Communication Electronics

Lecture 4:

Antennas and propagation of electromagnetic waves

Coordinate systems

Cartesian

$3D = (x, y, z)$

$-\infty < x[\mathrm{m}] < +\infty$

$-\infty < y[\mathrm{m}] < +\infty$

$-\infty < z[\mathrm{m}] < +\infty$

Orthogonal

$\vec{1}_x \perp \vec{1}_y \perp \vec{1}_z \perp \vec{1}_x$

$\vec{1}_r \perp \vec{1}_\Theta \perp \vec{1}_\Phi \perp \vec{1}_r$

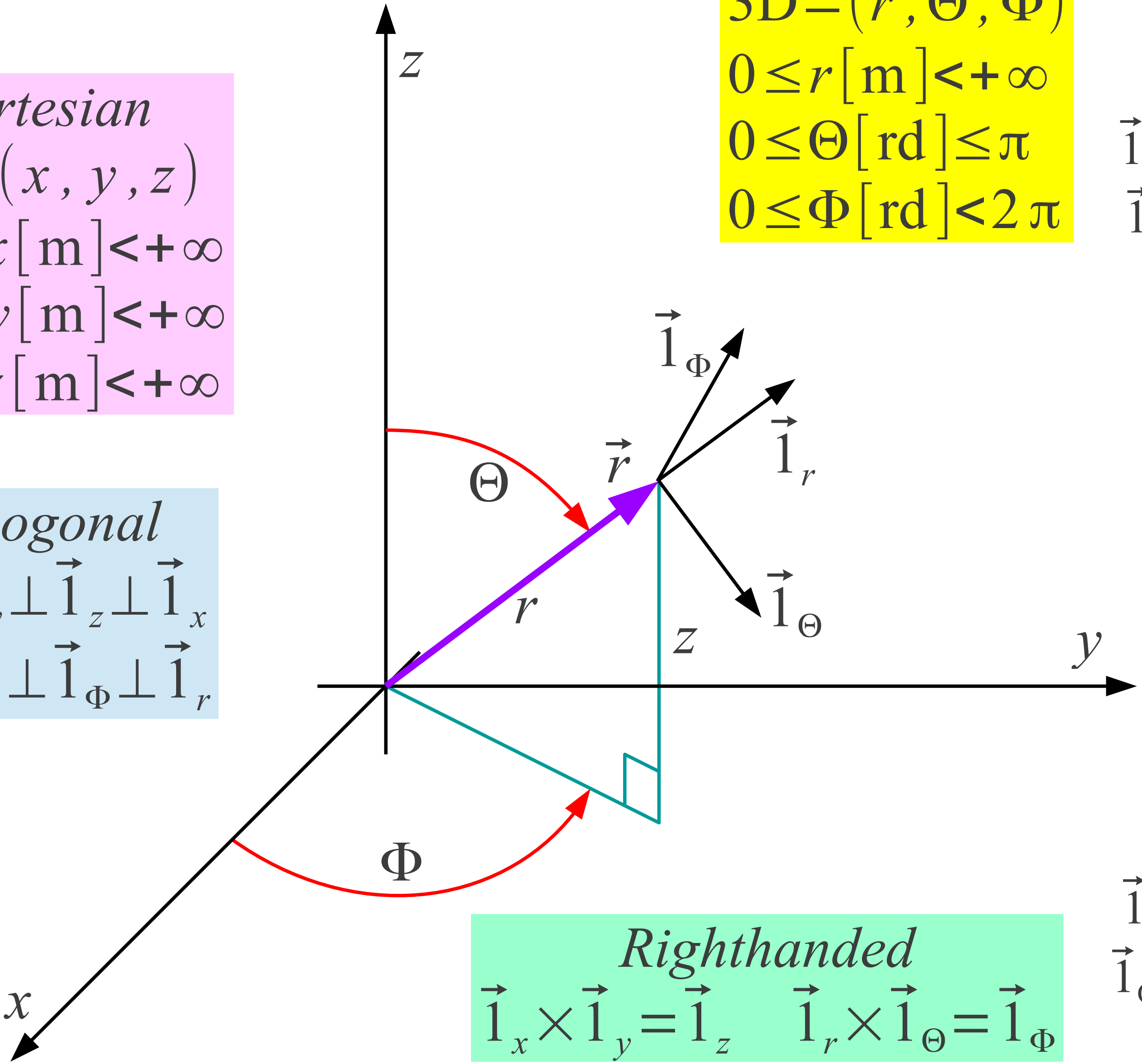
Spherical

$3D = (r, \Theta, \Phi)$

$0 \le r[\mathrm{m}] < +\infty$

$0 \le \Theta[\mathrm{rd}] \le \pi$

$0 \le \Phi[\mathrm{rd}] < 2\pi$



Righthanded

$\vec{1}_x \times \vec{1}_y = \vec{1}_z \qquad \vec{1}_r \times \vec{1}_\Theta = \vec{1}_\Phi$

Conversion $(r, \Theta, \Phi) \rightarrow (x, y, z)$

$$x = r \sin\Theta \cos\Phi$$
$$y = r \sin\Theta \sin\Phi$$
$$z = r \cos\Theta$$
$$\vec{1}_x = \vec{1}_r \sin\Theta \cos\Phi + \vec{1}_\Theta \cos\Theta \cos\Phi - \vec{1}_\Phi \sin\Phi$$
$$\vec{1}_y = \vec{1}_r \sin\Theta \sin\Phi + \vec{1}_\Theta \cos\Theta \sin\Phi + \vec{1}_\Phi \cos\Phi$$
$$\vec{1}_z = \vec{1}_r \cos\Theta - \vec{1}_\Theta \sin\Theta$$

$0 \le \Theta \le \pi \quad \rightarrow \quad \sin\Theta \ge 0$

Conversion $(x, y, z) \rightarrow (r, \Theta, \Phi)$

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\Theta = \arccos\left(z / \sqrt{x^2 + y^2 + z^2}\right)$$
$$\Phi = \arctan(y/x) \quad (\textit{quadrant ?})$$
$$\vec{1}_r = \vec{1}_x \sin\Theta \cos\Phi + \vec{1}_y \sin\Theta \sin\Phi + \vec{1}_z \cos\Theta$$
$$\vec{1}_\Theta = \vec{1}_x \cos\Theta \cos\Phi + \vec{1}_y \cos\Theta \sin\Phi - \vec{1}_z \sin\Theta$$
$$\vec{1}_\Phi = -\vec{1}_x \sin\Phi + \vec{1}_y \cos\Phi$$

Maxwell equations

Ampère	$\nabla\times\vec{H}=\vec{J}+j\omega\vec{D}$
Faraday	$\nabla\times\vec{E}=-j\omega\vec{B}$
Gauss	$\nabla\cdot\vec{D}=\rho$

Potentials

$V[\mathrm{V}]\equiv$ *scalar potential*

$\vec{A}[\mathrm{Vs/m}]\equiv$ *vector potential*

$$\vec{E}=-j\omega\vec{A}-\nabla V$$

$$\vec{B}=\nabla\times\vec{A}$$

Vector operations

$$\vec{A}=\vec{1}_x A_x+\vec{1}_y A_y+\vec{1}_z A_z$$

$$\vec{A}\cdot\vec{B}=A_x B_x+A_y B_y+A_z B_z$$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$\vec{B}[\mathrm{Vs/m^2}]\equiv$ *magnetic flux density*

$\vec{D}[\mathrm{As/m^2}]\equiv$ *electric displacement field*

$\vec{E}[\mathrm{V/m}]\equiv$ *electric field intensity*

$\vec{H}[\mathrm{A/m}]\equiv$ *magnetic field intensity*

$\vec{J}[\mathrm{A/m^2}]\equiv$ *conductive current density*

$\rho[\mathrm{As/m^3}]\equiv$ *electric charge density*

Wave equations (Lorenz gauge)

$$\Delta\vec{A}+k^2\vec{A}=-\mu\vec{J}$$

$$\Delta V+k^2 V=-\frac{\rho}{\epsilon}$$

Laplace $\quad \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

$k=\omega\sqrt{\mu\epsilon}=\frac{2\pi}{\lambda}\equiv$ *wavenumber*

Time−harmonic derivative $\quad \frac{\partial}{\partial t}=j\omega$

Matter

$$\vec{D}=\epsilon\vec{E}$$

$$\vec{B}=\mu\vec{H}$$

$$\vec{J}=\gamma\vec{E}$$

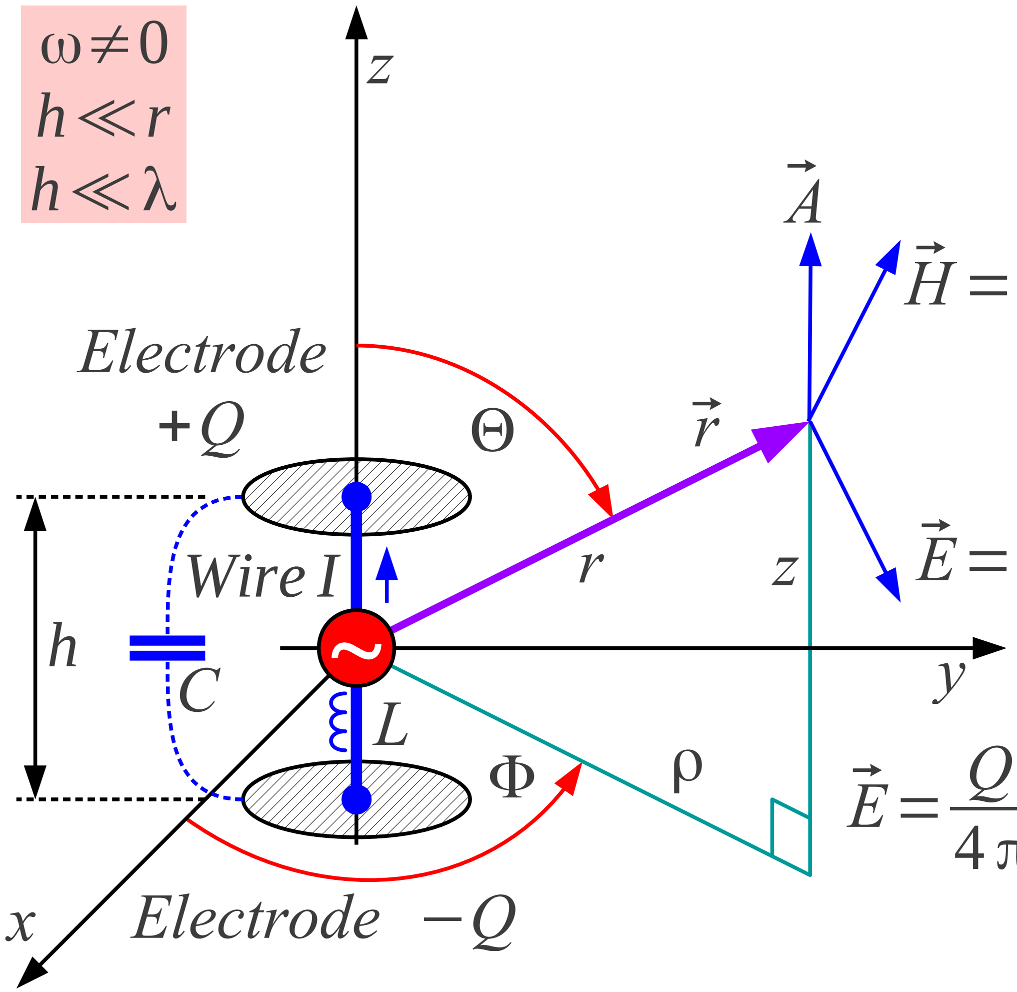
Retarded potentials

$$\vec{A}(\vec{r})=\frac{\mu}{4\pi}\int_{V'}\vec{J}(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dV'$$

$$V(\vec{r})=\frac{1}{4\pi\epsilon}\int_{V'}\rho(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dV'$$

