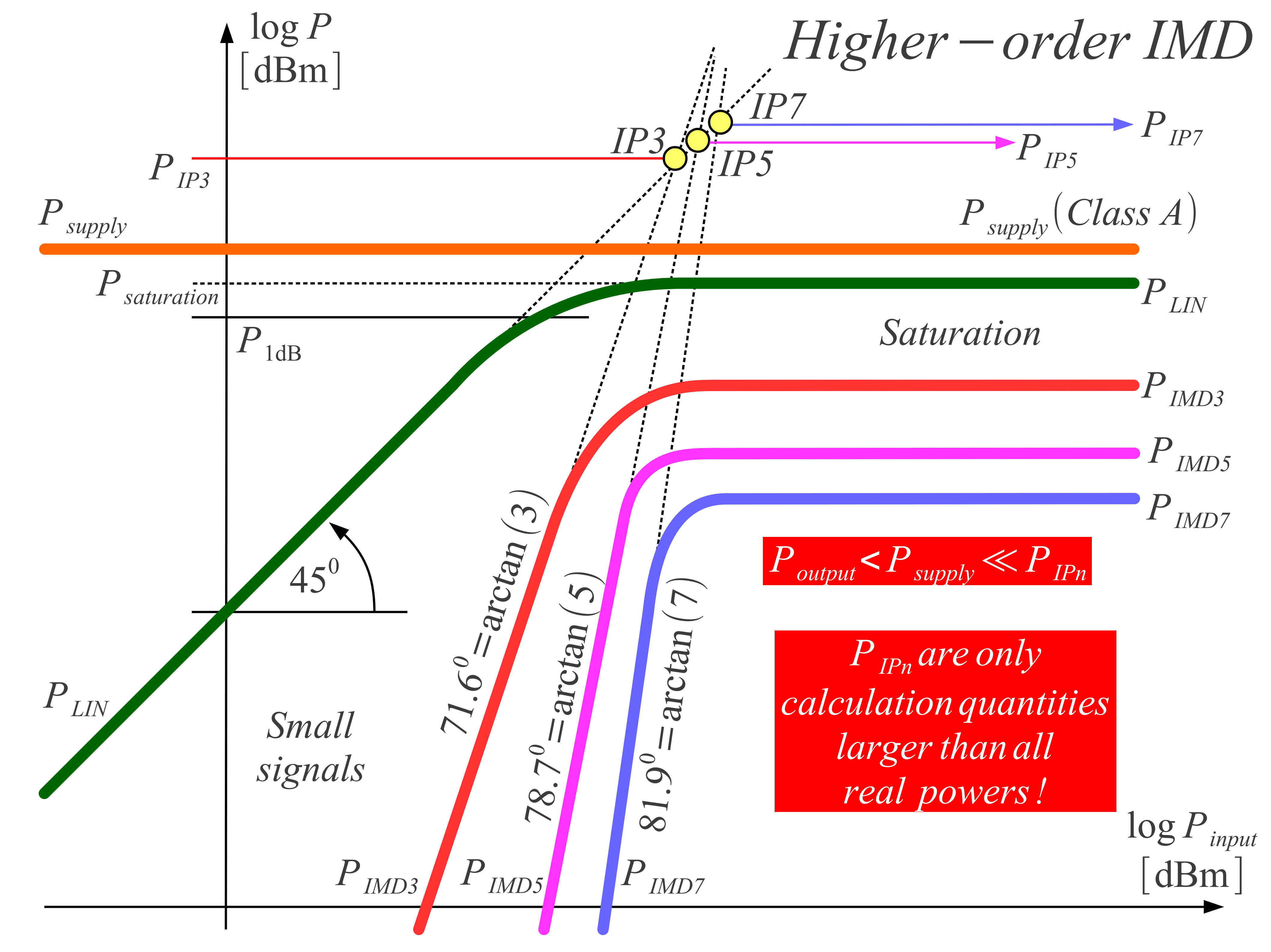
$$P_{IMDn} = \frac{P_{LIN}^n}{P_{IPn}^{n-1}}$$

