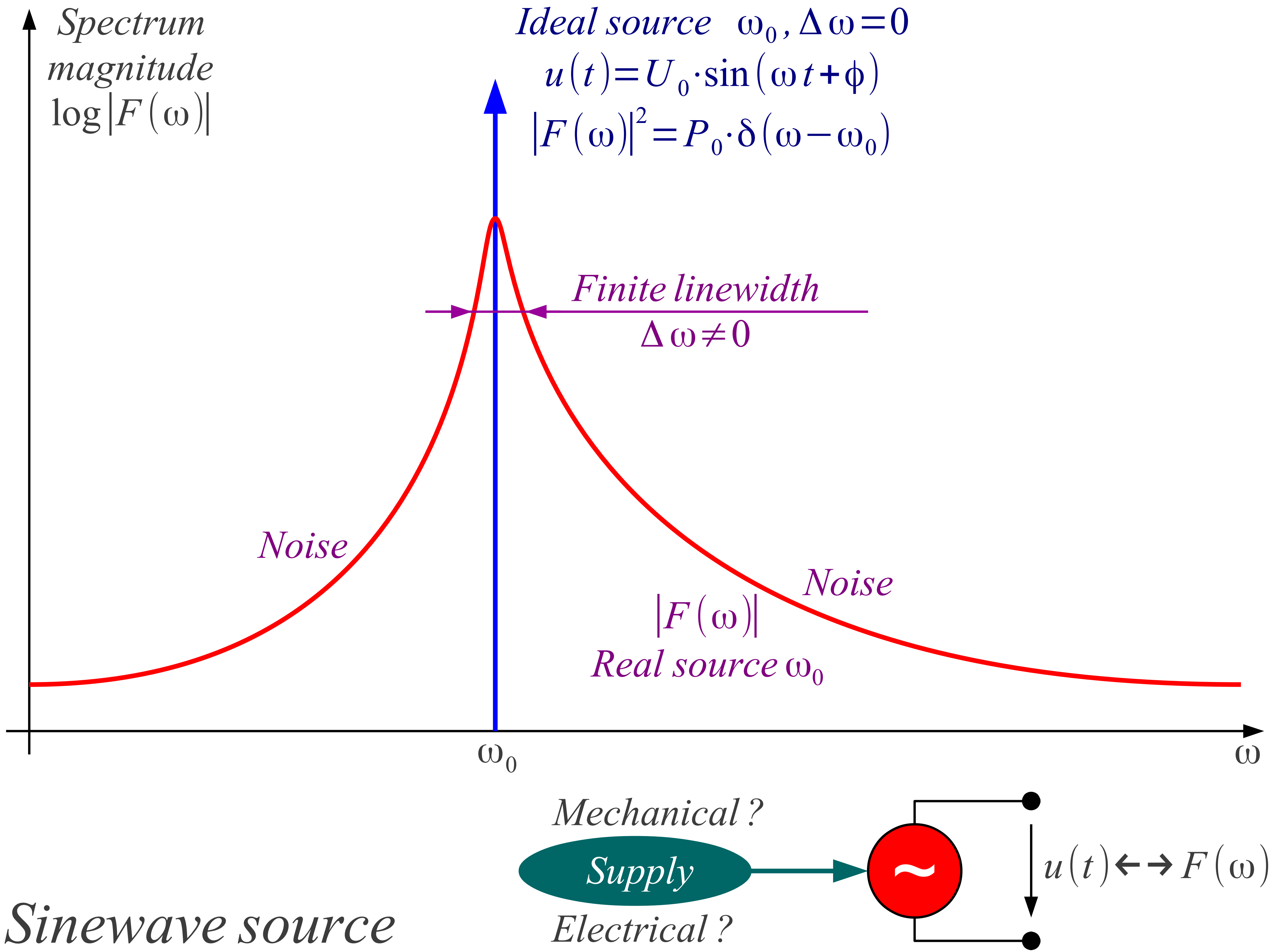
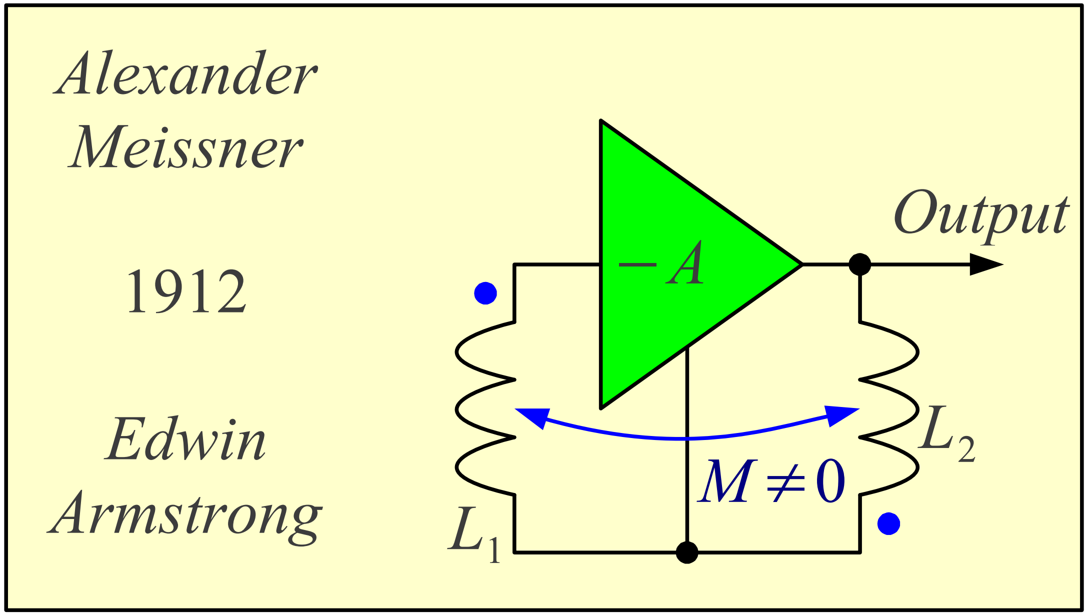
Communication Electronics

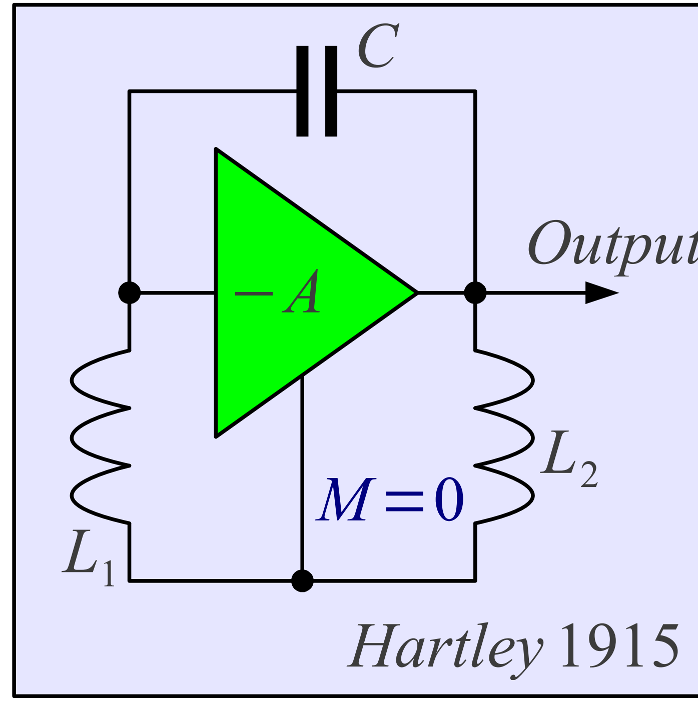
Lecture 14:

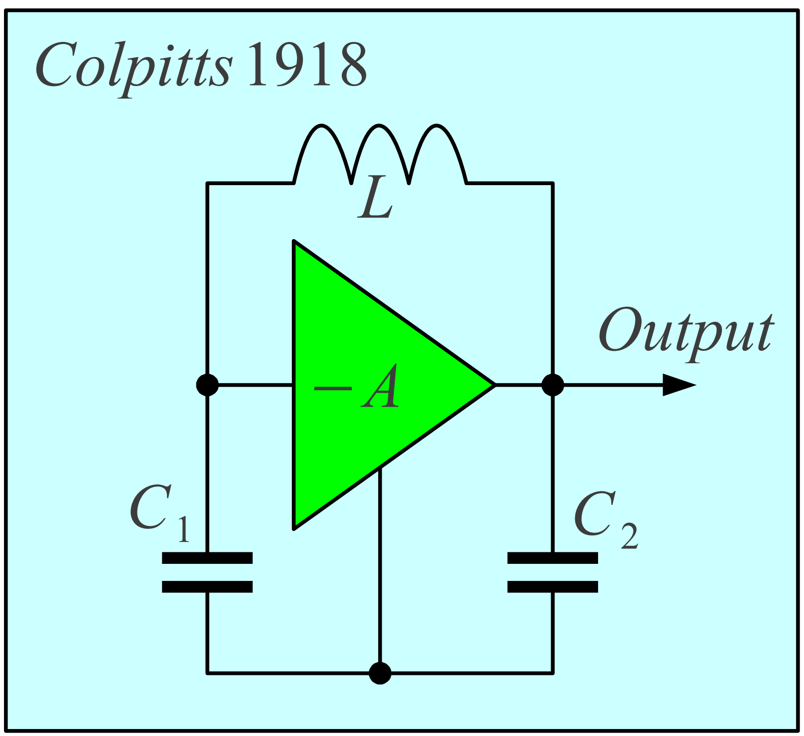
Electronic oscillator

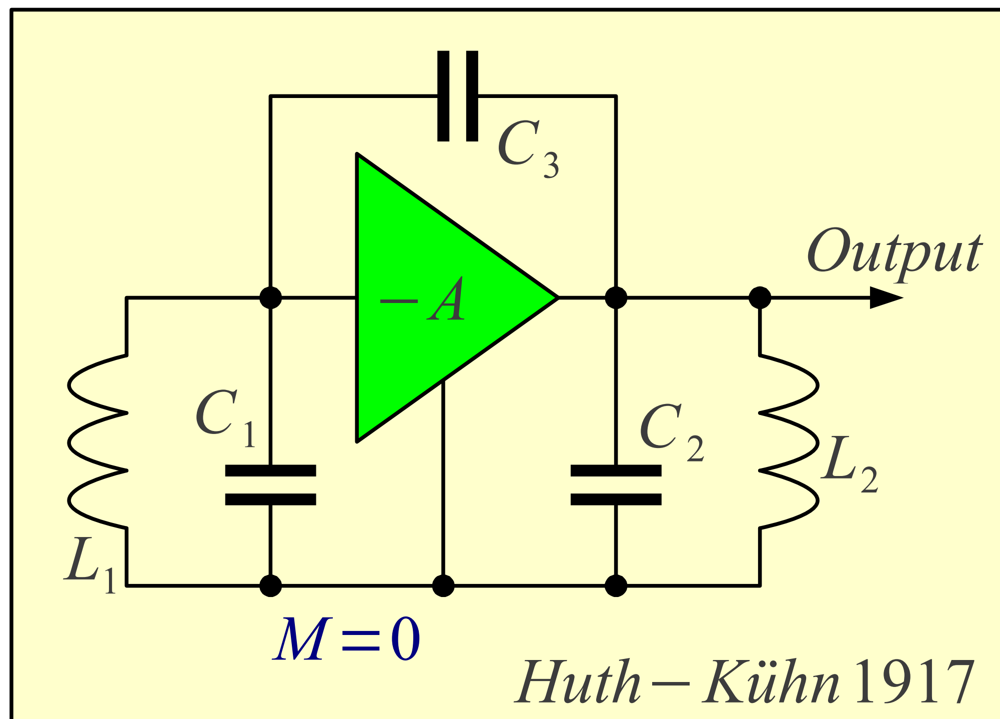

Spectrum magnitude
$\log|F(\omega)|$
Ideal source $\omega_0, \Delta\omega=0$
$u(t)=U_0\cdot\sin(\omega t+\phi)$
$|F(\omega)|^2=P_0\cdot\delta(\omega-\omega_0)$
Finite linewidth
$\Delta\omega\neq 0$
Noise
Noise
$|F(\omega)|$
Real source ω_0
ω_0
ω
Mechanical ?
Supply
Electrical ?
$u(t)\leftrightarrow F(\omega)$
Sinewave source



