Communication Electronics

Lecture 7:

Noise in electronics

Fifth Solvay International Conference on Electrons and Photons (October 1927). The leading figures Albert Einstein and Niels Bohr disagreed:

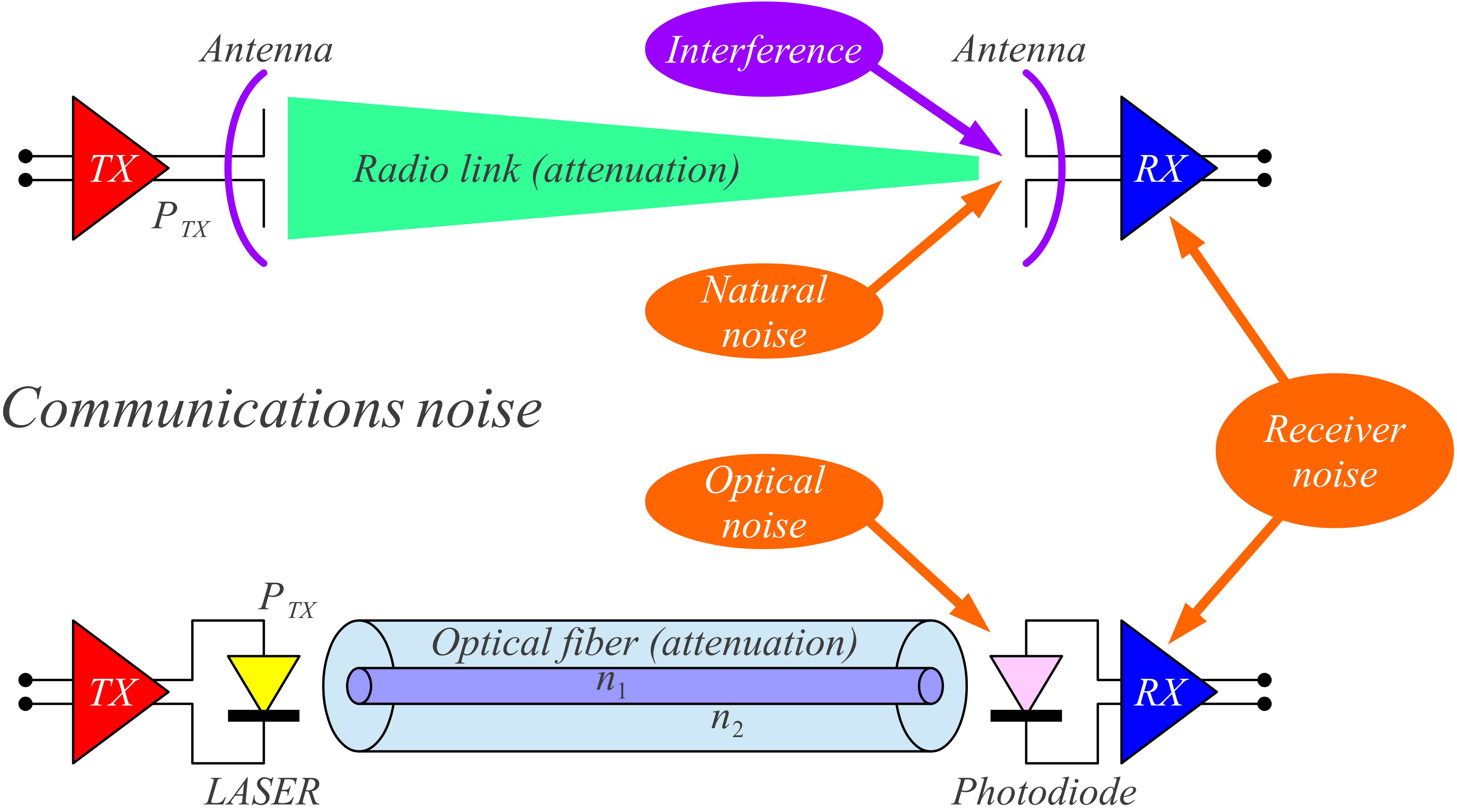
Albert Einstein: „God does not play dice!“

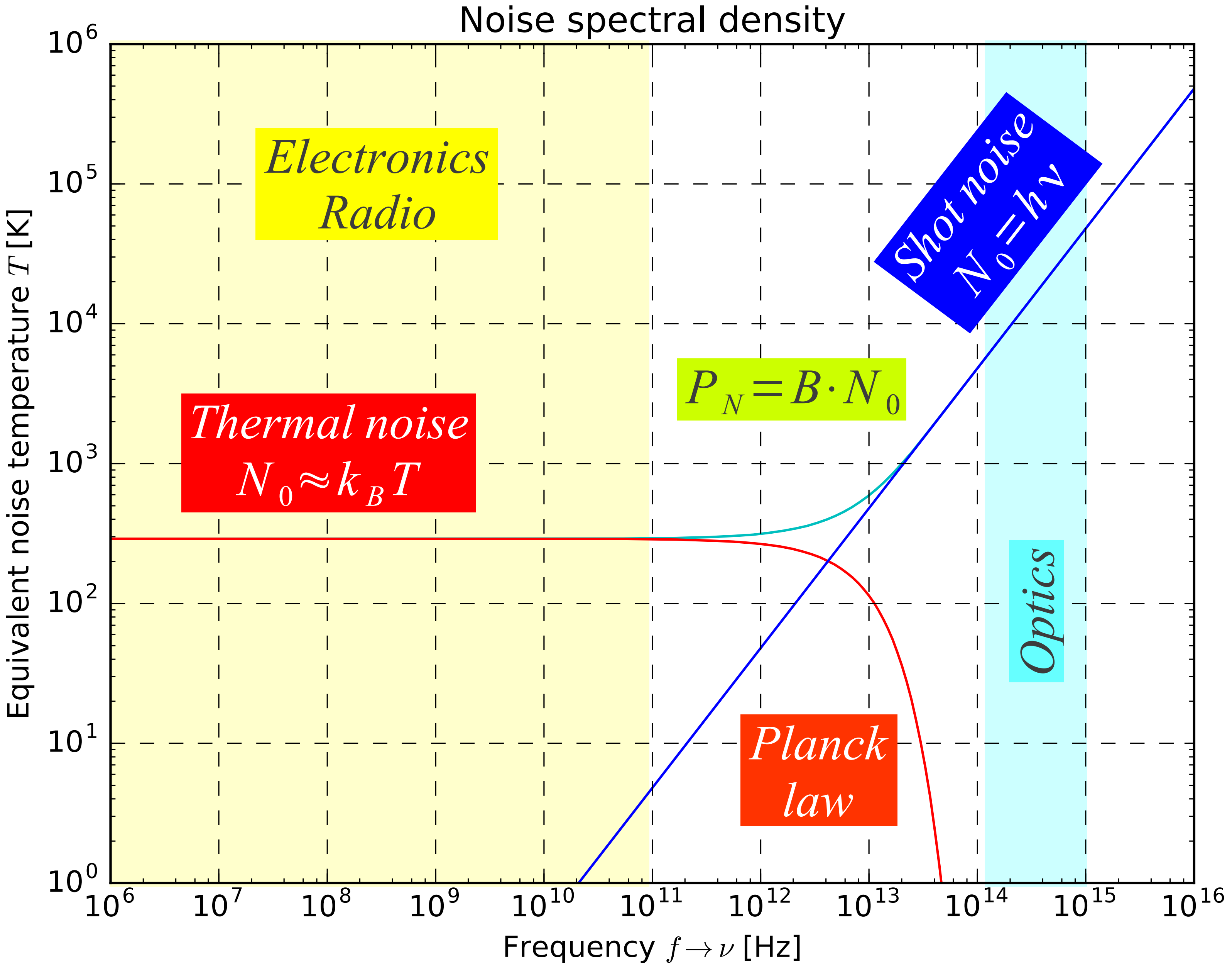
Niels Bohr: „Einstein, stop telling God what to do!“

In telecommunications random signals are called noise.
Noise impairs the performance of any communication link.

Noise is a macroscopic description of quantum effects!

Communications noise





Thermal noise:

$P_N = B\cdot k_B T$

Boltzmann constant [K]:

$k_B = 1.380649\cdot 10^{-23}$ J/K

Room temperature

$T_0 = 290$K

Shot noise:

