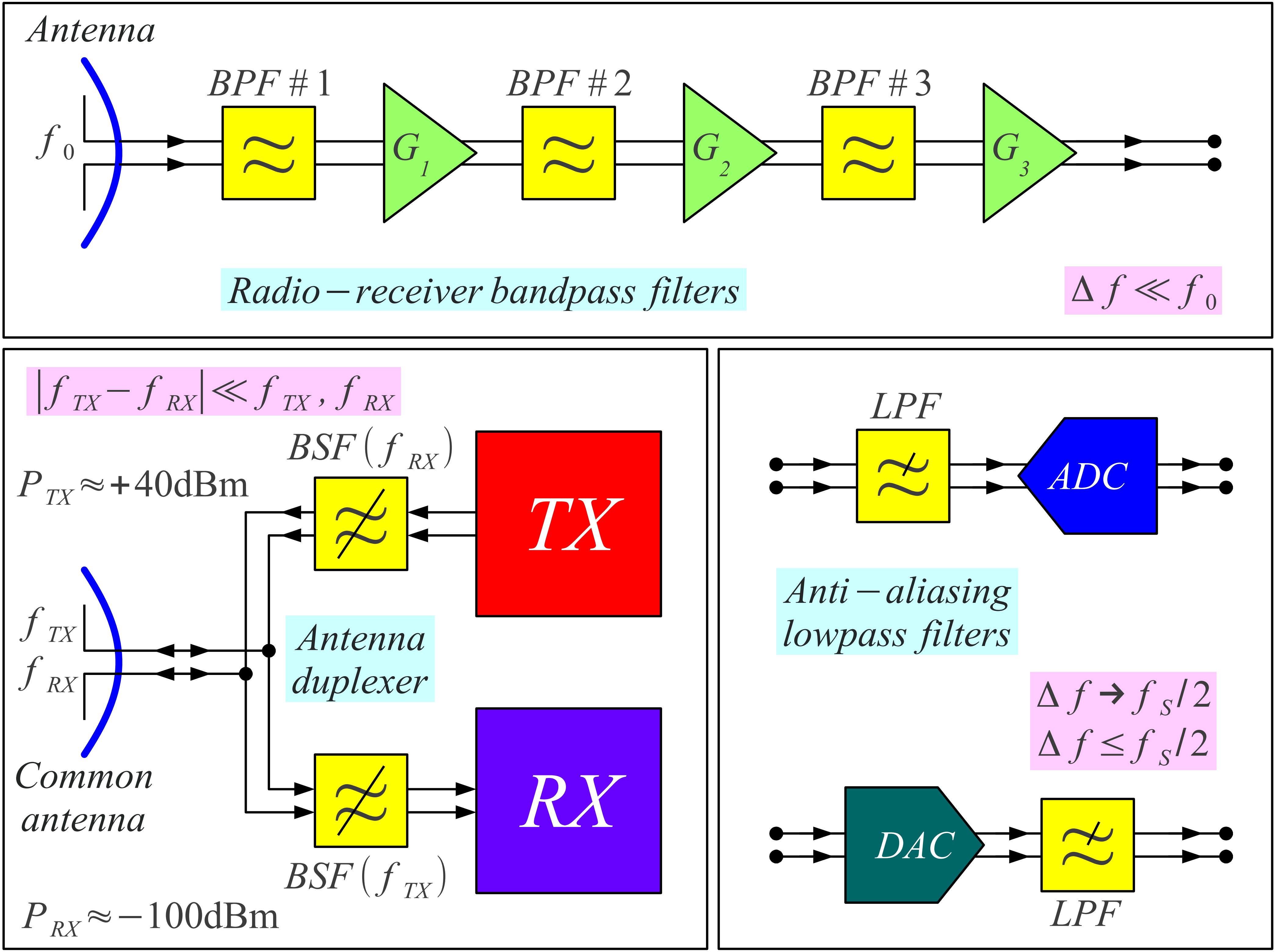
Communication Electronics

Lecture 8:

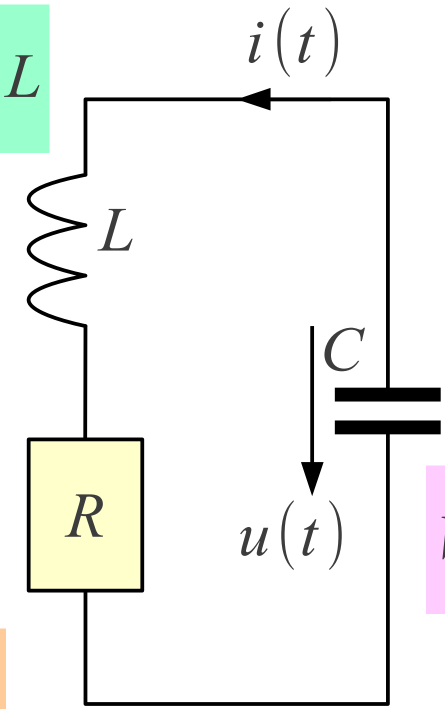
Analog filters in frequency domain

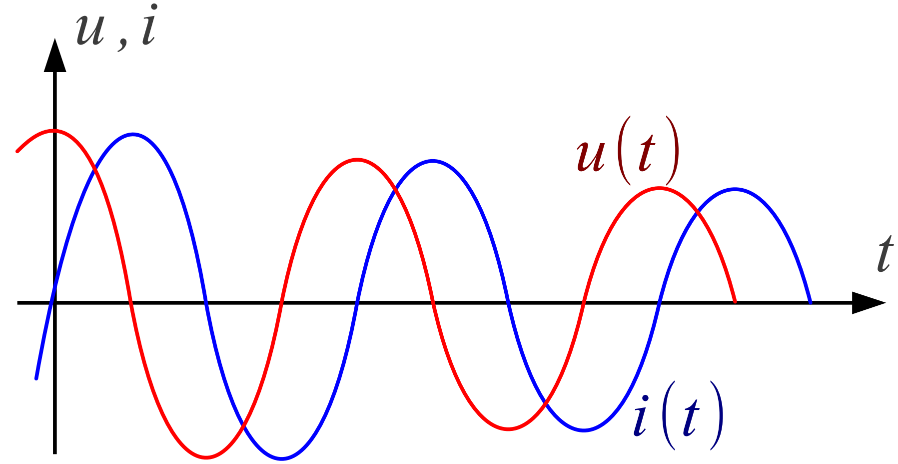


Antenna
f_0
BPF # 1
BPF # 2
BPF # 3
G_1
G_2
G_3
Radio − receiver bandpass filters
$\Delta f \ll f_0$
$|f_{TX} - f_{RX}| \ll f_{TX}, f_{RX}$
$BSF(f_{RX})$
$P_{TX} \approx +40\text{dBm}$
TX
f_{TX}
f_{RX}
Antenna duplexer
Common antenna
$BSF(f_{TX})$
RX
$P_{RX} \approx -100\text{dBm}$
LPF
ADC
Anti − aliasing lowpass filters
$\Delta f \to f_S/2$
$\Delta f \le f_S/2$
DAC
LPF

Electrical tuned circuit

$$W_m(t) = \frac{1}{2} i^2(t) L$$





$$W_e(t) = \frac{1}{2} u^2(t) C$$

$$P(t) = i^2(t) R$$

$$\langle P \rangle = \frac{1}{2} I^2_{MAX} R \equiv \textit{average power loss}$$

$$W = W_e + W_m = \frac{1}{2} I^2_{MAX} L = \frac{1}{2} U^2_{MAX} C \equiv \textit{stored energy}$$

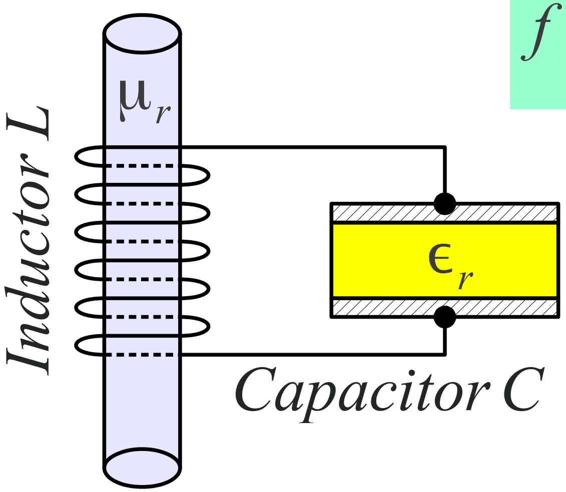
Resonator quality

$$\textit{General}: \quad Q = \omega \frac{W}{\langle P \rangle}$$

$$\textit{LC circuit}: \quad Q = \frac{\omega L}{R}$$

$$\frac{1}{Q} = \frac{1}{Q_{inductor}} + \frac{1}{Q_{capacitor}} = \frac{R_{winding}}{\omega L} + \tan \delta_{dielectric}$$

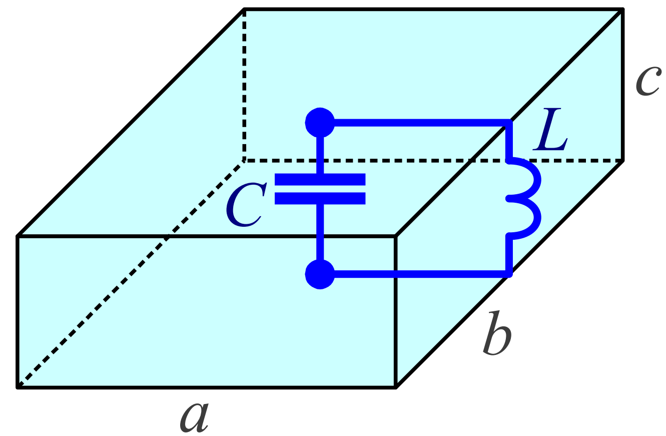
Lumped components L + C



$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$Q \approx 100$$

Rectangular cavity



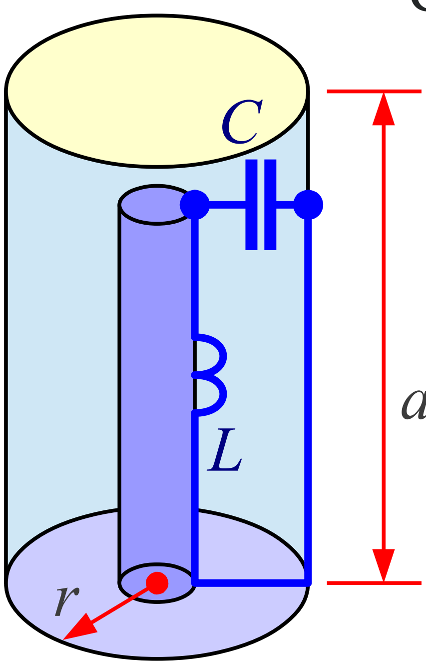
$$a = b = \frac{\lambda}{\sqrt{2}}$$

$$f_{110} = \frac{c_0}{a\sqrt{2}}$$

$$f_{lmn} = \frac{c_0}{2}\sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$$