Spatial derivatives $\quad \nabla=\vec{1}_x\frac{\partial}{\partial x}+\vec{1}_y\frac{\partial}{\partial y}+\vec{1}_z\frac{\partial}{\partial z}$

$\omega \neq 0$
$h \ll r$
$h \ll \lambda$



$$\vec{A} \approx \vec{1}_z \frac{\mu I h}{4\pi} \frac{e^{-jkr}}{r} = \left(\vec{1}_r \cos\Theta - \vec{1}_\Theta \sin\Theta\right) \frac{\mu I h}{4\pi} \frac{e^{-jkr}}{r}$$

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$$

$$\vec{H} = \vec{1}_\Phi \frac{I h}{4\pi} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2}\right) \sin\Theta$$

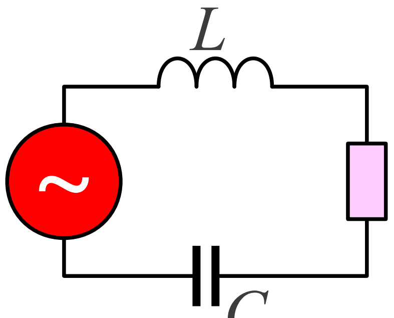
$$\vec{E} = \frac{\nabla \times \vec{H}}{j\omega\epsilon}$$

Continuity
$I = j\omega Q$

Radiation

$$\vec{E} = \frac{Q h}{4\pi\epsilon} e^{-jkr} \left[\vec{1}_r \left(\frac{jk}{r^2} + \frac{1}{r^3}\right) 2\cos\Theta + \vec{1}_\Theta \left(-\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3}\right) \sin\Theta\right]$$

$$\mathrm{Re}[\vec{S}] = \mathrm{Re}\left[\frac{1}{2} \vec{E} \times \vec{H}^*\right] = \vec{1}_r \frac{Z k^2}{32\pi^2} |I|^2 h^2 \frac{\sin^2\Theta}{r^2}$$



$$R = \frac{Z k^2 h^2}{6\pi} = \frac{2\pi Z}{3} \left(\frac{h}{\lambda}\right)^2$$

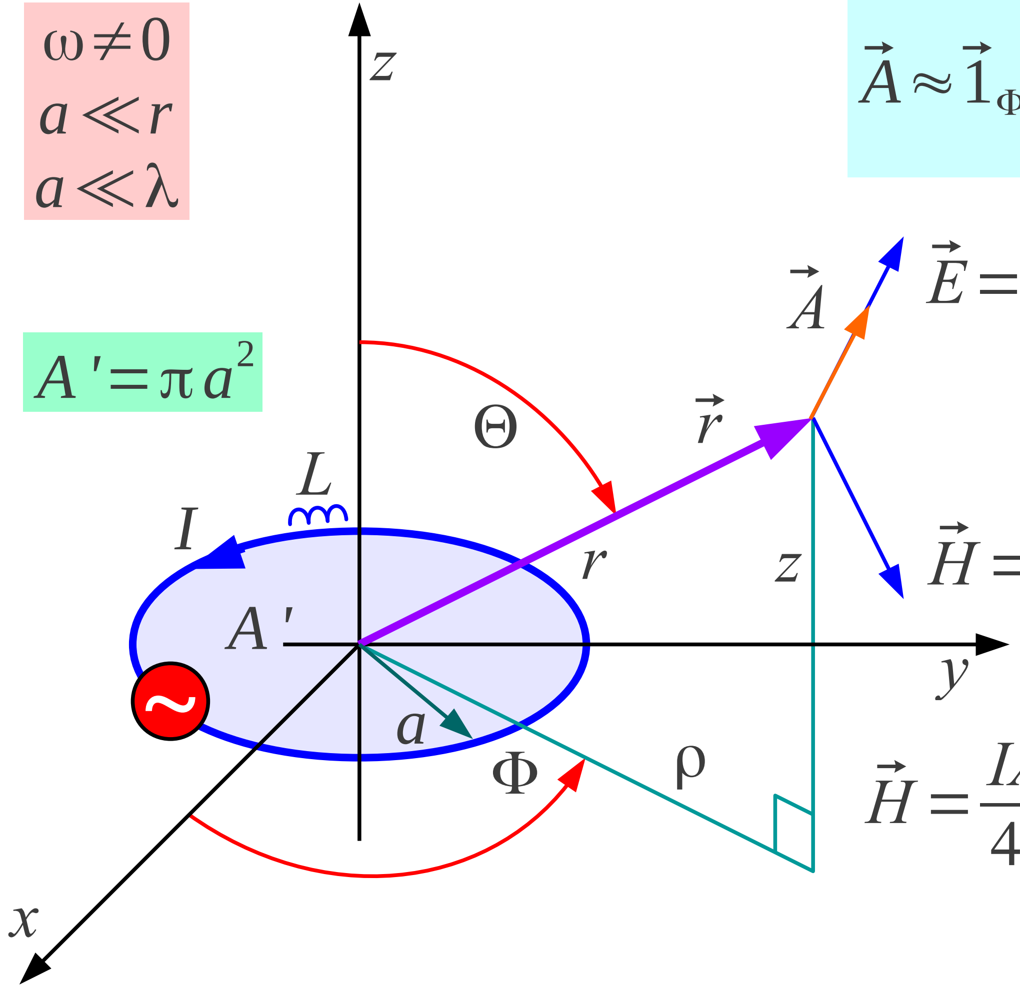
Small electric dipole

Small magnetic dipole

$\omega \neq 0$
$a \ll r$
$a \ll \lambda$

$A' = \pi a^2$

$Q = 0 \rightarrow V = 0$



$$\vec{A} \approx \vec{1}_\Phi \frac{\mu I (\pi a^2)}{4\pi} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin\Theta$$

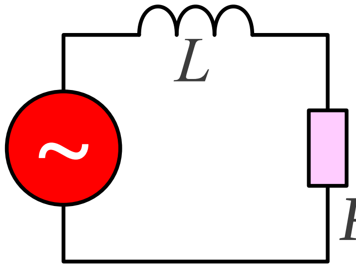
$$\vec{E} = -j\omega \vec{A} - \nabla V = \vec{1}_\Phi \frac{Z I A'}{4\pi} e^{-jkr} \left(\frac{k^2}{r} - \frac{jk}{r^2} \right) \sin\Theta$$

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$$

Radiation

$$\vec{H} = \frac{I A'}{4\pi} e^{-jkr} \left[\vec{1}_r \left(\frac{jk}{r^2} + \frac{1}{r^3} \right) 2\cos\Theta + \vec{1}_\Theta \left(-\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3} \right) \sin\Theta \right]$$

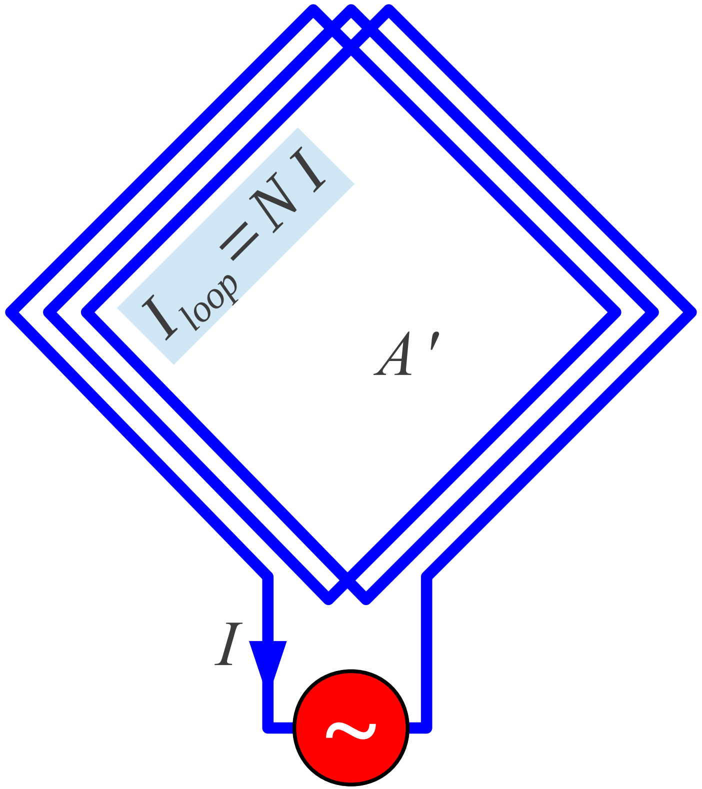
$$\mathrm{Re}[\vec{S}] = \mathrm{Re}\left[\frac{1}{2} \vec{E} \times \vec{H}^* \right] = \vec{1}_r \frac{Z k^4}{32\pi^2} |I|^2 (A')^2 \frac{\sin^2\Theta}{r^2}$$



$$R = \frac{Z k^4 (A')^2}{6\pi} = \frac{8\pi^3 Z}{3} \left(\frac{A'}{\lambda^2} \right)^2$$

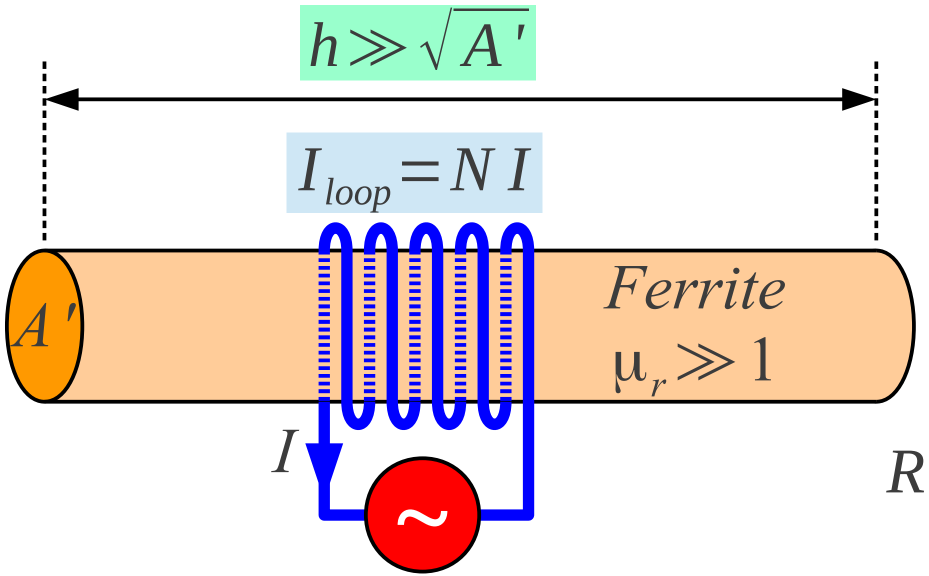
$$R = \frac{Z k^4 (N A')^2}{6\pi} = \frac{8\pi^3 Z}{3}\left(\frac{N A'}{\lambda^2}\right)^2$$