$$\log P_{IMDn} = n \log P_{LIN} - (n-1) \log P_{IPn}$$

# IMD power calculation

# *IMD3 spectrum calculation*



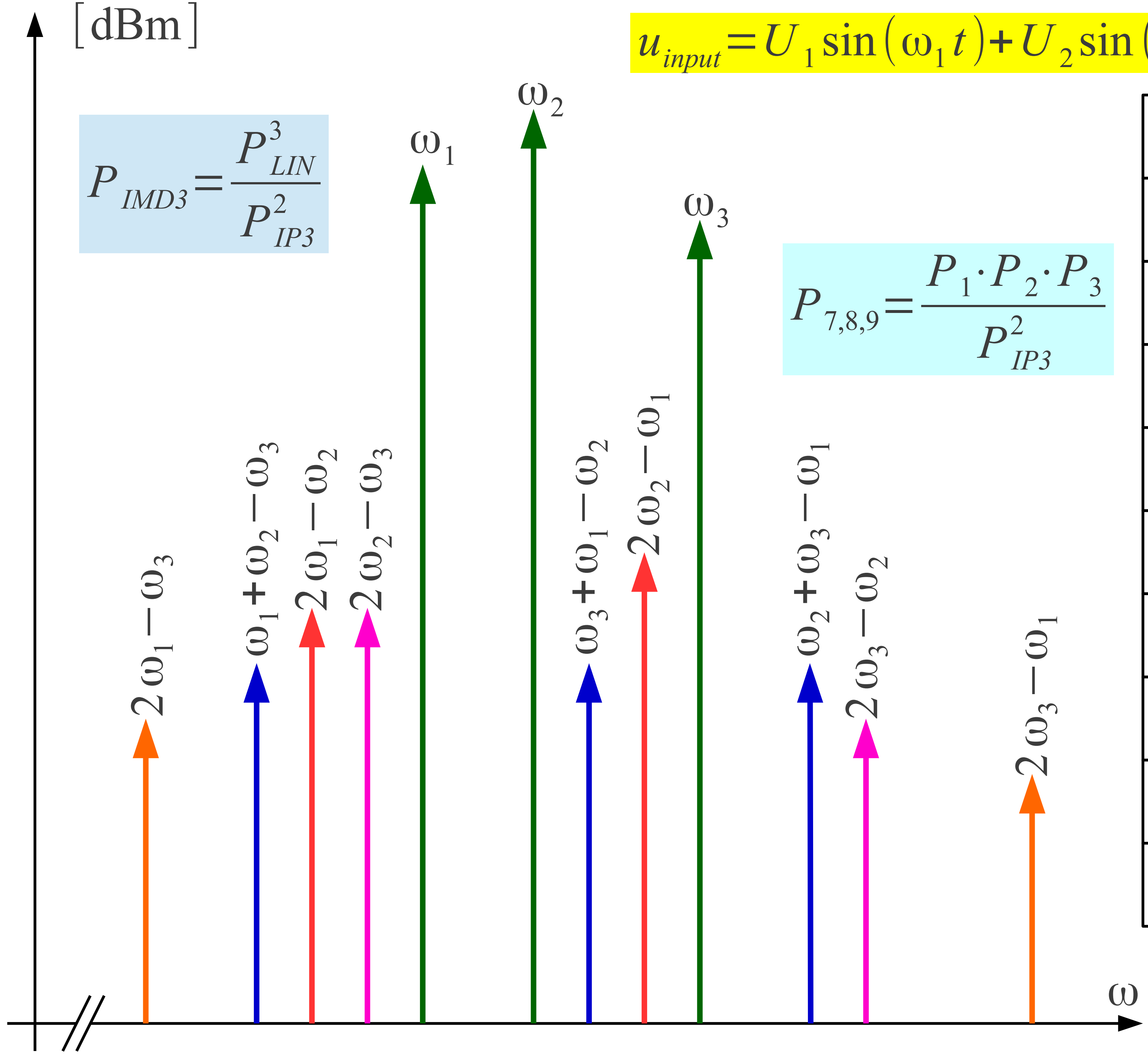

Higher − order IMD
log P
[dBm]
IP7
IP3
IP5
$P_{IP7}$
$P_{IP5}$
$P_{IP3}$
$P_{supply}$
$P_{supply}$ (Class A)
$P_{saturation}$
$P_{LIN}$
$P_{1dB}$
Saturation
$P_{IMD3}$
$P_{IMD5}$
$P_{IMD7}$
$71.6^0$ = arctan(3)
$78.7^0$ = arctan(5)
$81.9^0$ = arctan(7)
$45^0$
$P_{output} < P_{supply} \ll P_{IPn}$
$P_{IPn}$ are only calculation quantities larger than all real powers!
$P_{LIN}$
Small signals
$P_{IMD3}$
$P_{IMD5}$
$P_{IMD7}$
log $P_{input}$