$$Q \approx 10^4$$

Coaxial cavity



$$f_m = m \cdot \frac{c_0}{4a}$$

$$m = 1,3,5\ldots$$

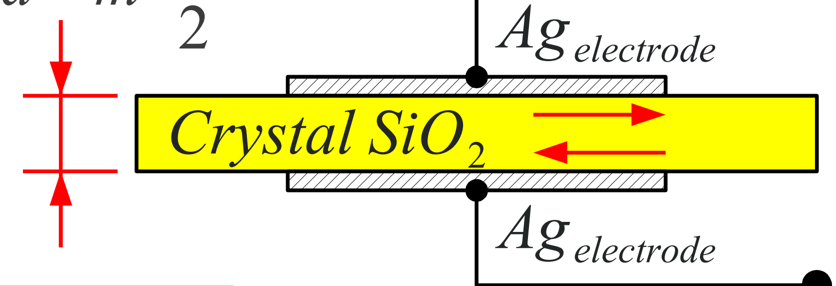
$$a = m \cdot \frac{\lambda}{4}$$

$$r \ll a$$

$$Q \approx 1000$$

Mechanical shear resonator + piezo

$$d = m \cdot \frac{\Lambda}{2}$$

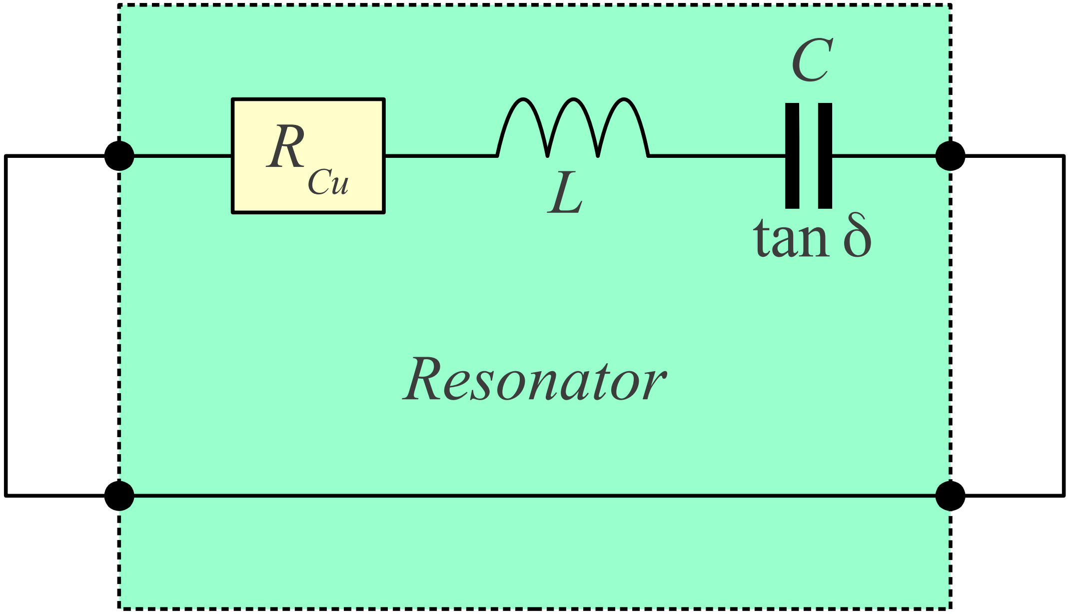


$$f_m = m \cdot \frac{v}{2d}$$

$$m = 1,3,5\ldots$$

$$\Lambda \cdot f = v \approx 3.3\,\mathrm{km/s}$$

$$Q \approx 10^5$$



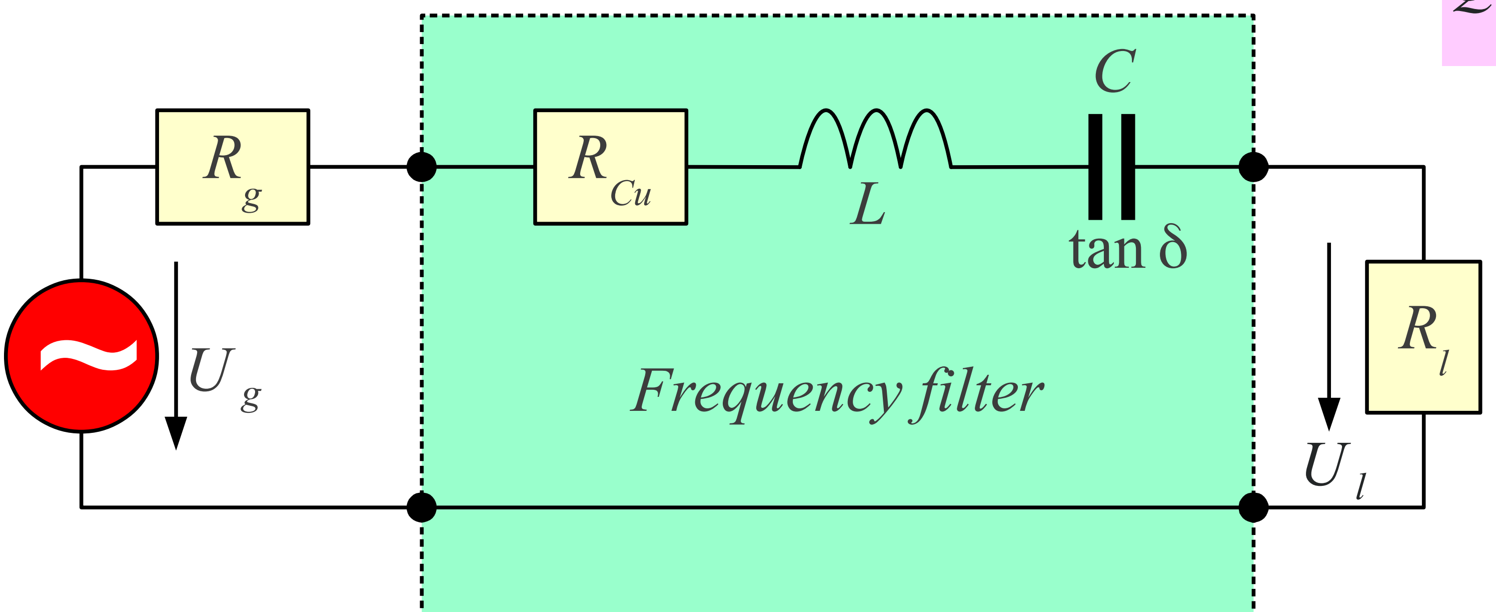
Unloaded quality

$$Q_U = \frac{\omega L}{R_{Cu}}$$

$$Q_L < Q_U$$

Loaded quality

$$Q_L = \frac{\omega L}{R_g + R_{Cu} + R_l}$$

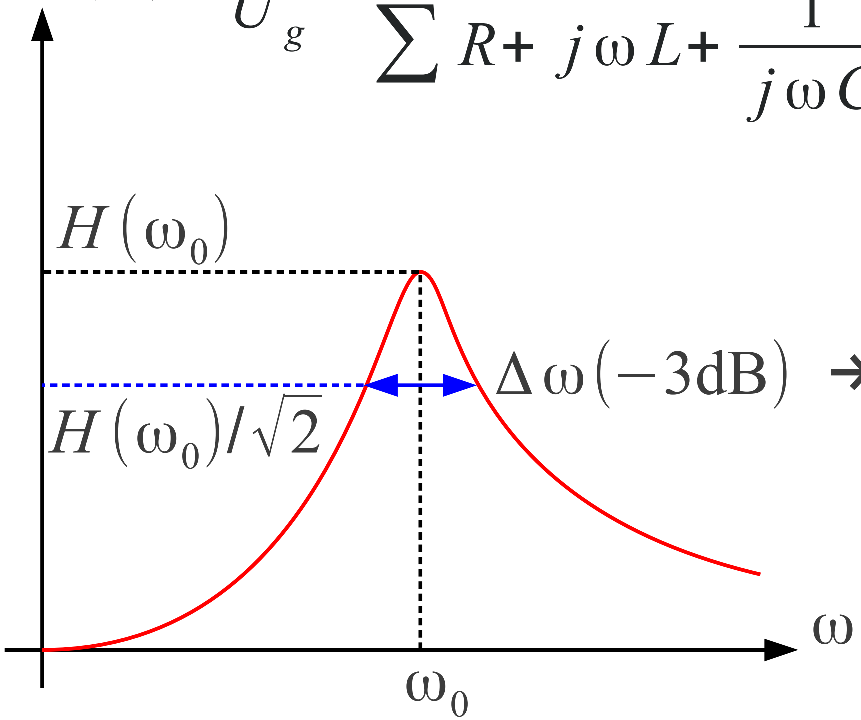


$$\sum R = R_g + R_{Cu} + R_l$$

$$Q_L = \frac{\omega L}{\sum R}$$

$$H(\omega)=\frac{U_l}{U_g}=\frac{R_l}{\sum R+j\omega L+\frac{1}{j\omega C}}\approx\frac{R_l}{\sum R\pm j\Delta\omega L}$$

$$Q_L=\frac{\omega_0 L}{\sum R}\gg 1$$



$$\Delta\omega(-3\text{dB})\ \rightarrow\ \sum R=\Delta\omega L$$

−3dB *bandwidth*

$$\Delta\omega=\frac{\sum R}{L}\qquad \sum R=\frac{\omega_0 L}{Q_L}$$

$$\Delta\omega=\frac{\omega_0}{Q_L}\qquad \Delta f=\frac{f_0}{Q_L}$$

$$j\omega_0 L+\frac{1}{j\omega_0 C}=0\ \rightarrow\ \omega_0=\frac{1}{\sqrt{LC}}$$

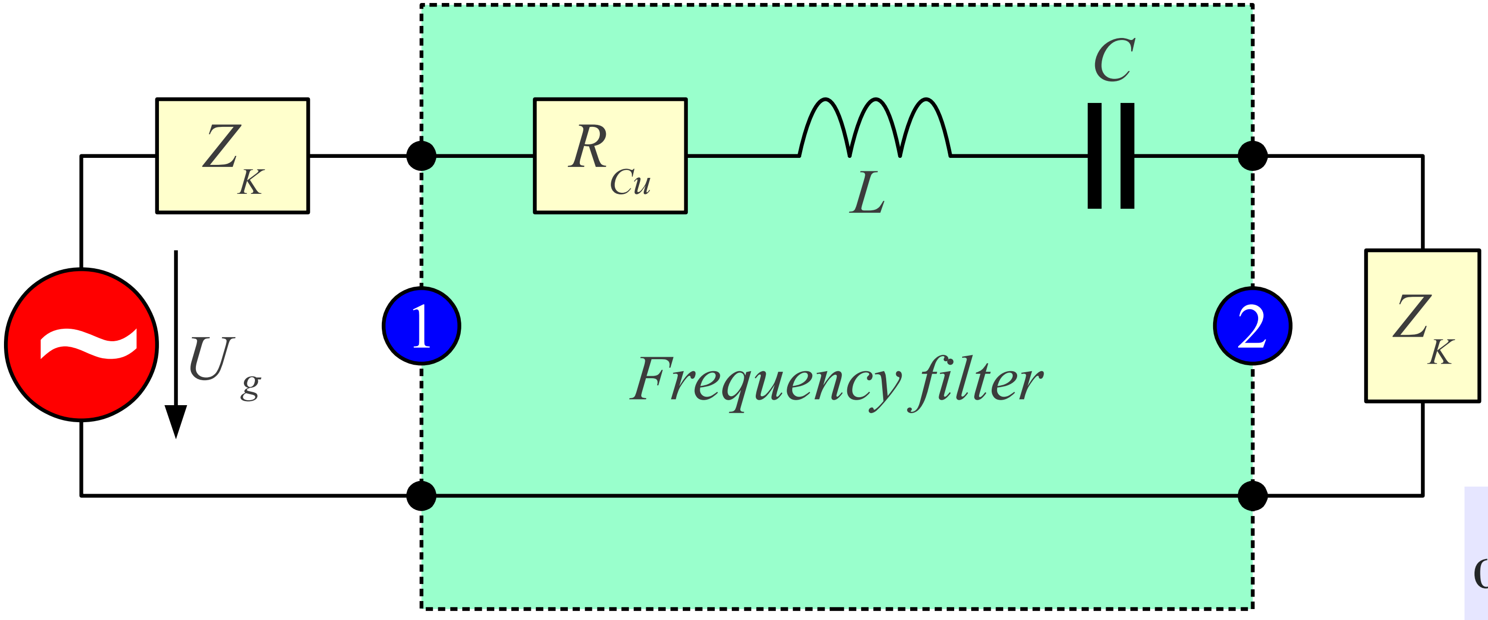
$$\frac{1}{1+x}\approx 1-x\quad @\quad x\ll 1$$

$$\omega=\omega_0\pm\frac{\Delta\omega}{2}$$

$$\frac{\Delta\omega}{2}\ll\omega_0$$

$$j\omega L+\frac{1}{j\omega C}=j\left(\omega_0\pm\frac{\Delta\omega}{2}\right)L+\frac{1}{j\omega_0\left(1\pm\frac{\Delta\omega}{2\omega_0}\right)C}\approx$$

$$\approx j\left(\omega_0\pm\frac{\Delta\omega}{2}\right)L+\frac{1}{j\omega_0 C}\left(1\mp\frac{\Delta\omega}{2\omega_0}\right)=\pm j\Delta\omega L$$



$$\sum R = R_{Cu} + 2\,Z_K$$

$$\frac{\omega_0 L}{Q_L} = \frac{\omega_0 L}{Q_U} + 2\,Z_K$$

$$\omega_0 L \left(\frac{1}{Q_L} - \frac{1}{Q_U} \right) = 2\,Z_K$$

Unmatched flter $S_{11} = S_{22} \neq 0 \; @ \; \omega = \omega_0$

$$Q_L \sum R = \omega_0 L = \frac{2\,Z_K\,Q_L\,Q_U}{Q_U - Q_L}$$

$$Z = R_{Cu} + j\,\omega\,L + \frac{1}{j\,\omega\,C} \neq 0$$

$$S_{11} = S_{22} = \frac{Z}{Z + 2\,Z_K} = \frac{R_{Cu} + j\,\omega\,L + \dfrac{1}{j\,\omega\,C}}{\sum R + j\,\omega\,L + \dfrac{1}{j\,\omega\,C}}$$

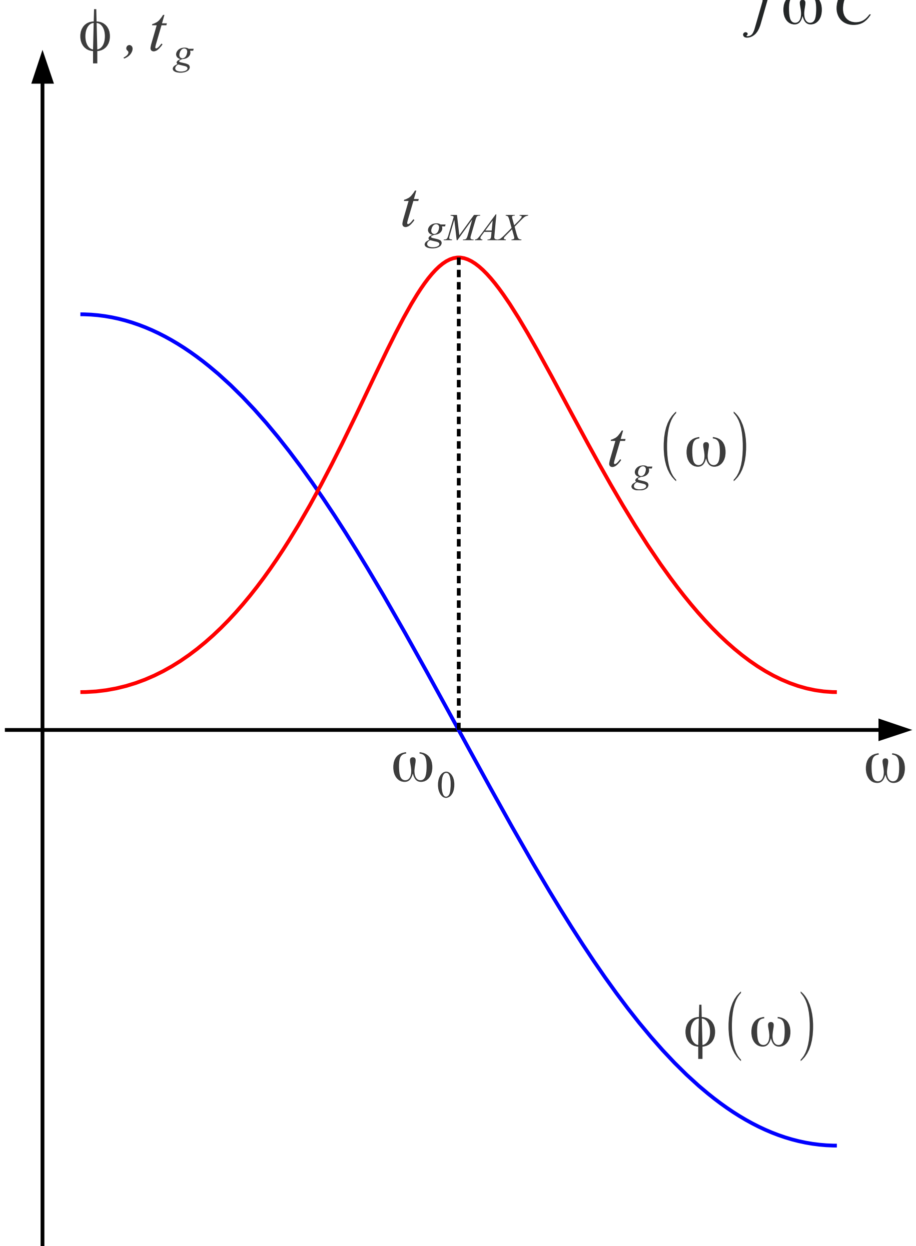
$$S_{12} = S_{21} = \frac{2\,Z_K}{Z + 2\,Z_K} = \frac{2\,Z_K}{\sum R + j\,\omega\,L + \dfrac{1}{j\,\omega\,C}}$$

Passband insertion loss $@ \; \omega = \omega_0$

$$S_{12} = S_{21} = \frac{2\,Z_K}{\sum R} = 1 - \frac{Q_L}{Q_U}$$

$$a = |S_{21}|^2 = \left(1 - \frac{Q_L}{Q_U} \right)^2$$

$$H(\omega)=\frac{U_l}{U_g}=\frac{R_l}{\sum R+j\omega L+\frac{1}{j\omega C}} \rightarrow \phi(\omega)=-\arctan\frac{\omega L-\frac{1}{\omega C}}{\sum R}\equiv \textit{phase angle}$$