$P_N = B\cdot h\nu$

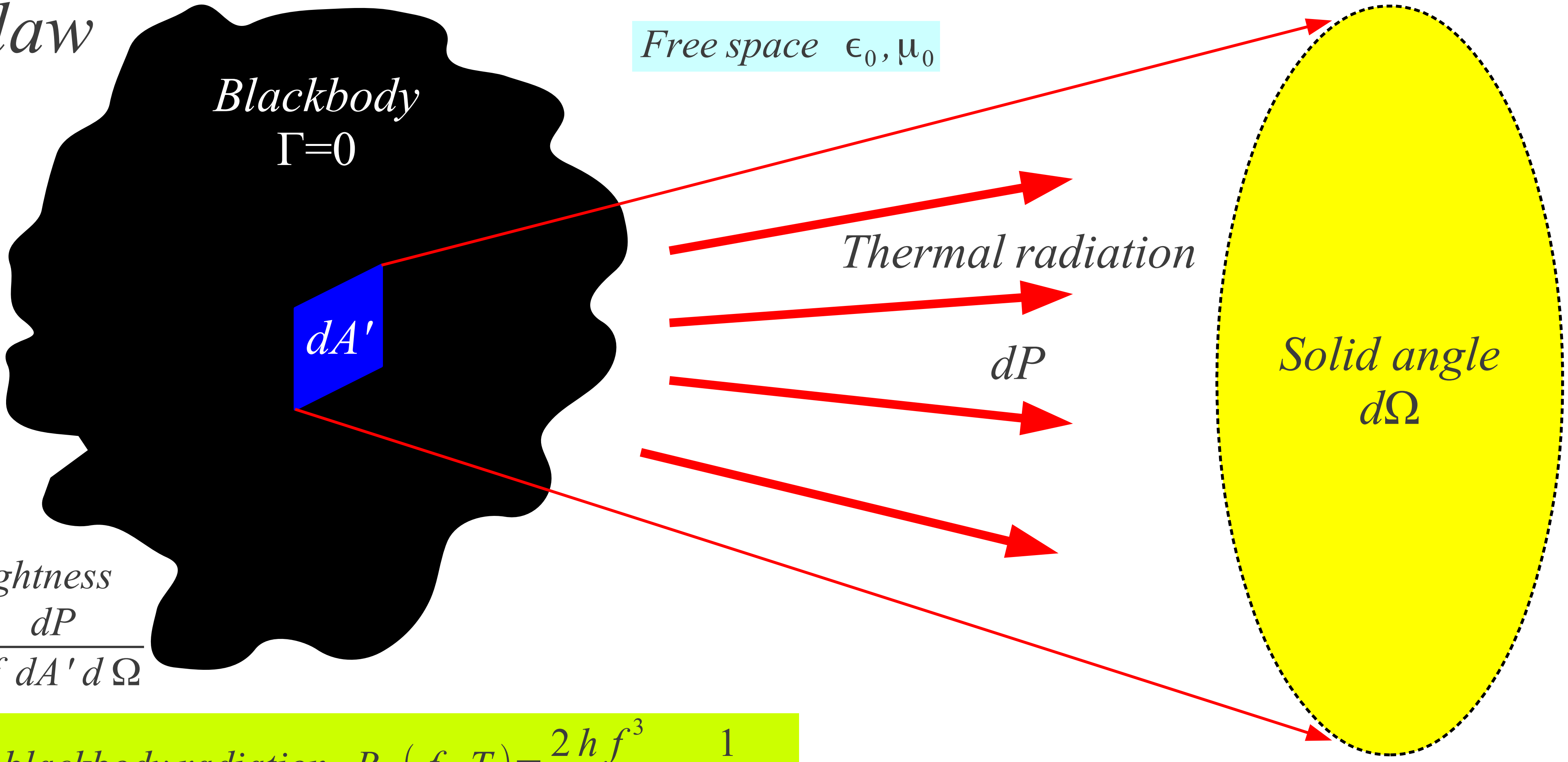
Planck constant [kg]:

$h = 6.62607015\cdot 10^{-34}$ Js

Equivalent $T = h\nu / k_B$

Natural noise

Planck law



Spectral brightness

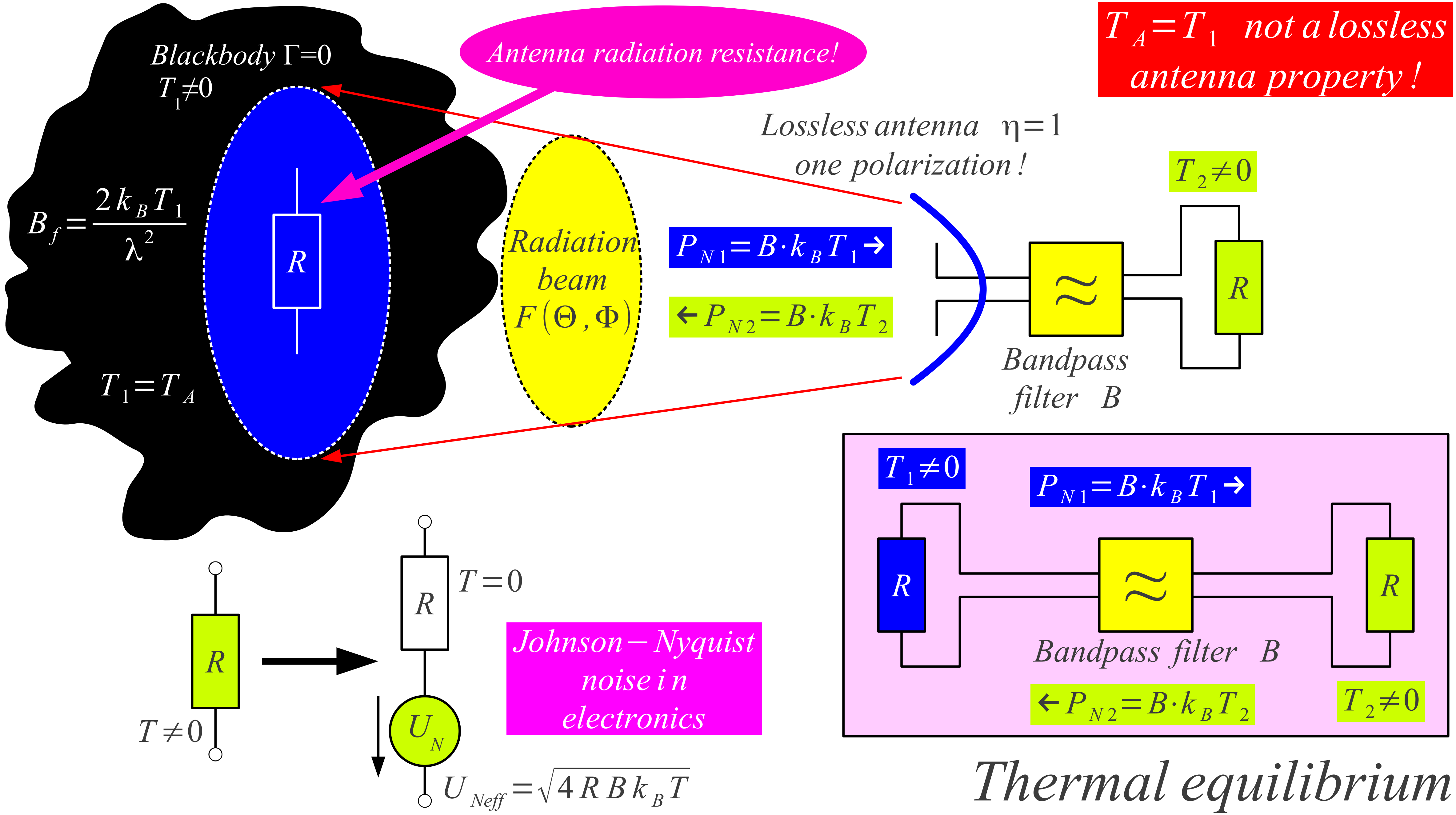
$$B_f(f,T)=\frac{dP}{df\; dA'\, d\Omega}$$

Planck law of blackbody radiation $B_f(f,T)=\frac{2hf^3}{c_0^2}\cdot\frac{1}{e^{\frac{hf}{k_BT}}-1}$

Radio
Electronics

$h f \ll k_B T$ → Rayleigh − Jeans approximation $B_f(f,T)\approx\frac{2k_BTf^2}{c_0^2}=\frac{2k_BT}{\lambda^2}$

Thermal equilibrium



$T_A = T_1$ not a lossless antenna property !

Blackbody $\Gamma = 0$

$T_1 \neq 0$

$B_f = \frac{2 k_B T_1}{\lambda^2}$

$T_1 = T_A$

Antenna radiation resistance!

Radiation beam $F(\Theta, \Phi)$

Lossless antenna $\eta = 1$

one polarization !