Loop antenna ~ 1920



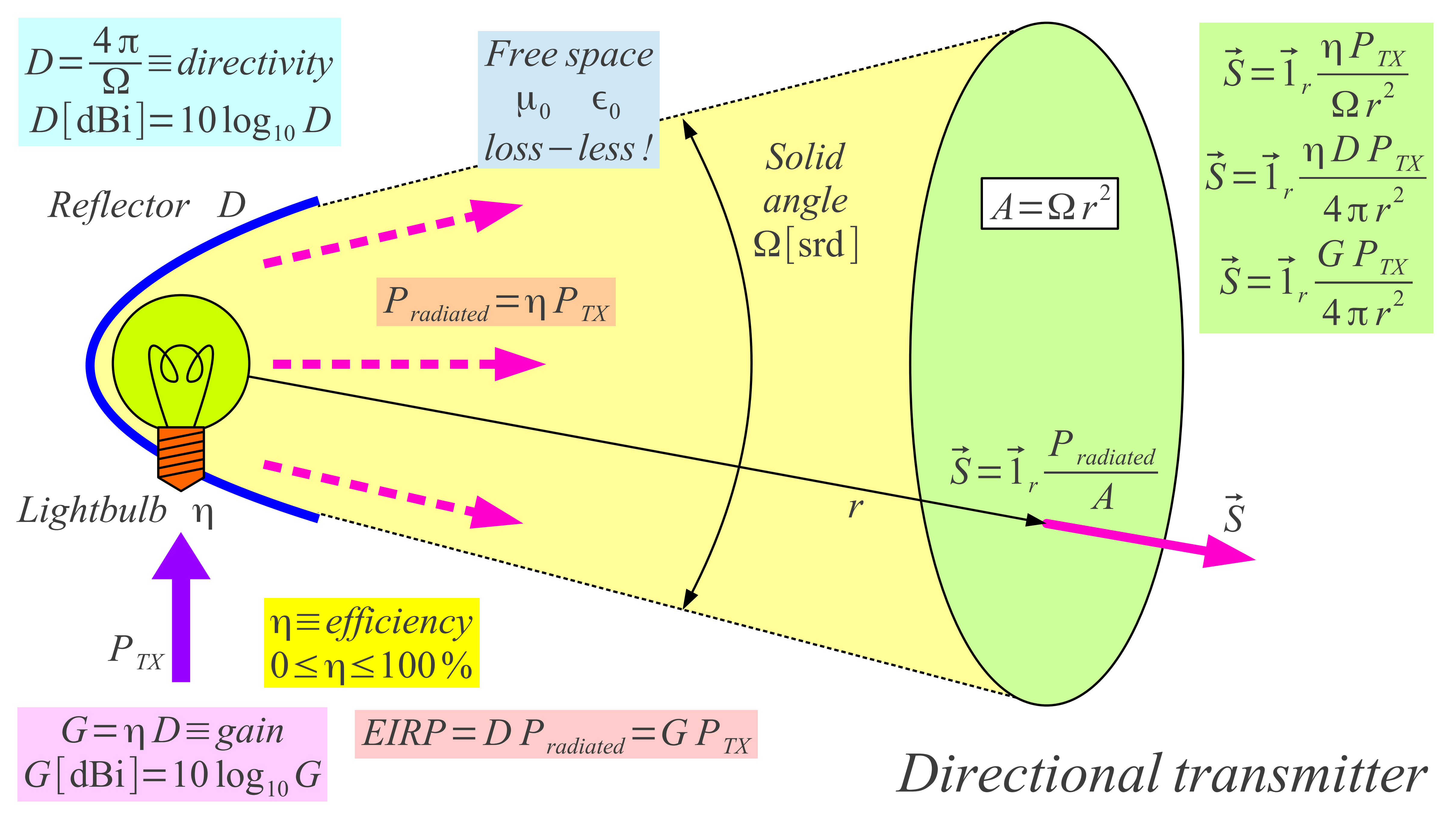
$f \approx 300\,\text{kHz}$
$A' \approx 1\,\text{m}^2$
$N \approx 10$
$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\,\Omega$
$\lambda = c_0 / f = 1\,\text{km}$
$R_S \approx 3.1\,\mu\Omega$

Ferrite antenna ~ 1970

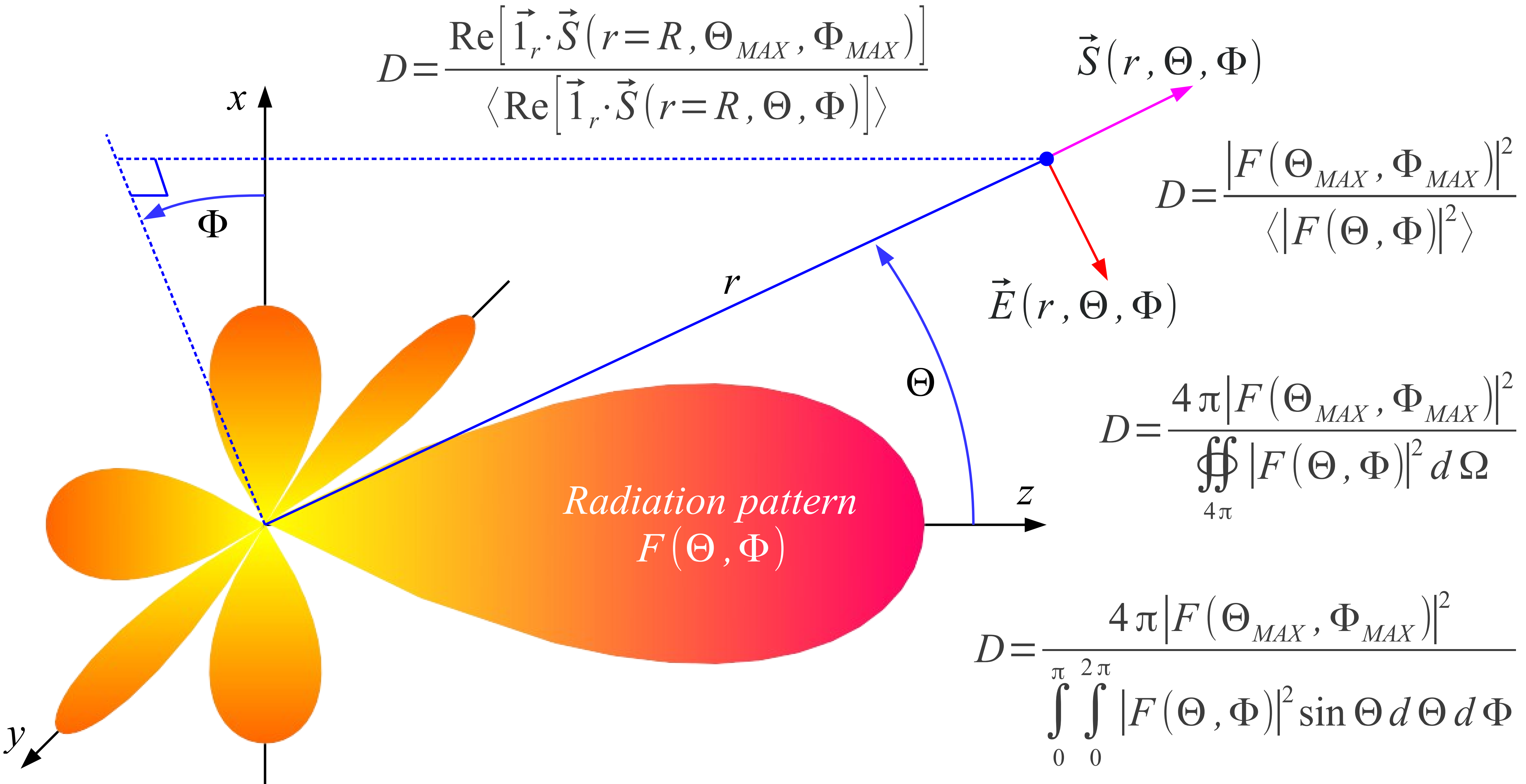


$$R = \frac{Z k^4 (\mu_r N A')^2}{6\pi} = \frac{8\pi^3 Z}{3}\left(\frac{\mu_r N A'}{\lambda^2}\right)^2$$

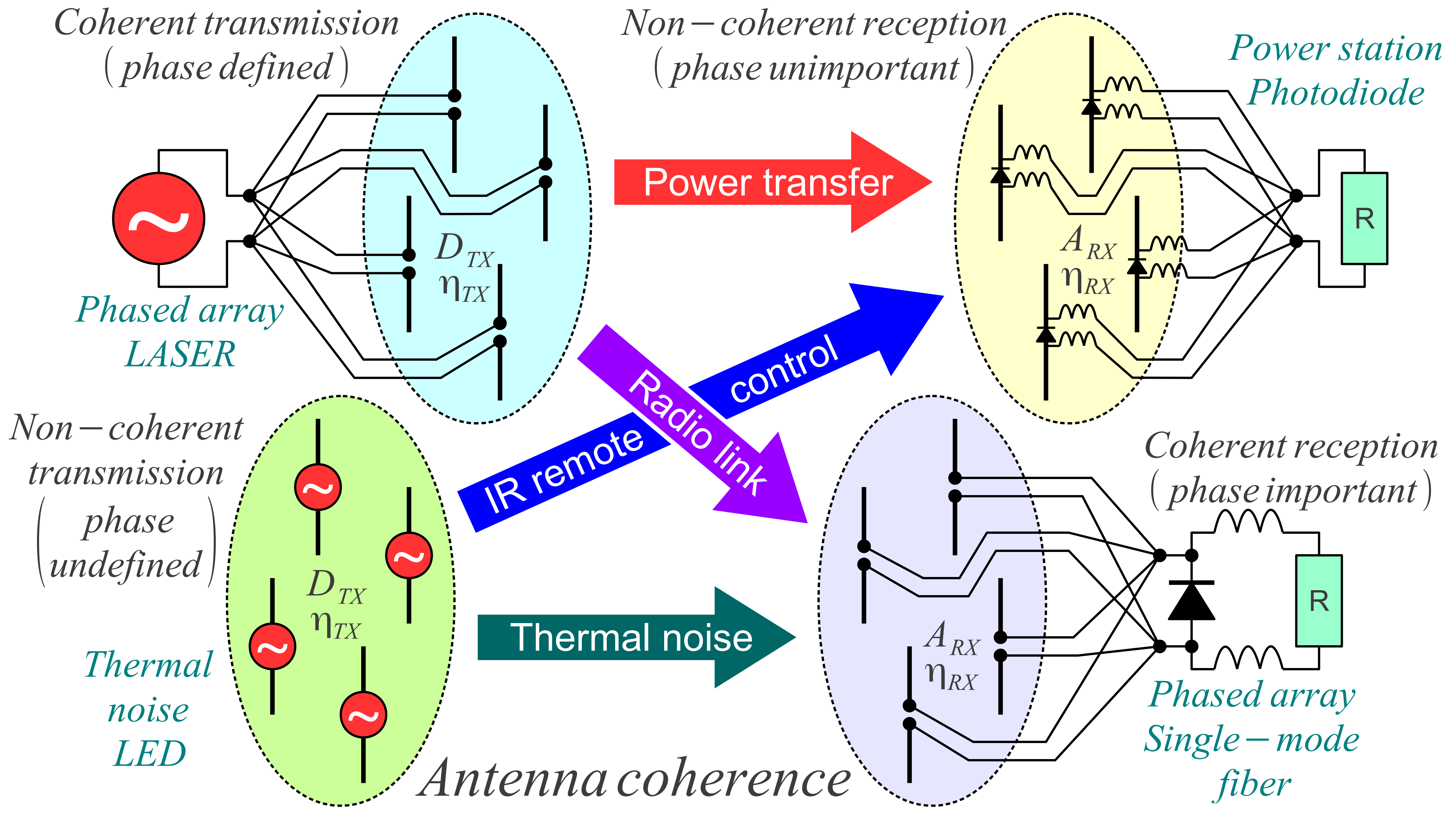
$f \approx 1\,\text{MHz}$
$A' \approx 1\,\text{cm}^2$
$h \approx 20\,\text{cm}$
$\mu_r \approx 100$
$N \approx 30$
$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\,\Omega$
$\lambda = c_0 / f = 300\,\text{m}$
$R \approx 0.35\,\mu\Omega$