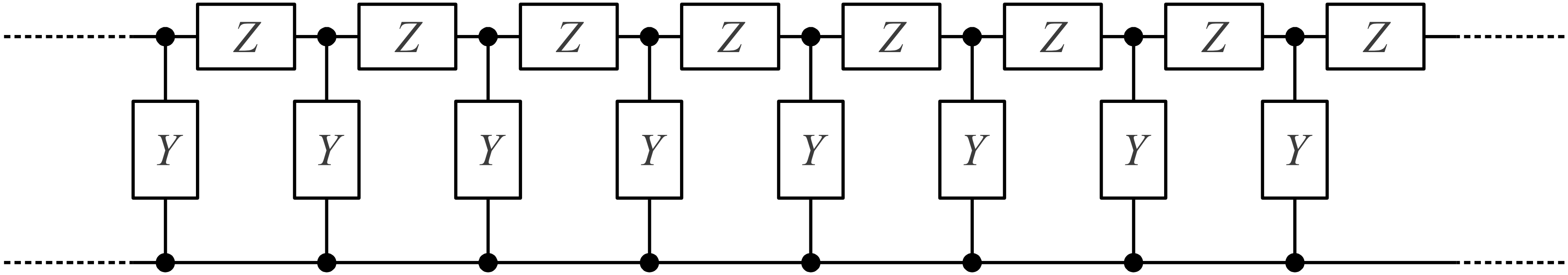


Group delay

$$t_g(\omega)=-\frac{d\phi}{d\omega}=\frac{1}{1+\left(\frac{\omega L-\frac{1}{\omega C}}{\sum R}\right)^2}\cdot\frac{L+\frac{1}{\omega^2 C}}{\sum R}$$

$$t_{gMAX}=t_g(\omega=\omega_0)=\frac{L+\frac{1}{\omega_0^2 C}}{\sum R}=\frac{2L}{\sum R}$$

$$t_{gMAX}=\frac{2Q_L}{\omega_0}=\frac{Q_L}{\pi f_0}$$



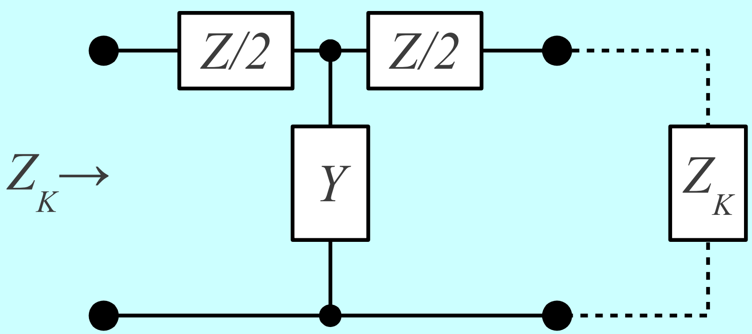
$Z, Y \equiv$ *reactive components*

$Z_K \equiv$ *real* → *passband*

$Z_K \equiv$ *imaginary* → *stopband*

T element

$$Z_K = Z/2 + \cfrac{1}{Y + \cfrac{1}{Z/2 + Z_K}}$$

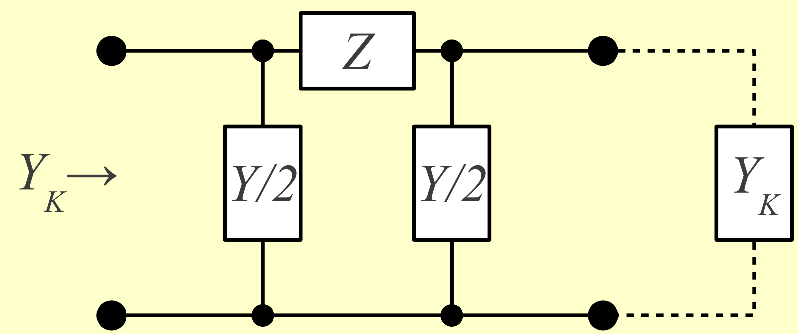


Characteristic impedance

$$Z_K = \sqrt{\frac{Z}{Y} + \left(\frac{Z}{2}\right)^2}$$

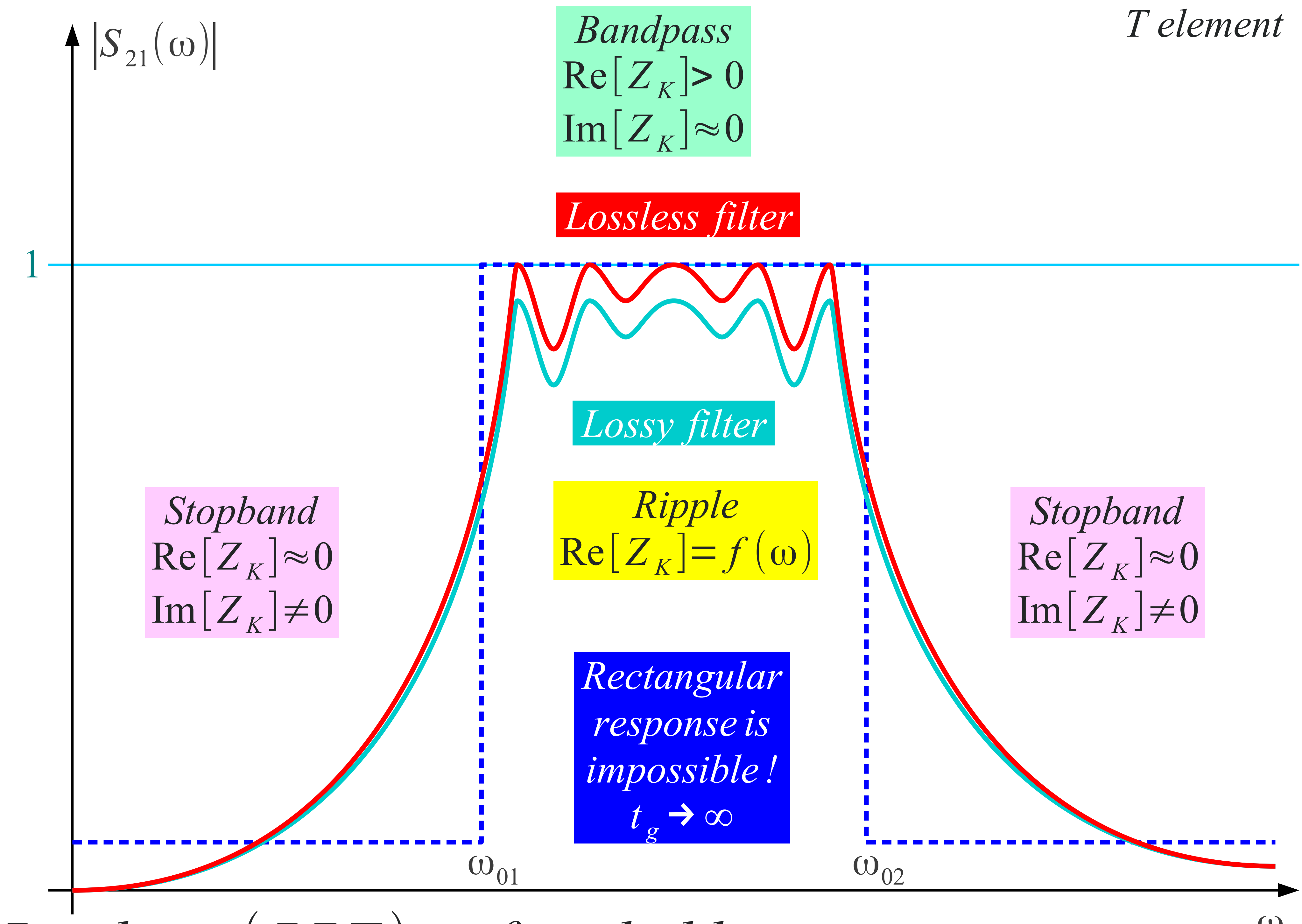
π element

$$Y_K = Y/2 + \cfrac{1}{Z + \cfrac{1}{Y/2 + Y_K}}$$



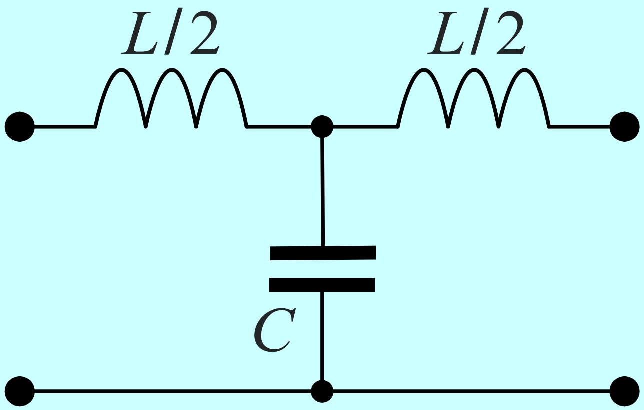
Characteristic admittance

$$Y_K = \sqrt{\frac{Y}{Z} + \left(\frac{Y}{2}\right)^2} = \frac{1}{Z_K}$$



Bandpass (BPF) uniform ladder response

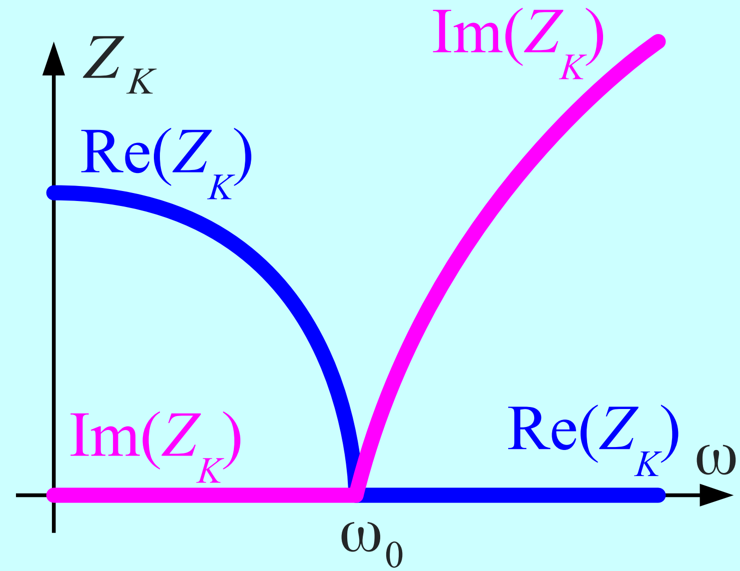
Lowpass filter (*LPF*)



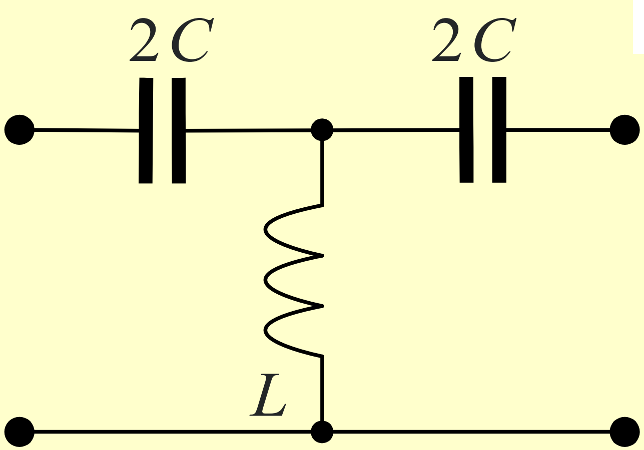
$$Z = j\omega L$$
$$Y = j\omega C$$

$$Z_K = \sqrt{\frac{L}{C} - \left(\frac{\omega L}{2}\right)^2}$$

$$\omega_0 = \frac{2}{\sqrt{LC}}$$



Highpass filter (*HPF*)

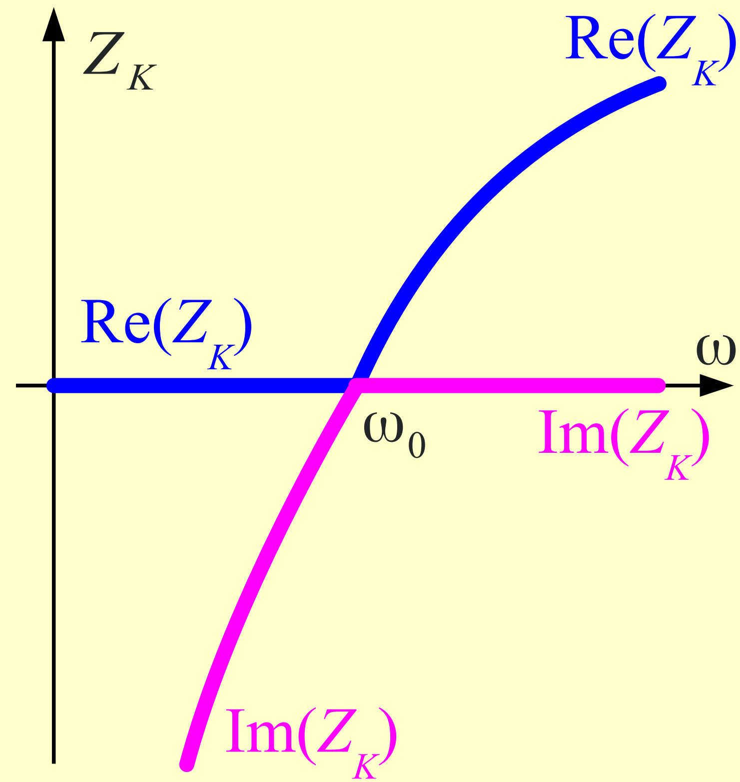


$$Z = \frac{1}{j\omega C}$$

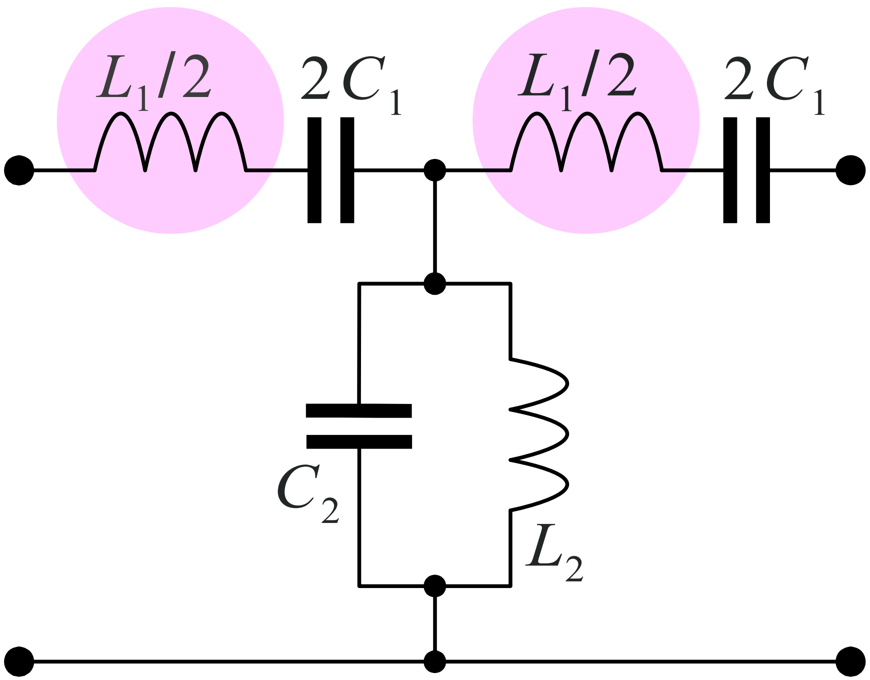
$$Y = \frac{1}{j\omega L}$$

$$Z_K = \sqrt{\frac{L}{C} - \left(\frac{1}{2\omega C}\right)^2}$$

$$\omega_0 = \frac{1}{2\sqrt{LC}}$$



$(Non)\ implementable\ ?$



$$Z = j\omega L_1 + \frac{1}{j\omega C_1}$$

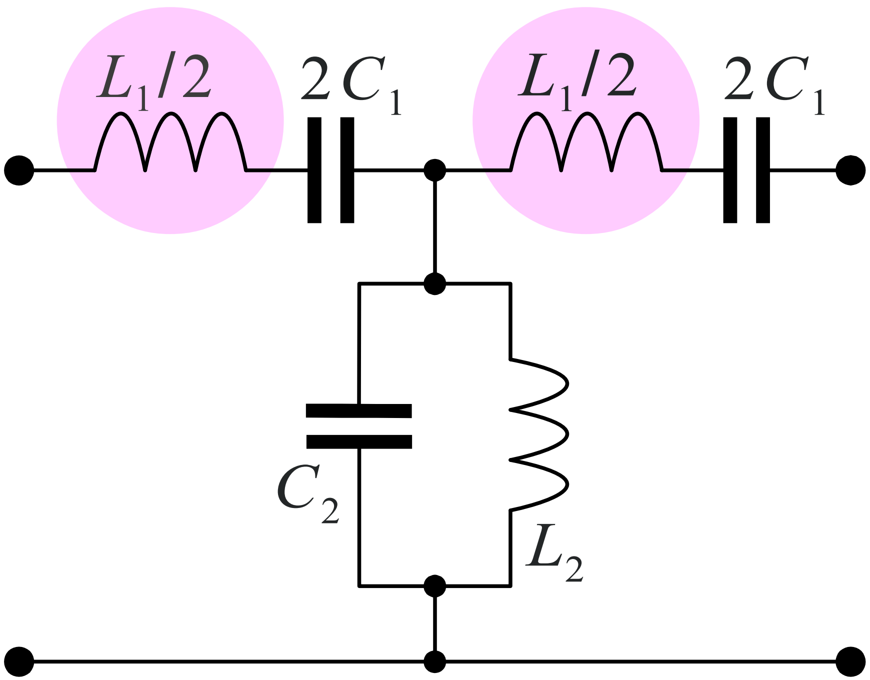
$$Y = j\omega C_2 + \frac{1}{j\omega L_2}$$