[dBm]

# *Three − tone IMD*

$$u_{input} = U_1 \sin(\omega_1 t) + U_2 \sin(\omega_2 t) + U_3 \sin(\omega_3 t)$$

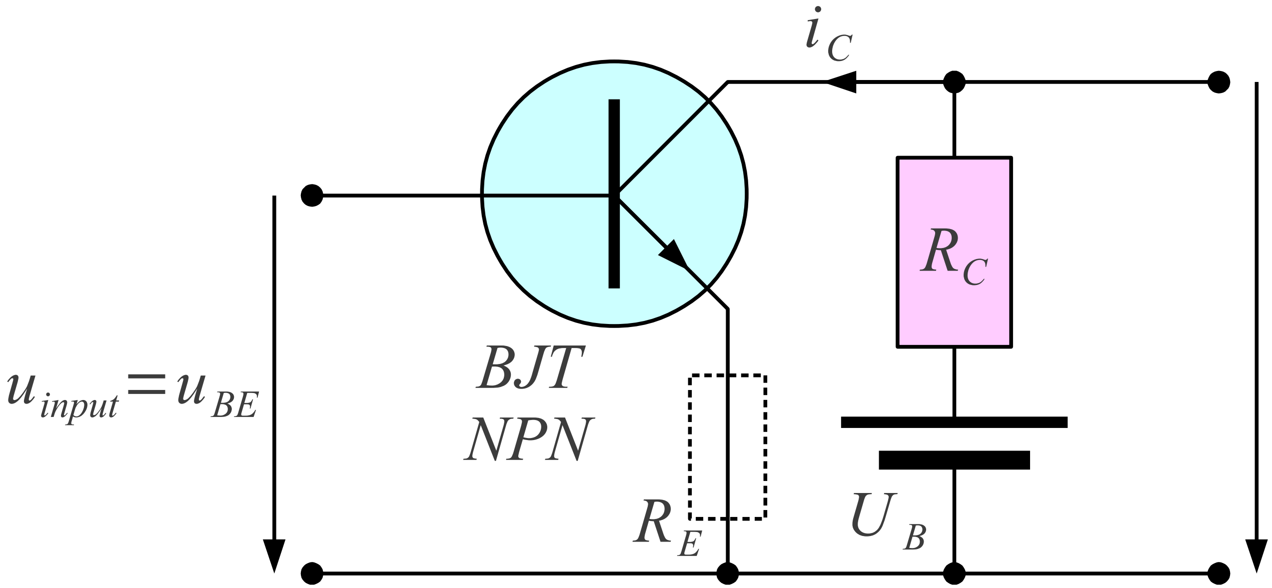
$$P_{IMD3} = \frac{P_{LIN}^3}{P_{IP3}^2}$$

$$P_{7,8,9} = \frac{P_1 \cdot P_2 \cdot P_3}{P_{IP3}^2}$$




| IMD3 products | |
|---|---|
| 1 | $2\omega_1 - \omega_2$ |
| 2 | $2\omega_2 - \omega_1$ |
| 3 | $2\omega_2 - \omega_3$ |
| 4 | $2\omega_3 - \omega_2$ |
| 5 | $2\omega_1 - \omega_3$ |
| 6 | $2\omega_3 - \omega_1$ |
| 7 | $\omega_1 + \omega_2 - \omega_3$ |
| 8 | $\omega_3 + \omega_1 - \omega_2$ |
| 9 | $\omega_2 + \omega_3 - \omega_1$ |