$P_{N1} = B \cdot k_B T_1 \rightarrow$

$\leftarrow P_{N2} = B \cdot k_B T_2$

$T_2 \neq 0$

Bandpass filter B

$T \neq 0$

$T = 0$

$U_{Neff} = \sqrt{4 R B k_B T}$

Johnson – Nyquist noise in electronics

$T_1 \neq 0$

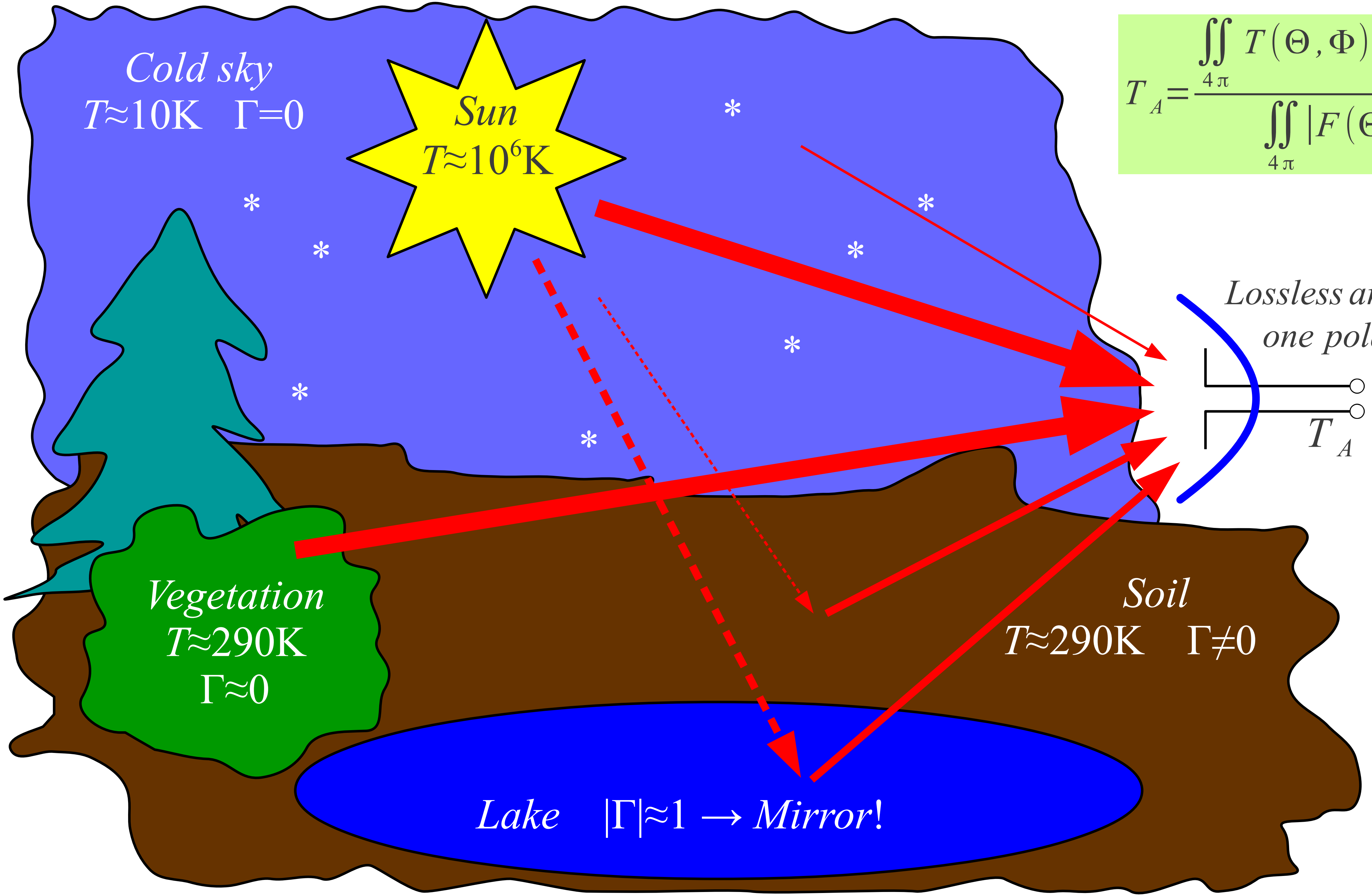
$P_{N1} = B \cdot k_B T_1 \rightarrow$

Bandpass filter B

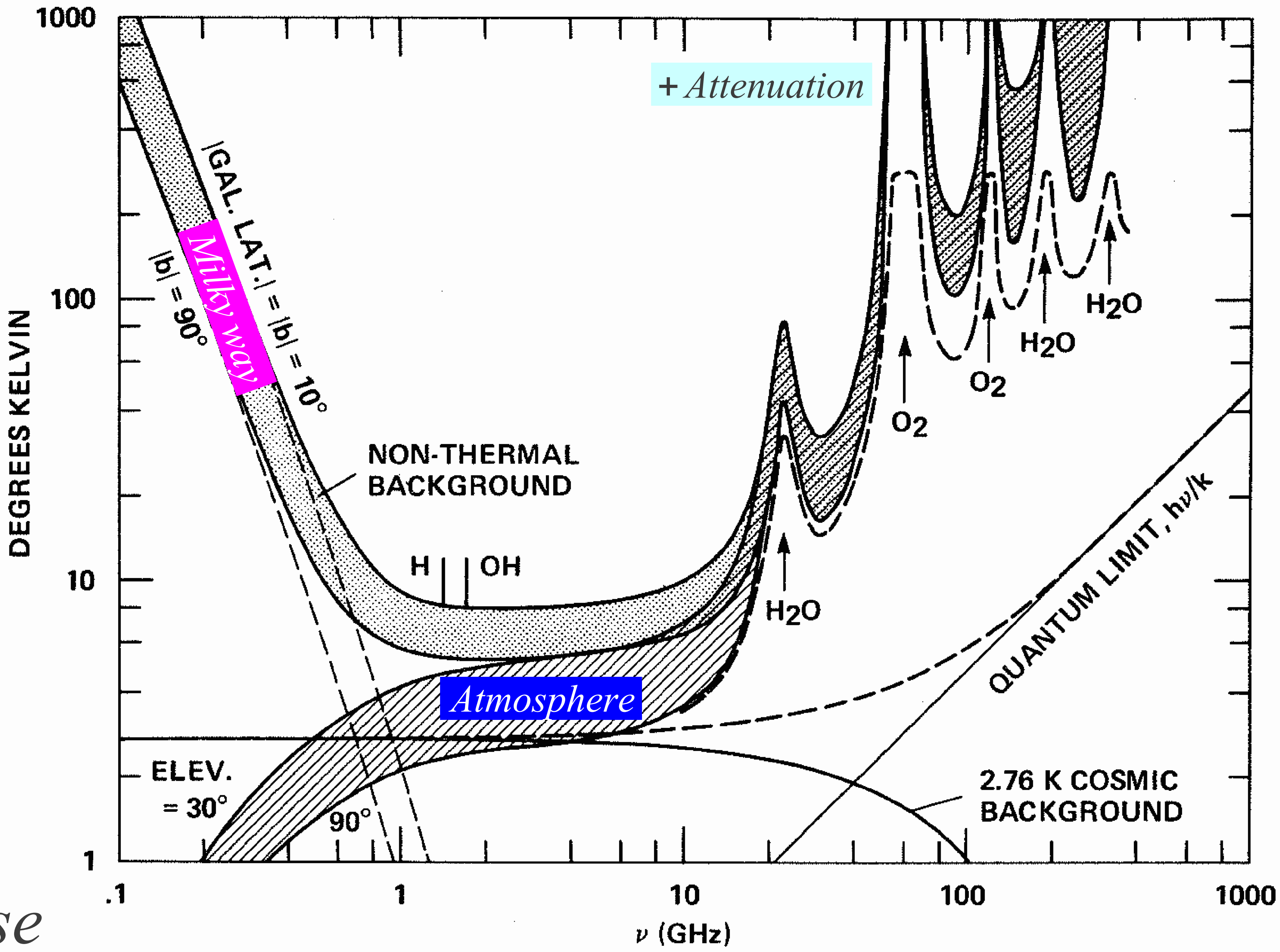
$\leftarrow P_{N2} = B \cdot k_B T_2$

$T_2 \neq 0$

$$T_A=\frac{\iint_{4\pi} T(\Theta,\Phi)\left|F(\Theta,\Phi)\right|^2 d\Omega}{\iint_{4\pi}\left|F(\Theta,\Phi)\right|^2 d\Omega}$$



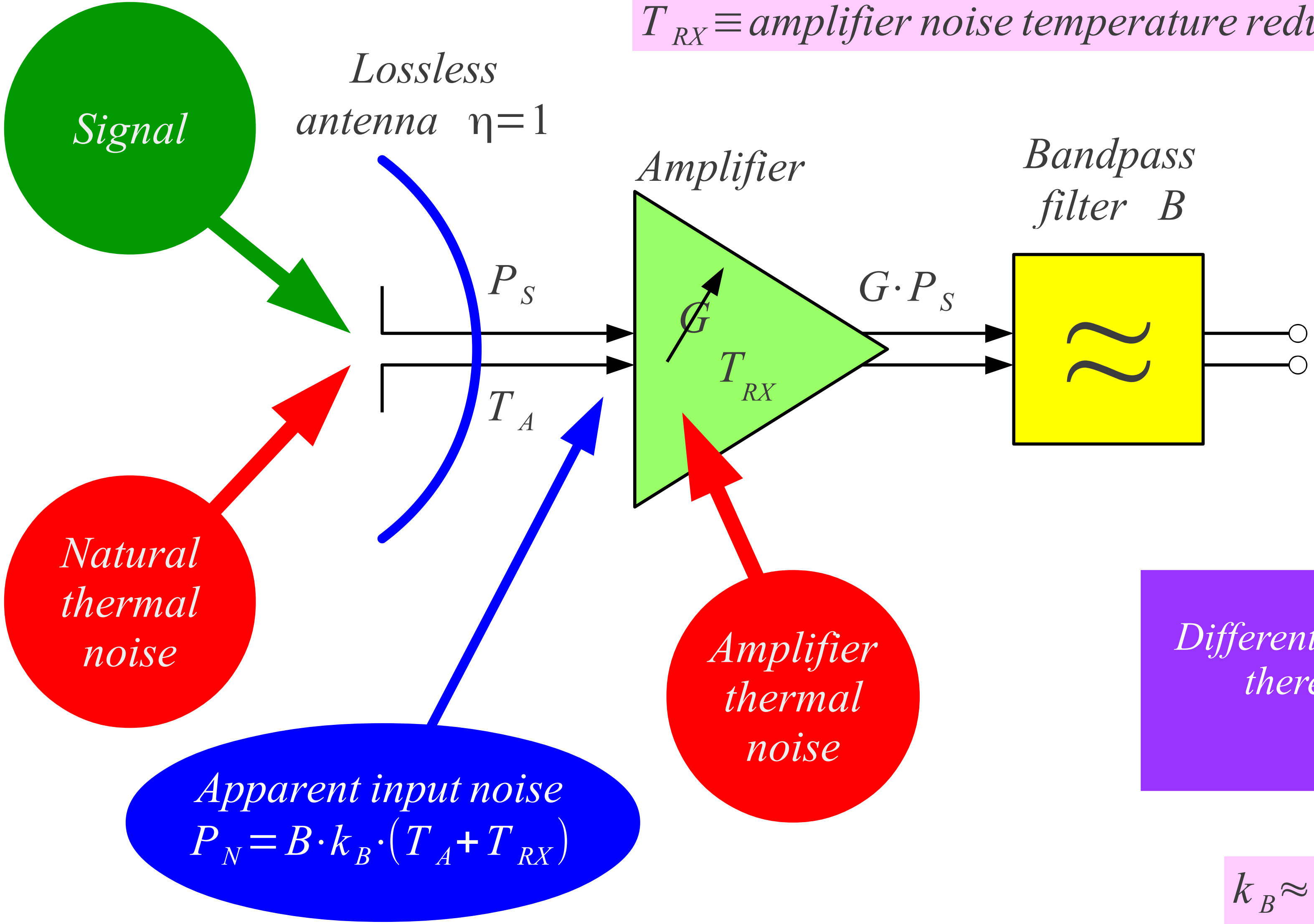
Noise sources


+ Attenuation
Milky way
Atmosphere
|GAL. LAT.| = |b| = 10°
|b| = 90°
NON-THERMAL
BACKGROUND
H
OH
H2O
O2
O2
H2O
H2O
QUANTUM LIMIT, hν/k
ELEV.
= 30°
90°
2.76 K COSMIC
BACKGROUND
DEGREES KELVIN
1000
100
10
1
.1
1
10
100
1000
ν (GHz)

Sky noise

Signal / noise ratio

$T_{RX} \equiv$ amplifier noise temperature reduced t o the input !