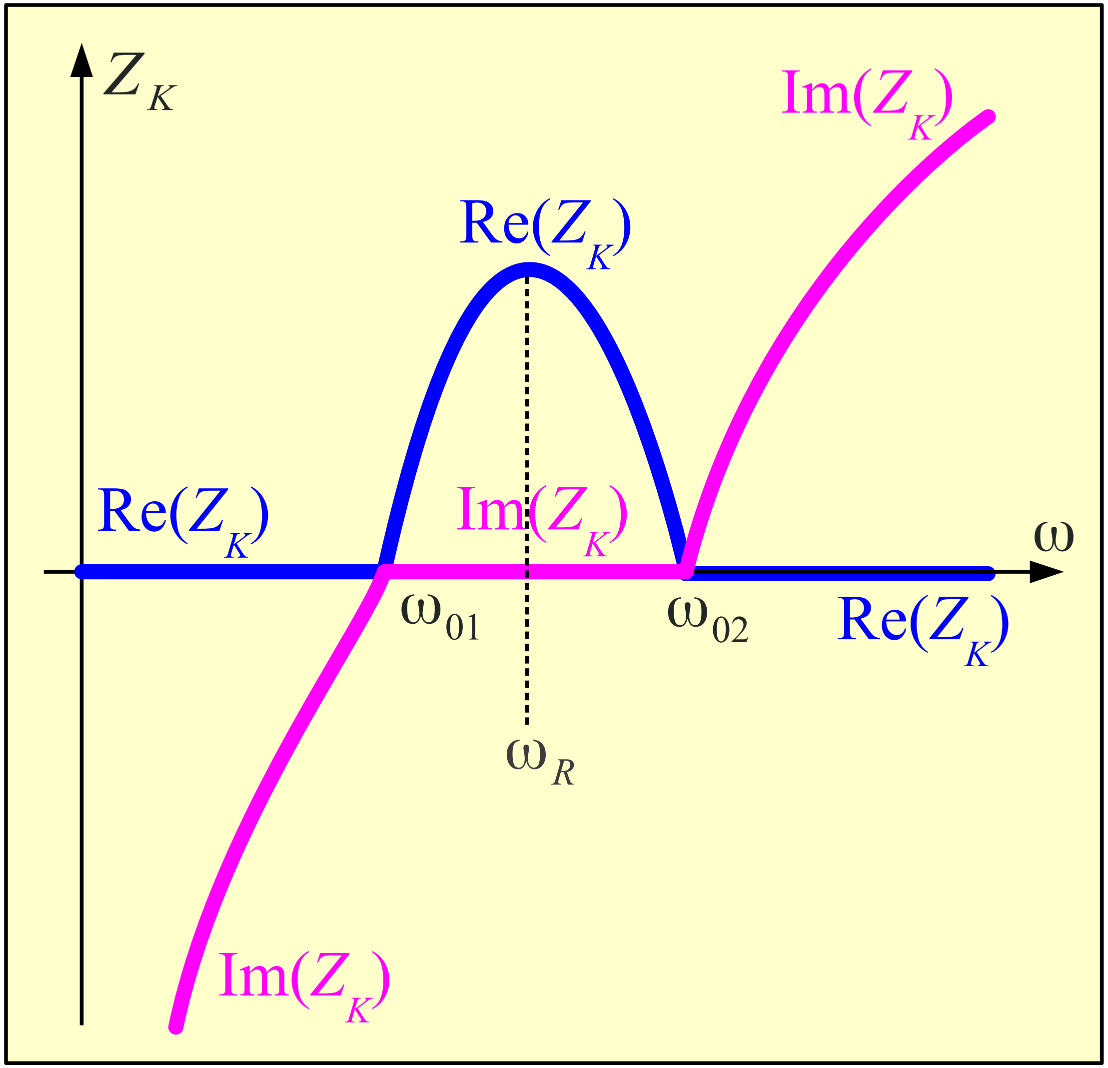
$$Z_K = \sqrt{\frac{\omega L_1 - \frac{1}{\omega C_1}}{\omega C_2 - \frac{1}{\omega L_2}} - \left(\frac{\omega L_1 - \frac{1}{\omega C_1}}{2}\right)^2}$$

$$\omega_{01}, \omega_{02} = \sqrt{\frac{\left(\frac{L_1}{L_2} + \frac{C_2}{C_1} + 4\right) \pm \sqrt{\left(\frac{L_1}{L_2} + \frac{C_2}{C_1} + 4\right)^2 - 4\frac{L_1 C_2}{L_2 C_1}}}{2 L_1 C_2}}$$

Bandpass filter (BPF)

(Non) implementable?





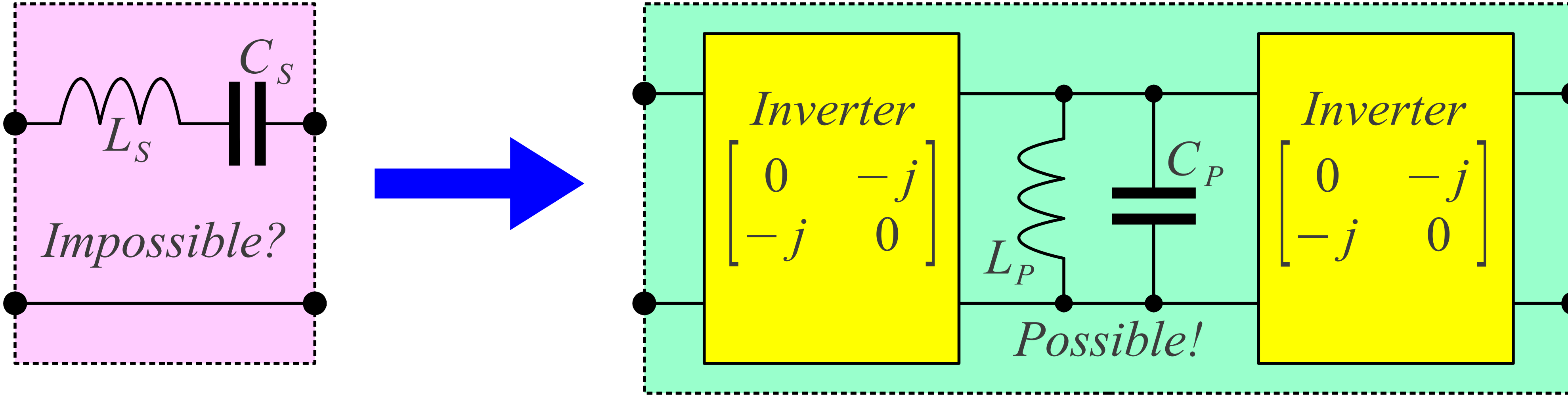
$$m=\frac{L_1}{L_2}=\frac{C_2}{C_1}$$

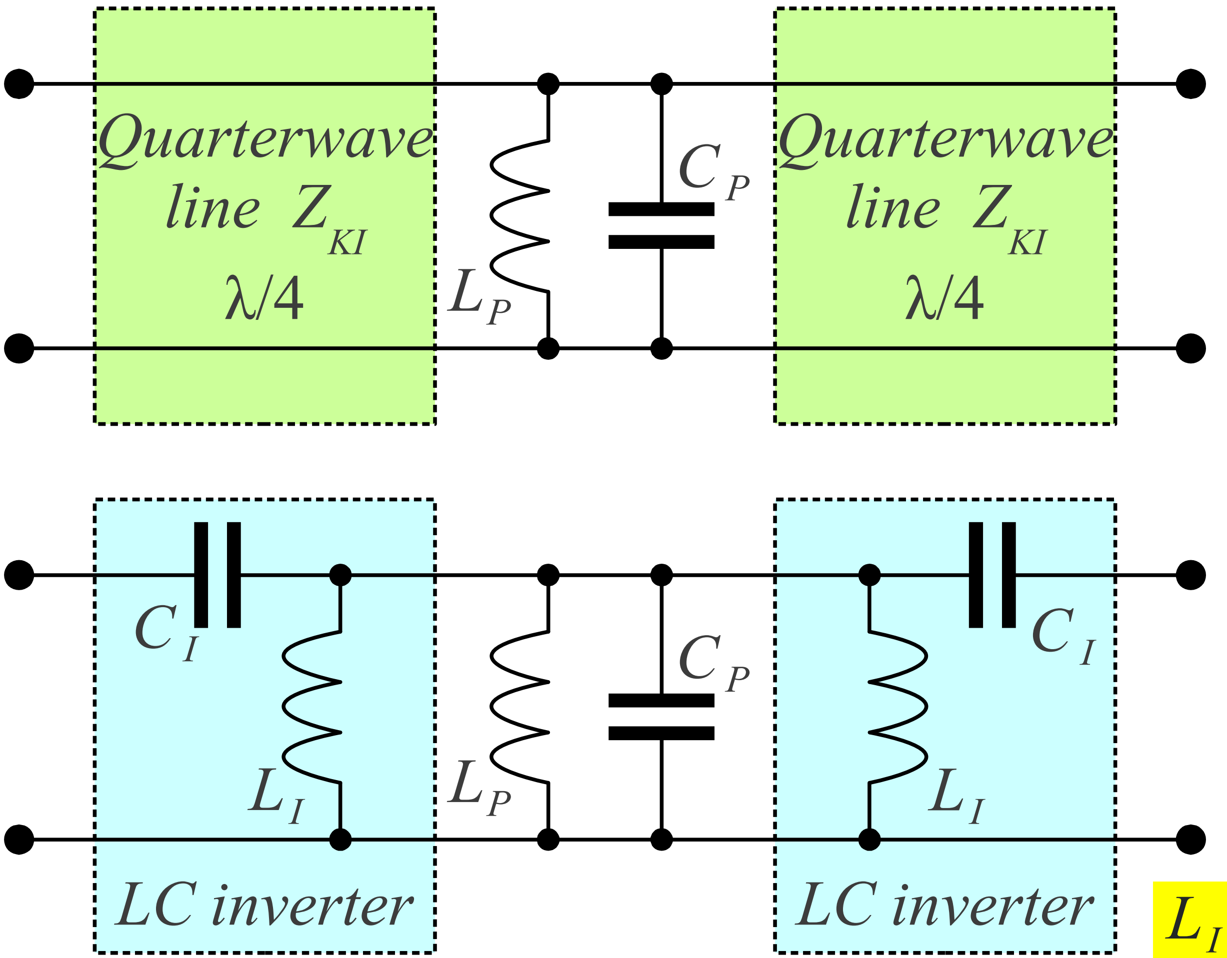
$$Z_K=\sqrt{\frac{L_1}{C_2}-\left(\frac{\omega L_1-\frac{1}{\omega C_1}}{2}\right)^2}$$

$$\omega_R=\frac{1}{\sqrt{L_1 C_1}}=\frac{1}{\sqrt{L_2 C_2}}$$

$$\omega_{01},\omega_{02}=\omega_R\sqrt{\frac{m+2\pm 2\sqrt{m+1}}{m}}$$

Sensible choice for BPF





Impedance inverter :

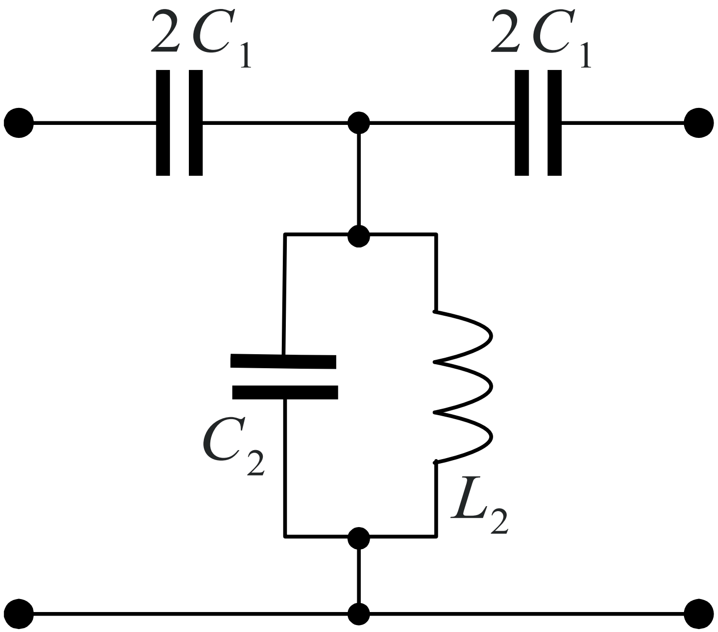
$$Z_{output} = \frac{Z_{KI}^2}{Z_{input}}$$