Triode 1907 *Lee DeForest*





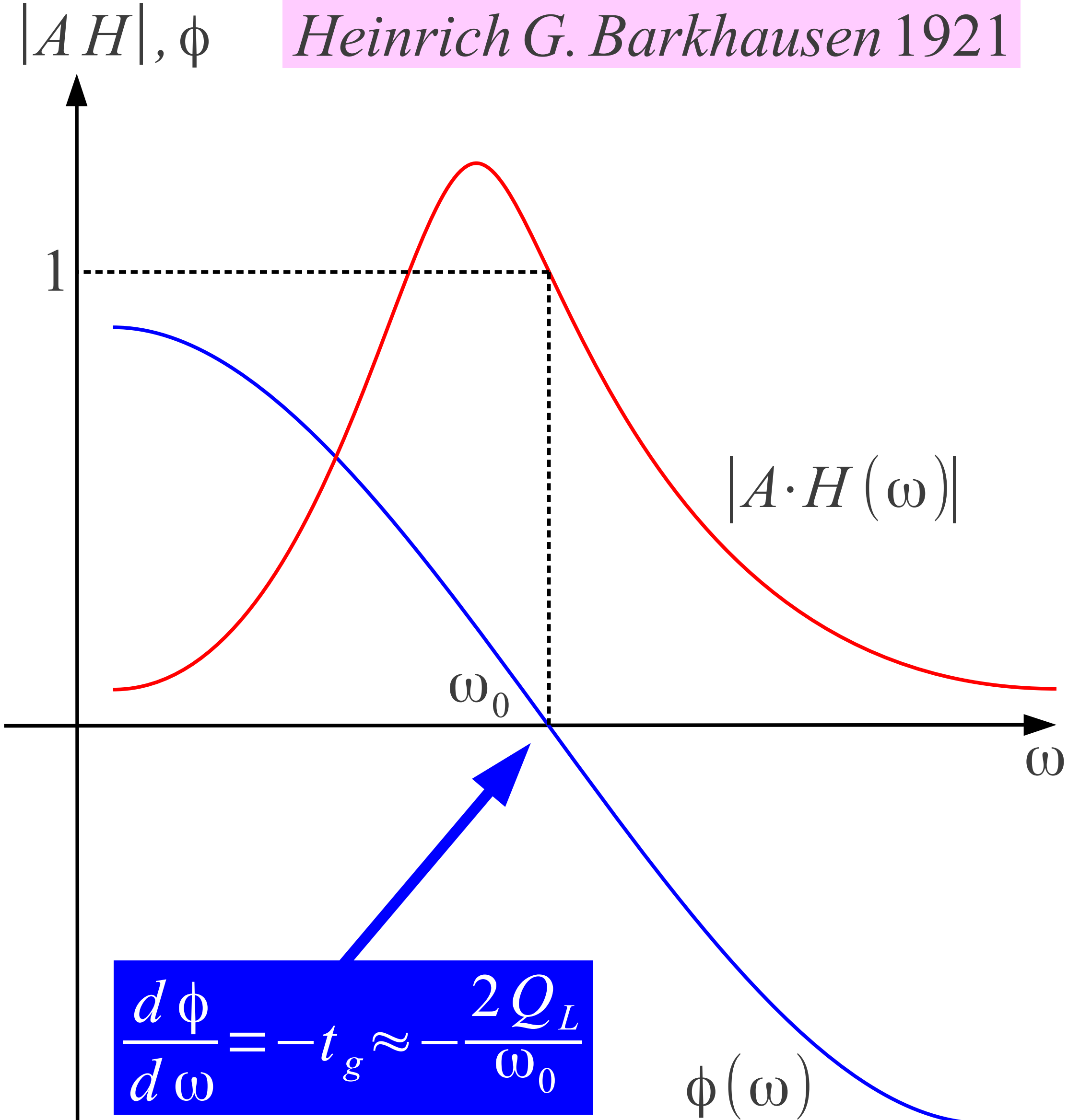


RF oscillators

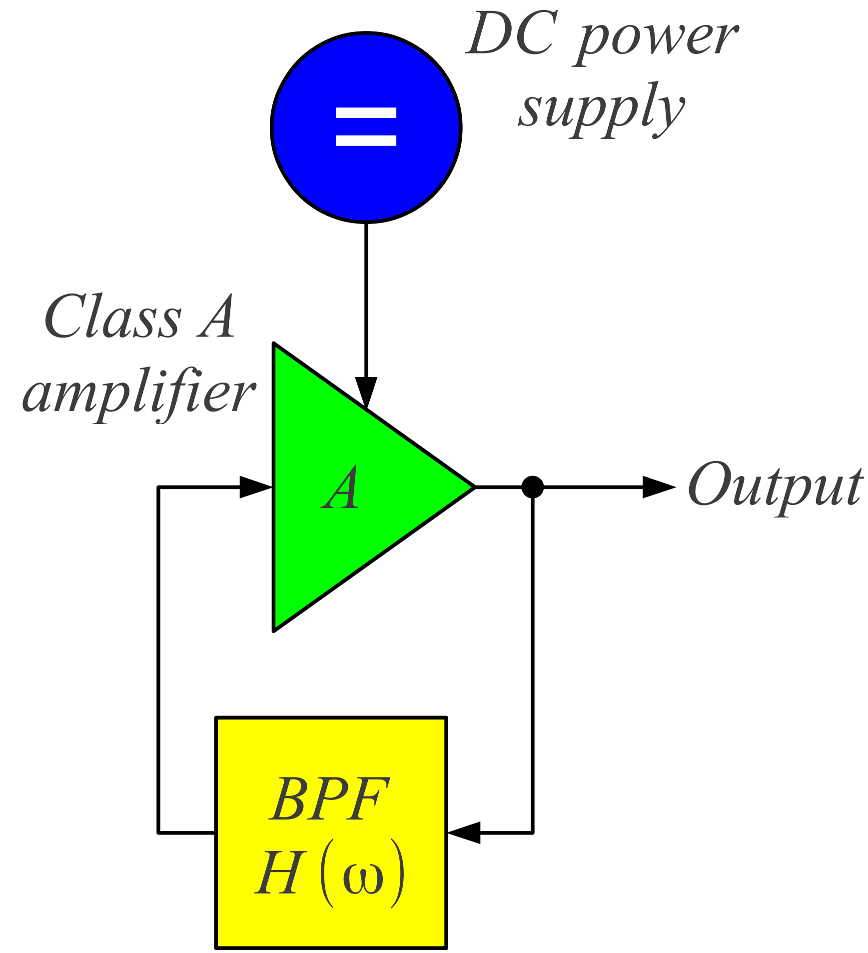
$$\left|A \cdot H(\omega_0)\right| = 1$$