*Higher − order IMD neglected !*




$$i_C = \beta \cdot I_S \cdot \left( e^{\frac{|Q_e|}{n\, k_B\, T} \cdot u_{BE}} - 1 \right)$$

$$u_{output} = U_B - i_C \cdot R_C$$

$$u_{output} = \alpha_0 + \alpha_1 \cdot u_{input} + \alpha_2 \cdot u_{input}^2 + \alpha_3 \cdot u_{input}^3 + \alpha_4 \cdot u_{input}^4 + \ldots$$

$$P_{IP3} \approx 10 \cdot P_{1\mathrm{dB}}$$

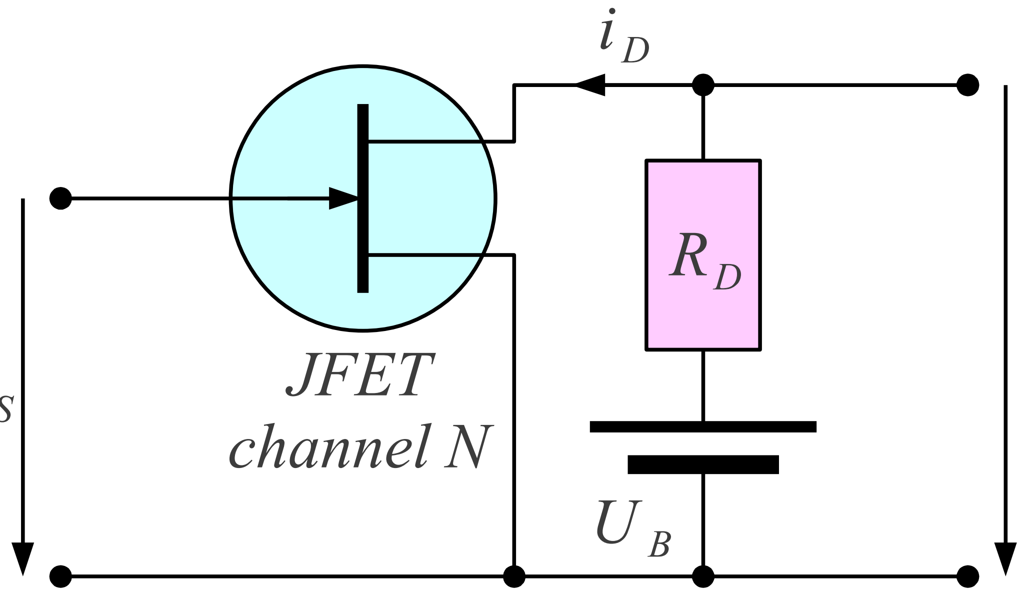
$$\log P_{IP3} \approx \log P_{1\mathrm{dB}} + 10\mathrm{dB}$$

*Bipolar − transistor*
*$P_{IP3}$ estimate*
*without feedback*

$R_E$ (*negative feedback*) → $\log P_{IP3} \approx \log P_{1\mathrm{dB}} + 15\mathrm{dB}$

# *BJT $P_{IP3}$ estimate*

# FET $P_{IP3}$ estimate




$$u_{input} = u_{GS}$$

$$i_D = I_{DSS} \cdot \left( \frac{u_{GS} - U_T}{U_T} \right)^2$$

$$u_{output} = U_B - i_D \cdot R_D$$

$$u_{output} = \alpha_0 + \alpha_1 \cdot u_{input} + \alpha_2 \cdot u_{input}^2$$