Apparent input noise $P_N = B \cdot k_B \cdot (T_A + T_{RX})$

$$P_S' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S'}{P_N'} = \frac{P_S}{B \cdot k_B \cdot (T_A + T_{RX})}$$

$$P_N' = G \cdot B \cdot k_B \cdot (T_A + T_{RX})$$

Different noise sources are uncorrelated, therefore summing noise powers (noise temperatures)!

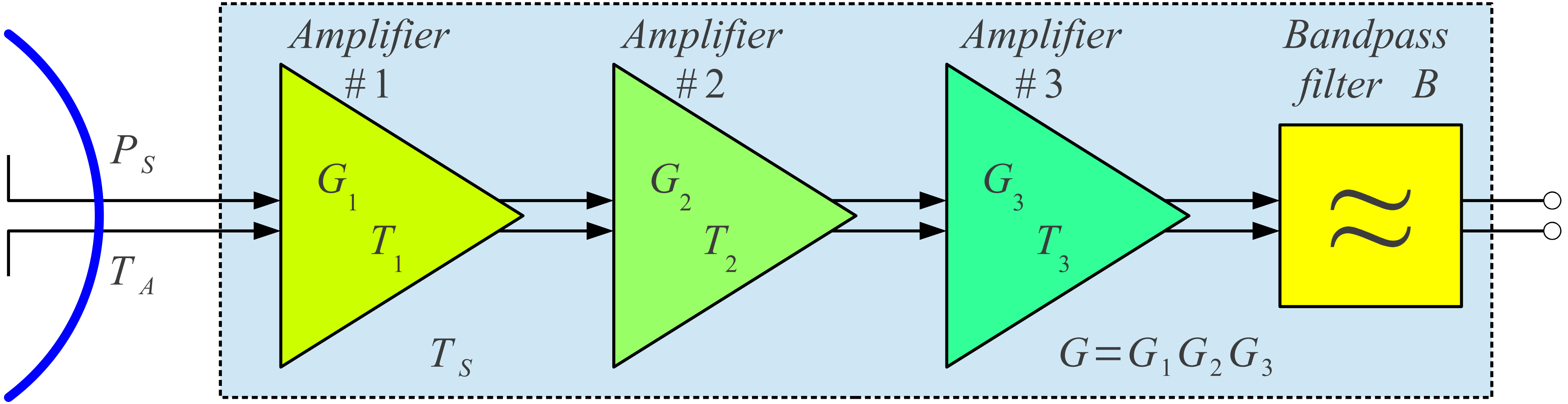
$k_B \approx 1.38 \cdot 10^{-23}\ \mathrm{J/K} \qquad T_0 = 290\mathrm{K} \approx 17^0\mathrm{C}$

$$10 \log_{10} \frac{k_B T_0}{1\mathrm{mJ}} \approx -174\ \mathrm{dBm/Hz}$$

All impedances mathched $Z_{input} \approx Z_{output} \approx Z_K = 50\,\Omega$

Amplifier chain

$$P_S' = G_3 G_2 G_1 P_S$$

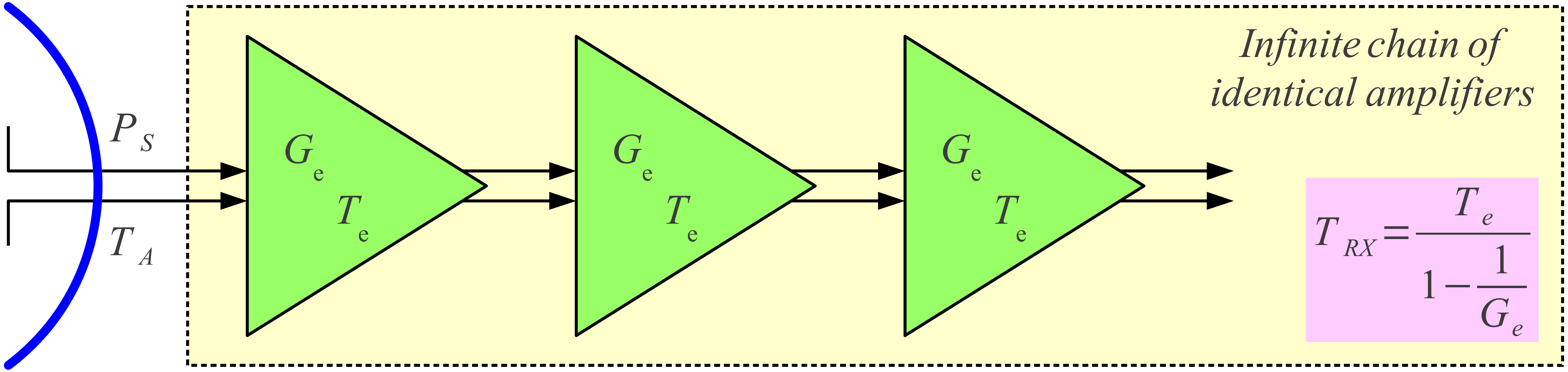


Lossless antenna $\eta = 1$

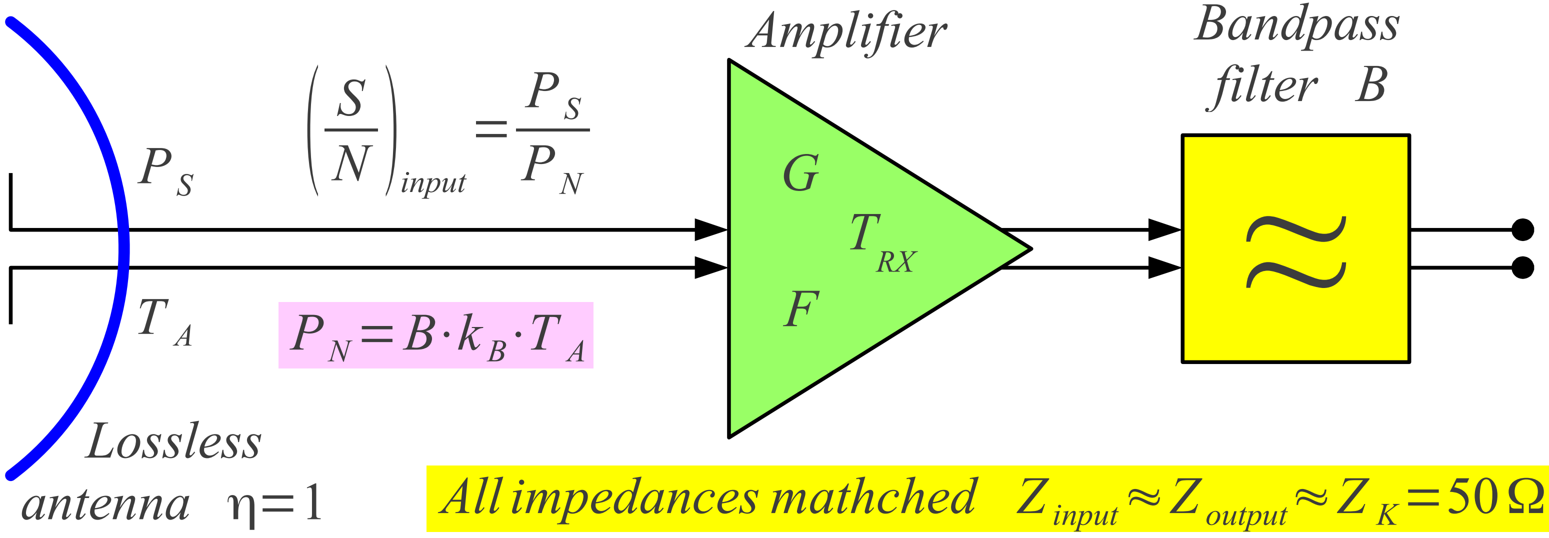
$$T_{RX} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$

$$P_N' = B k_B \left[G_3 G_2 G_1 (T_A + T_1) + G_3 G_2 T_2 + G_3 T_3 \right]$$

All impedances mathched $Z_{input} \approx Z_{output} \approx Z_K = 50\,\Omega$



$$T_{RX} = \frac{T_e}{1 - \frac{1}{G_e}}$$



$$\left(\frac{S}{N}\right)_{input}=\frac{P_S}{P_N}$$

$$P_N=B\cdot k_B\cdot T_A$$

$$P_S{}'=G\cdot P_S$$

$$\left(\frac{S}{N}\right)_{output}=\frac{P_S{}'}{P_N{}'}=\frac{P_S}{B\cdot k_B\cdot(T_A+T_{RX})}$$

$$P_N{}'=G\cdot B\cdot k_B\cdot(T_A+T_{RX})$$

All impedances mathched $Z_{input}\approx Z_{output}\approx Z_K=50\,\Omega$

Nonsense definition of the noise figure:

$$F=\frac{\left(\frac{S}{N}\right)_{input}}{\left(\frac{S}{N}\right)_{output}}=\frac{\dfrac{P_S}{B\,k_B\,T_A}}{\dfrac{G\,P_S}{G\,B\,k_B(T_A+T_{RX})}}=\frac{T_A+T_{RX}}{T_A}=1+\frac{T_{RX}}{T_A}$$