$$\phi(\omega_0) = m \cdot 2\pi \qquad m = 0, 1, 2, 3 \ldots$$

Heinrich G. Barkhausen 1921

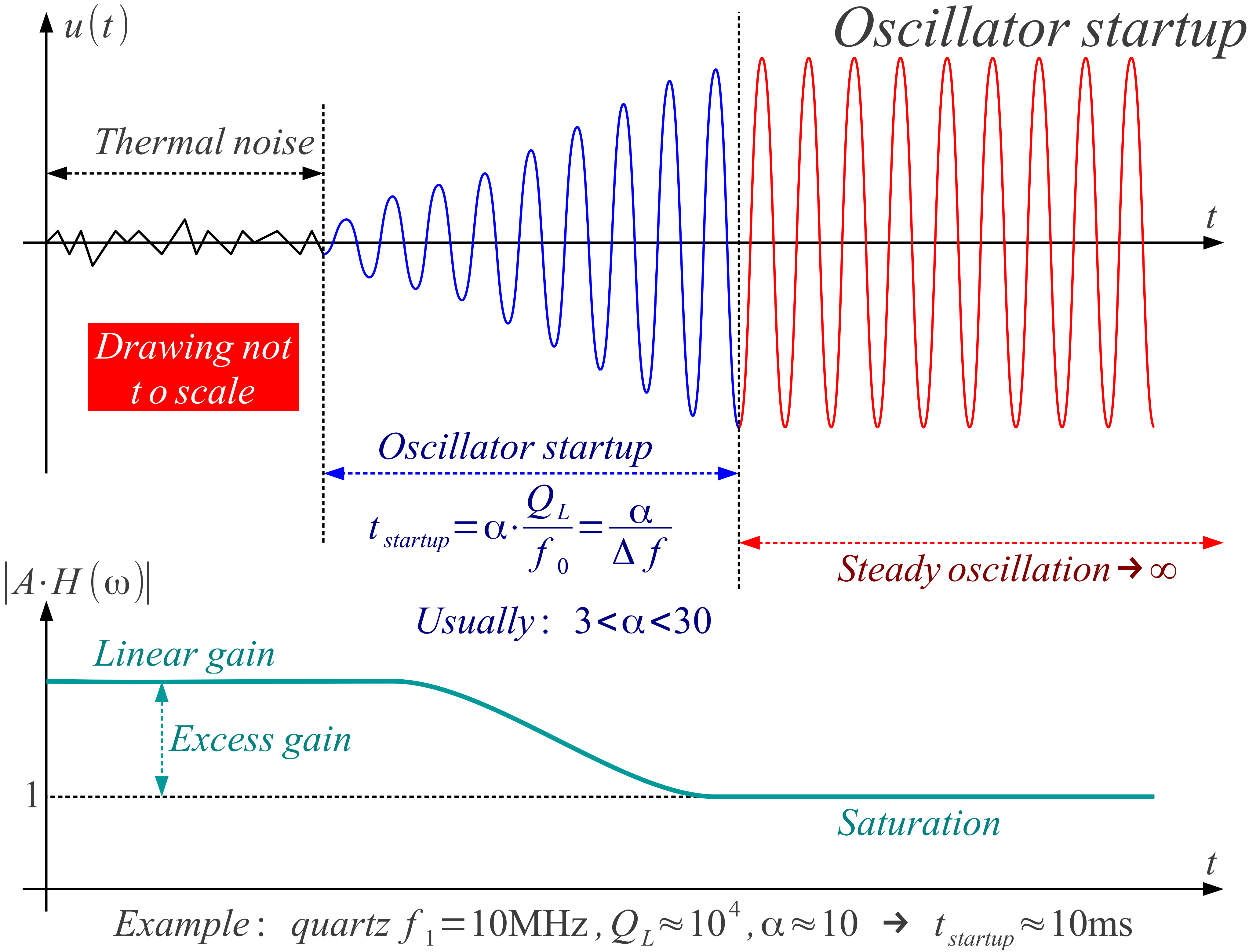


$$\frac{d\phi}{d\omega} = -t_g \approx -\frac{2 Q_L}{\omega_0}$$

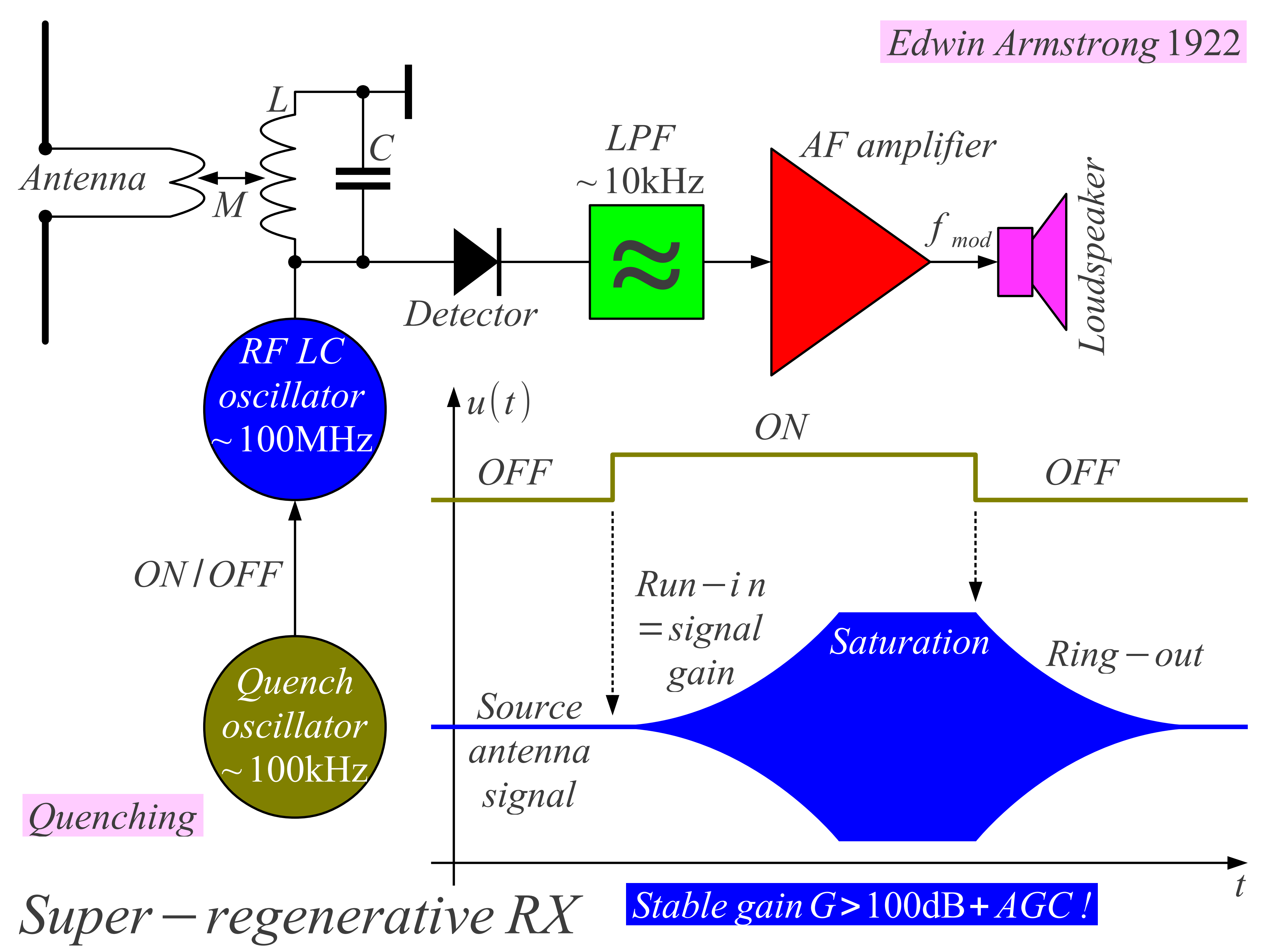


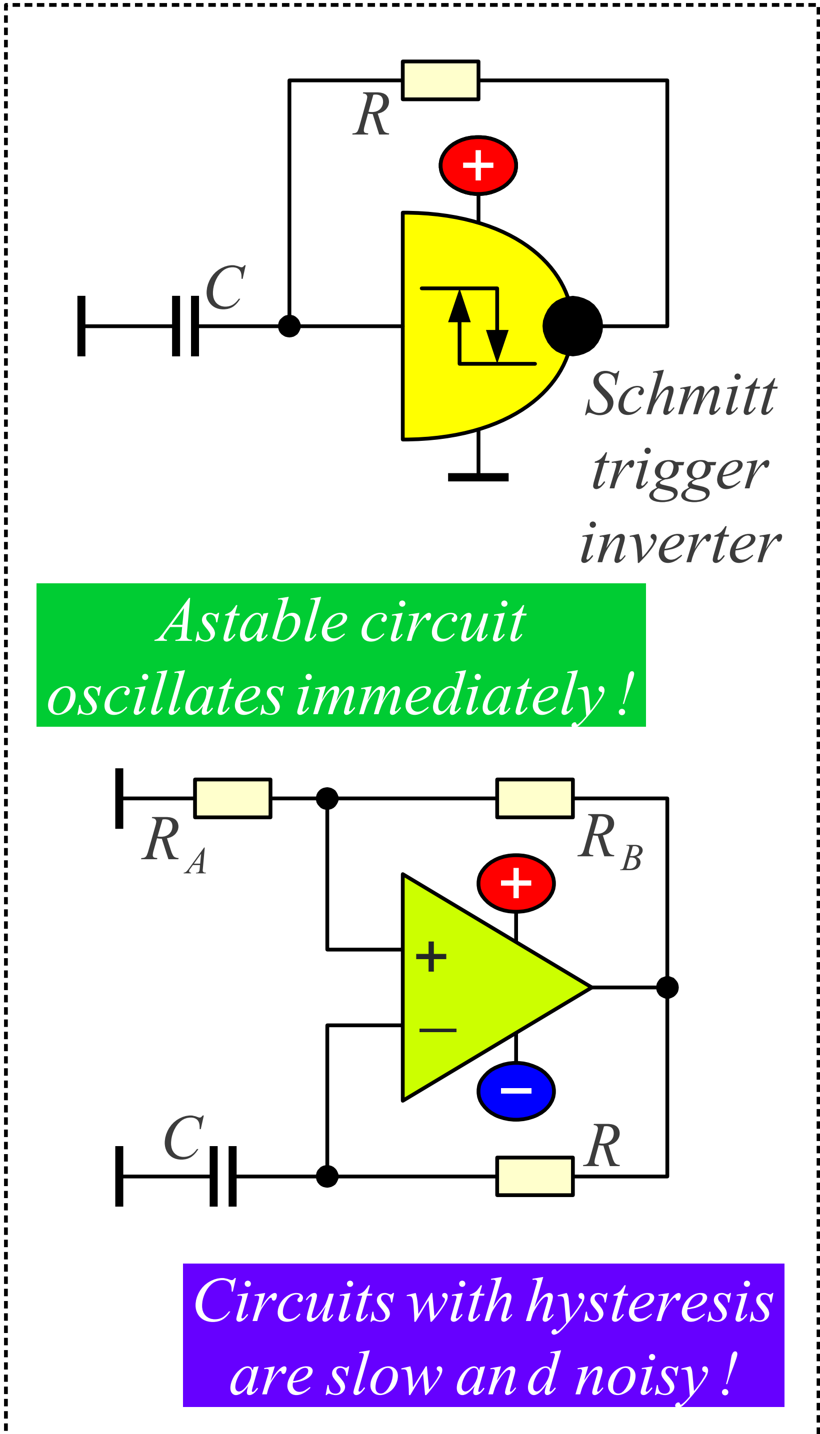
$$A \cdot H(\omega) = \left|A \cdot H(\omega)\right| \cdot e^{j\phi(\omega)}$$

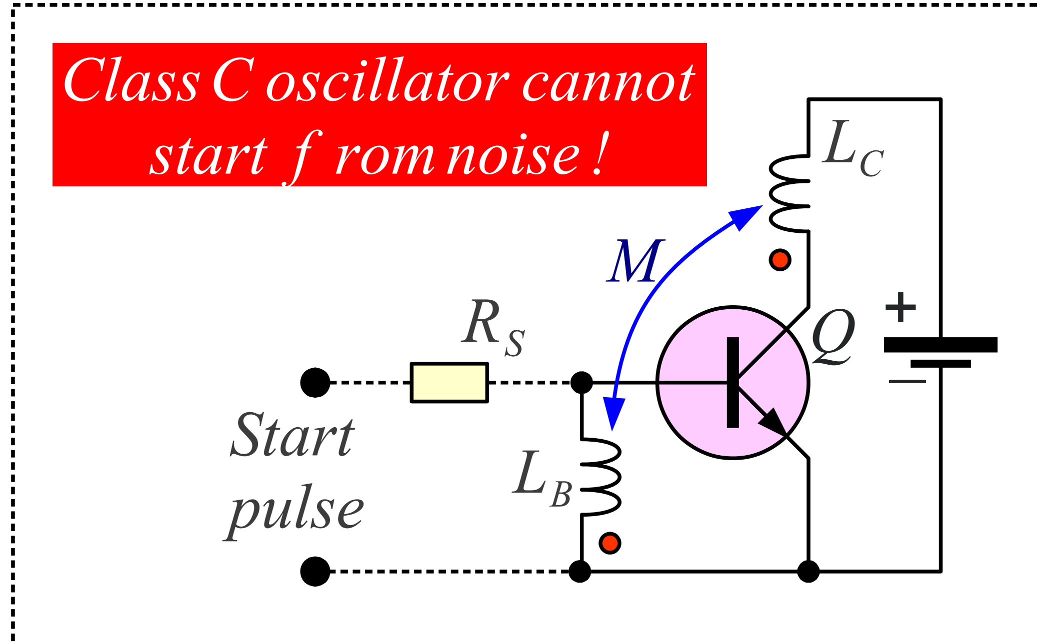
Barkhausen criterion

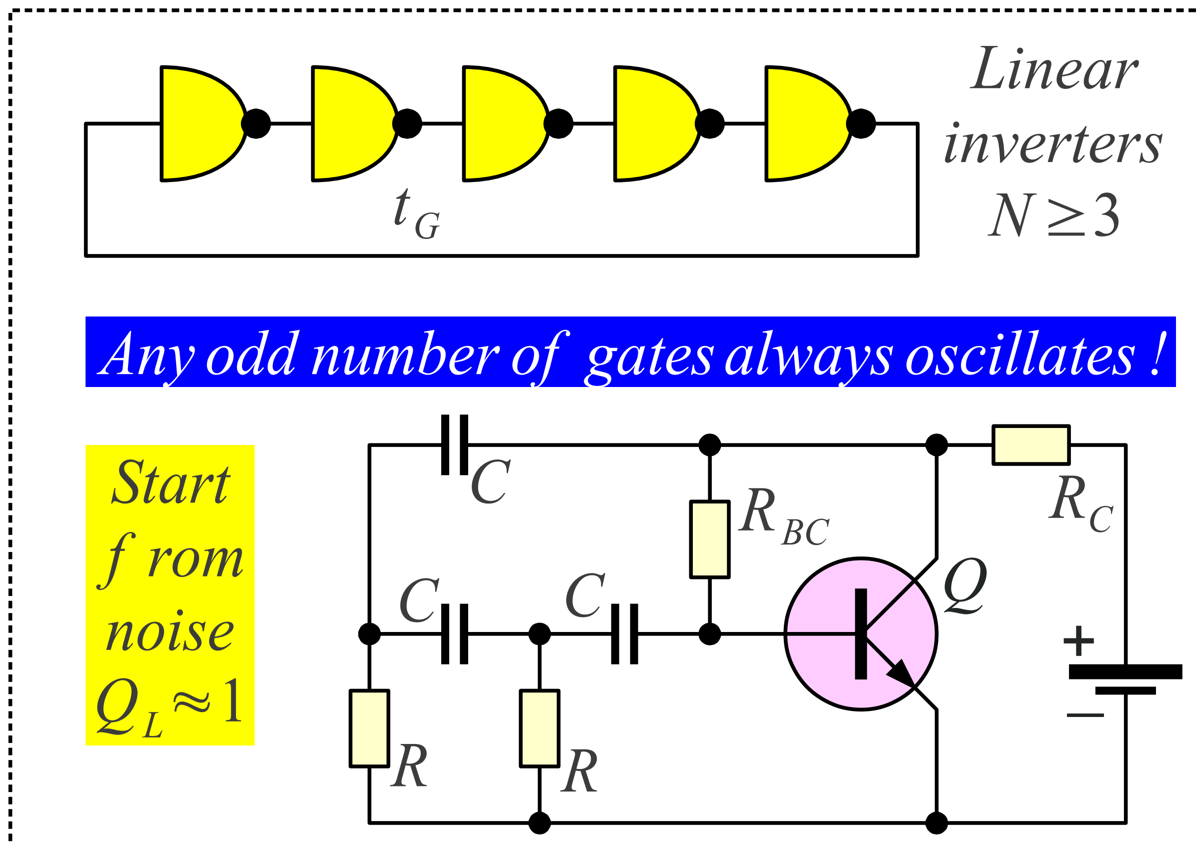


Oscillator startup
u(t)
t
Thermal noise
Drawing not to scale
Oscillator startup
$t_{startup} = \alpha \cdot \frac{Q_L}{f_0} = \frac{\alpha}{\Delta f}$
Steady oscillation → ∞
Usually: $3 < \alpha < 30$
$|A \cdot H(\omega)|$
Linear gain
Excess gain
1
Saturation
t
Example: quartz $f_1 = 10\text{MHz}, Q_L \approx 10^4, \alpha \approx 10$ → $t_{startup} \approx 10\text{ms}$

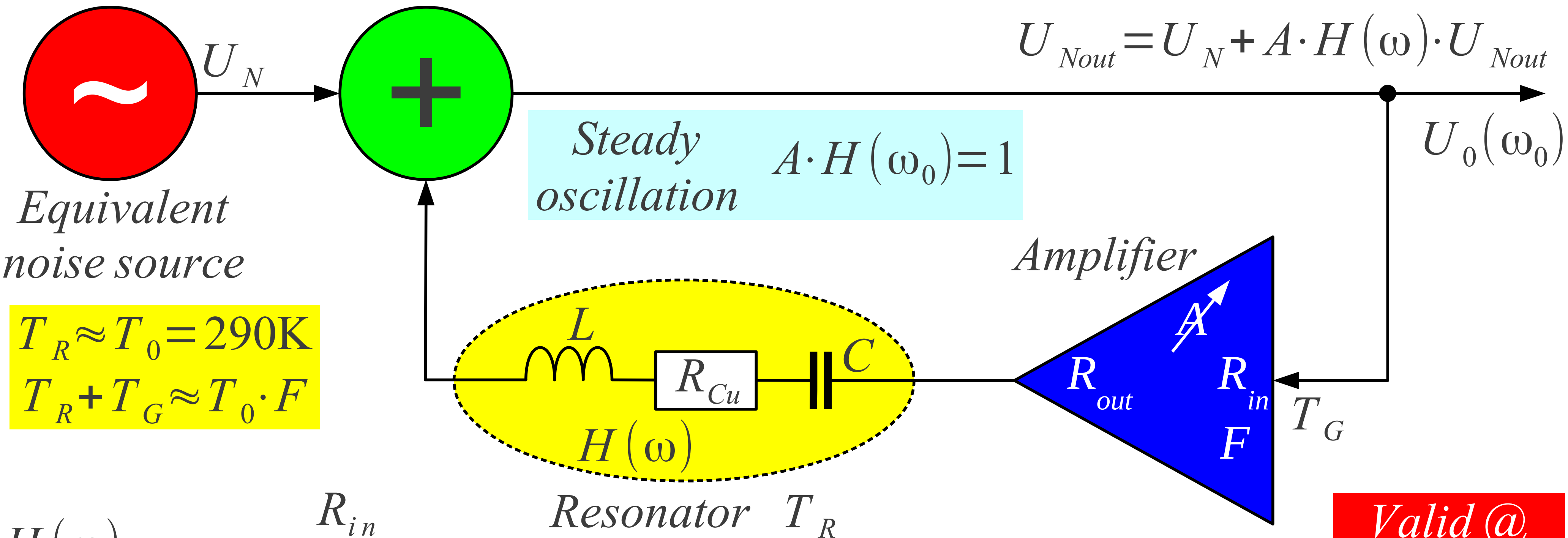

Edwin Armstrong 1922
L
C
Antenna
M
LPF
~10kHz
AF amplifier
f mod
Loudspeaker
Detector
RF LC oscillator ~100MHz
u(t)
ON
OFF
OFF
ON/OFF
Run−in =signal gain
Saturation
Ring−out
Source antenna signal
Quench oscillator ~100kHz
Quenching
t
Super−regenerative RX
Stable gain G>100dB+AGC !