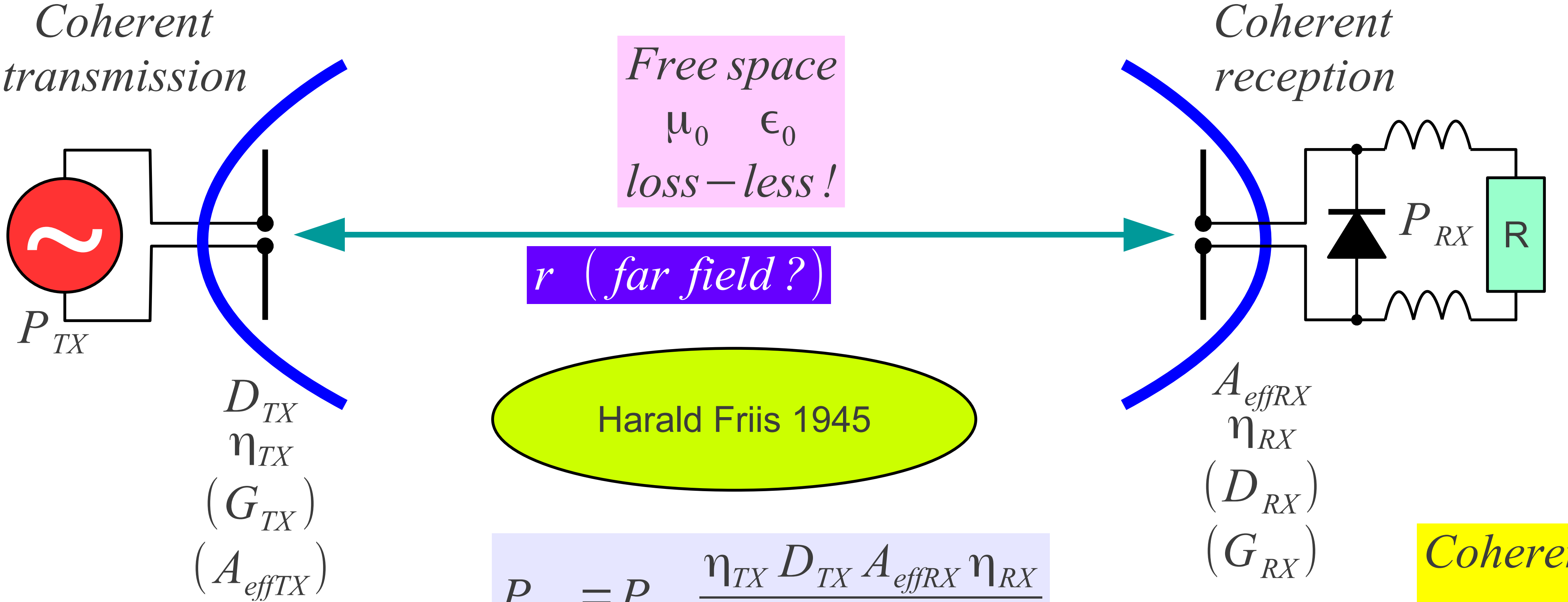
$D = \frac{4\pi}{\Omega} \equiv directivity$
$D[\text{dBi}] = 10 \log_{10} D$
Free space
$\mu_0 \quad \epsilon_0$
loss − less !
$\vec{S} = \vec{1}_r \frac{\eta P_{TX}}{\Omega r^2}$
$\vec{S} = \vec{1}_r \frac{\eta D P_{TX}}{4\pi r^2}$
$\vec{S} = \vec{1}_r \frac{G P_{TX}}{4\pi r^2}$
Reflector D
Solid angle
$\Omega[\text{srd}]$
$A = \Omega r^2$
$P_{radiated} = \eta P_{TX}$
$\vec{S} = \vec{1}_r \frac{P_{radiated}}{A}$
Lightbulb η
r
$\vec{S}$
P_{TX}
$\eta \equiv efficiency$
$0 \leq \eta \leq 100\%$
$G = \eta D \equiv gain$
$G[\text{dBi}] = 10 \log_{10} G$
$EIRP = D P_{radiated} = G P_{TX}$
Directional transmitter



Directivity for arbitrary radiation patterns



Coherent transmission
(phase defined)
Non − coherent reception
(phase unimportant)
Power station
Photodiode
Power transfer
R
D_{TX}
η_{TX}
A_{RX}
η_{RX}
Phased array
LASER
control
Radio link
IR remote
Non − coherent
transmission
(phase
undefined)
Coherent reception
(phase important)
R
D_{TX}
η_{TX}
Thermal noise
A_{RX}
η_{RX}
Thermal
noise
LED
Phased array
Single − mode
fiber
Antenna coherence



$$P_{RX} = P_{TX} \frac{\eta_{TX} D_{TX} A_{effRX} \eta_{RX}}{4 \pi r^2}$$

Antenna gains: $$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4 \pi r} \right)^2$$

Antenna sizes: $$P_{RX} = P_{TX} \frac{\eta_{TX} A_{effTX} A_{effRX} \eta_{RX}}{\lambda^2 r^2}$$

Reciprocity !

Coherent antenna

$$D = \frac{4 \pi}{\lambda^2} A_{eff}$$

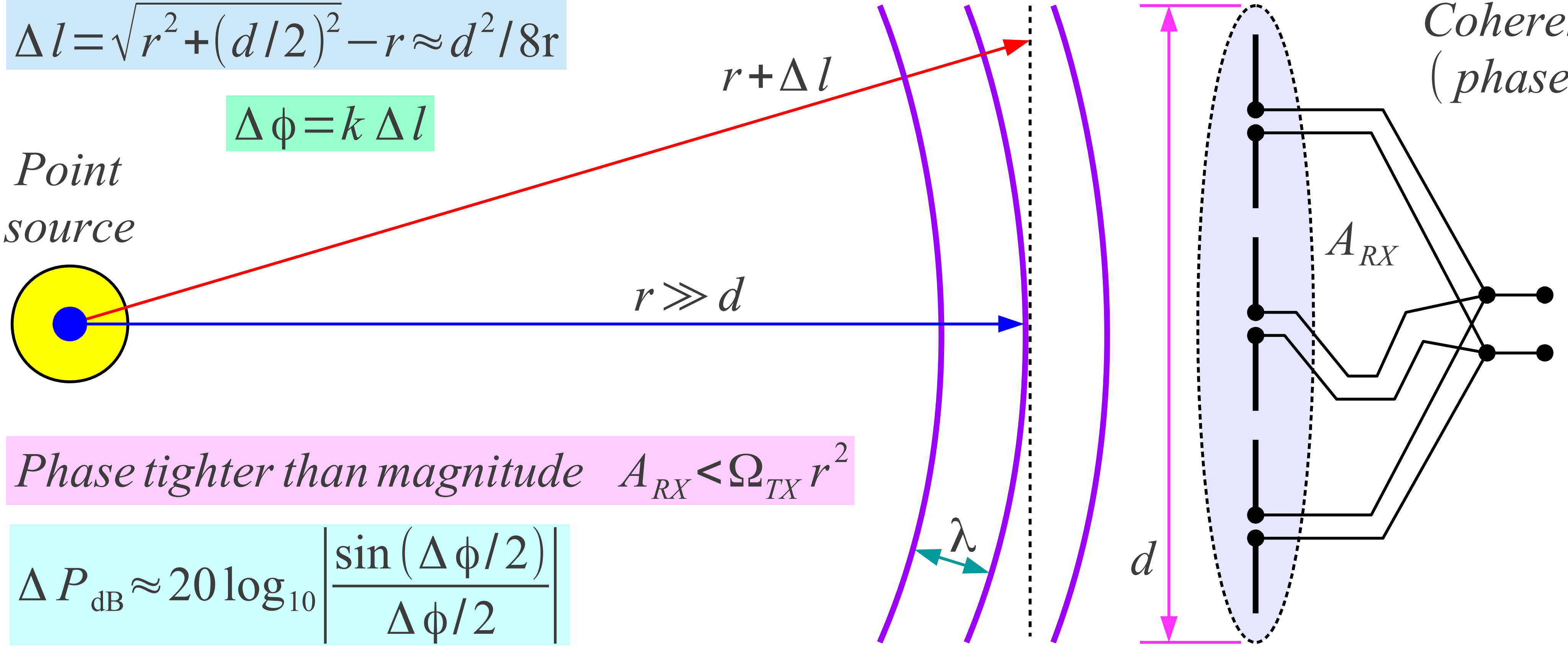
$$G = \frac{4 \pi}{\lambda^2} \eta A_{eff}$$

Friis equation

Rayleigh distance

$$\Delta l = \sqrt{r^2 + (d/2)^2} - r \approx d^2/8r$$

$$\Delta\phi = k\,\Delta l$$



Phase tighter than magnitude $A_{RX} < \Omega_{TX} r^2$

$$\Delta P_{dB} \approx 20\log_{10}\left|\frac{\sin(\Delta\phi/2)}{\Delta\phi/2}\right|$$