$$\Gamma_{output} = -\Gamma_{input}$$

$$[S] = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

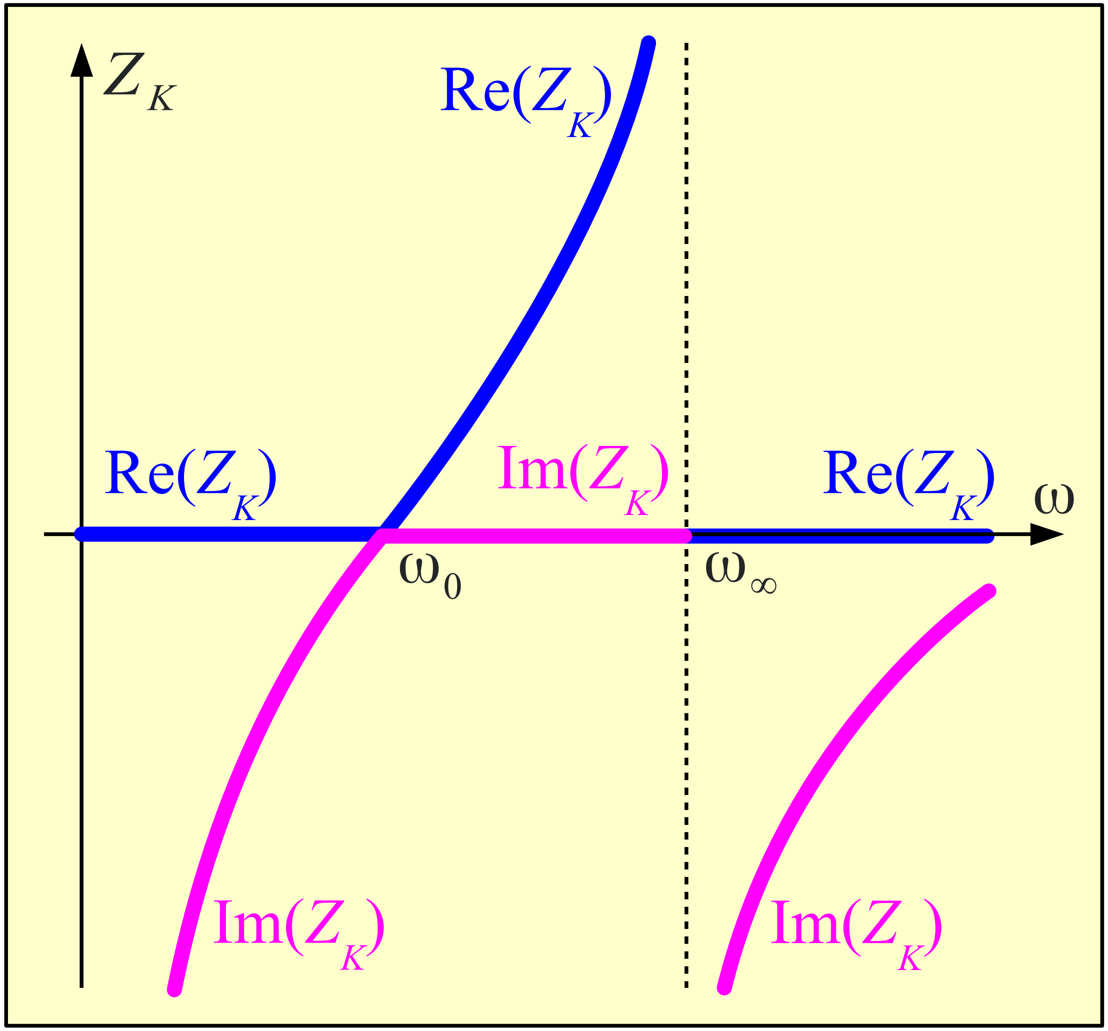
Choice of $Z_K = ?$

$L_I \| L_P \| L_I \equiv$ *implementable !*



$$Z=\frac{1}{j\omega C_1}$$

$$Y=j\omega C_2+\frac{1}{j\omega L_2}$$

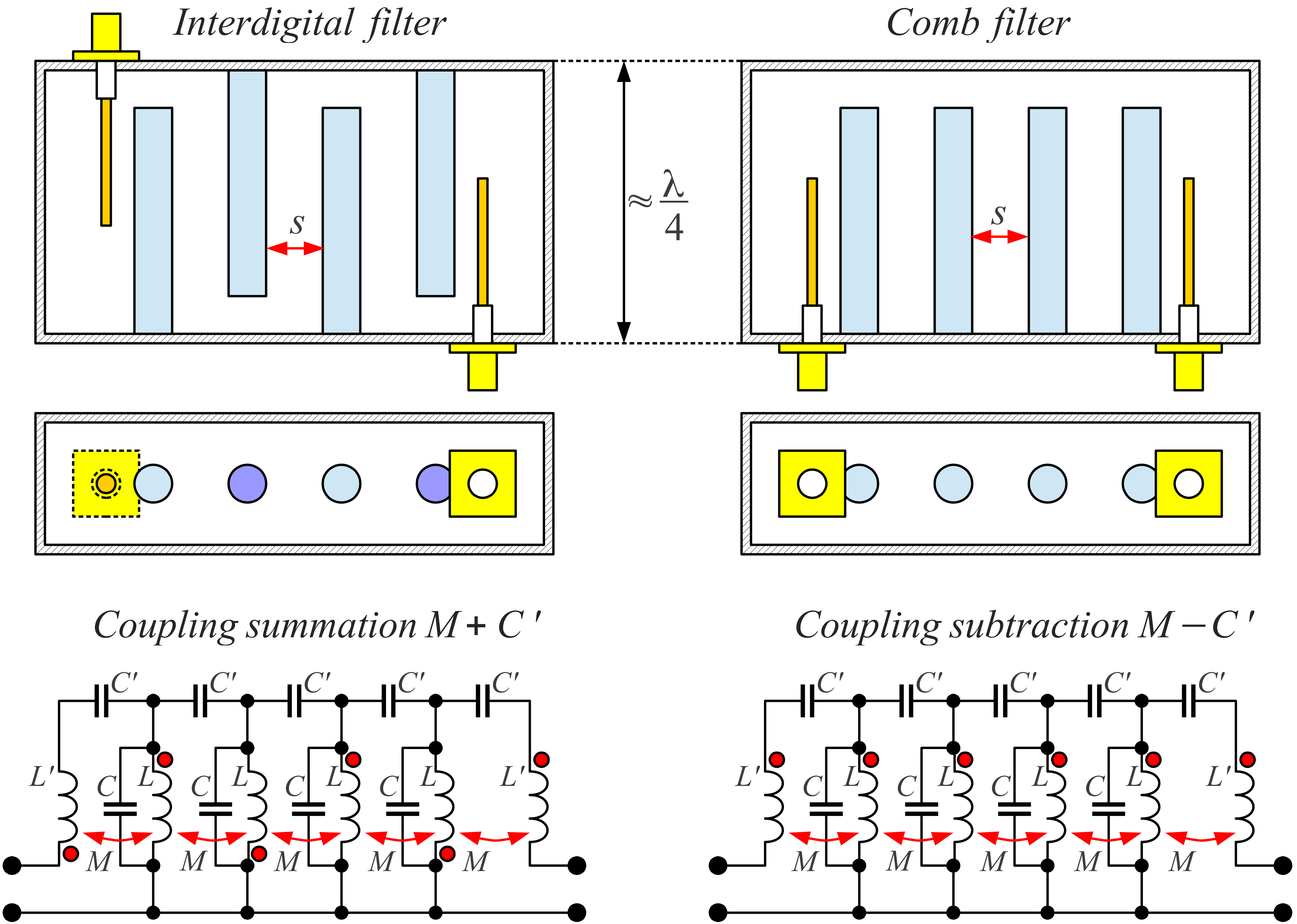


$$Z_K=\sqrt{\frac{1}{\omega C_1\left(\frac{1}{\omega L_2}-\omega C_2\right)}-\left(\frac{1}{2\omega C_1}\right)^2}$$

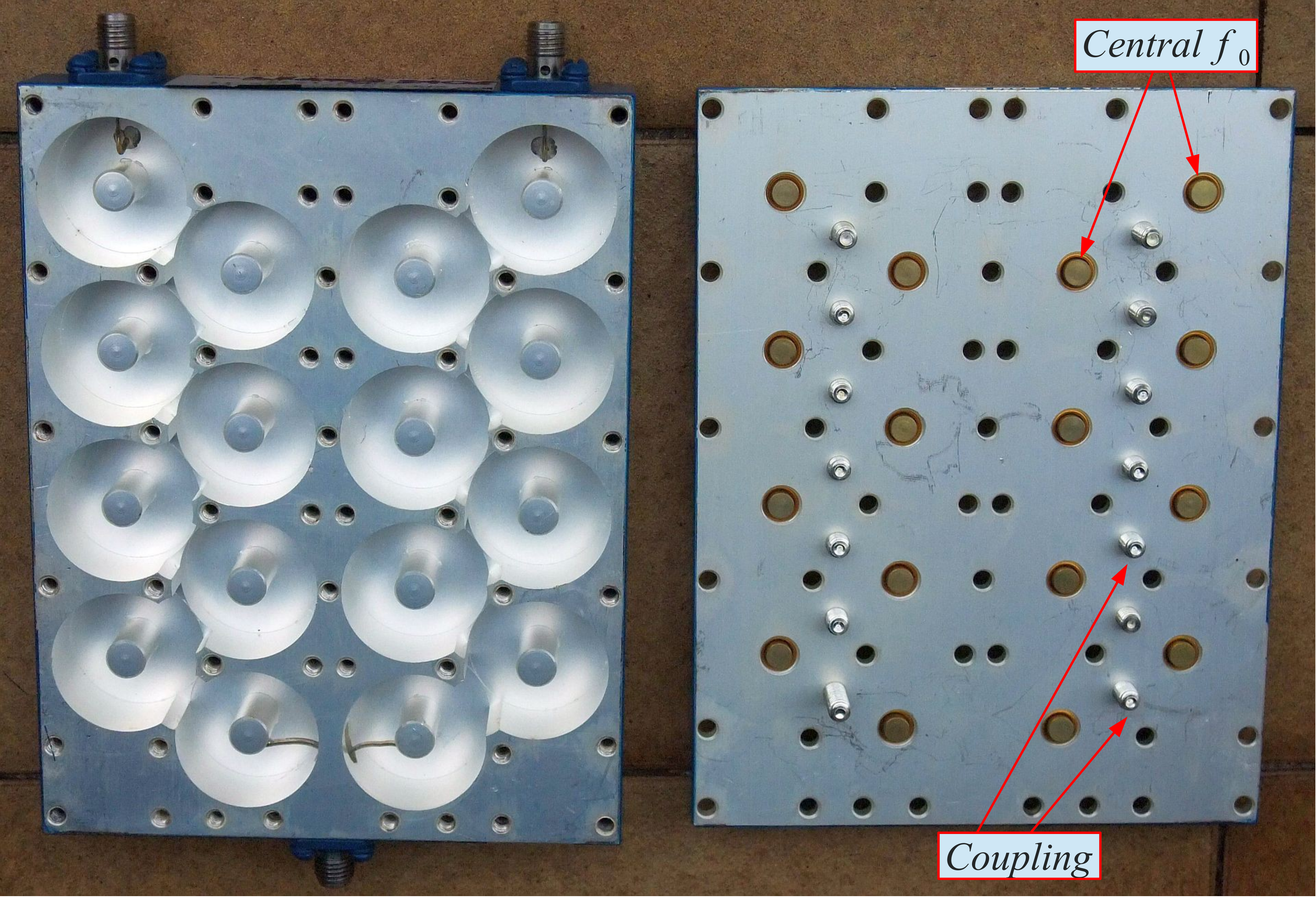
$$\omega_0=\frac{1}{\sqrt{L_2\left(4C_1+C_2\right)}}\qquad \omega_\infty=\frac{1}{\sqrt{L_2C_2}}$$

BPF without non−implementable (too large) inductors

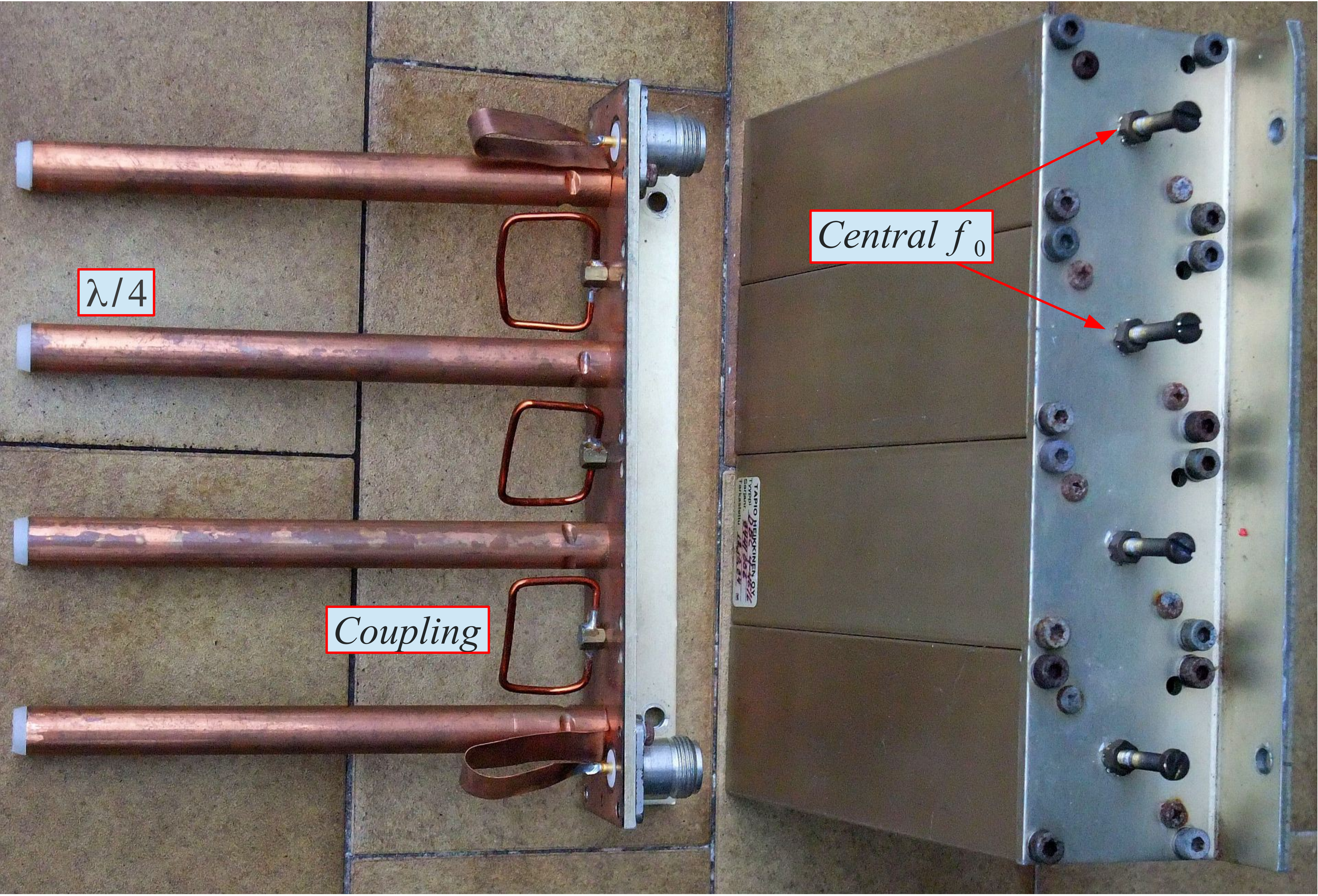

λ/4 cavity for 450MHz with
adjustable capacitive coupling
λ/4



Interdigital filter
Comb filter
s
s
≈ λ/4
Coupling summation M + C′
Coupling subtraction M − C′
C′
L′
C
L
M



Duplexer for 3.4GHz *with comb BPFs with adjustable couplings*

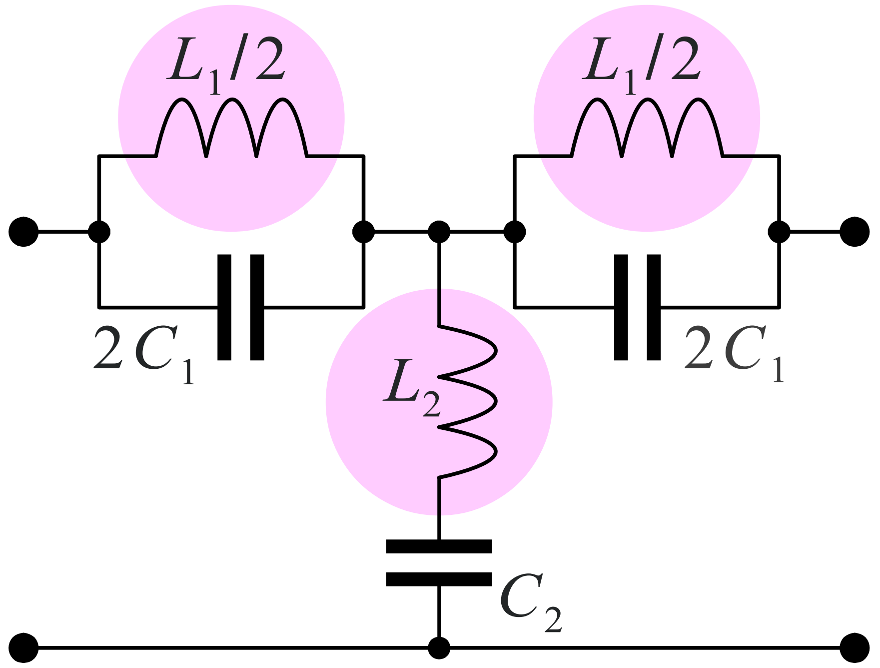


BPF with $\lambda/4$ *resonators for* 400MHz *, adjustable inductive couplings*

Comb filters for $f \approx 3.4$GHz & $\lambda/4$ *resonators for* $f \approx 450$MHz

Silver − plated (*Ag*) *ceramics based on* TiO_2

$\epsilon_r \approx 20 \ldots 100$ $\tan\delta \approx 0.0003$



Difficult to implement?