Different oscillators



$$T_R \approx T_0 = 290\mathrm{K}$$
$$T_R + T_G \approx T_0 \cdot F$$

Valid @ $U_{Nout} \ll U_0$

$$H(\omega) = \frac{R_{in}}{\Sigma R + j\omega L + \dfrac{1}{j\omega C}}$$

$$Q_L = \frac{\omega_0 L}{\Sigma R}$$

$$\Sigma R = R_{out} + R_{Cu} + R_{in}$$

$$\Delta\omega = \omega - \omega_0$$

$$U_{Nout} = \frac{U_N}{1 - A \cdot H(\omega)}$$

$$A \cdot H(\omega) = \frac{\Sigma R}{\Sigma R + j\omega L + \dfrac{1}{j\omega C}} \approx \frac{1}{1 + j2Q_L \dfrac{\Delta\omega}{\omega_0}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$U_{Nout} \approx \frac{U_N}{1 - \dfrac{1}{1 + j2Q_L \dfrac{\Delta\omega}{\omega_0}}} = U_N \cdot \left(1 + \frac{\omega_0}{j2Q_L\Delta\omega}\right)$$

Oscillator noise

$$U_{Nout} \approx U_N \cdot \left(1 + \frac{\omega_0}{j2Q_L\Delta\omega}\right)$$

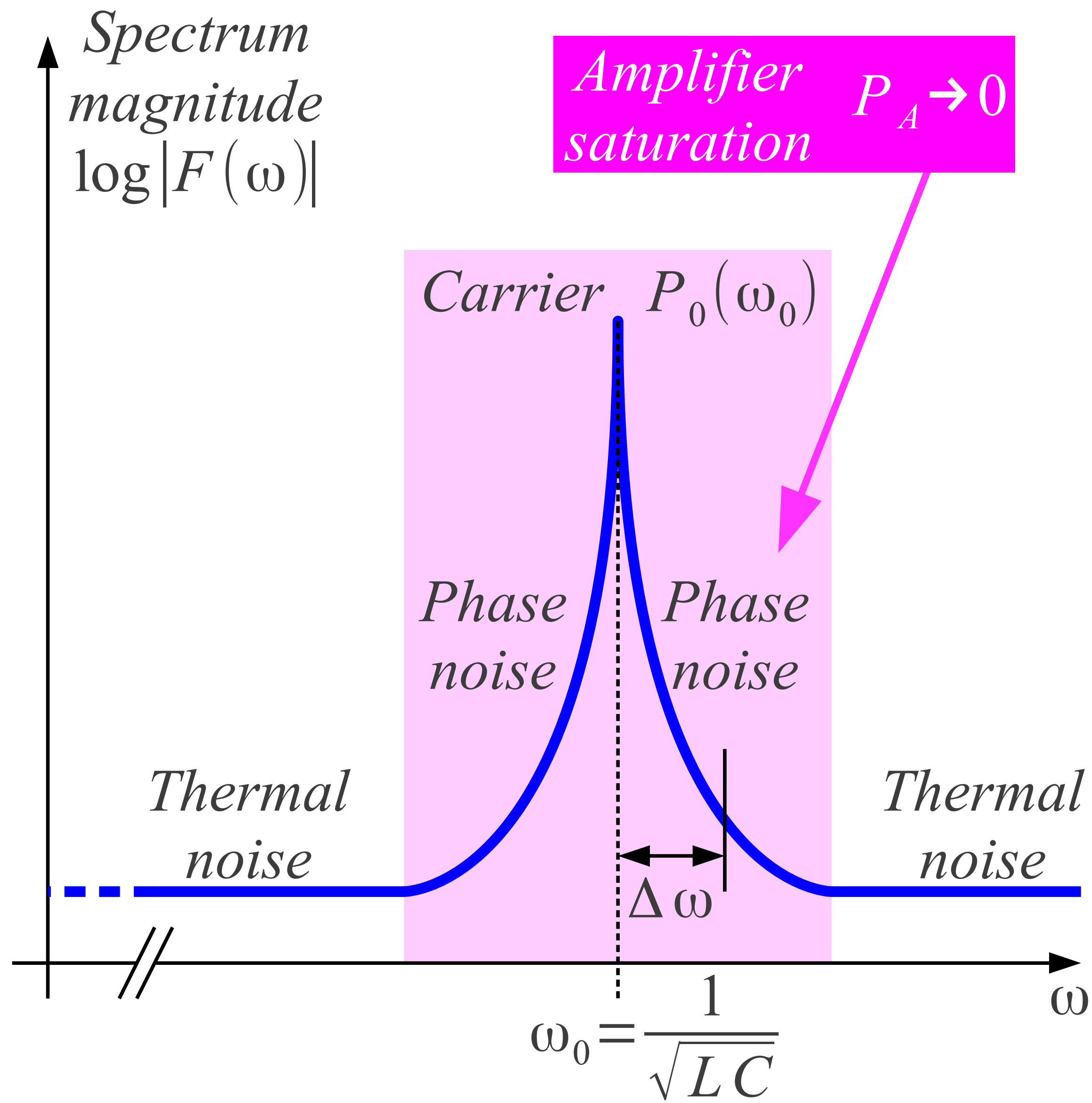
$P = \alpha |U|^2$ $|a \pm jb|^2 = a^2 + b^2$

$$P_{Nout} \approx P_N \cdot \left[1 + \left(\frac{\omega_0}{2Q_L\Delta\omega}\right)^2\right]$$

$\omega = 2\pi f \rightarrow \Delta f = f - f_0$

$$P_{Nout} \approx P_N \cdot \left[1 + \left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]$$

$P_{Nout} \equiv$ *total noise power*
$P_A \equiv$ *amplitude* − *noise power*
$P_\phi \equiv$ *phase* − *noise power*



$$P_\phi = P_A = \frac{P_{Nout}}{2} \approx \frac{1}{2}\left[1 + \left(\frac{f_0}{2Q_L\Delta f}\right)^2\right] \cdot P_N$$

Amplitude and phase noise

Relative phase−noise density

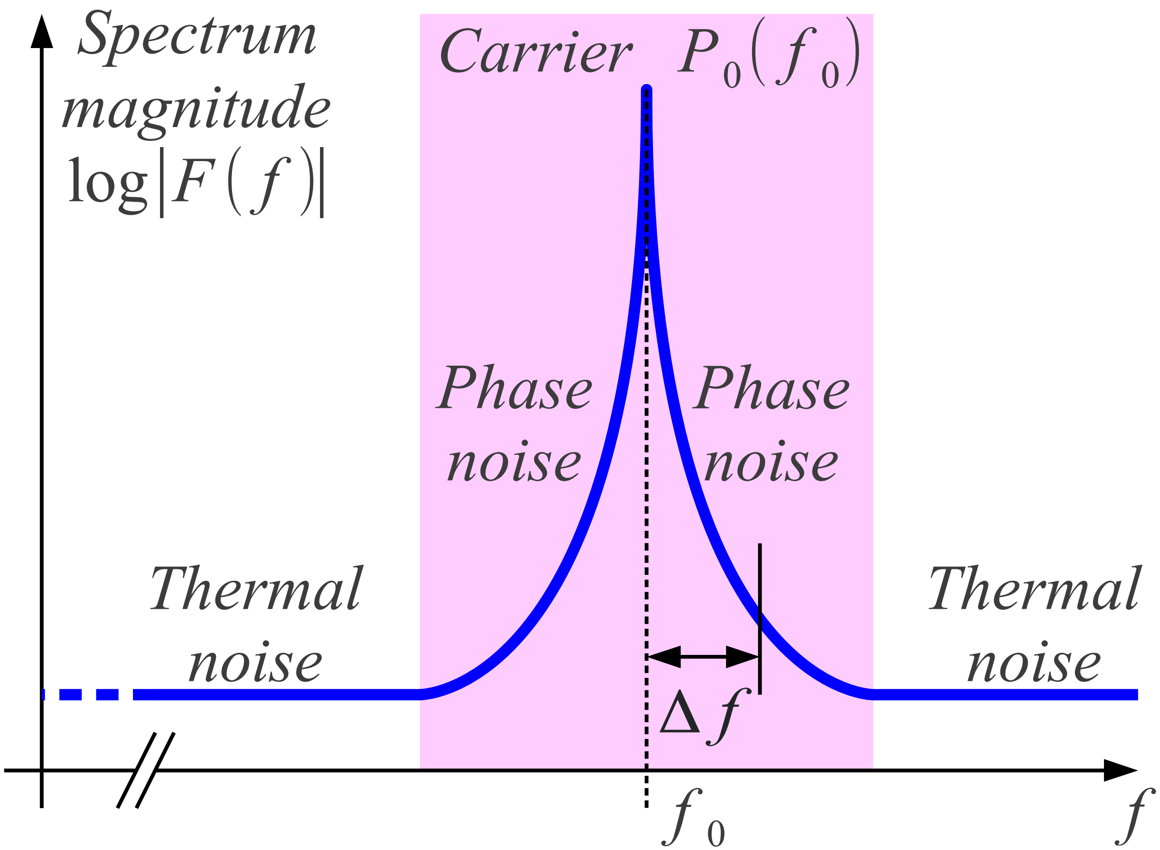
$$L(\Delta f)=\frac{1}{P_0}\cdot\frac{dP_\phi}{df}\quad [\mathrm{Hz}^{-1}]$$

$$\frac{dP_N}{df}=N_0=k_B\cdot(T_R+T_G)\approx k_B T_0 F$$

David B. Leeson 1966

Valid @ $L(\Delta f)\cdot\Delta f\ll 1$

$$L(\Delta f)=\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}$$

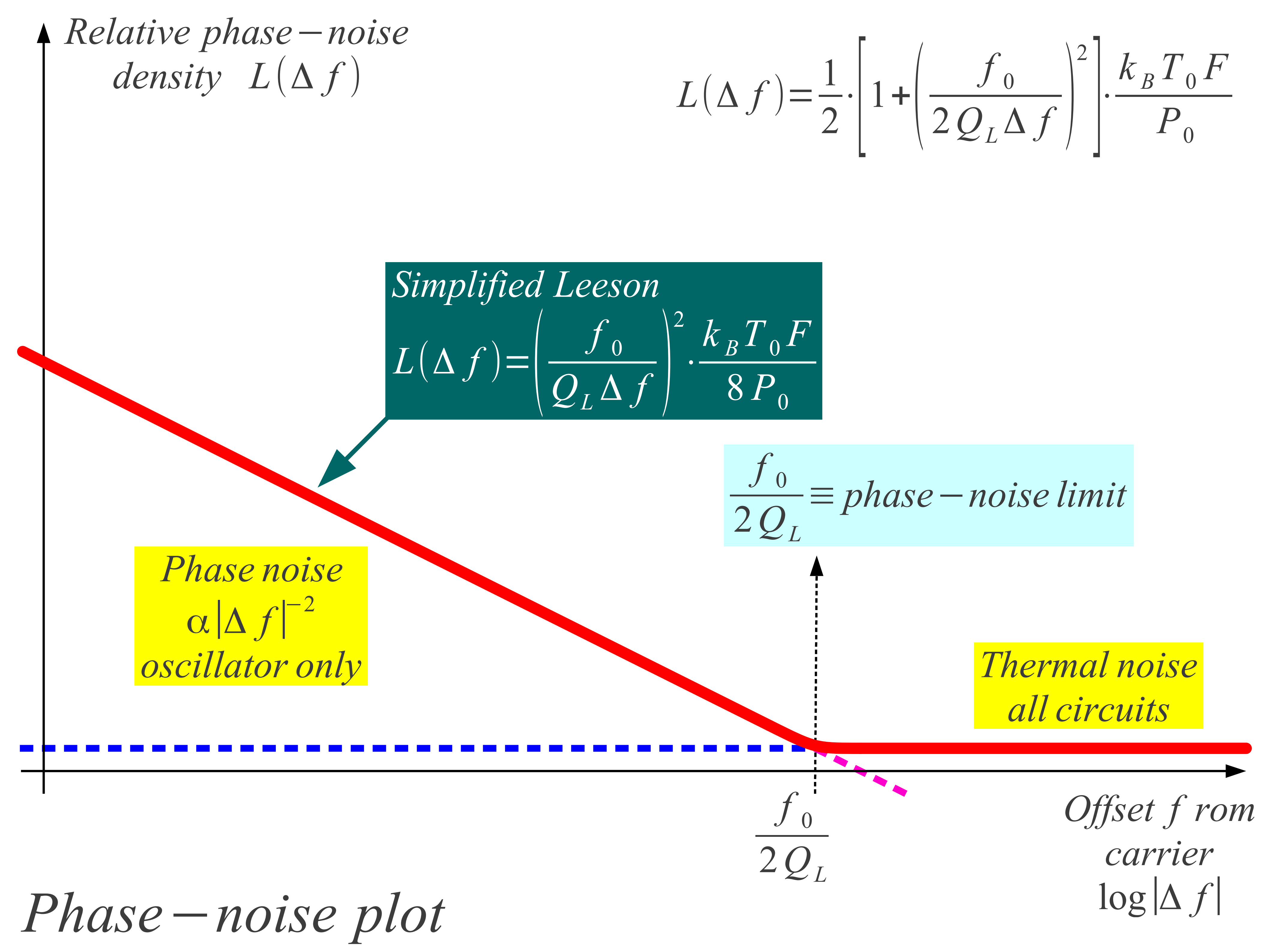


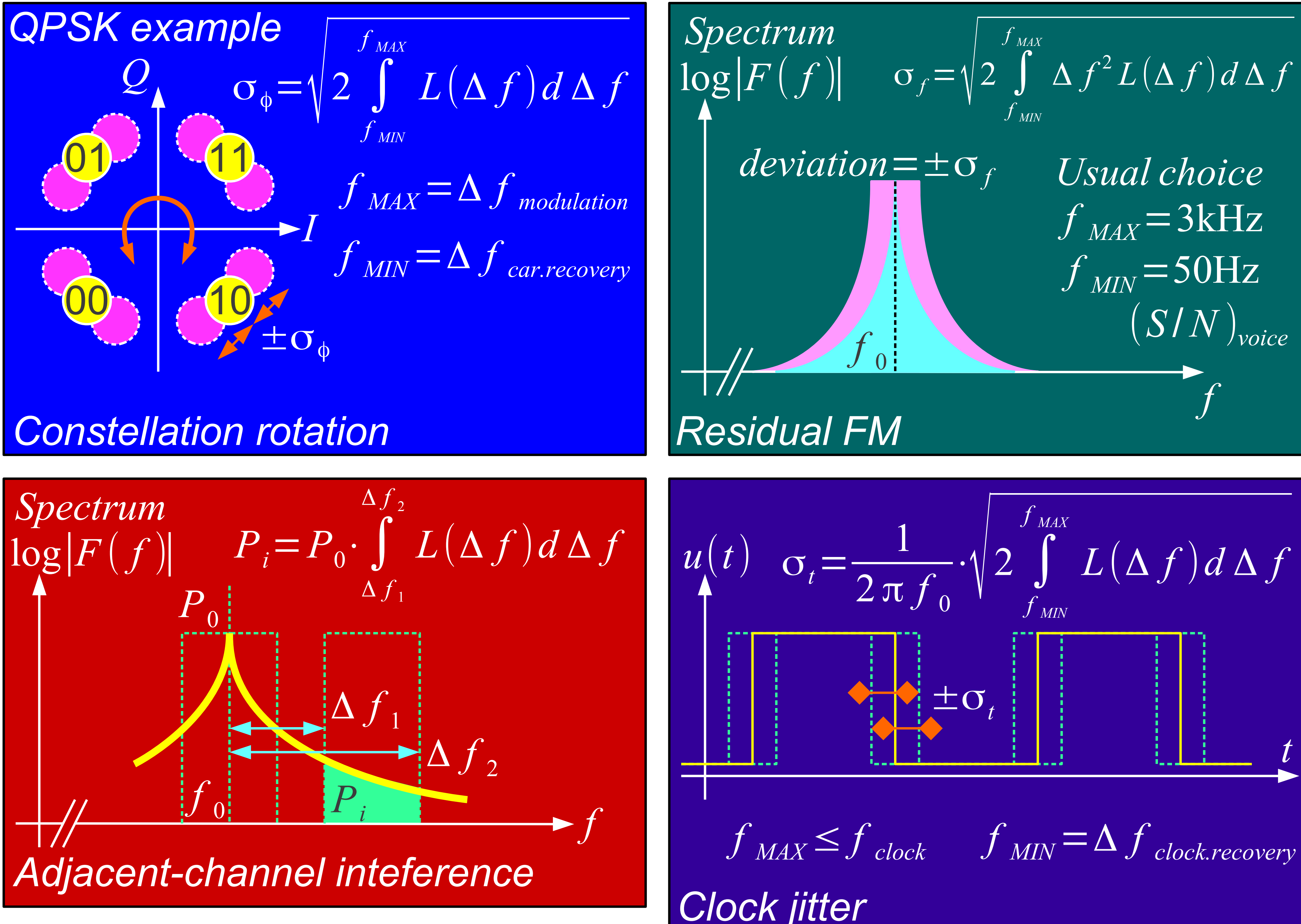
- $Q_L\equiv$ resonator loaded Q
- $k_B\approx 1.38\cdot 10^{-23}\ \mathrm{J/K}\equiv$ Boltzmann constant
- $T_0\approx 290\mathrm{K}\equiv$ circuit temperature
- $F\equiv$ amplifier noise figure @ P_0
- $P_0\equiv$ carrier power @ f_0

$$\log L(\Delta f)_{\mathrm{dBc/Hz}}=10\log_{10}[L(\Delta f)\cdot 1\mathrm{Hz}]$$