Antenna far field

$$r \geq \frac{2d^2}{\lambda}$$

$F(\Theta,\Phi)$

D G

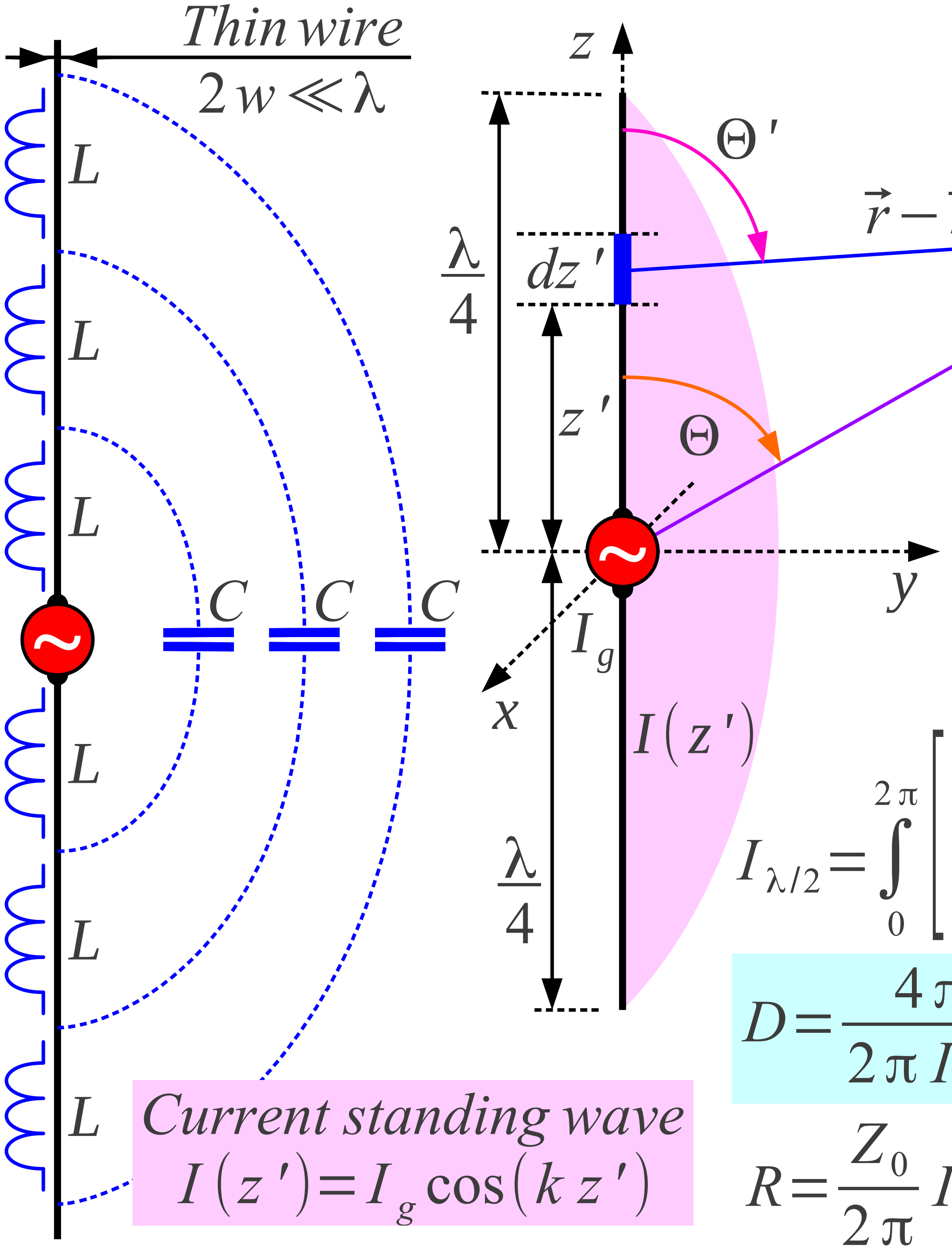
Δl	$\Delta\phi$[rd]	ΔP[dB]	$r\geq$	Usage
$\lambda/2$	π	−3.922	$d^2/4\lambda$	Photo depth of field
$\lambda/4$	$\pi/2$	−0.912	$d^2/2\lambda$	Lord Rayleigh 1891
$\lambda/8$	$\pi/4$	−0.224	d^2/λ	
$\lambda/16$	$\pi/8$	−0.056	$2d^2/\lambda$	Antenna measurements

Example photo camera

$Lens \equiv d = 2\text{mm}$

$\lambda = 0.5\,\mu\text{m}$

$DoF \equiv d^2/4\lambda = 2\text{m}$



Far field $d\vec{E} \approx \vec{1}_{\Theta'} \dfrac{jkZ_0}{4\pi} I(z')\,dz' \dfrac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} \sin\Theta'$

$$d\vec{E} \approx \vec{1}_{\Theta} \frac{jkZ_0}{4\pi} I(z')\,dz' \frac{e^{-jkr}}{r} e^{jkz'\cos\Theta} \sin\Theta$$

$$\vec{E} = \int_{-\lambda/4}^{\lambda/4} d\vec{E} = \vec{1}_{\Theta} \frac{jZ_0}{2\pi} I_g \frac{e^{-jkr}}{r} \frac{\cos\left(\frac{\pi}{2}\cos\Theta\right)}{\sin\Theta}$$

$$F(\Theta,\Phi) = \frac{\cos\left(\frac{\pi}{2}\cos\Theta\right)}{\sin\Theta}$$

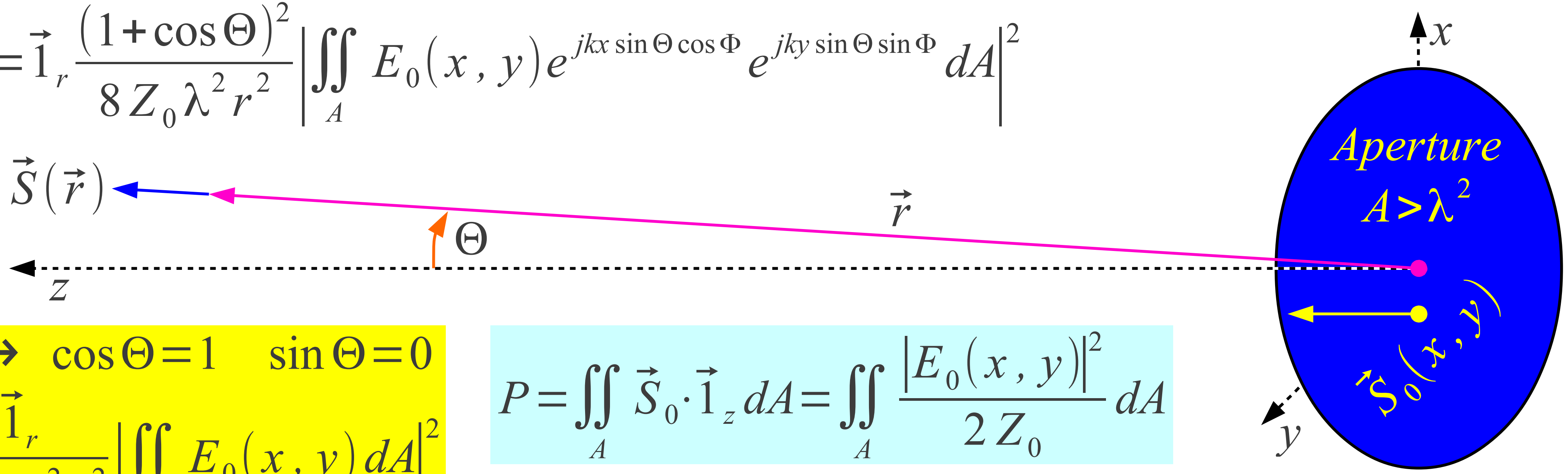
$$I_{\lambda/2} = \int_0^{2\pi} \left[\frac{\cos\left(\frac{\pi}{2}\cos\Theta\right)}{\sin\Theta}\right]^2 \sin\Theta\, d\Theta \approx 1.218827$$

$$D = \frac{4\pi}{2\pi I_{\lambda/2}} \approx \frac{2}{1.218827} \approx 1.64 \approx 2.15\mathrm{dBi}$$

$$R = \frac{Z_0}{2\pi} I_{\lambda/2} \approx 60\,\Omega\, I_{\lambda/2} \approx 73.1\,\Omega$$

Half − wave dipole

$$\vec{S} = \vec{1}_r \frac{|E|^2}{2Z_0} = \vec{1}_r \frac{(1+\cos\Theta)^2}{8Z_0\lambda^2 r^2} \left| \iint_A E_0(x,y) e^{jkx \sin\Theta \cos\Phi} e^{jky \sin\Theta \sin\Phi} dA \right|^2$$



$$\Theta_{MAX} = 0 \;\rightarrow\; \cos\Theta = 1 \quad \sin\Theta = 0$$

$$\vec{S}_{MAX} = \frac{\vec{1}_r}{2Z_0\lambda^2 r^2} \left| \iint_A E_0(x,y)\, dA \right|^2$$

$$P = \iint_A \vec{S}_0 \cdot \vec{1}_z \, dA = \iint_A \frac{|E_0(x,y)|^2}{2Z_0} dA$$

$$A_{eff} = \eta_0 A \equiv \textit{effective area}$$

$$D = \frac{|\vec{S}_{MAX}|}{P/(4\pi r^2)} = \frac{4\pi \left| \iint_A E_0(x,y)\, dA \right|^2}{\lambda^2 \iint_A |E_0(x,y)|^2 dA}$$

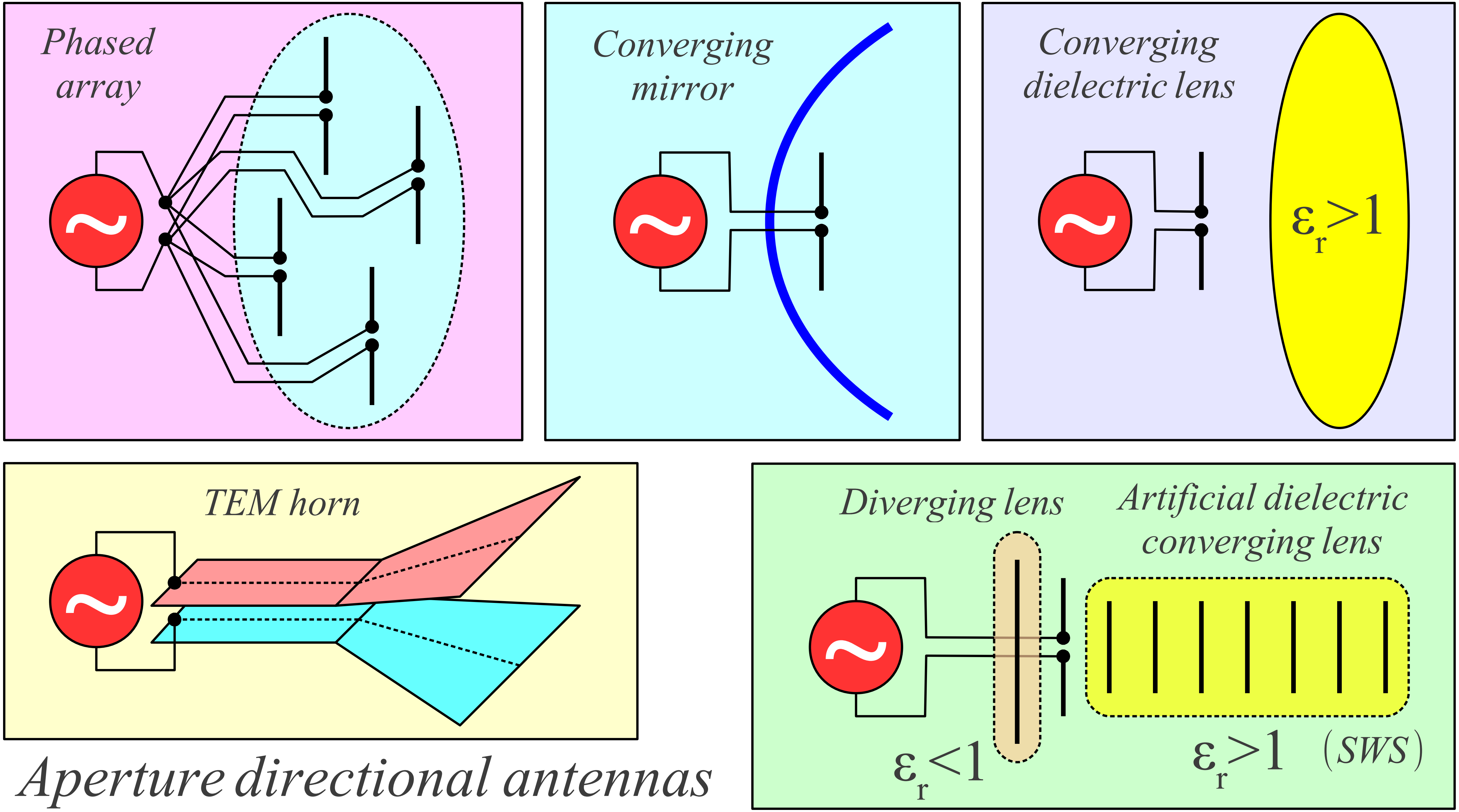
$$D = \frac{4\pi}{\lambda^2} A_{eff} = \frac{4\pi}{\lambda^2} \eta_0 A$$

$$A_{eff} = \frac{\left| \iint_A E_0(x,y)\, dA \right|^2}{\iint_A |E_0(x,y)|^2 dA}$$