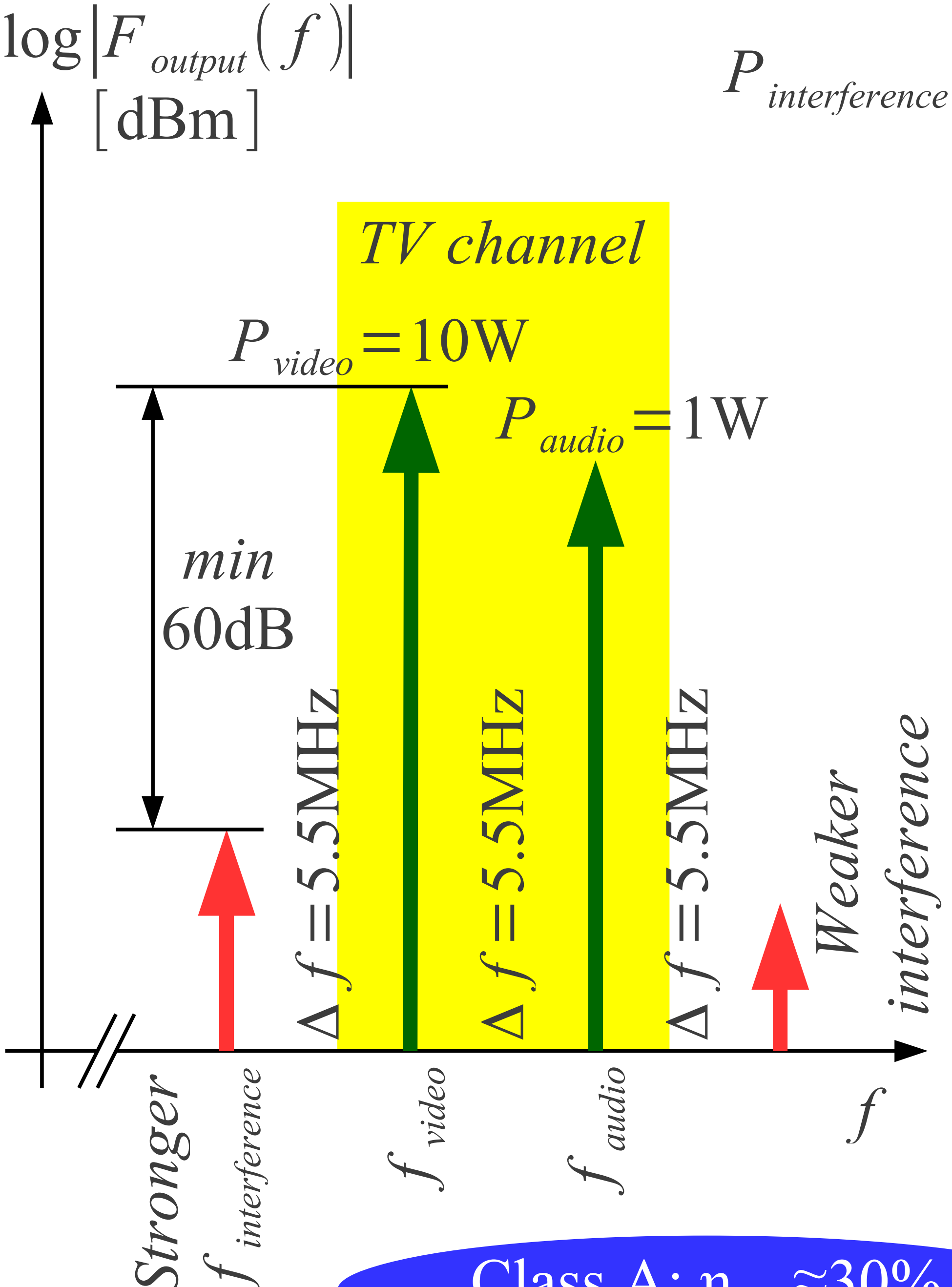
No higher terms!

*Parasitics generate higher-order terms!*

$$P_{IP3} \approx 100 \cdot P_{1\mathrm{dB}}$$

$$\log P_{IP3} \approx \log P_{1\mathrm{dB}} + 20\mathrm{dB}$$

*Field-effect-transistor $P_{IP3}$ estimate (any amplifier with strong feedback)*