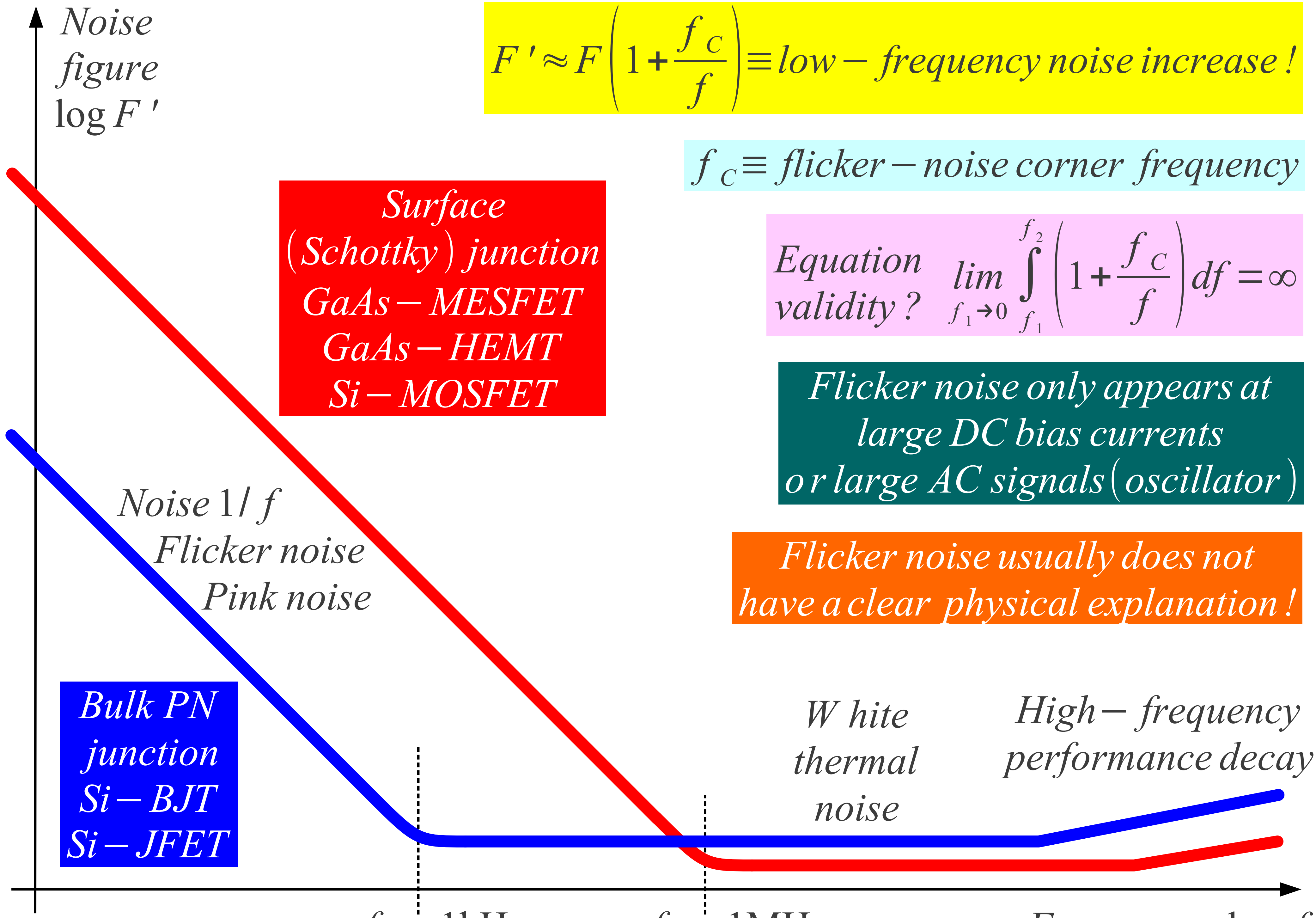
$$\log L(\Delta f)_{\mathrm{dBc/Hz}}=10\log_{10}\left\{\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot 1\mathrm{Hz}\right\}$$

Leeson's equation


Relative phase−noise density $L(\Delta f)$
$L(\Delta f)=\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}$
Simplified Leeson
$L(\Delta f)=\left(\frac{f_0}{Q_L\Delta f}\right)^2\cdot\frac{k_B T_0 F}{8P_0}$
$\frac{f_0}{2Q_L}\equiv$ phase−noise limit
Phase noise
$\alpha|\Delta f|^{-2}$
oscillator only
Thermal noise
all circuits
$\frac{f_0}{2Q_L}$
Offset f rom
carrier
$\log|\Delta f|$
Phase−noise plot



Phase – noise consequences



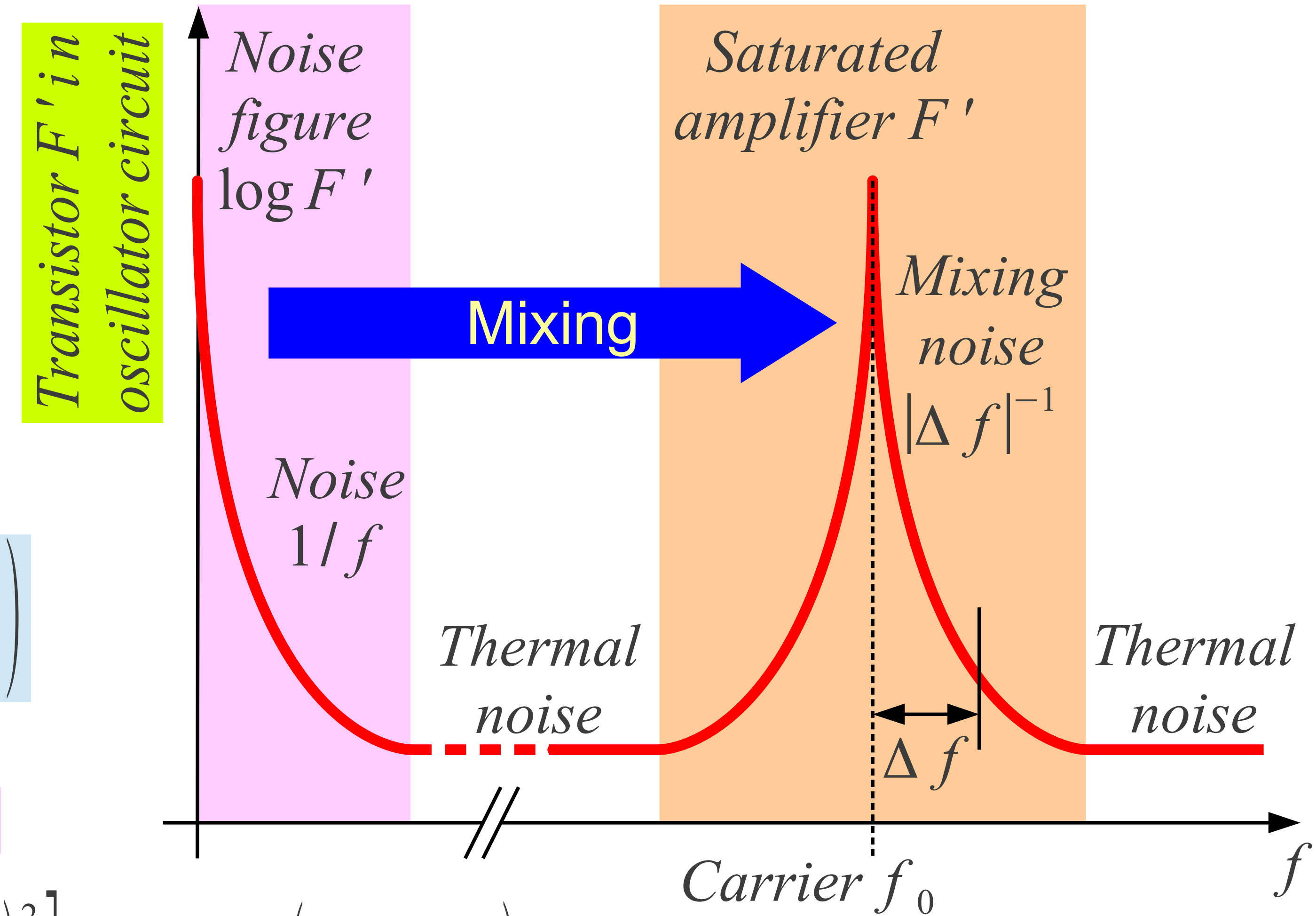
Flicker noise

Including noise $1/f$

$$F' = F \cdot \left(1 + \frac{f_C}{|\Delta f|}\right)$$

$$\frac{dP_N}{df} \approx k_B \cdot T_0 \cdot F'$$

$$\frac{dP_N}{df} \approx k_B \cdot T_0 \cdot F \cdot \left(1 + \frac{f_C}{|\Delta f|}\right)$$



Leeson with flicker noise

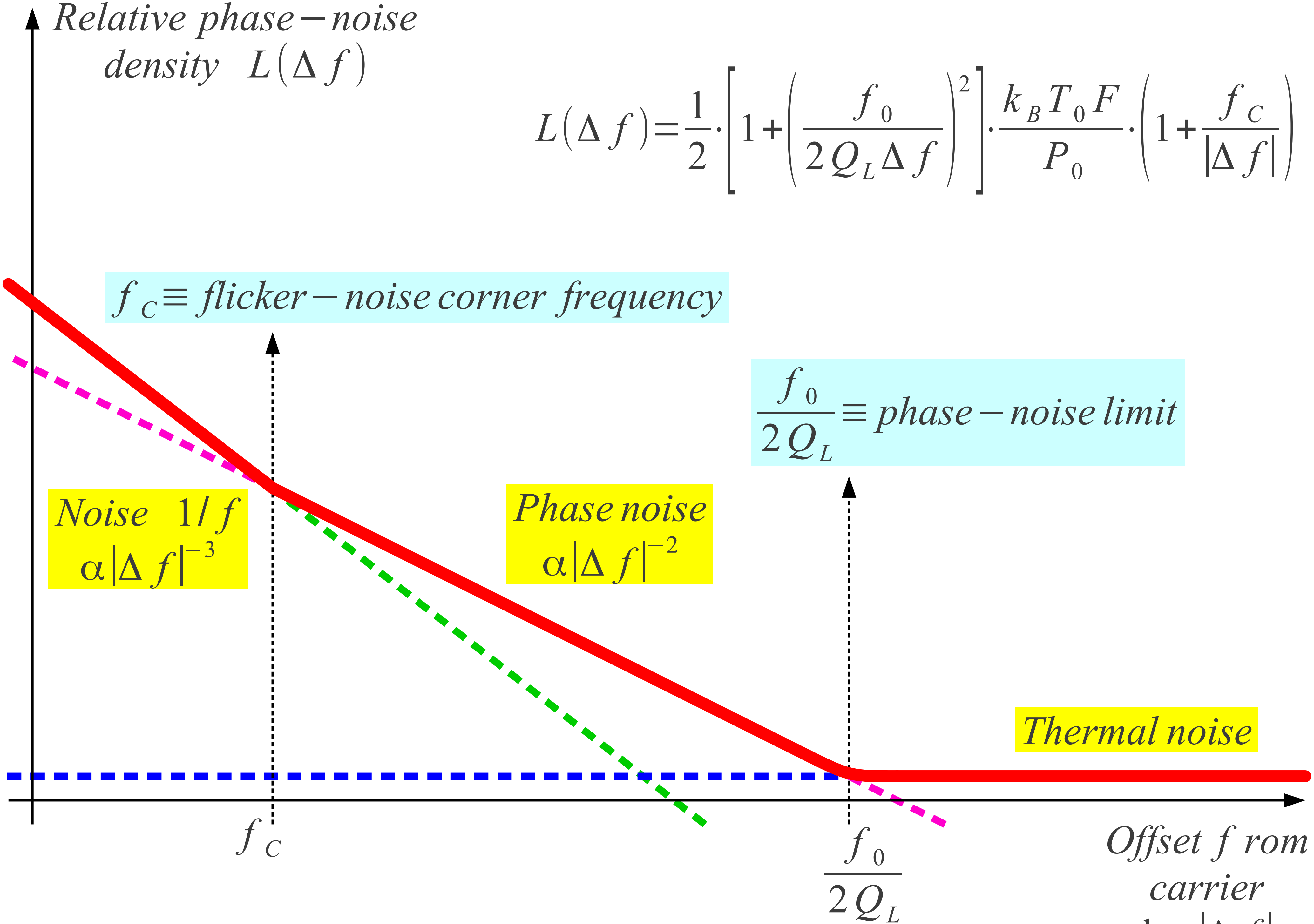
$$L(\Delta f) = \frac{1}{2} \cdot \left[1 + \left(\frac{f_0}{2 Q_L \Delta f}\right)^2\right] \cdot \frac{k_B T_0 F}{P_0} \cdot \left(1 + \frac{f_C}{|\Delta f|}\right)$$