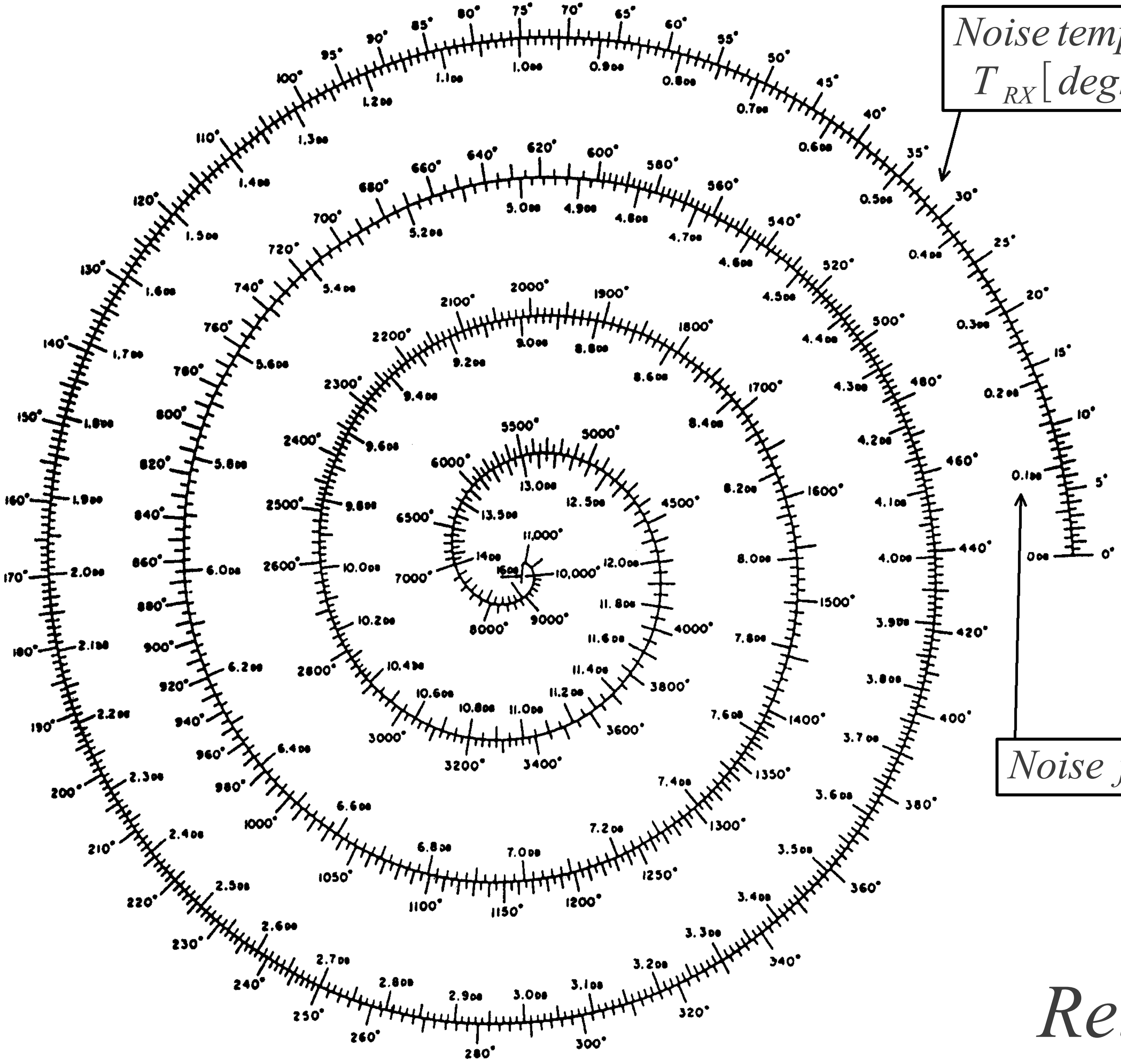
A property of an amplifier can not be a function of T_A*!*

Sensible definition $F=1+\dfrac{T_{RX}}{T_0}$ @ $T_0=290\text{K}$ ←→ $T_{RX}=T_0(F-1)$

Noise figure $F>1$ $F_{\text{dB}}>0$

Logarithmic units $F_{\text{dB}}=10\log_{10}F=10\log_{10}\left(1+\dfrac{T_{RX}}{T_0}\right)$ ←→ $T_{RX}=T_0\left(10^{\frac{F_{\text{dB}}}{10}}-1\right)$

Noise figure



Noise temperature
T_{RX} [*degrees K*]

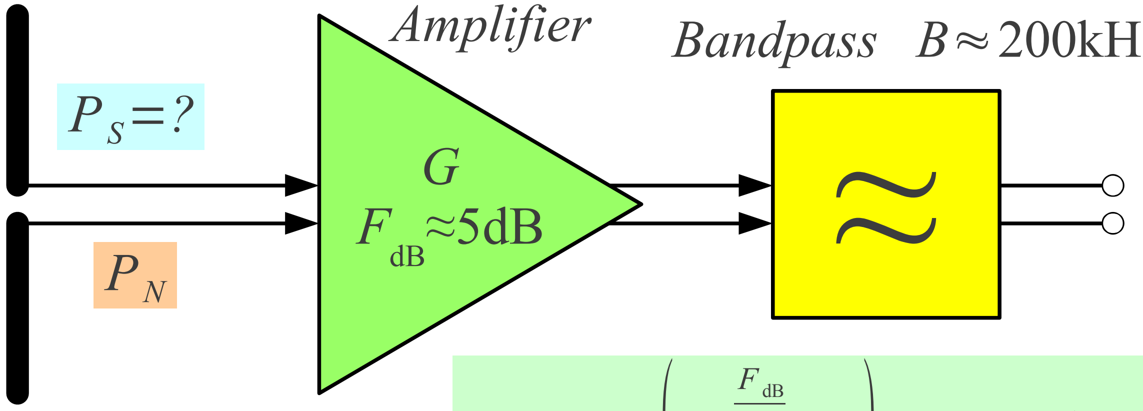
Noise figure F_{dB}

$$F_{\mathrm{dB}} = 10\log_{10}\left(1 + \frac{T_{RX}}{290\mathrm{K}}\right)$$

Agreement :
$T_0 = 290\mathrm{K}$

$$T_{RX} = 290\mathrm{K}\left(10^{\frac{F_{\mathrm{dB}}}{10}} - 1\right)$$

Relationship $T_{RX} \leftrightarrow F_{\mathrm{dB}}$



Omnidirectional antenna
$T_A \approx T_0 = 290\text{K}$

$P_S = ?$

P_N

Amplifier

G

$F_{\text{dB}} \approx 5\text{dB}$

Bandpass $B \approx 200\text{kHz}$ (*GSM phone*)

$\left(\frac{S}{N}\right)_{\text{dB}} \geq 10\text{dB}$

$\left(\frac{S}{N}\right) = 10^{\frac{\left(\frac{S}{N}\right)_{\text{dB}}}{10}} \geq 10$

$k_B \approx 1.38 \cdot 10^{-23}\,\text{J/K}$

$T_{RX} = T_0 \cdot \left(10^{\frac{F_{\text{dB}}}{10}} - 1\right) = 290\text{K} \cdot (3.162 - 1) = 627\text{K}$

$P_N = B \cdot k_B \cdot (T_A + T_{RX}) = 200\text{kHz} \cdot 1.38 \cdot 10^{-23}\,\text{J/K} \cdot (290\text{K} + 627\text{K}) = 2.531 \cdot 10^{-15}\,\text{W}$

$P_S = P_N \cdot \left(\frac{S}{N}\right) = 2.531 \cdot 10^{-15}\,\text{W} \cdot 10 = 2.531 \cdot 10^{-14}\,\text{W}$

$P_{S\,\text{dBm}} = 10 \log_{10} \frac{P_S}{1\text{mW}} = -106\text{dBm}$

Simplified calculation valid only for $T_A \approx T_0 = 290\text{K}$

$P_{S\,\text{dBm}} \approx (S/N)_{\text{dB}} + (B)_{\text{dB}\cdot\text{Hz}} + (k_B T_0)_{\text{dBm/Hz}} + F_{\text{dB}}$

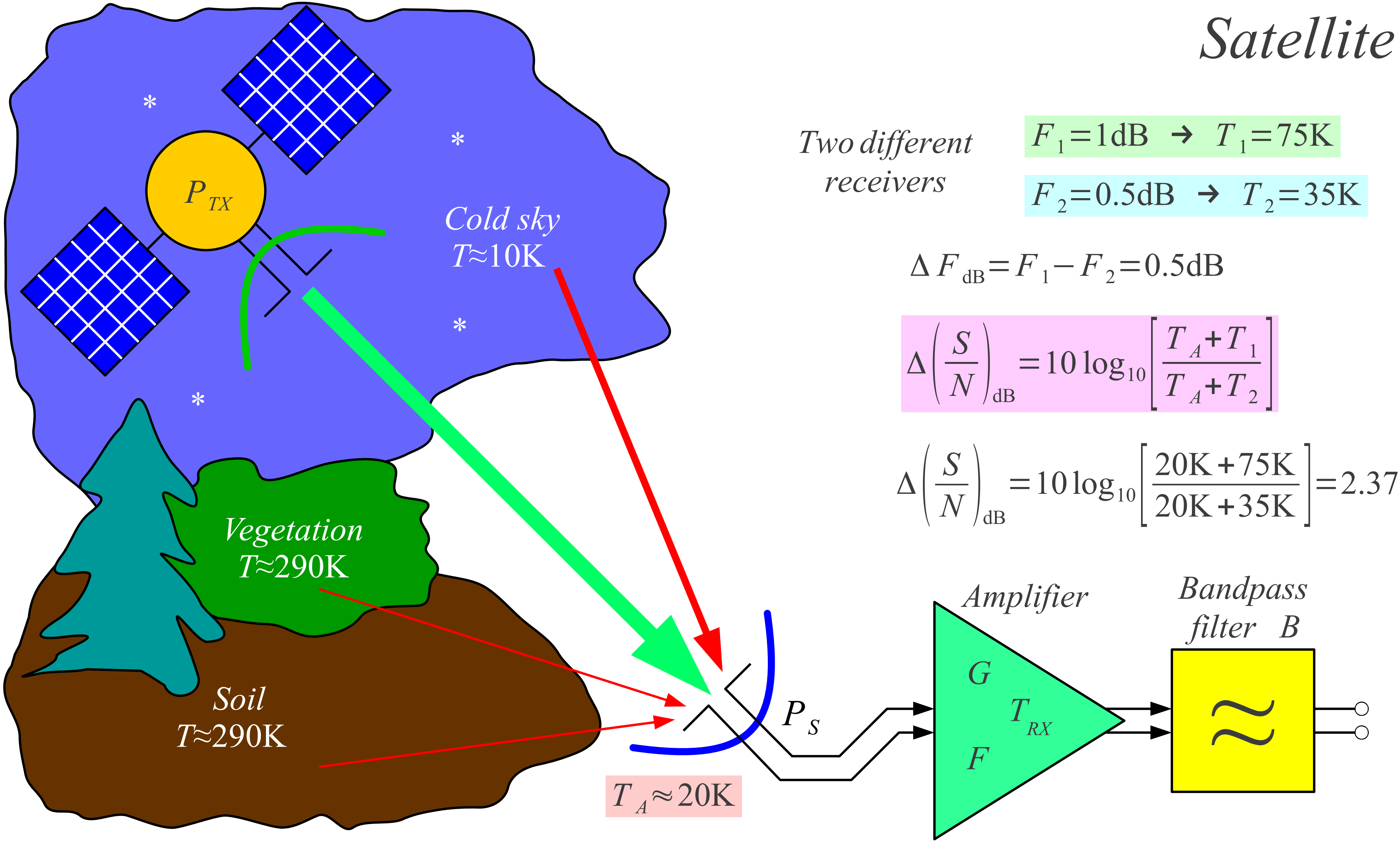
$(B)_{\text{dB}\cdot\text{Hz}} = 10 \log_{10}\left(\frac{B}{1\text{Hz}}\right) = 53\text{dB}\cdot\text{Hz}$