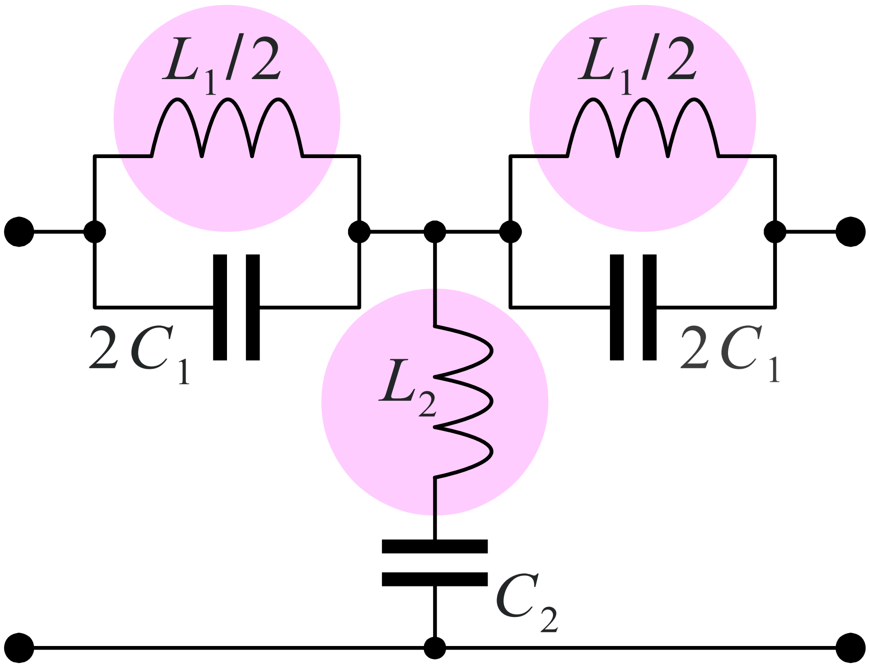
$$Z=\frac{1}{j\omega C_1+\frac{1}{j\omega L_1}}$$

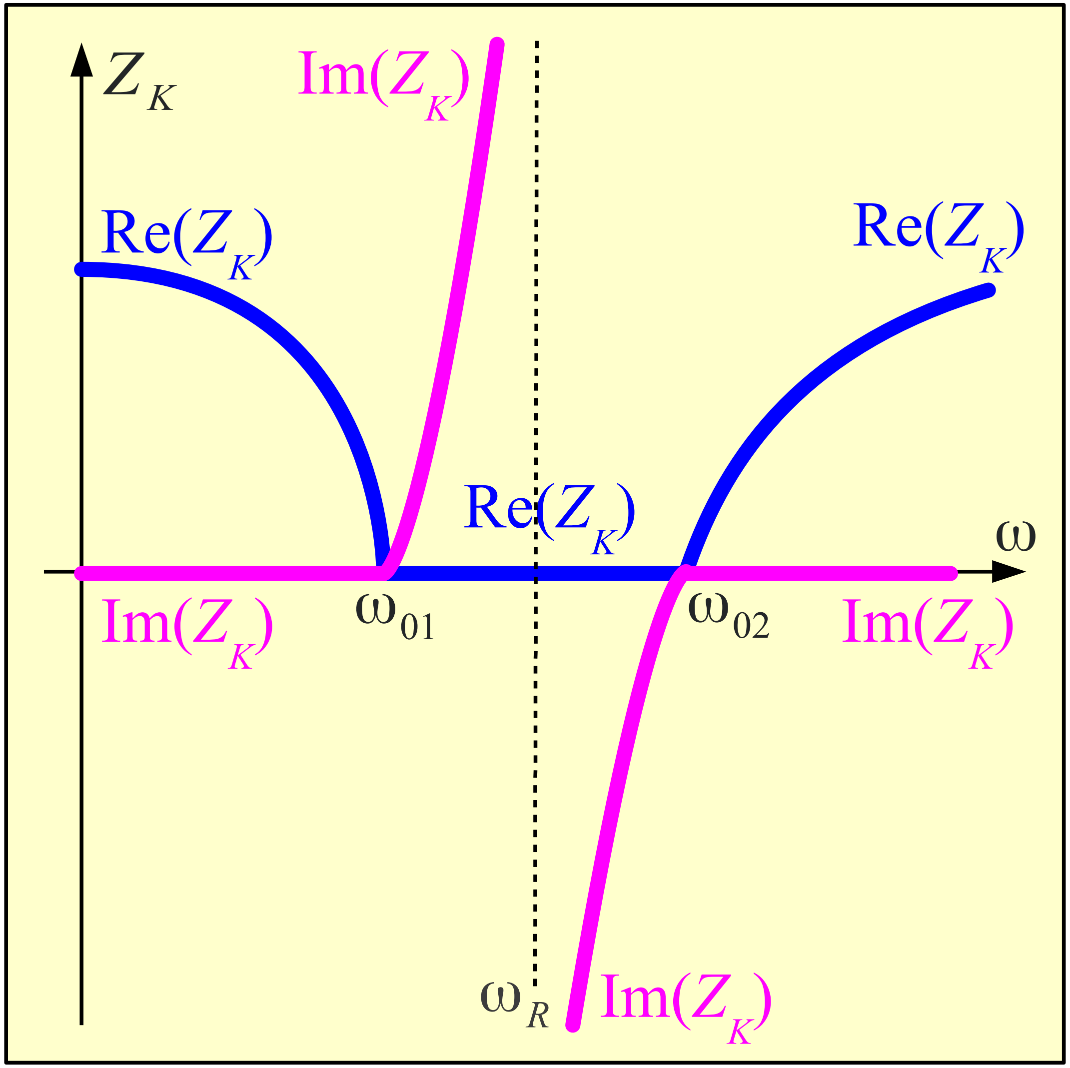
$$Y=\frac{1}{j\omega L_2+\frac{1}{j\omega C_2}}$$

$$Z_K=\sqrt{\frac{\omega L_2-\frac{1}{\omega C_2}}{\omega C_1-\frac{1}{\omega L_1}}-\left(\frac{1}{2\omega C_1-\frac{2}{\omega L_1}}\right)^2}$$

$$\omega_{01},\omega_{02}=\sqrt{\frac{\left(4\frac{C_1}{C_2}+4\frac{L_2}{L_1}+1\right)\pm\sqrt{\left(4\frac{C_1}{C_2}+4\frac{L_2}{L_1}+1\right)^2-64\frac{L_2C_1}{L_1C_2}}}{8L_2C_1}}$$

Bandstop filter (BSF)





Difficult to implement?

$$m = \frac{L_1}{L_2} = \frac{C_2}{C_1}$$

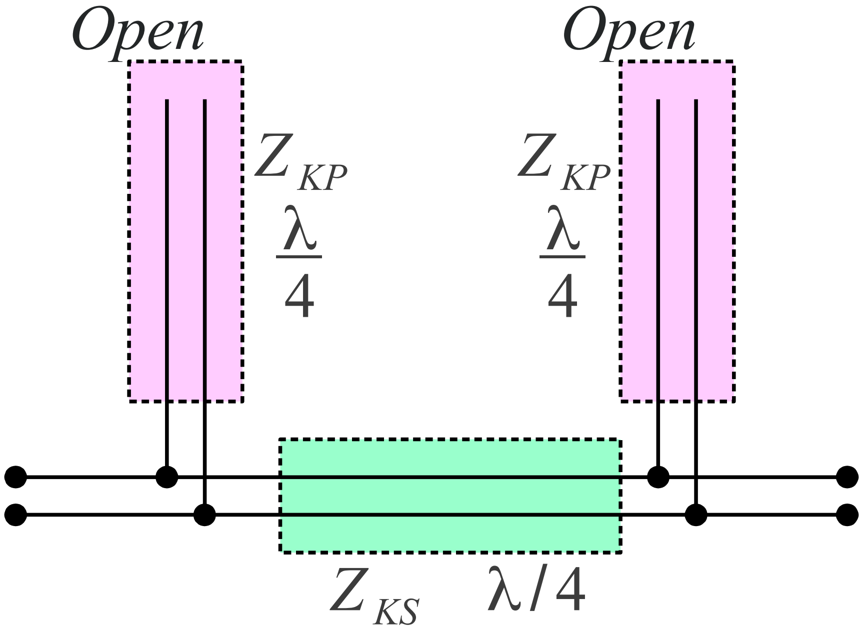
$$Z_K = \sqrt{\frac{L_1}{C_2} - \left(\frac{1}{2\omega C_1 - \frac{2}{\omega L_1}} \right)^2}$$

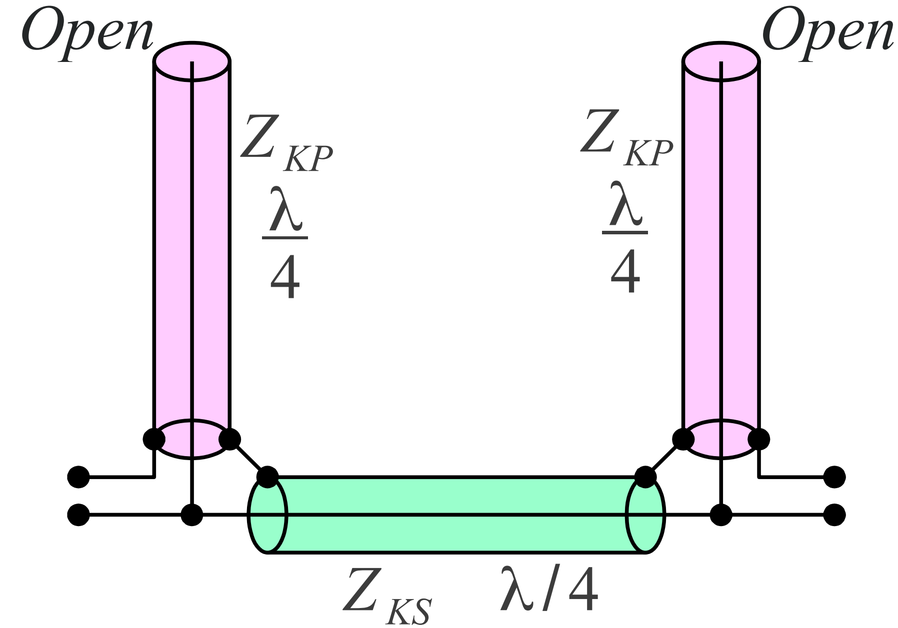
$$\omega_R = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}$$

$$\omega_{01}, \omega_{02} = \omega_R \sqrt{1 + \frac{m}{8} \pm \sqrt{\frac{m}{4} + \frac{m^2}{64}}}$$