Valid @ $L(\Delta f) \cdot \Delta f \ll 1$

BulkPN junction
$Si-BJT$
$Si-JFET$

$$\log L(\Delta f)_{\mathrm{dBc/Hz}} = 10 \log_{10} \left\{ \frac{1}{2} \cdot \left[1 + \left(\frac{f_0}{2 Q_L \Delta f}\right)^2\right] \cdot \frac{k_B T_0 F}{P_0} \cdot \left(1 + \frac{f_C}{|\Delta f|}\right) \cdot 1\mathrm{Hz} \right\}$$

Extended oscillator noise

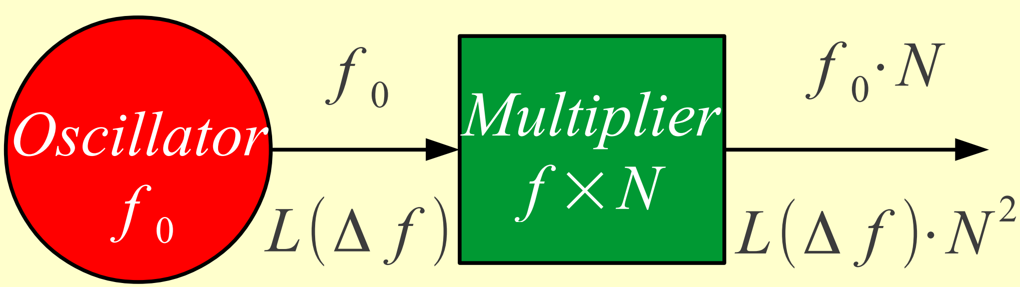


Phase−noise plot including flicker

The loaded-resonator quality Q_L is the most important quantity for phase noise!

$$L(\Delta f)=\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)$$

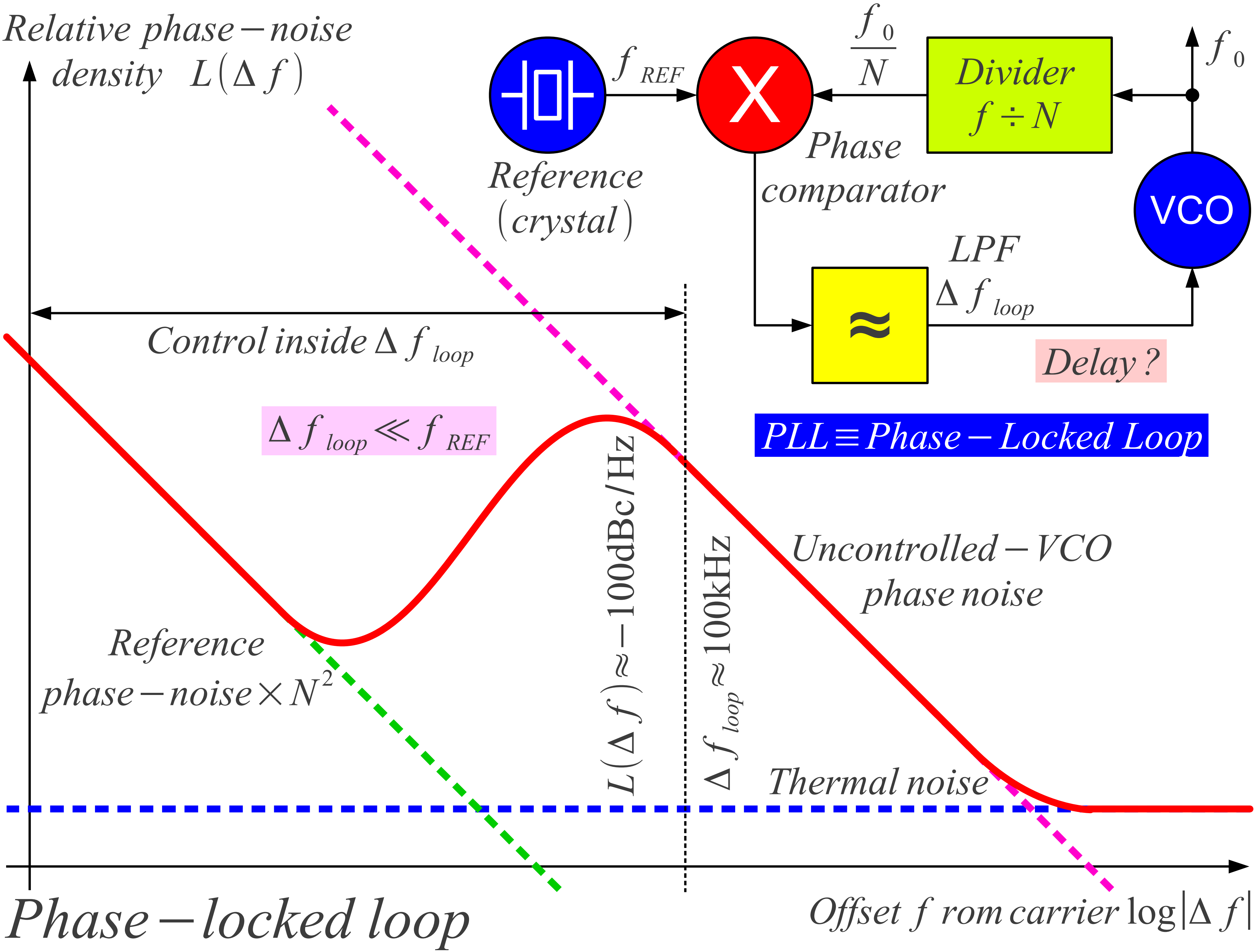
Variable-frequency oscillators	Q_L
RC VCO	~1
BWO tube	~1
LC varactor VCO	10↔30
YIG ($Y_3Fe_5O_{12}$) oscillator	300↔1000



Phase-noise power multiplies with the square of the frequency! The role of Q_L stays unchanged!

Fixed-frequency oscillators	Q_L
RC oscillator	~1
LC tuned circuit	30↔100
Cavity resonator	1000↔3000
Ceramic dielectric resonator	1000↔3000
AT-cut quartz crystal (fundamental)	3000↔10000
AT-cut quartz crystal (3rd/5th overtone)	10000↔30000
Electro-optical delay line ($)	$\sim 10^5$
Sapphire dielectric resonator ($$$)	$\sim 3\cdot 10^5$
Red HeNe LASER	$\sim 10^8$

Loaded – resonator quality


Relative phase−noise density L(Δf)
f_REF
f_0 / N
Divider f ÷ N
f_0
Reference (crystal)
Phase comparator
VCO
LPF Δf_loop
≈
Delay ?
PLL ≡ Phase − Locked Loop
Control inside Δf_loop
Δf_loop ≪ f_REF
L(Δf) ≈ −100dBc/Hz
Δf_loop ≈ 100kHz
Uncontrolled − VCO phase noise
Reference phase − noise × N²
Thermal noise
Offset from carrier log|Δf|
Phase − locked loop

Oscilator with noise $A\cdot H(\omega_0)=1-\epsilon \qquad 0<\epsilon\ll 1$

$$A\cdot H(\omega)\approx\frac{1-\epsilon}{1+j2Q_L\frac{\Delta\omega}{\omega_0}}$$

$$U_{Nout}=\frac{U_N}{1-A\cdot H(\omega)}\approx\frac{U_N}{1-\frac{1-\epsilon}{1+j2Q_L\frac{\Delta\omega}{\omega_0}}}=U_N\frac{1+j2Q_L\frac{\Delta\omega}{\omega_0}}{j2Q_L\frac{\Delta\omega}{\omega_0}-\epsilon}$$

$$\frac{dP_N}{df}\approx k_BT_0F$$

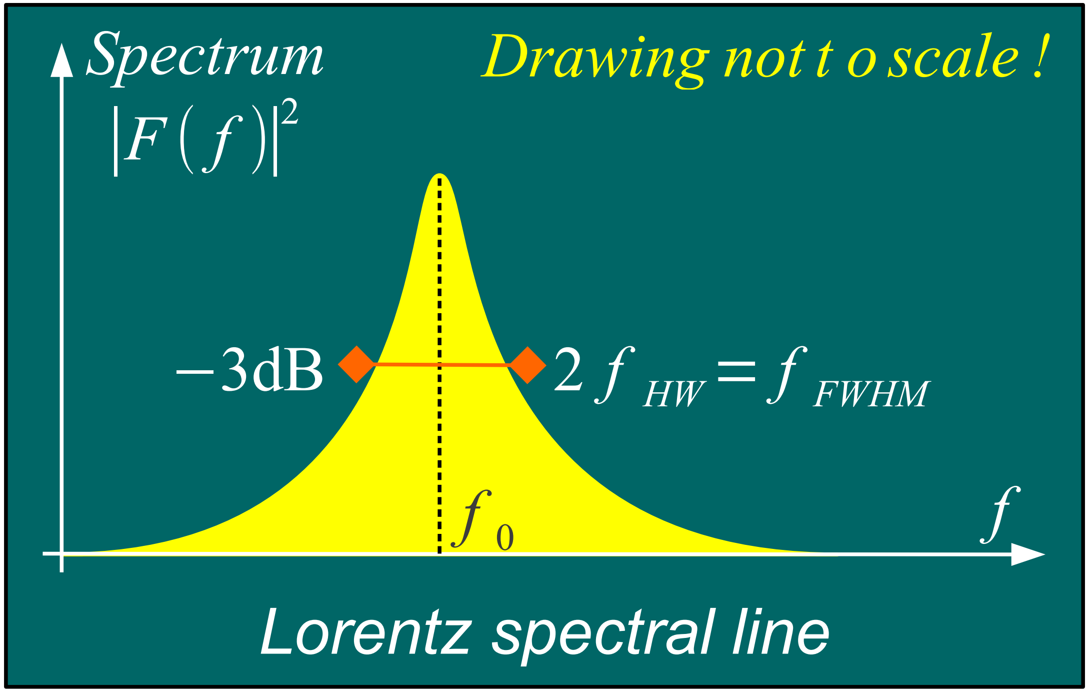
Near ω_0 $\rightarrow \left|2Q_L\frac{\Delta\omega}{\omega_0}\right|\ll 1 \rightarrow U_{Nout}\approx\frac{U_N}{j2Q_L\frac{\Delta\omega}{\omega_0}-\epsilon} \rightarrow P_{Nout}\approx\frac{P_N}{\epsilon^2+\left(2Q_L\frac{\Delta\omega}{\omega_0}\right)^2}$

$$P_\phi=\frac{P_{Nout}}{2}\approx\frac{P_N/2}{\epsilon^2+\left(2Q_L\frac{\Delta f}{f_0}\right)^2}=\frac{P_Nf_0^2}{8Q_L^2}\cdot\frac{1}{\left(\frac{\epsilon f_0}{2Q_L}\right)^2+\Delta f^2}$$

Half width $f_{HW}=\frac{\epsilon f_0}{2Q_L}$

$$L(\Delta f)=\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{1}{f_{HW}^2+\Delta f^2}\cdot\frac{k_BT_0F}{8P_0}=\frac{C}{f_{HW}^2+\Delta f^2}\equiv \textit{Lorentz spectral line}$$