$(k_B T_0)_{\text{dBm/Hz}} = 10 \log_{10} \frac{k_B T_0}{1\text{mJ}} \approx$
$\approx -174\,\text{dBm/Hz}$

$P_{S\,\text{dBm}} \approx 10\text{dB} + 53\text{dB}\cdot\text{Hz} - 174\text{dBm/Hz} + 5\text{dB} = -106\text{dBm}$

GSM phone

Satellite TV



Two different receivers

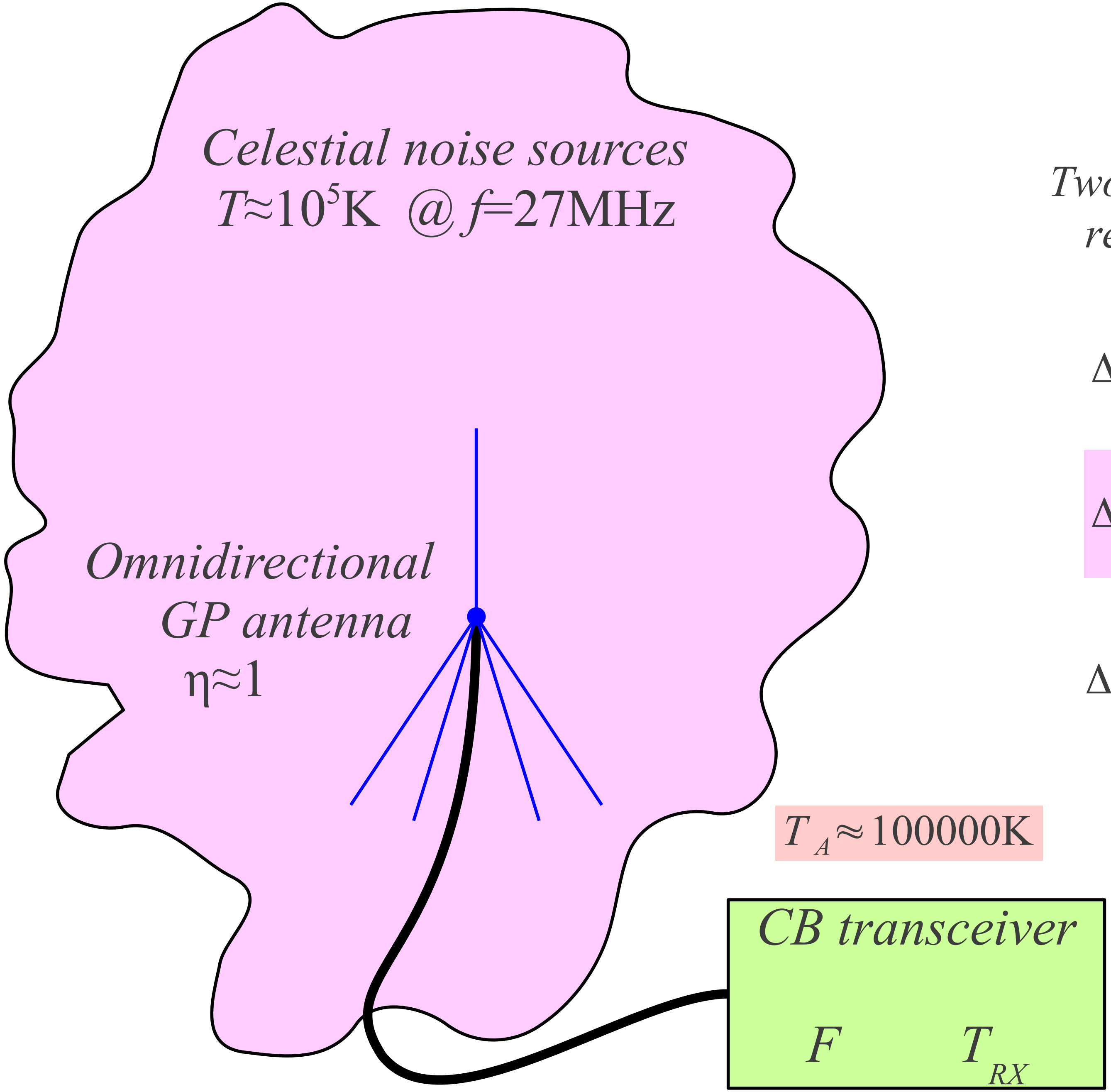
$F_1 = 1\mathrm{dB} \rightarrow T_1 = 75\mathrm{K}$

$F_2 = 0.5\mathrm{dB} \rightarrow T_2 = 35\mathrm{K}$

$$\Delta F_{\mathrm{dB}} = F_1 - F_2 = 0.5\mathrm{dB}$$

$$\Delta\left(\frac{S}{N}\right)_{\mathrm{dB}} = 10\log_{10}\left[\frac{T_A + T_1}{T_A + T_2}\right]$$

$$\Delta\left(\frac{S}{N}\right)_{\mathrm{dB}} = 10\log_{10}\left[\frac{20\mathrm{K} + 75\mathrm{K}}{20\mathrm{K} + 35\mathrm{K}}\right] = 2.37\mathrm{dB}$$



Celestial noise sources

$T \approx 10^5\text{K}$ @ $f = 27\text{MHz}$

Omnidirectional GP antenna

$\eta \approx 1$

$T_A \approx 100000\text{K}$

CB transceiver

F T_{RX}

Two different receivers

$F_1 = 10\text{dB} \rightarrow T_1 = 2610\text{K}$

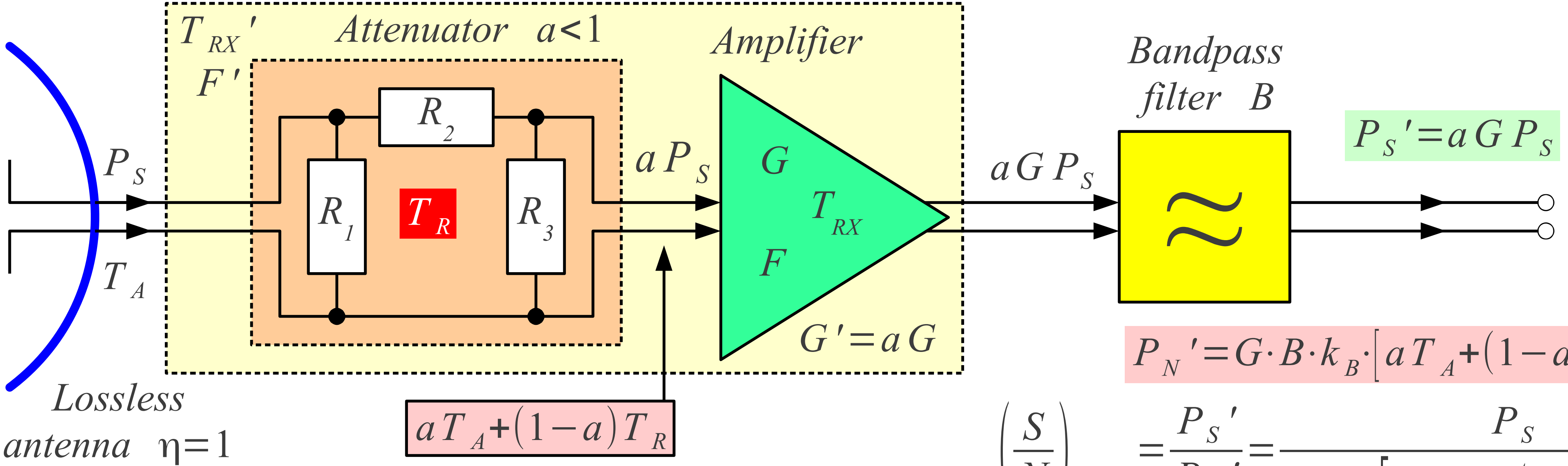
$F_2 = 5\text{dB} \rightarrow T_2 = 627\text{K}$

$$\Delta F_{\text{dB}} = F_1 - F_2 = 5\text{dB}$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{T_A + T_1}{T_A + T_2}\right]$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{100000\text{K} + 2610\text{K}}{100000\text{K} + 627\text{K}}\right] = 0.085\text{dB}$$