Example $E_0(x,y) = \textit{const.} \;\rightarrow\; D = \frac{4\pi}{\lambda^2} A$

$$\eta_0 = \frac{\left| \iint_A E_0(x,y)\, dA \right|^2}{A \iint_A |E_0(x,y)|^2 dA} \equiv \textit{illumination efficiency}$$

Aperture radiation


Phased array
Converging mirror
Converging dielectric lens
$\varepsilon_r > 1$
TEM horn
Diverging lens
Artificial dielectric converging lens
$\varepsilon_r < 1$
$\varepsilon_r > 1$ (SWS)
Aperture directional antennas

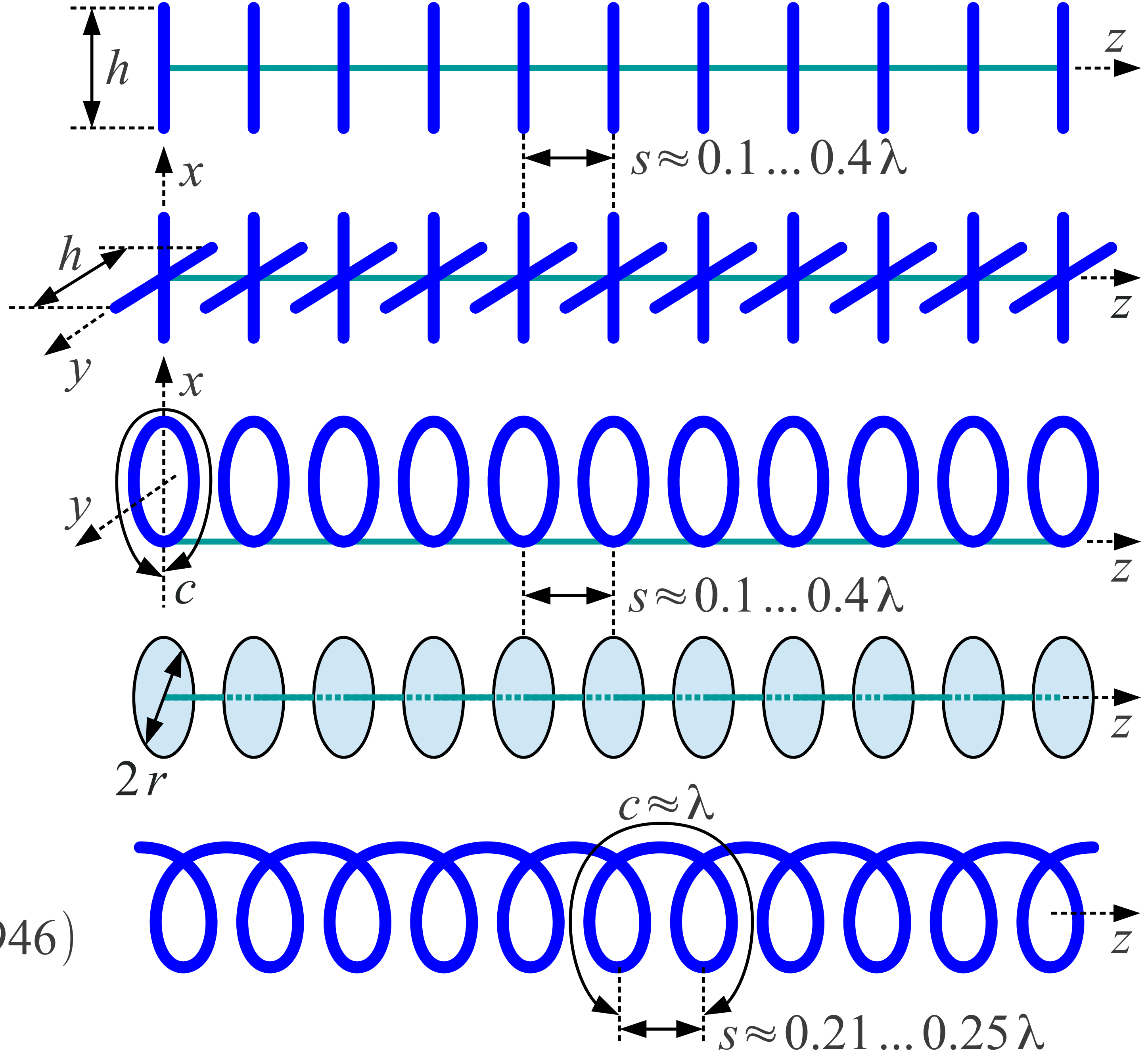
Metal rods $h \approx 0.45\lambda$
(*Shintaro Uda* 1926)

Crossed metal rods
(*both polarizations*)

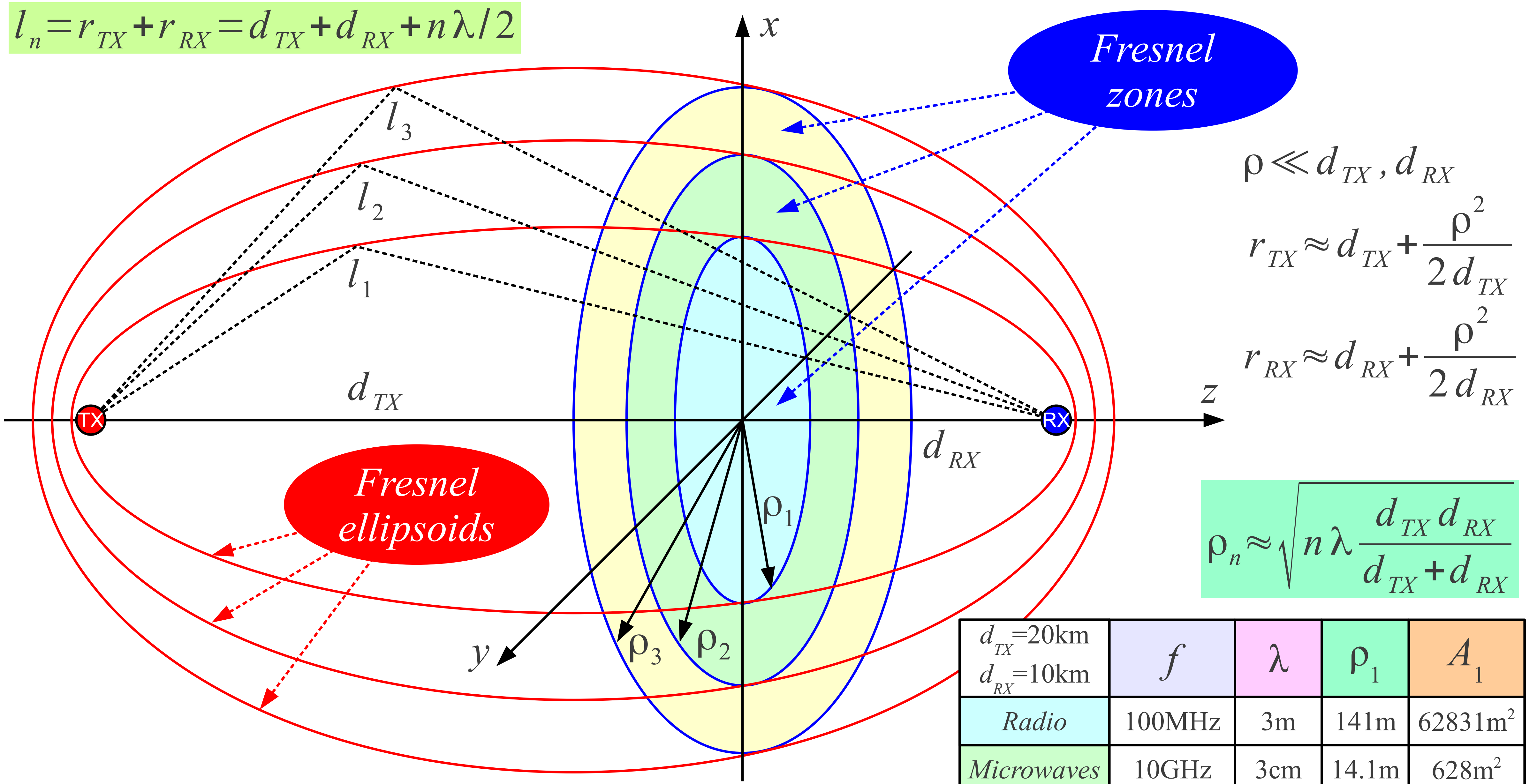
Wire loops $c \approx 0.9\lambda$
(*different shapes*, *loop−Yagi*)

Metal disks $2r \approx 0.3\lambda$
(*both polarizations*, *disk−Yagi*)
(*J.C.Simon* & *G.Weil* 1953)

Helix $0.75\lambda < c < 1.33\lambda$
(*circular polarization*, *J.Kraus* 1946)



Slow−wave structures



d_{TX}=20km d_{RX}=10km	f	λ	ρ_1	A_1
Radio	100MHz	3m	141m	62831m^2
Microwaves	10GHz	3cm	14.1m	628m^2
Light	600THz	0.5µm	5.8cm	0.0105m^2