Derivation of the Lorentz spectral line



$$L(\Delta f)=\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{1}{f_{HW}^2+\Delta f^2}\cdot\frac{k_B T_0 F}{8P_0}$$

$$f_{HW}\equiv f_{HALF-WIDTH}$$
$$f_{FWHM}\equiv f_{FULL-WIDTH-HALF-MAXIMUM}$$

$$L(\Delta f)=\frac{C}{f_{HW}^2+\Delta f^2}$$

$$\epsilon=\frac{2Q_L f_{HW}}{f_0}$$

$$\int_{-f_0}^{\infty}L(\Delta f)\,d\Delta f=1\approx\int_{-\infty}^{\infty}\frac{C}{f_{HW}^2+\Delta f^2}\,d\Delta f=\left[\frac{C}{f_{HW}}\cdot\arctan\frac{\Delta f}{f_{HW}}\right]_{\Delta f=-\infty}^{\Delta f=\infty}=\frac{\pi C}{f_{HW}}$$

$$f_{HW}\approx\pi C=\frac{\pi k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2$$

$$C=\frac{k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2\approx\frac{f_{HW}}{\pi}$$

$$L(\Delta f)\approx\frac{f_{HW}/\pi}{f_{HW}^2+\Delta f^2}$$

Example $f_0=3\text{GHz}$

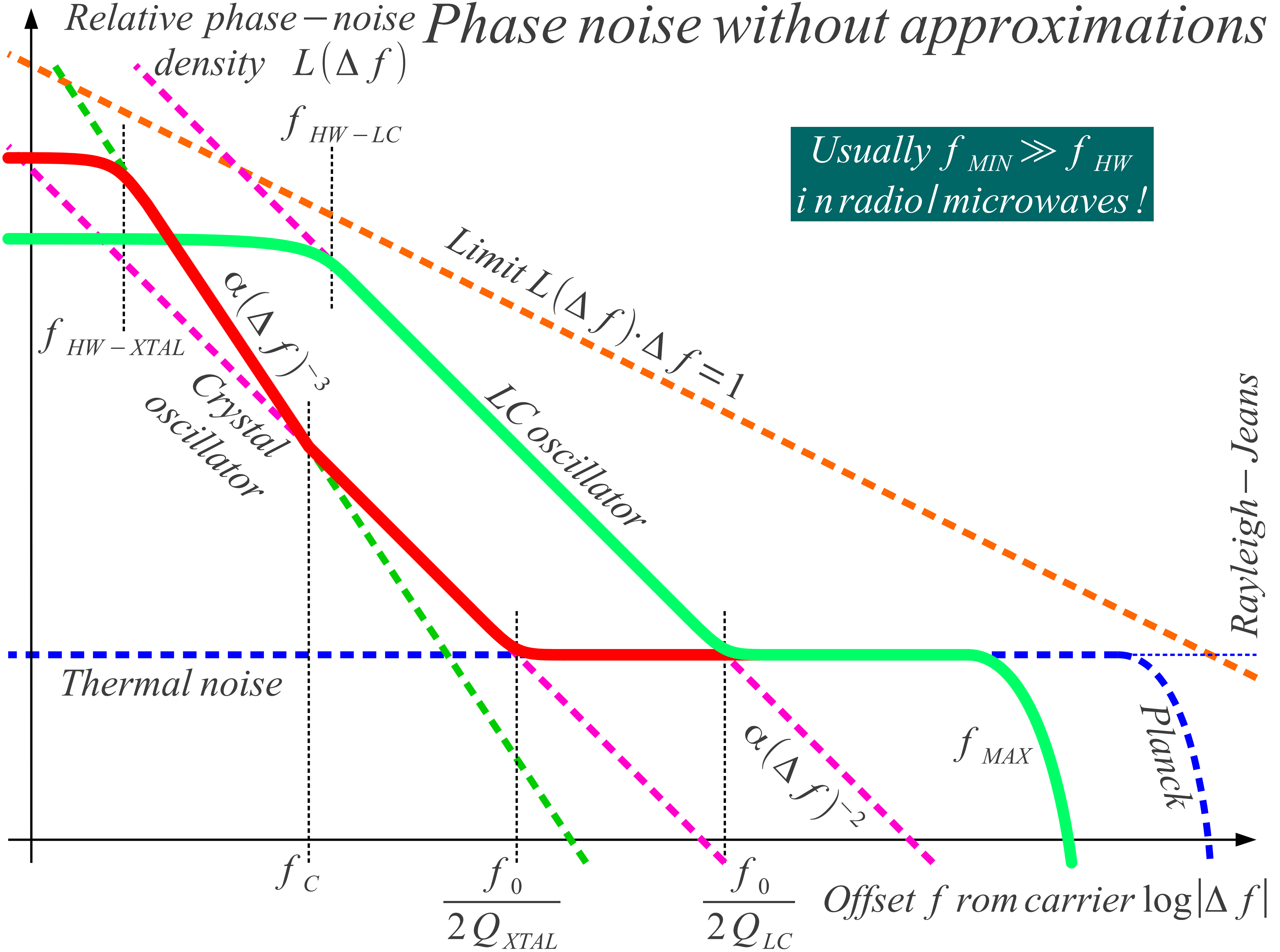
$Q_L=10$ $P_0=0.1\text{mW}$ $F=10\text{dB}$

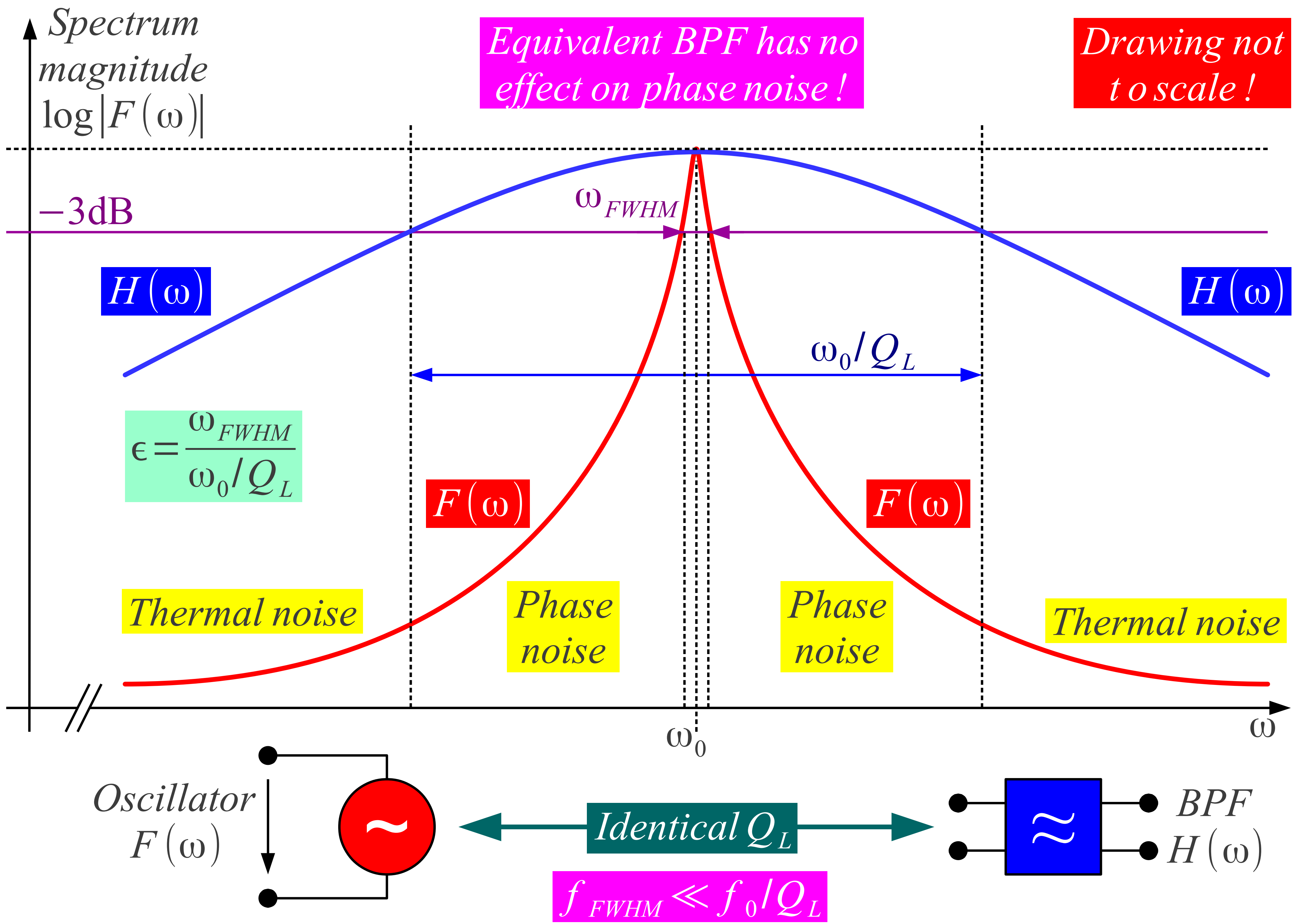
$f_{HW}\approx14\text{Hz}$ $f_{FWHM}\approx28\text{Hz}$ $\epsilon\approx10^{-7}$

without flicker noise !

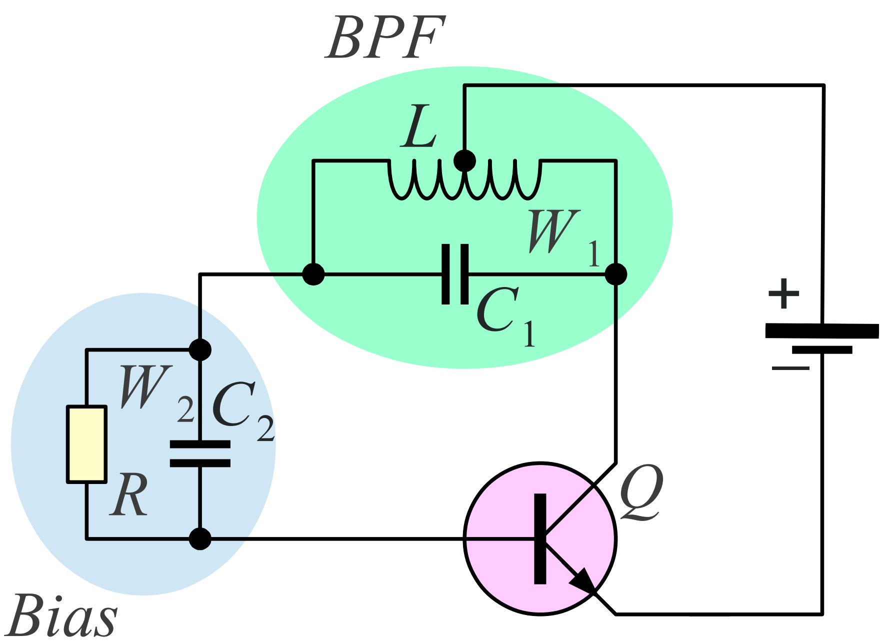
Lorentz spectral linewidth

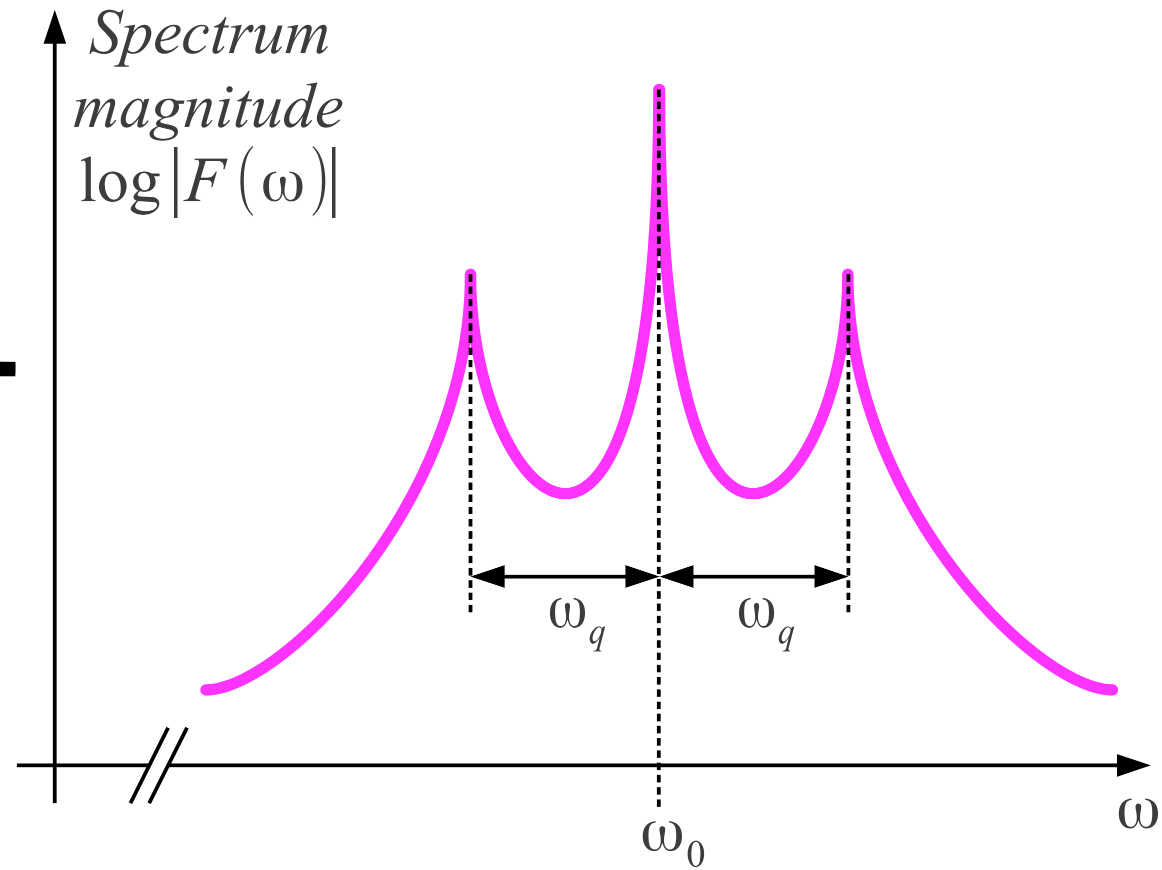
Phase noise without approximations





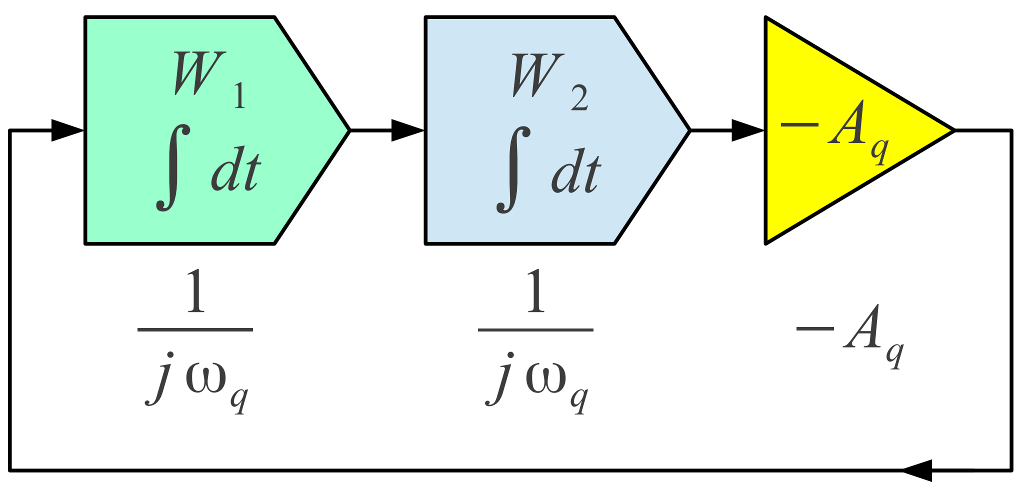
Oscillator / BPF comparison





Quenching ?