HF transceiver



$$P_N'=G\cdot B\cdot k_B\cdot\left[aT_A+(1-a)T_R+T_{RX}\right]$$

$$\left(\frac{S}{N}\right)_{output}=\frac{P_S'}{P_N'}=\frac{P_S}{B\cdot k_B\cdot\left[T_A+T_R\left(\frac{1}{a}-1\right)+\frac{T_{RX}}{a}\right]}$$

All impedances mathched $Z_{input}\approx Z_{output}\approx Z_K=50\,\Omega$

$$F'=1+\frac{T_{RX}'}{T_0}=1+\frac{T_R}{T_0}\left(\frac{1}{a}-1\right)+\frac{T_{RX}}{aT_0}$$

$$T_{RX}'=T_R\left(\frac{1}{a}-1\right)+\frac{T_{RX}}{a}$$

Frequent case $T_R\approx T_0=290\text{K}$

$$F'\approx\frac{1}{a}+\frac{T_{RX}}{aT_0}=\frac{1}{a}\left(1+\frac{T_{RX}}{T_0}\right)=\frac{F}{a}$$

$F_{\mathrm{dB}}'\approx F_{\mathrm{dB}}-a_{\mathrm{dB}}$ $a_{dB}=10\log_{10}a<0$

Attenuator examples $T_R\approx T_0=290\text{K}$

(1) lossy antenna $\eta_{\mathrm{dB}}=10\log_{10}\eta<0$

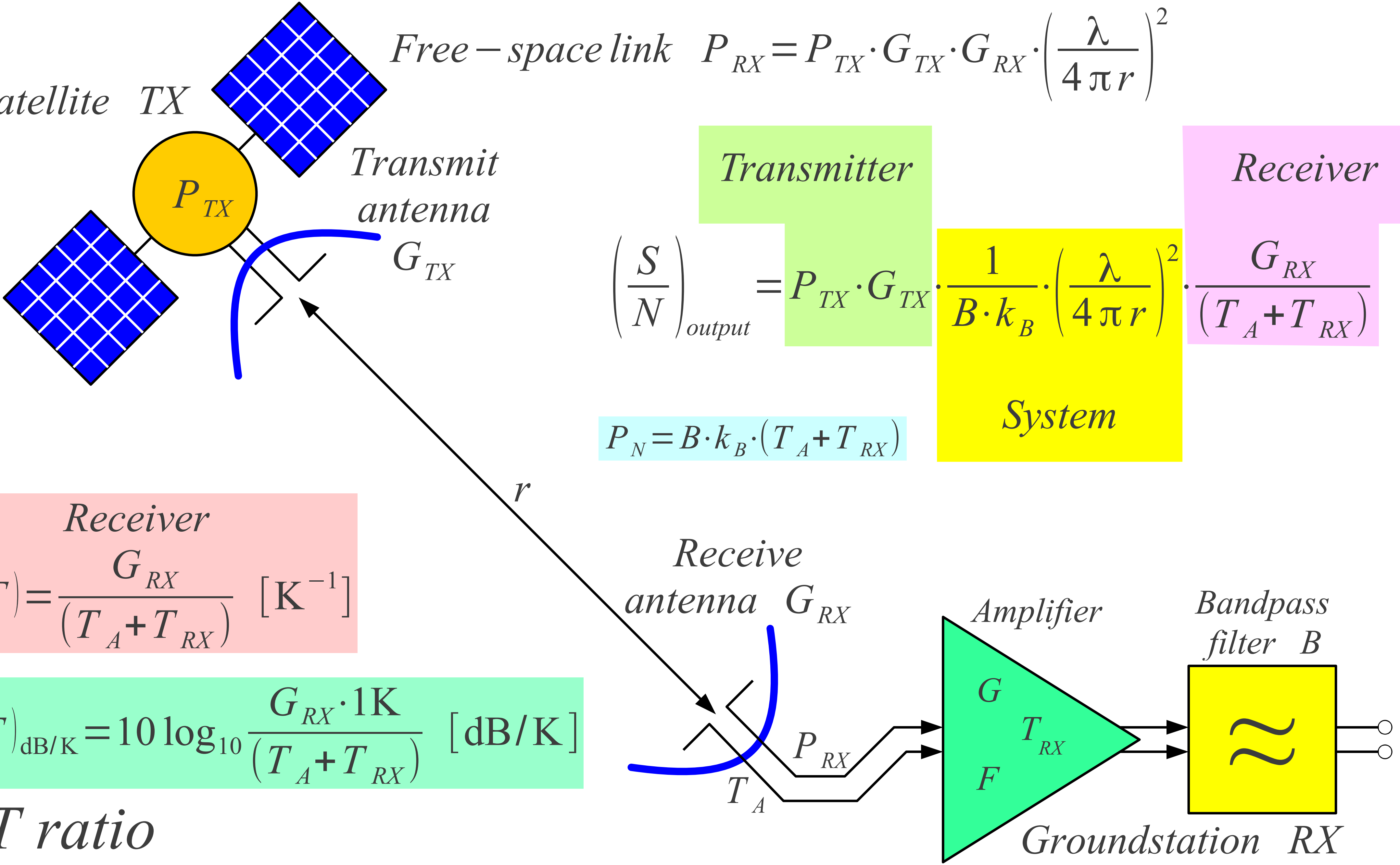
(2) lossy transmission line $a_{\mathrm{dB}}<0$

(3) filter bandpass loss $a_{\mathrm{dB}}<0$

(4) mixer insertion loss $a_{\mathrm{dB}}<0$

Attenuator noise

G/T ratio



Free-space link $P_{RX} = P_{TX} \cdot G_{TX} \cdot G_{RX} \cdot \left(\frac{\lambda}{4\pi r}\right)^2$

$$\left(\frac{S}{N}\right)_{output} = \overbrace{P_{TX} \cdot G_{TX}}^{\text{Transmitter}} \cdot \underbrace{\frac{1}{B \cdot k_B} \cdot \left(\frac{\lambda}{4\pi r}\right)^2}_{\text{System}} \cdot \overbrace{\frac{G_{RX}}{(T_A + T_{RX})}}^{\text{Receiver}}$$

$$P_N = B \cdot k_B \cdot (T_A + T_{RX})$$

$$(G/T) = \overbrace{\frac{G_{RX}}{(T_A + T_{RX})}}^{\text{Receiver}} \quad [\mathrm{K}^{-1}]$$

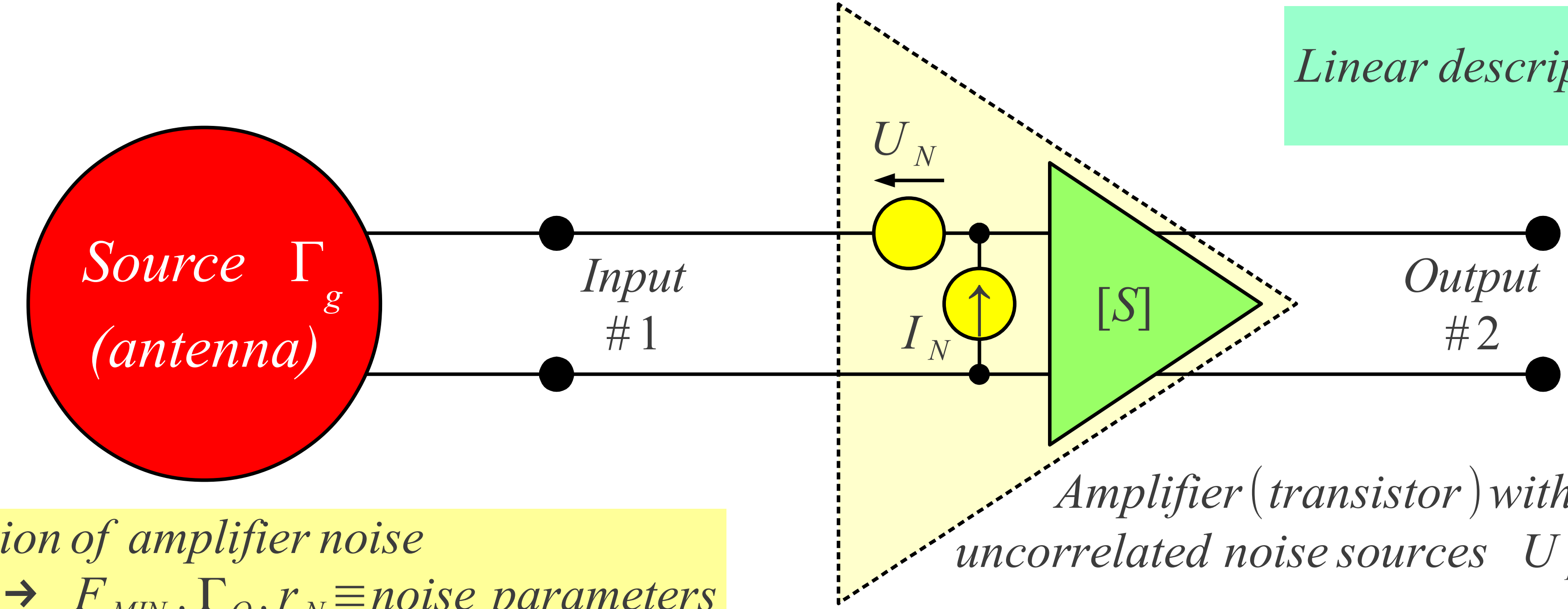
$$(G/T)_{\mathrm{dB/K}} = 10 \log_{10} \frac{G_{RX} \cdot 1\mathrm{K}}{(T_A + T_{RX})} \quad [\mathrm{dB/K}]$$