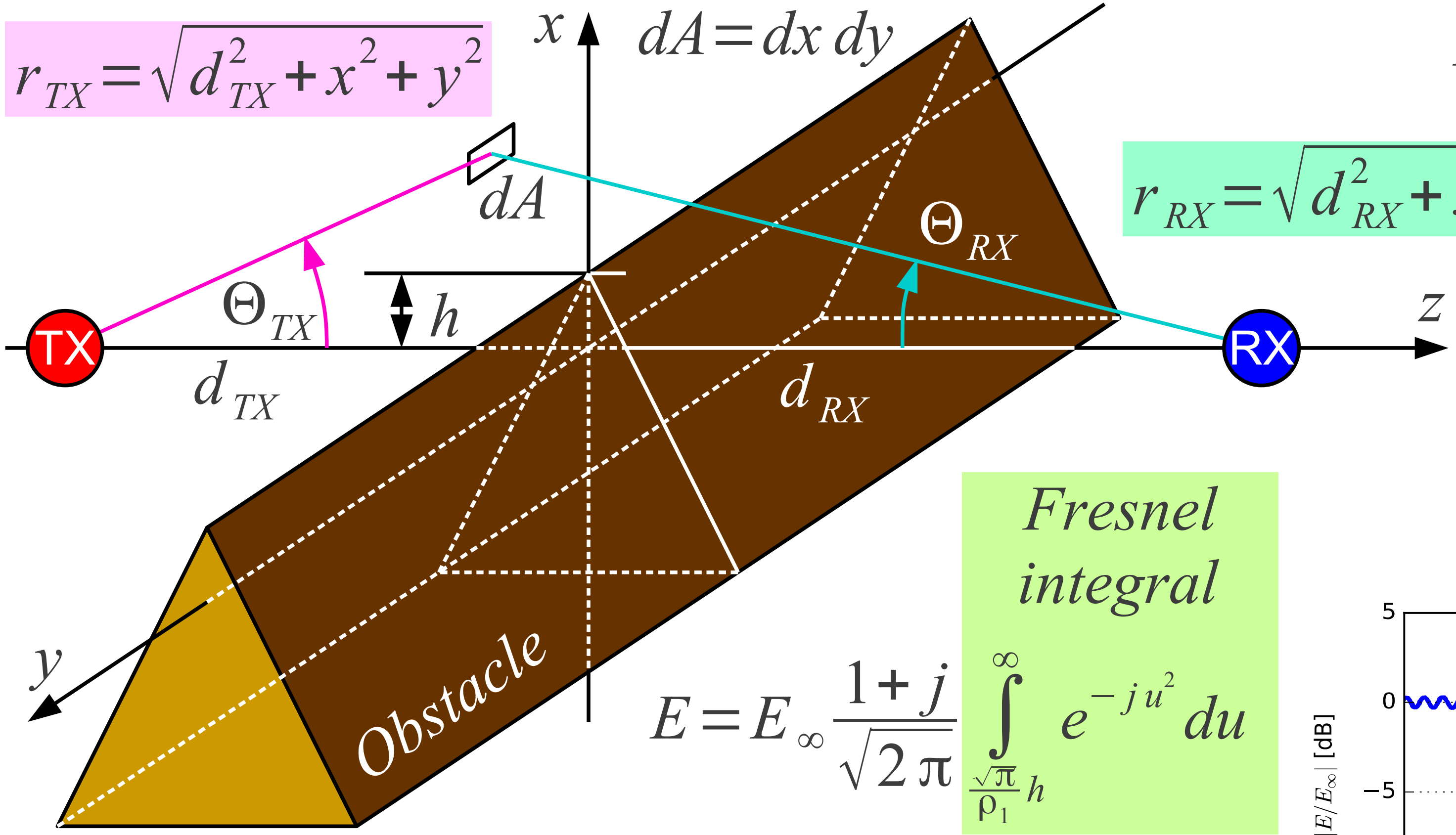
Fresnel ellipsoids

Knife–edge diffraction

$$r_{TX}=\sqrt{d_{TX}^2+x^2+y^2}$$

$$r_{RX}=\sqrt{d_{RX}^2+x^2+y^2}$$

$$\rho_1\approx\sqrt{\lambda\frac{d_{TX}d_{RX}}{d_{TX}+d_{RX}}}$$



$$E=E_\infty\frac{j}{\rho_1^2}\int_h^\infty e^{-j\pi\frac{x^2}{\rho_1^2}}dx\int_{-\infty}^{\infty}e^{-j\pi\frac{y^2}{\rho_1^2}}dy$$

Fresnel integral

$$E=E_\infty\frac{1+j}{\sqrt{2\pi}}\int_{\frac{\sqrt{\pi}}{\rho_1}h}^{\infty}e^{-ju^2}du$$

$$x,y\ll d_{TX},d_{RX}$$

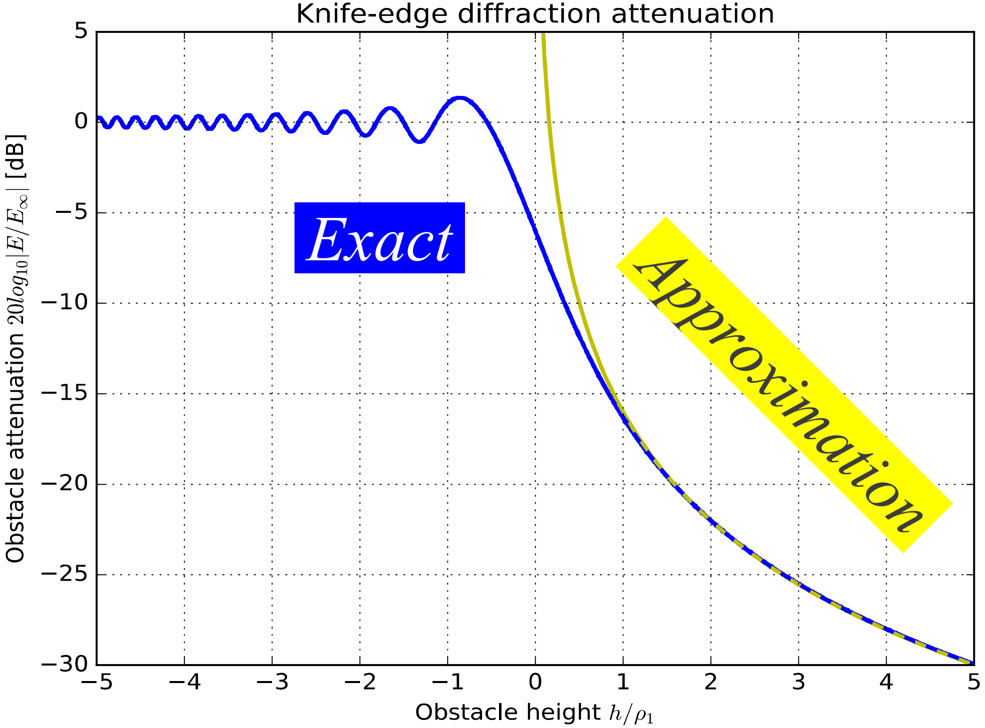
$$\cos\Theta_{TX}\approx1$$

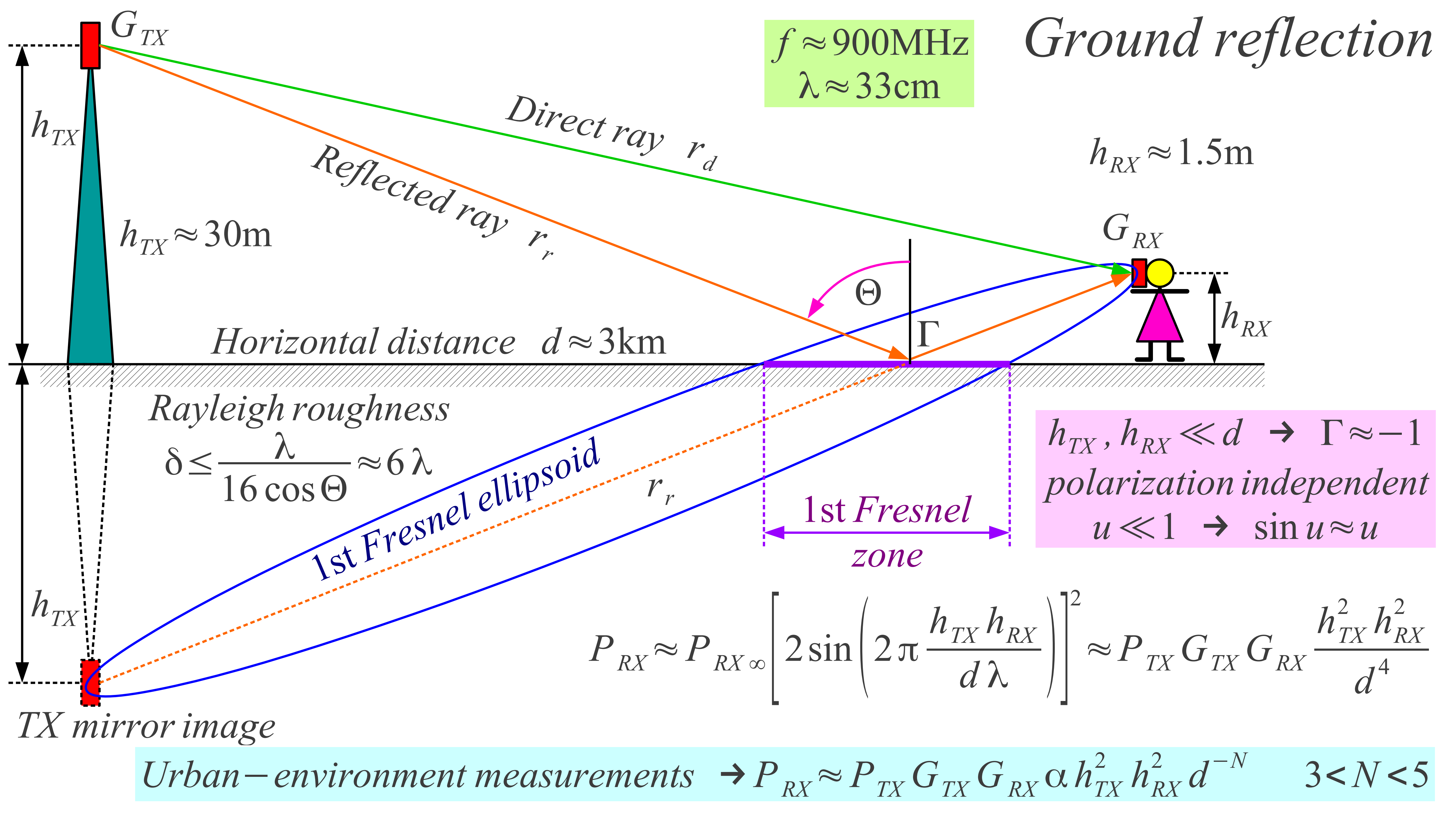
$$\cos\Theta_{RX}\approx1$$

$$a_{\mathrm{dB}}=20\log_{10}\frac{1}{\sqrt{\pi}}\left|\int_{\frac{\sqrt{\pi}}{\rho_1}h}^{\infty}e^{-ju^2}du\right|$$