Sensible choice for BSF

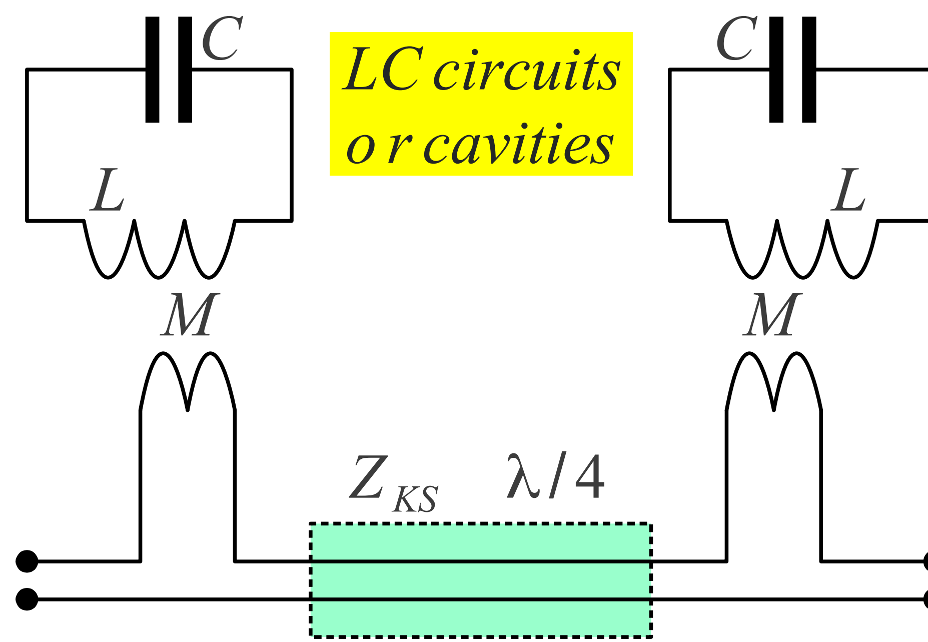
Implementation of BSFs



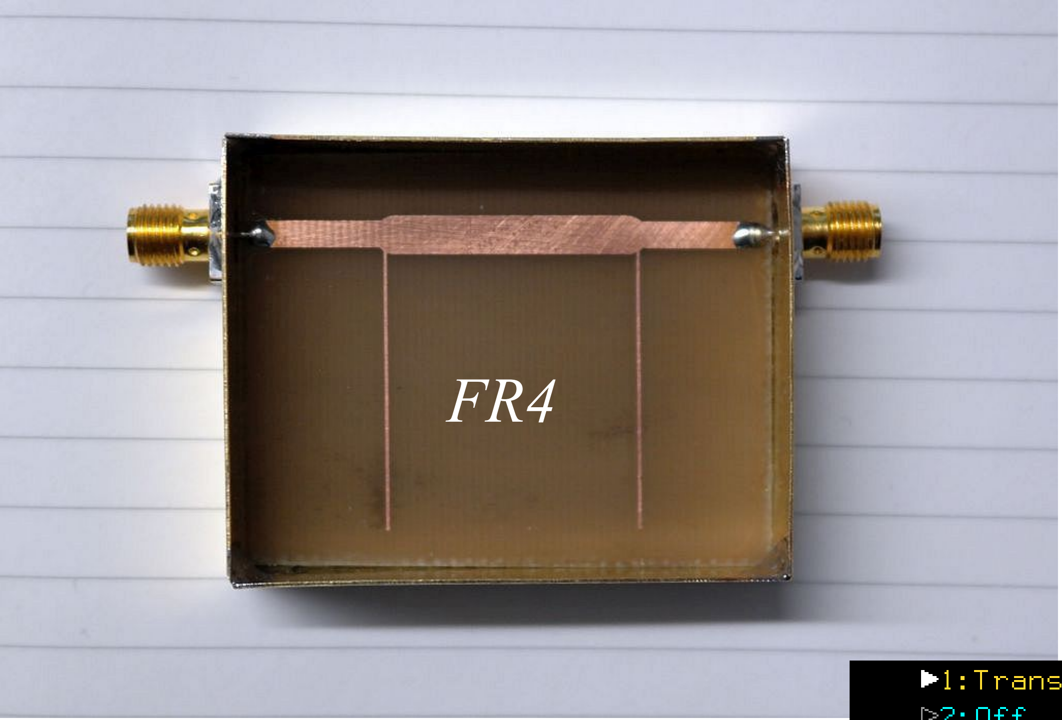


Narrow bandstop $\Delta f \ll f_0$ → *too high* Z_{KP}

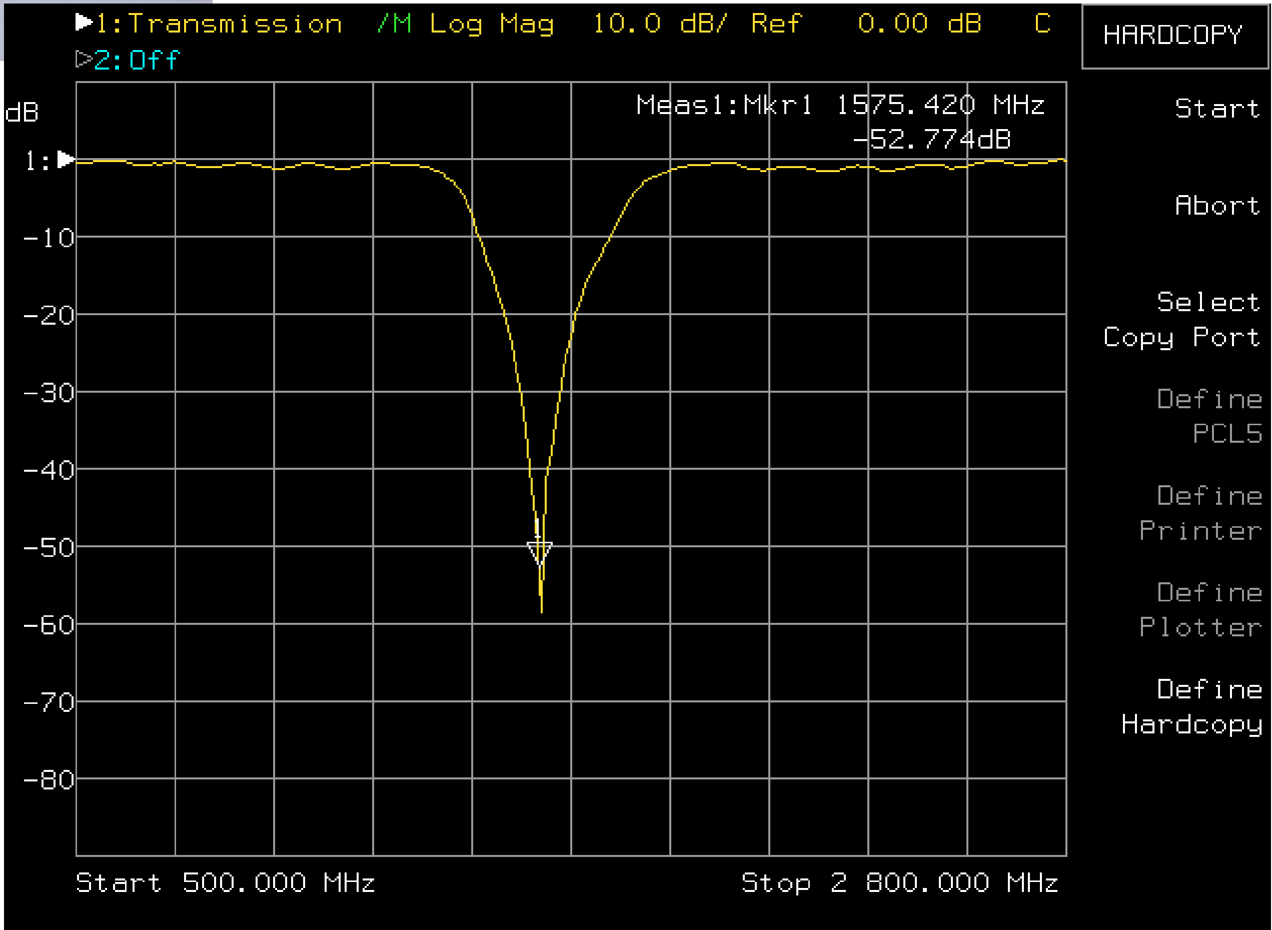
$$Z_{coax} \approx \frac{60\,\Omega}{\sqrt{\epsilon_r}} \ln \frac{R_{shield}}{R_{central}}$$



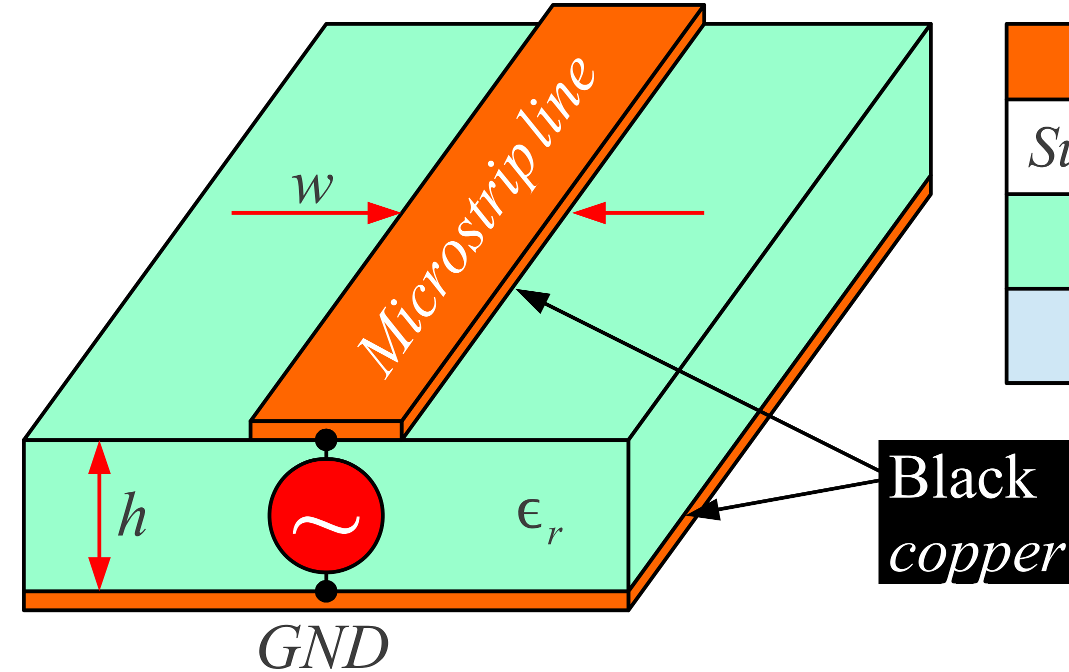
Duplexer for a GSM base station with bandstop filters



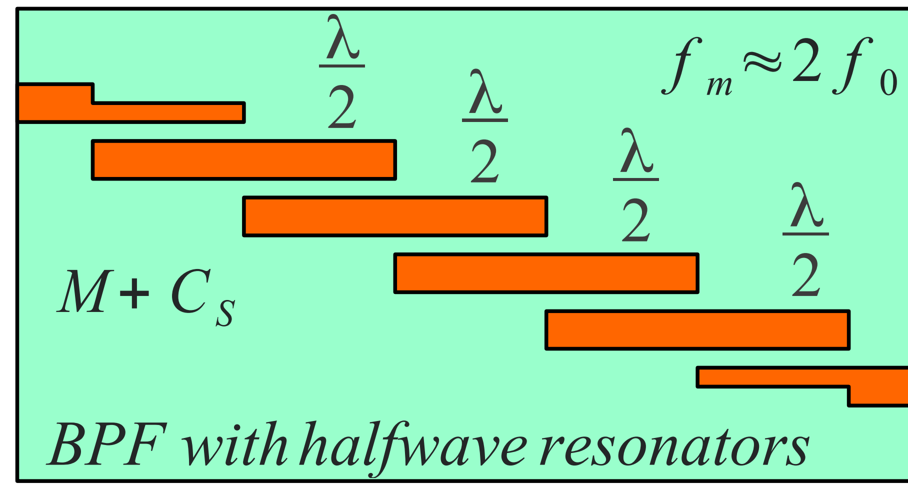
Microstrip bandstop
for GPS 1575.42MHz

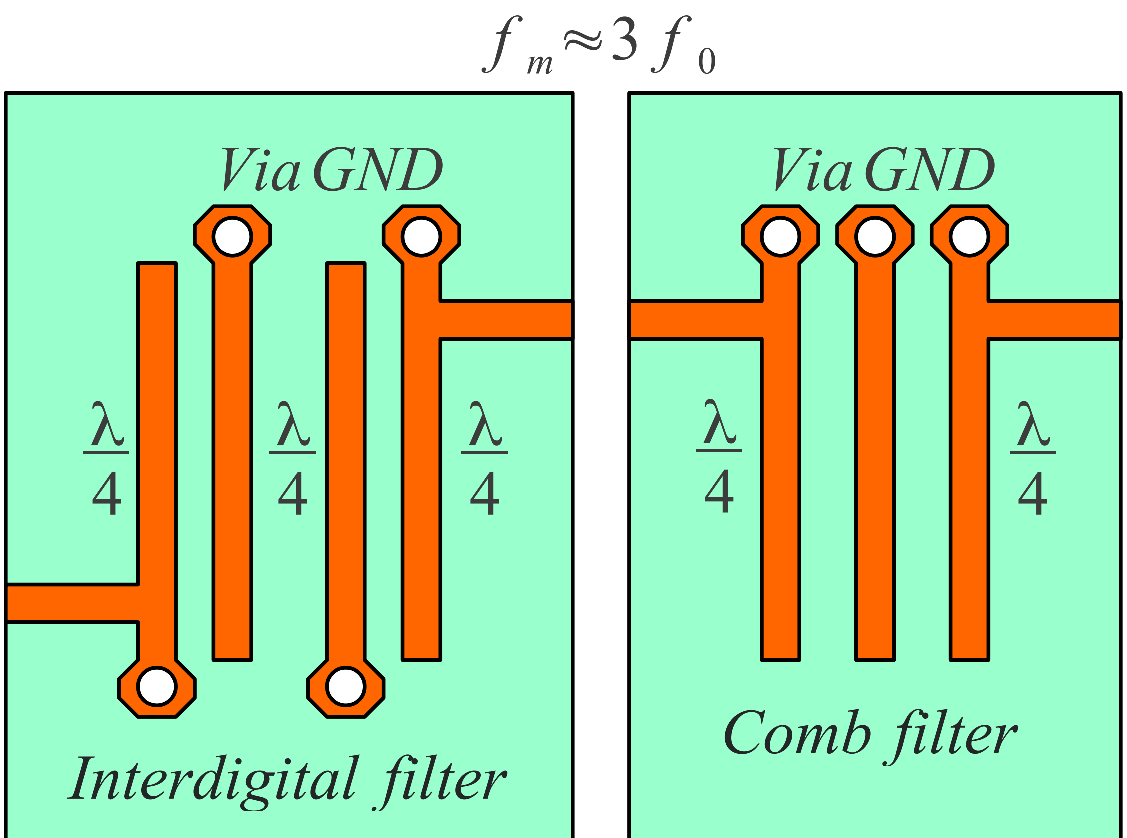


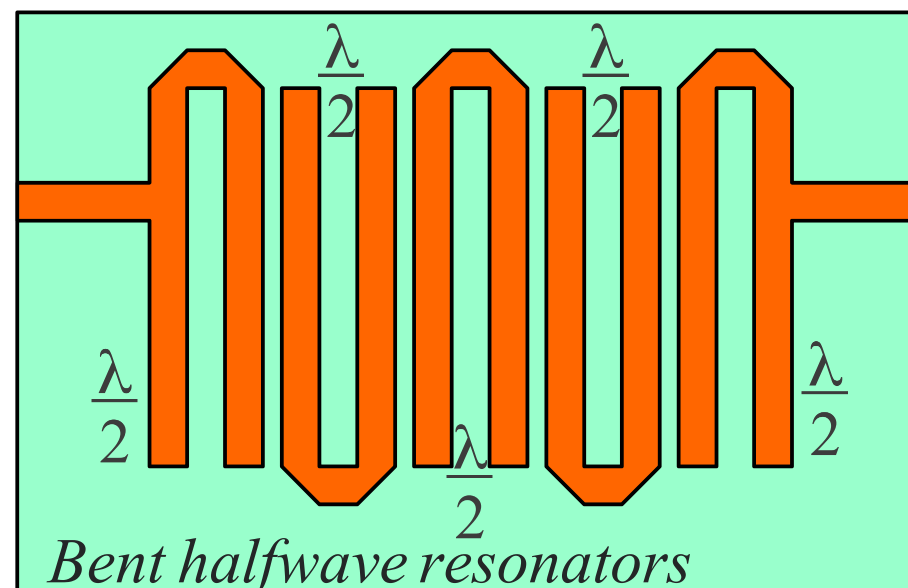
Engineering task :
avoid interference of
a wideband transmitter
t o an onboard
GPS receiver

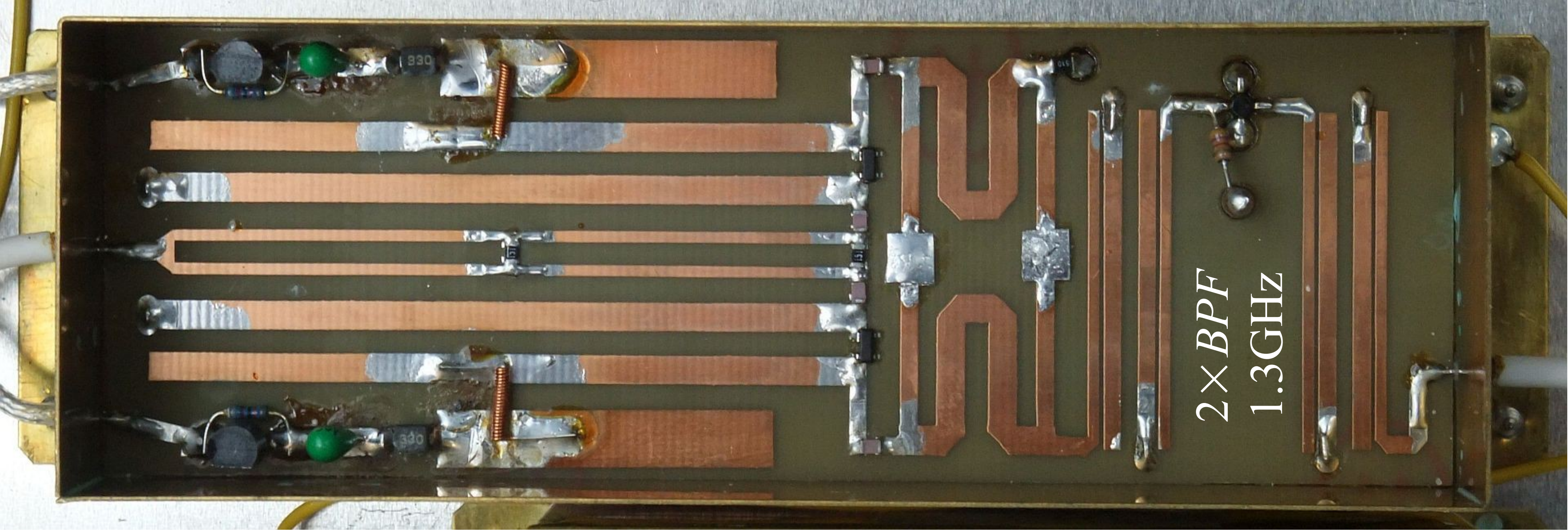


Microstrip resonator			
Substrate	ϵ_r	tan δ	Q_U
FR4	~4.3	0.02	~30
Teflon	~2.4	0.001	~200