Amplifier device	*Gain* G [dB]	*Noise temperature* T_{RX} [K]	*Noise figure* F_{dB} [dB]
Vacuum tube with control grid (triode, pentode)	10↔20	1600↔9000	8↔15
Vacuum tube with speed modulation (klystron, TWT)	20↔50	3000↔30000	10↔20
Parametric amplifier (varactor room temperature)	10↔15	75↔300	1↔3
Si BJT or JFET or MOSFET (room temperature)	10↔20	75↔300	1↔3
GaAs FET or HEMT (room temperature)	10↔15	20↔120	0.3↔1.5
GaAs FET or HEMT (cooled liquid-nitrogen 77K)	10↔15	7↔35	0.1↔0.5
Si or SiGe or InGaP MMIC amplifier	10↔25	170↔1600	2↔8
Operational amplifier	40↔100	10^4↔10^9	16↔66

Active − device noise

Noise parameters

Linear description $[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$



Amplifier (transistor) with uncorrelated noise sources U_N, I_N

Description of amplifier noise
$U_N, I_N \rightarrow F_{MIN}, \Gamma_O, r_N \equiv$ noise parameters

$$F = F_{MIN} + 4\frac{R_N}{Z_K}\cdot\frac{|\Gamma_g-\Gamma_O|^2}{(1-|\Gamma_g|^2)\cdot|1+\Gamma_O|^2} = F_{MIN} + 4r_N\cdot\frac{|\Gamma_g-\Gamma_O|^2}{(1-|\Gamma_g|^2)\cdot|1+\Gamma_O|^2}$$

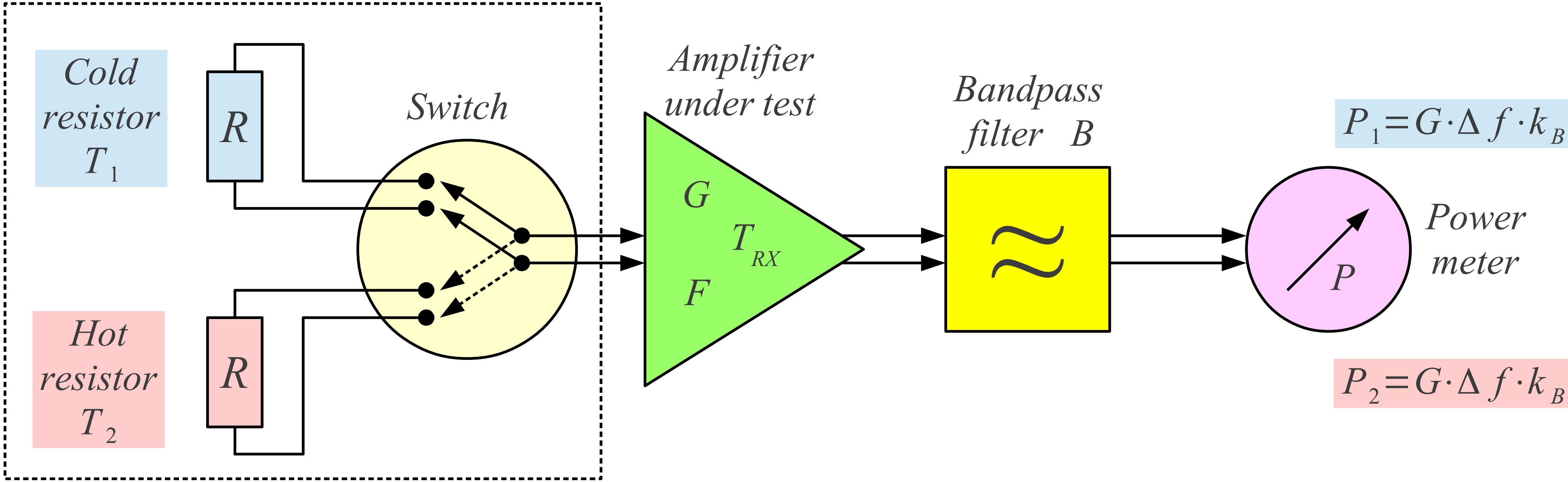
Impedance mismatch
$Z_{source} \neq Z_K = 50\,\Omega$
$Z_{input} \neq Z_K = 50\,\Omega$

$F_{MIN} \equiv$ minimum noise figure at $\Gamma_g = \Gamma_O$ linear units (not dB!)

$\Gamma_O \equiv$ optimum source reflection for F_{MIN} (uncorrelated with $[S]$!)

$R_N[\Omega] \equiv$ noise resistance $\qquad r_N = \frac{R_N}{Z_K} \equiv$ normalized noise resistance (usually $Z_K = 50\,\Omega$)

Hot / cold method



$$P_1 = G \cdot \Delta f \cdot k_B \cdot (T_1 + T_S)$$

$$P_2 = G \cdot \Delta f \cdot k_B \cdot (T_2 + T_S)$$

$$Y = \frac{P_2}{P_1} = \frac{G \cdot B \cdot k_B \cdot (T_2 + T_{RX})}{G \cdot B \cdot k_B \cdot (T_1 + T_{RX})} = \frac{T_2 + T_{RX}}{T_1 + T_{RX}}$$

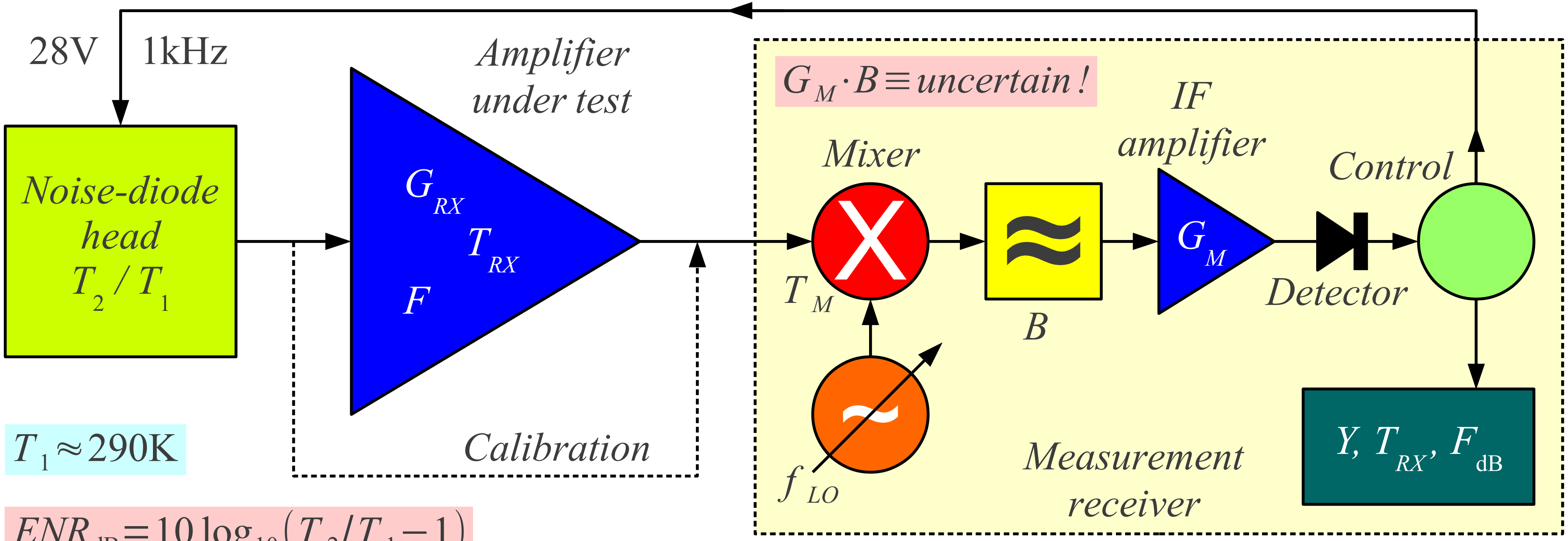
$$Y_{\mathrm{dB}} = 10 \log_{10} \frac{T_2 + T_{RX}}{T_1 + T_{RX}}$$

$$T_{RX} = \frac{T_2 - Y \cdot T_1}{Y - 1}$$

$$F_{\mathrm{dB}} = 10 \log_{10} \left[1 + \frac{T_2 - Y \cdot T_1}{(Y - 1) \cdot T_0} \right]$$

Resistor type	*Noise temperature*
Antenna into cold sky	~20K
R cooled liquid nitrogen	~77K
Antenna into absorber	~290K
R room temperature	~290K
Lightbulb filament as R	~2000K
Ionized gas as R	~10^4K
Diode avalanche breakdown	~10^6K

Noise – figure meter



$T_1 \approx 290\text{K}$

$$ENR_{\text{dB}} = 10\log_{10}(T_2/T_1 - 1)$$

Two measurements without calibration

$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_{RX} + T_M/G_{RX}}{T_1 + T_{RX} + T_M/G_{RX}}$$

$$T_{RX} = \frac{T_2 - Y \cdot T_1}{Y - 1} - \frac{T_M}{G_{RX}} \leftarrow G_{RX} \quad known$$

$$F_{\text{dB}} = 10\log_{10}\left[1 + \frac{1}{T_0} \cdot \left(\frac{T_2 - Y \cdot T_1}{Y - 1} - \frac{T_M}{G_{RX}}\right)\right]$$

Four measurements with calibration:

$$(1) \quad P_1 = G_M G_{RX} B k_B (T_1 + T_{RX} + T_M/G_{RX})$$

$$(2) \quad P_2 = G_M G_{RX} B k_B (T_2 + T_{RX} + T_M/G_{RX})$$

$$(3) \quad P_3 = G_M B k_B (T_1 + T_M)$$

$$(4) \quad P_4 = G_M B k_B (T_2 + T_M)$$

Four equations for four unknowns:

$T_{RX}, \quad G_{RX}, \quad T_M \quad \& \quad (G_M \cdot B \cdot k_B)$