$$P_{interference} \le P_{video} \cdot 10^{-60dB/10} = 10\mu W$$

$$P_{interference} = \frac{P^2_{video} \cdot P_{audio}}{P^2_{IP3}} \le 10\mu W$$

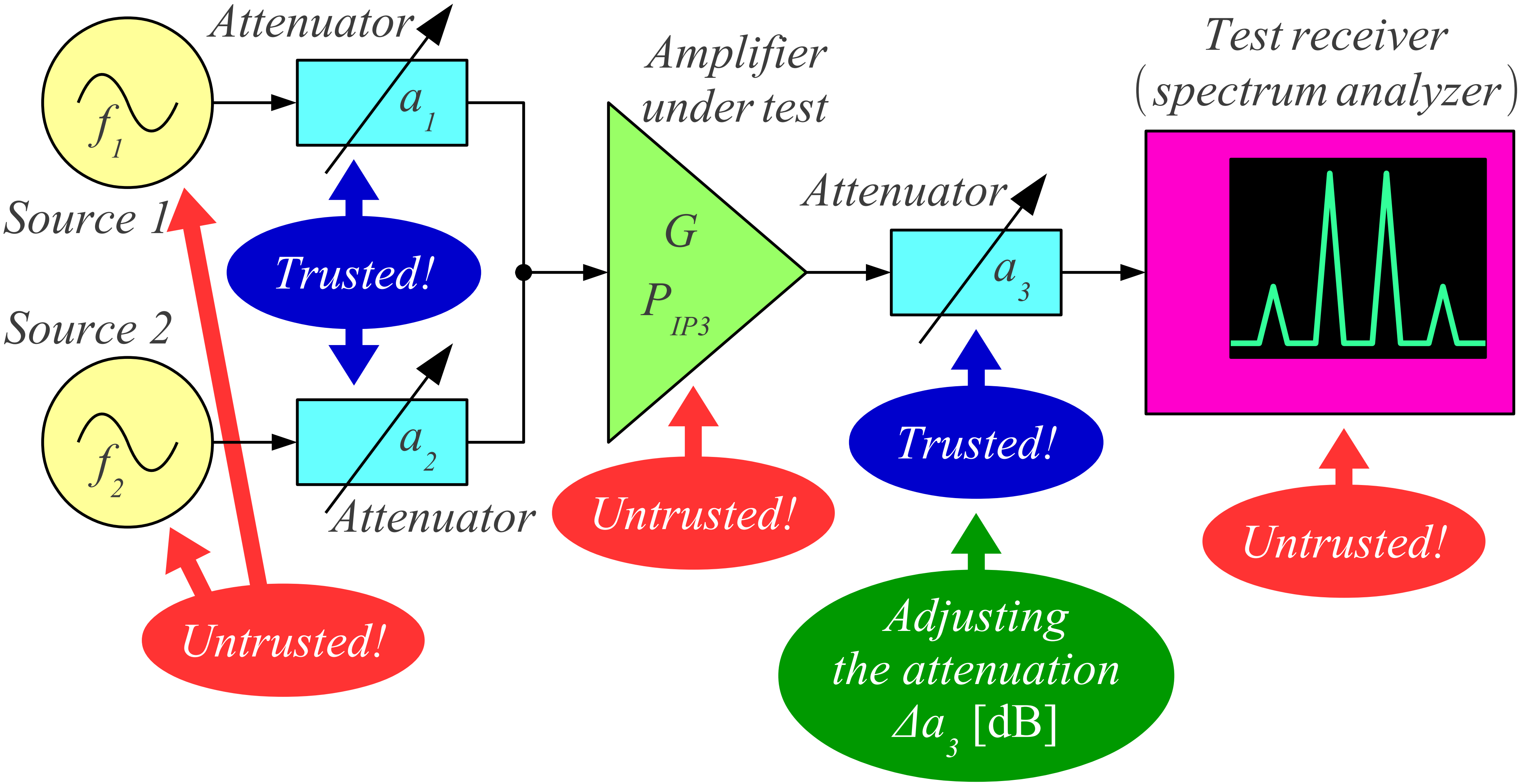
$$P_{IP3} = \sqrt{\frac{P^2_{video} \cdot P_{audio}}{P_{interference}}} = 3.16kW$$

Class A: $\eta_{1dB} \approx 30\%$

| Device | $P_{1dB}$ | $P_{supply}$ |
|---|---|---|
| BJT | 316W | 1.05kW |
| FET | 31.6W | 105W |