Approximation $h\geq\rho_1$

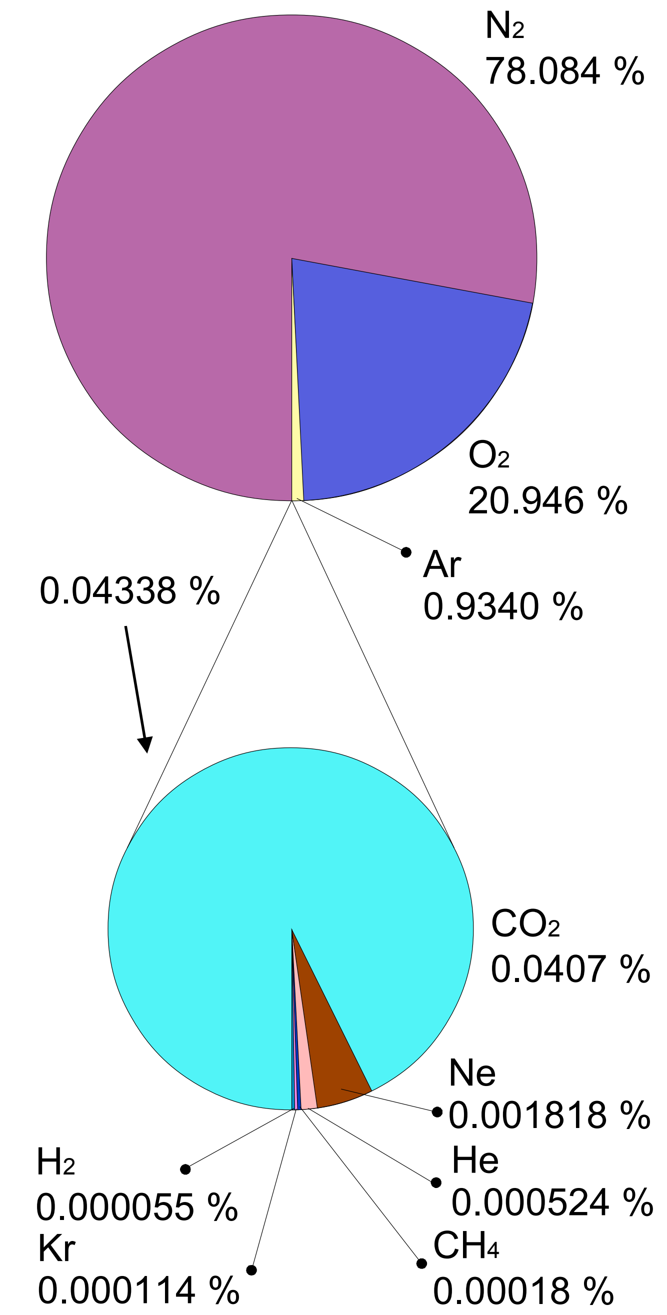
$$a_{\mathrm{dB}}\approx-16-20\log_{10}\frac{h}{\rho_1}$$



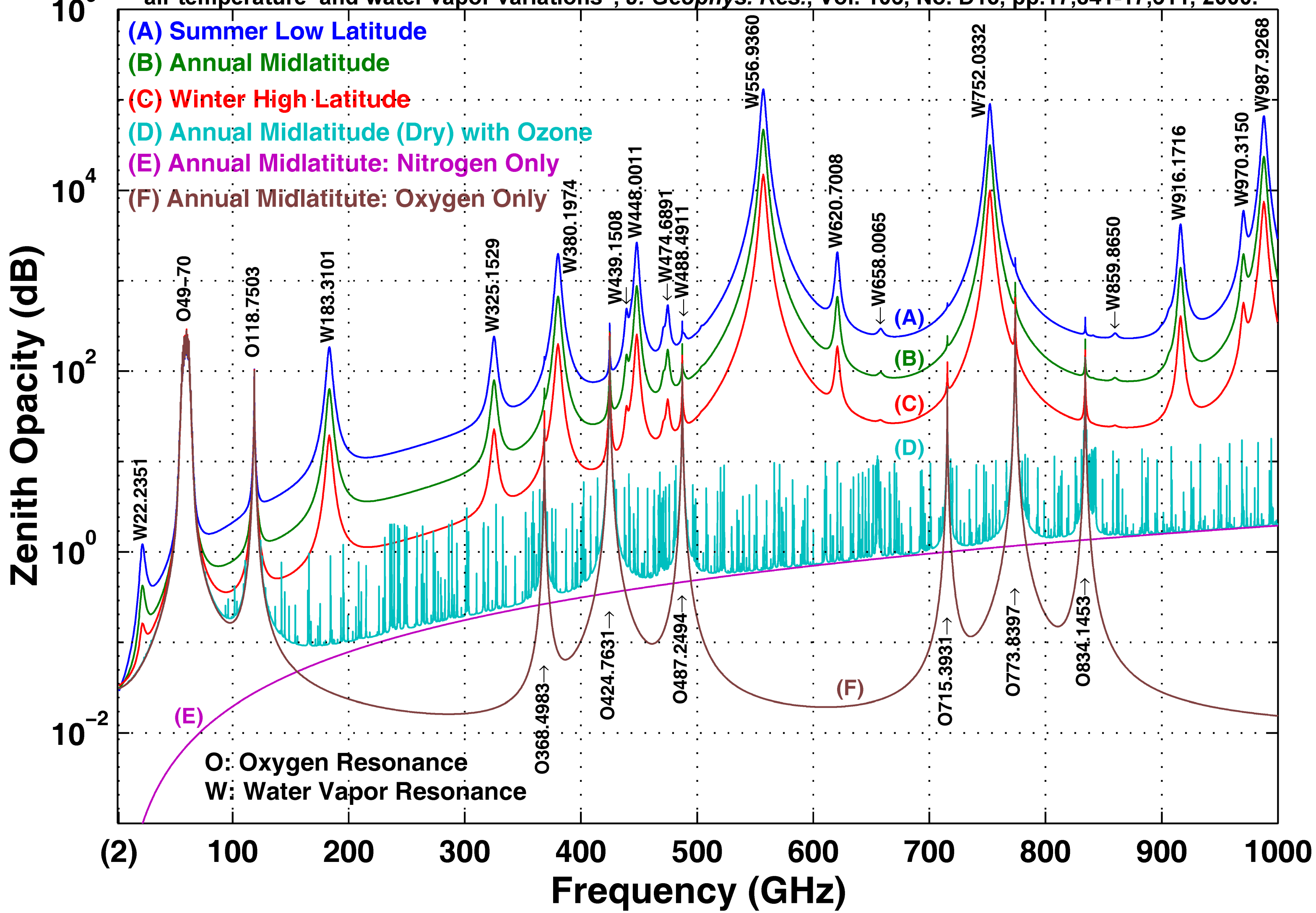


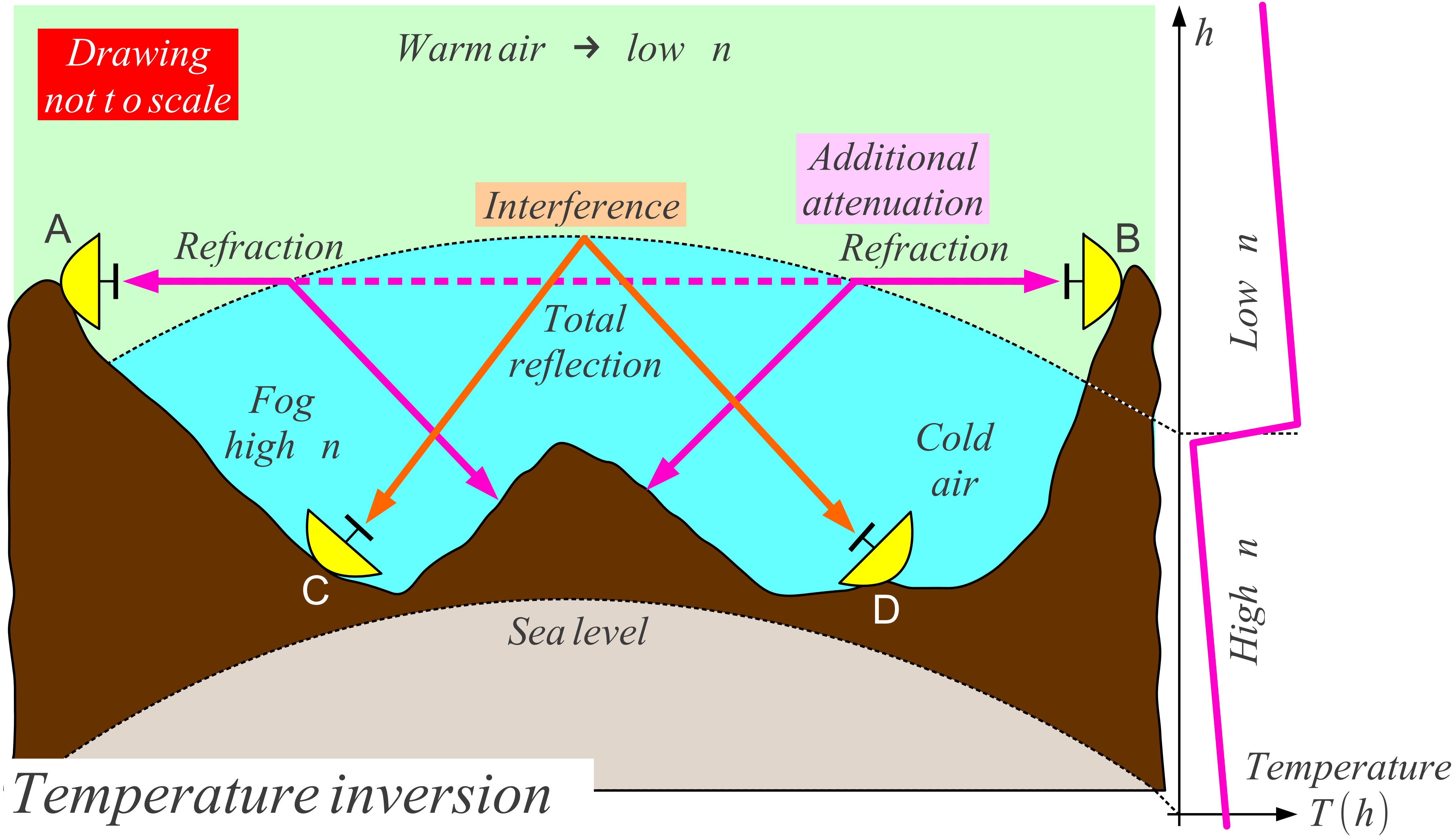
Ground reflection
$f \approx 900\text{MHz}$
$\lambda \approx 33\text{cm}$
G_{TX}
h_{TX}
$h_{TX} \approx 30\text{m}$
Direct ray r_d
Reflected ray r_r
$h_{RX} \approx 1.5\text{m}$
G_{RX}
Θ
Γ
h_{RX}
Horizontal distance $d \approx 3\text{km}$
Rayleigh roughness
$\delta \le \frac{\lambda}{16 \cos\Theta} \approx 6\lambda$
1st Fresnel ellipsoid
r_r
1st Fresnel zone
$h_{TX}, h_{RX} \ll d \rightarrow \Gamma \approx -1$
polarization independent
$u \ll 1 \rightarrow \sin u \approx u$
h_{TX}
TX mirror image
$P_{RX} \approx P_{RX\infty} \left[2 \sin\left(2\pi \frac{h_{TX} h_{RX}}{d \lambda} \right) \right]^2 \approx P_{TX} G_{TX} G_{RX} \frac{h_{TX}^2 h_{RX}^2}{d^4}$
Urban-environment measurements $\rightarrow P_{RX} \approx P_{TX} G_{TX} G_{RX} \alpha h_{TX}^2 h_{RX}^2 d^{-N}$ $3 < N < 5$

Atmospheric attenuation



Klein, M., A.J. Gasiewski: "Nadir sensitivity of passive millimeter and submillimeter wave channels to clear air temperature and water vapor variations", *J. Geophys. Res.*, Vol. 105, No. D13, pp.17,841-17,511, 2000.




Drawing not t o scale
Warm air → low n
Additional attenuation
Interference
A
Refraction
Refraction
B
Total reflection
Fog high n
Cold air
C
D
Sea level
h
Low n
High n
Temperature T(h)
Temperature inversion