Saturation feedback

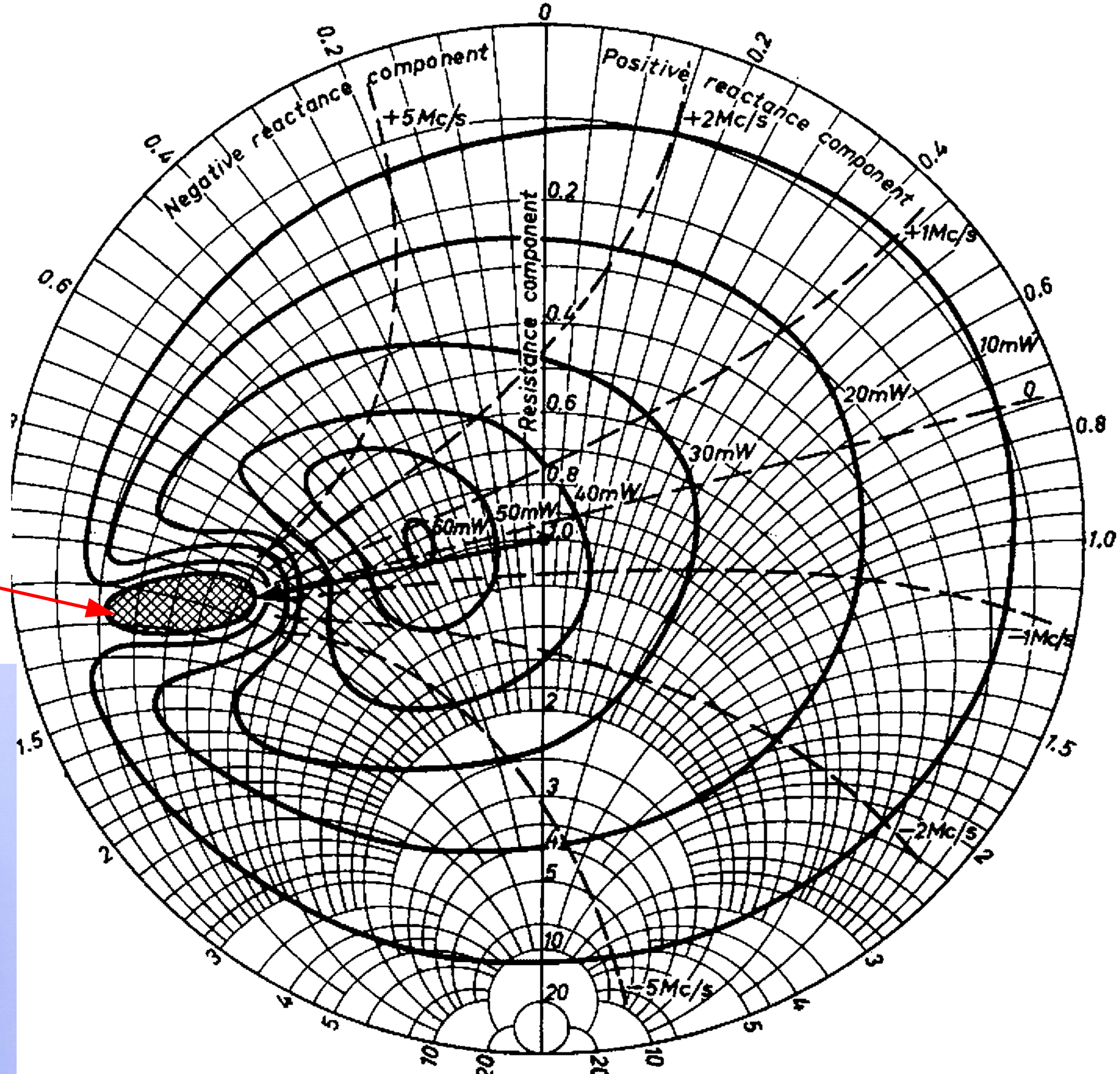


Unsuitable (large) C_2

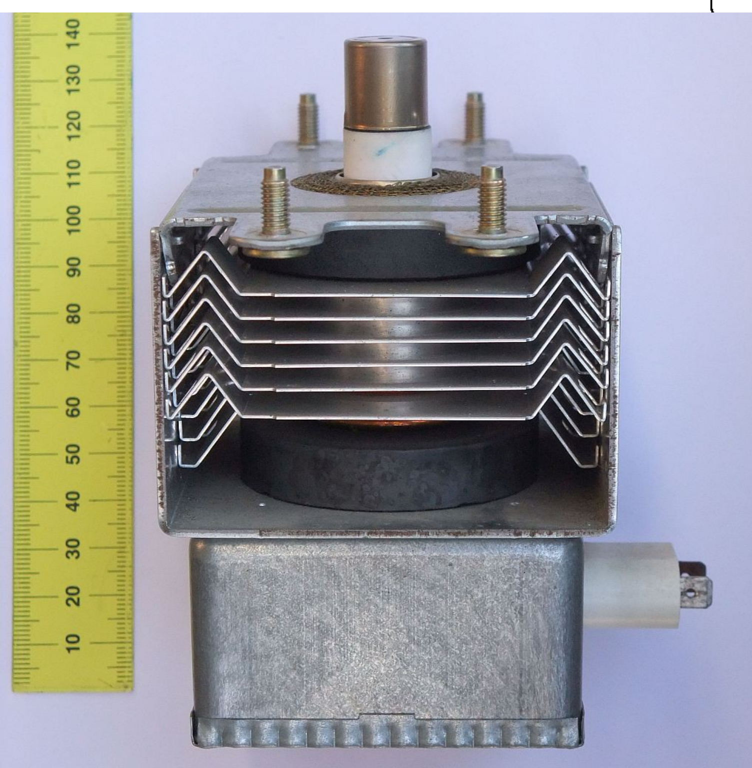
$$\frac{1}{j\omega_q} \cdot \frac{1}{j\omega_q} \cdot (-A_q) = 1$$

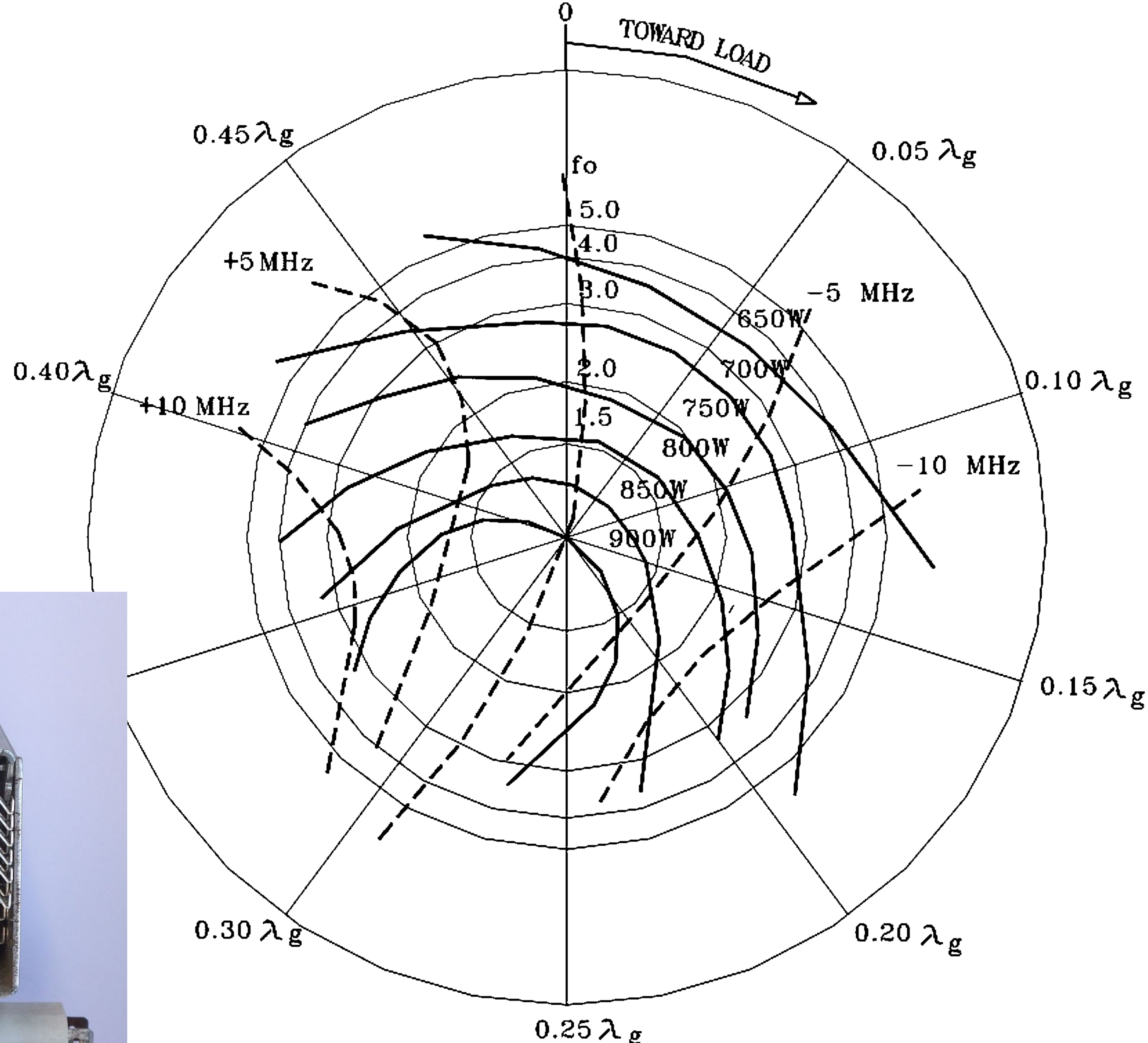
$$\omega_q = \sqrt{A_q}$$

Unstable saturation

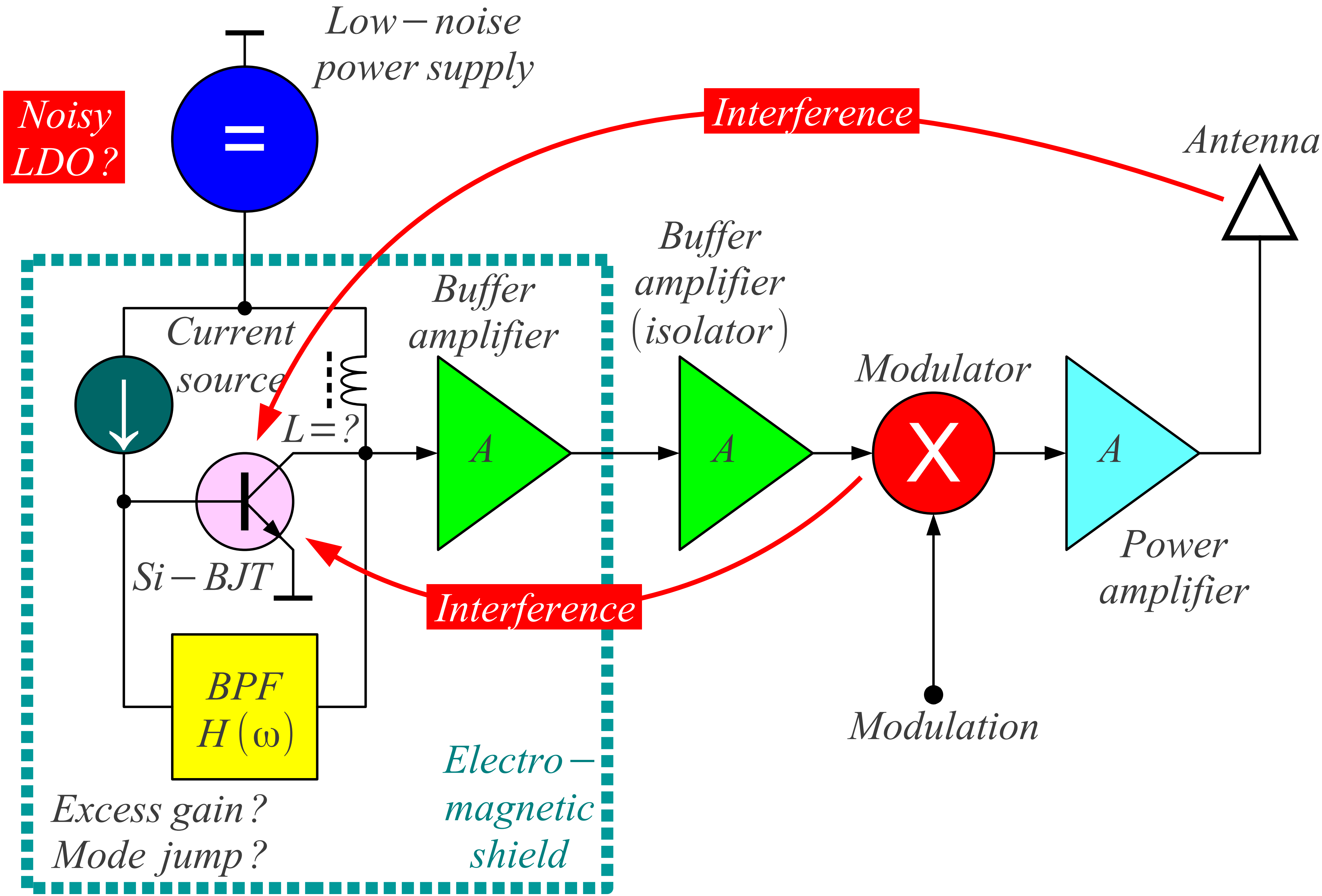


Klystron 2K25 Rieke diagram





Magnetron 2M214 *Rieke diagram*



Oscillator design rules