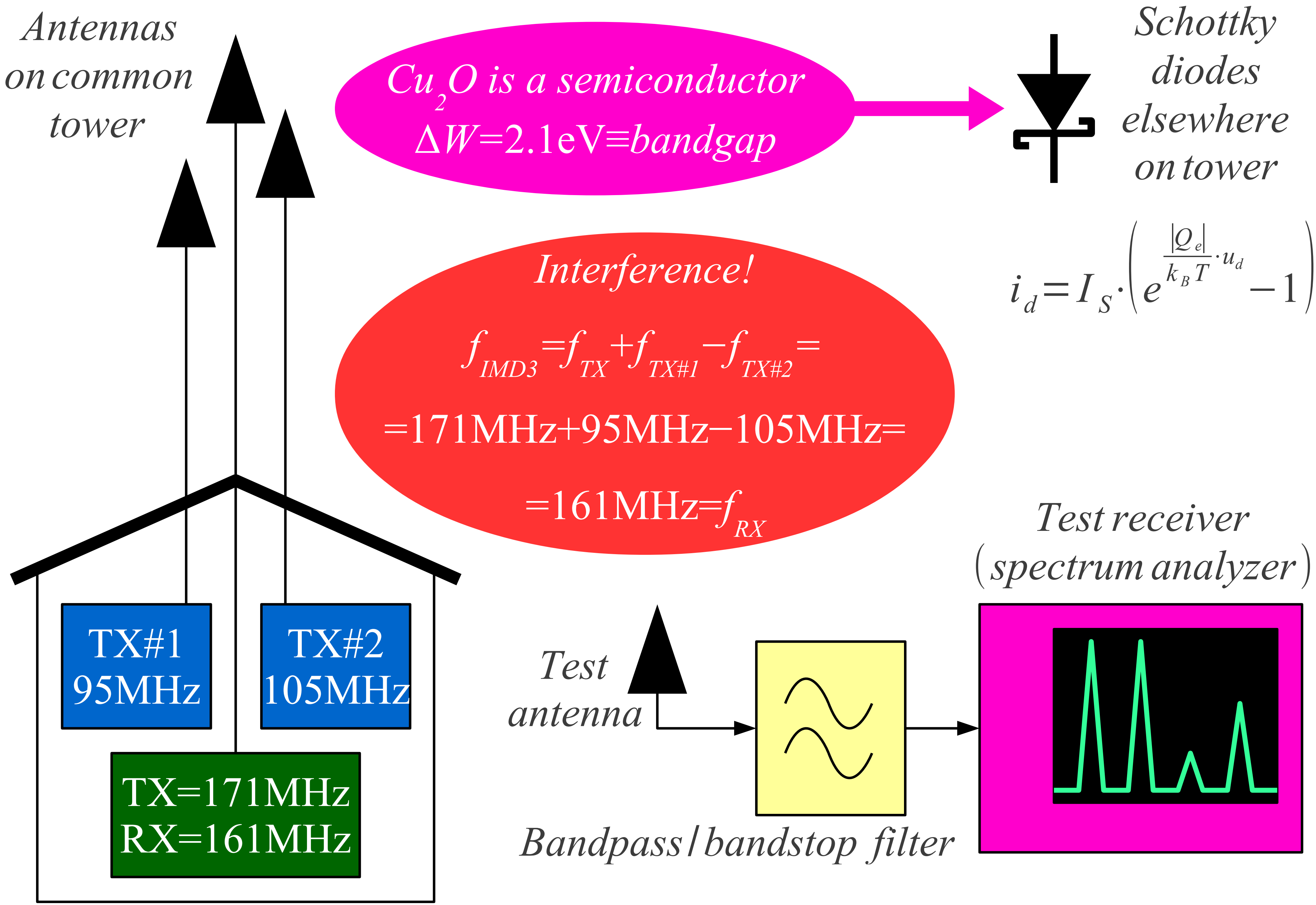
# *Analog TV transmitter example*

*Analog TV → UMTS*

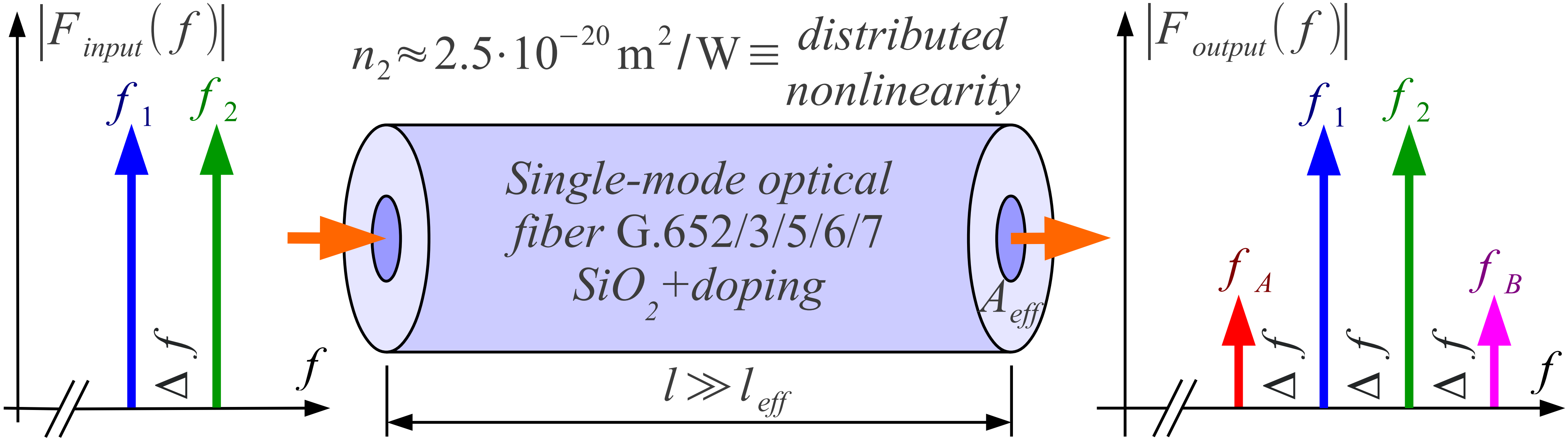



| | |
|---|---|
| $\Delta logP_{IMD3} \approx \Delta a_3$[dB] | *IMD3 source is BEFORE the attenuator (amplifier)* |
| $\Delta logP_{IMD3} \approx 3\Delta a_3$[dB] | *IMD3 source is AFTER the attenuator (SA)* |

# *IMD measurement*




*Passive intermodulation (PIM) example*




$$\Delta\beta\left[\frac{\mathrm{rd}}{\mathrm{m}}\right] = \beta_2 + \beta_A - 2\beta_1 \approx -\frac{2\pi\lambda_0^2 D}{c_0}\cdot(\Delta f)^2 \equiv \textit{phase mismatch}$$

$$P_A = \frac{P_1^2 P_2}{P_{IP3}^2}$$

$$P_B = \frac{P_1 P_2^2}{P_{IP3}^2}$$

*NZDSF* G.655
$A_{eff} \approx 80\,\mu m^2$
$\lambda_0 \approx 1.55\,\mu\mathrm{m}$
$a/l \approx 0.2\mathrm{dB/km}$
$D \approx +5\mathrm{ps/(nm.km)}$

$$\alpha\left[\frac{\mathrm{Np}}{\mathrm{m}}\right] = \frac{\ln 10}{20}\, a/l\left[\frac{\mathrm{dB}}{\mathrm{m}}\right] \equiv \textit{specific attenuation}$$

$$l_{eff}[\mathrm{m}] = \frac{1}{\sqrt{(2\alpha)^2 + (\Delta\beta)^2}} \equiv \textit{effective length}$$

$$l_{eff} \approx \frac{1}{|\Delta\beta|} \approx 0.4\mathrm{km}$$

$$P_{IP3}[\mathrm{W}] = \frac{\lambda_0 A_{eff}}{2\pi n_2 l_{eff}}$$

$\Delta f = 100\mathrm{GHz}$
$\Delta\beta \approx -2.52\mathrm{rd/km} \gg 2\alpha$

$$P_{IP3} \approx 2\mathrm{W} = +33\mathrm{dBm}$$

# *Four−wave mixing (FWM ≡ IMD in optical fibers)*