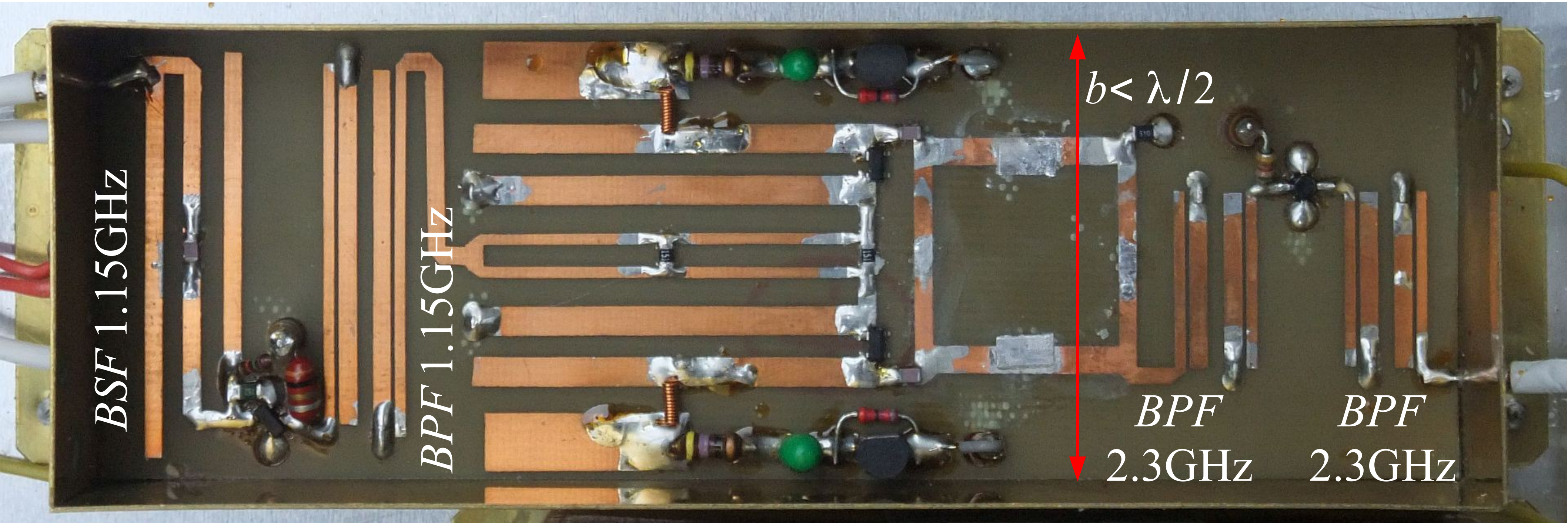


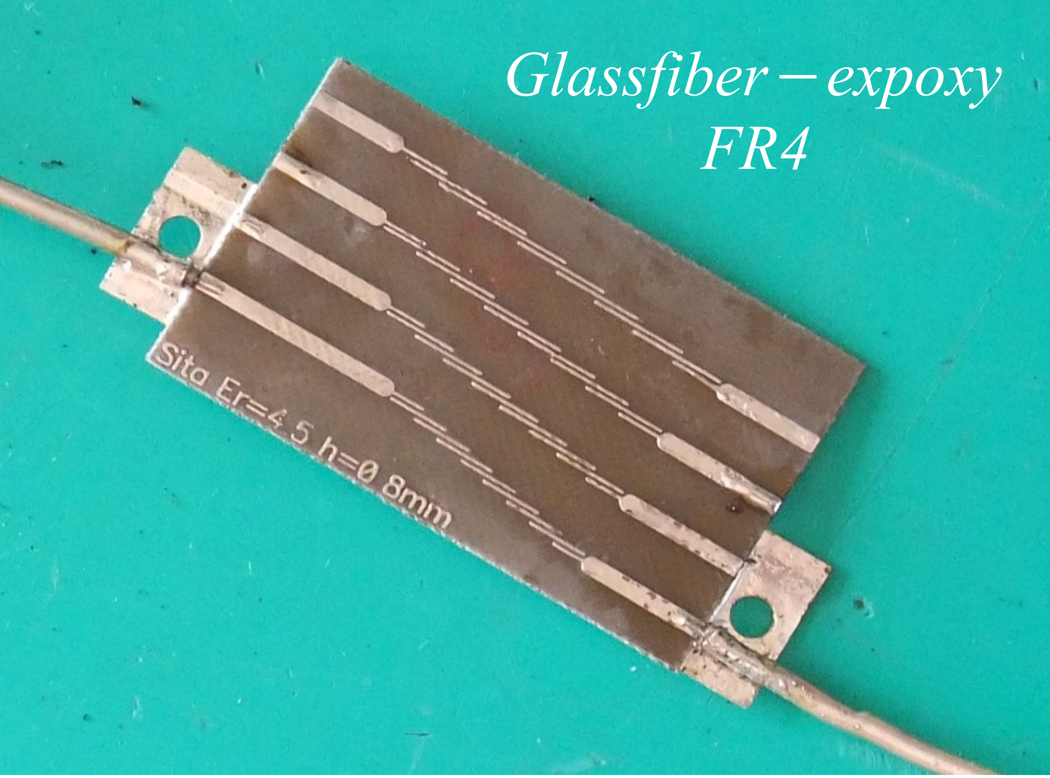


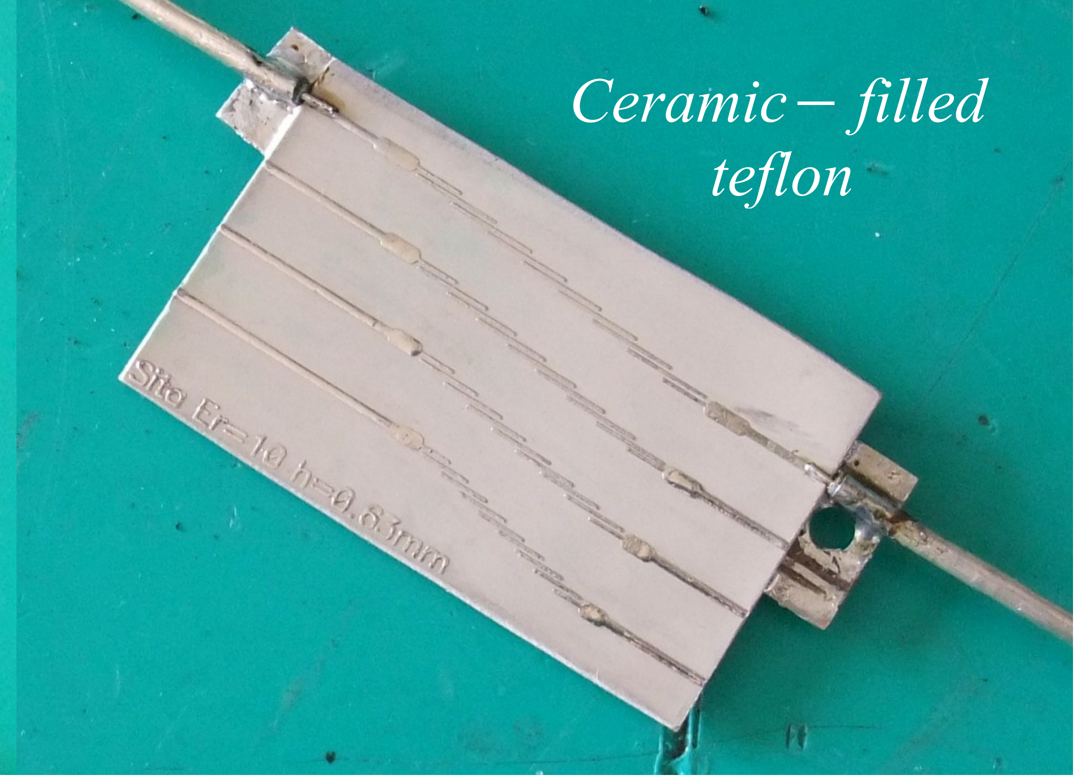




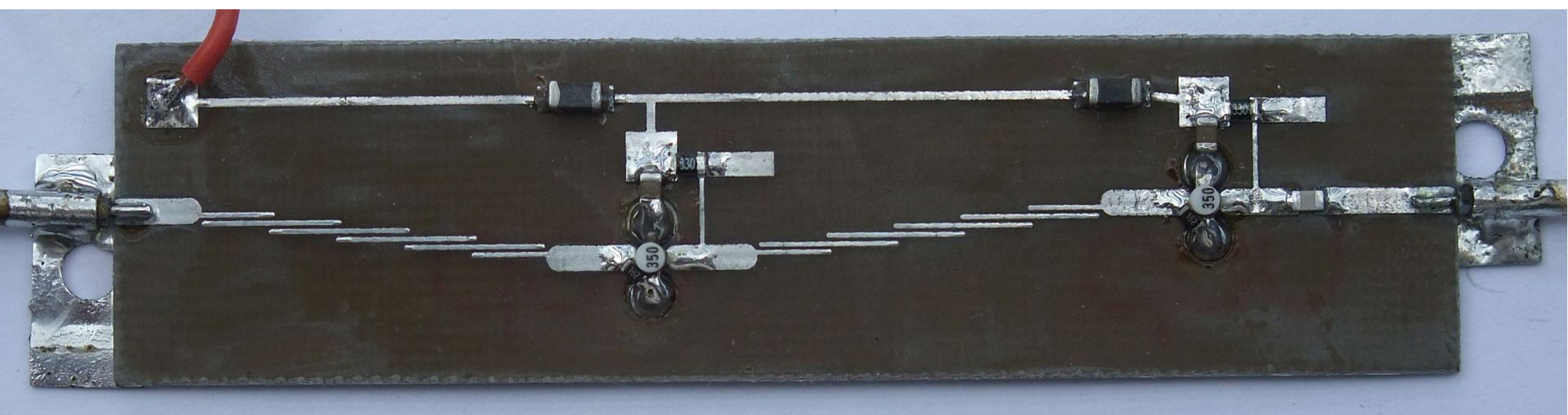
Quarterwave interdigital bandpass filters for 1.3GHz & 2.3GHz

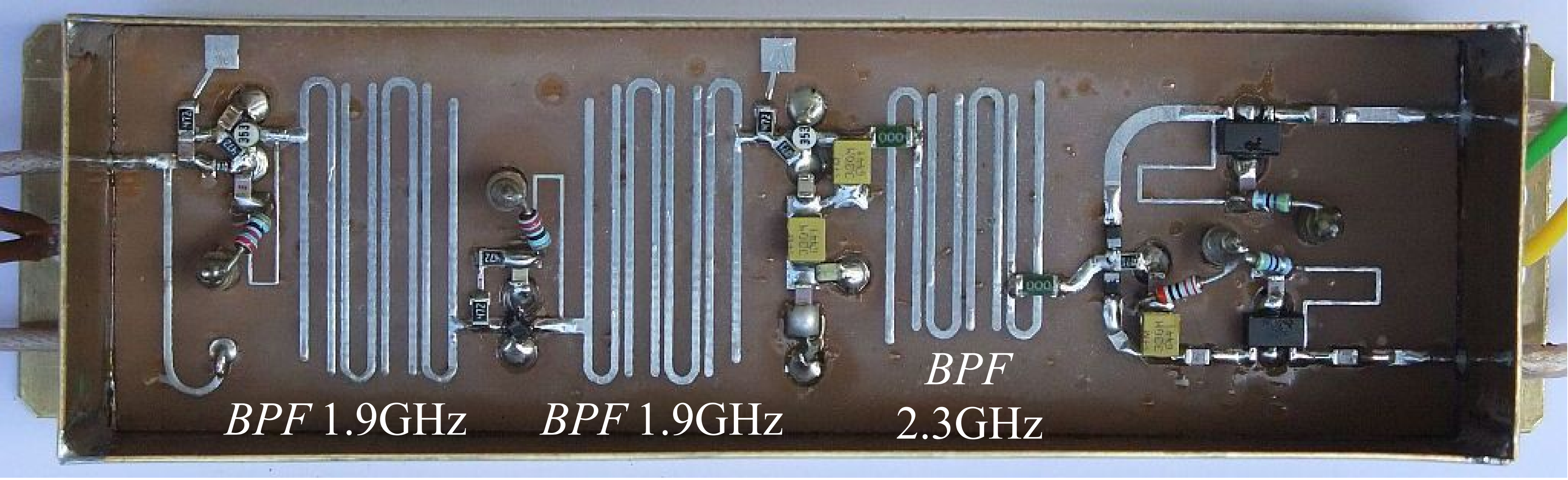




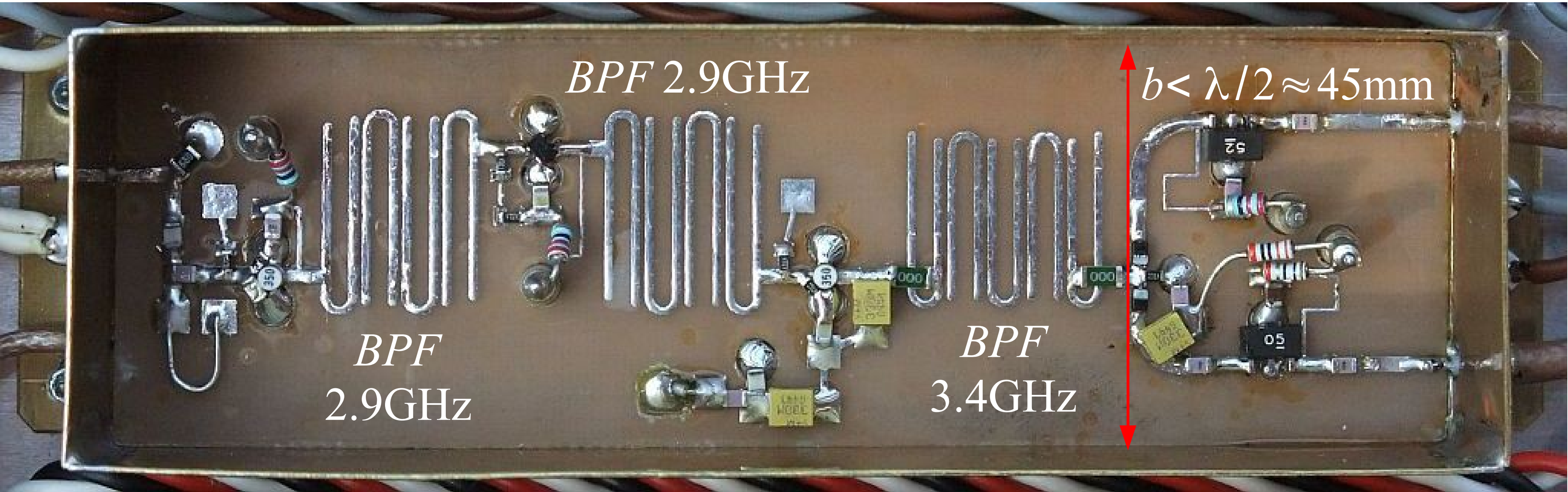


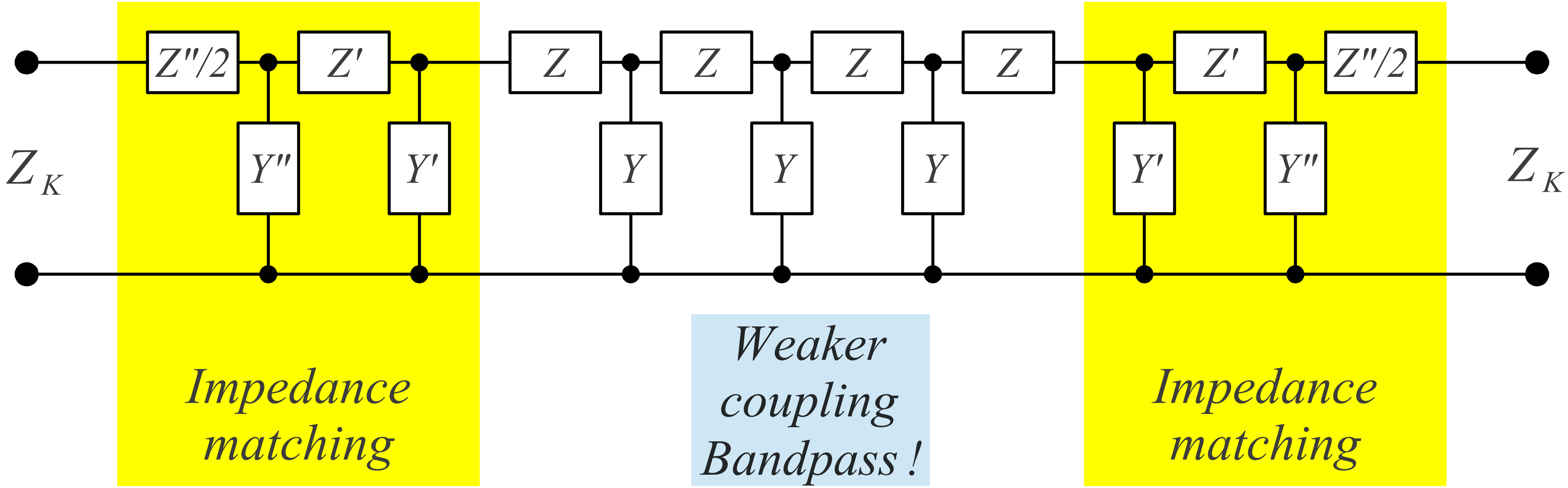
Development of a 8GHz ... 12.5GHz *microstrip halfwave BPF*





Receive/transmit mixer for 2.3GHz & 3.4GHz
with LO multiplier for 1.9GHz(×4) & 2.9GHz(×6)





Improving input / output filter impedance matching